Some features of the radiation from sources moving in refractive media

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Some effects associated with the observation of radiation emitted by a source moving through a medium with a velocity exceeding that of light in the medium are considered. It is shown that there is a certain critical angle at which the first flash from the source can be observed. For a medium without dispersion this angle is just the Vavilov–Cerenkov emission angle, whether the medium be isotropic or anisotropic. The parts of the radiaton corresponding to the normal and anomalous Doppler effects appear to the observer to come from two different sources moving in opposite directions. Peculiar effects associated with the observation of the Vavilov–Cerenkov effect in dispersive media are also discussed.

The theories of the Vavilov-Cerenkov and Doppler effects for a source moving in a refractive medium are well known (see, e.g., the reviews by Bolotovskiĭ[1, 2] and Ginzburg[3]). Here we want to call attention to a peculiar aspect of these effects, associated with the observation of radiation from a source moving with superluminal velocity. Let us consider a refractive medium (refractive index n) with no losses, which may be isotropic or anisotropic, and let us consider a point source of light moving through this medium in a straight line with constant velocity v, beginning its motion at the point A at time t = 0 and ending at point B at $t = l/v$ (Fig. 1). Let there be an observer at the point $M(x_0, y_0)$ who can scan the entire trajectory AB of the moving source. We shall want to know at what point on the trajectory will the observer first see a flash of light. The simple answer that the observer will first see the flash at point A and will then see the luminous point move from A to B, where it disappears, is not correct in the general case of a source moving with superlight velocity. The point is that here the dependence of the delay time on the propagation velocity of the radiation is important. Generally speaking, the radiation will require different lengths of time to reach the observation point from different parts of the trajectory.

To clarify this question let us calculate the time at which the radiation emitted at the intermediate point C will reach the observer. This time is obviously given by the equation

$$t(\alpha)=\frac{x_0}{v}-\frac{y_0}{v\,\mathrm{tg}\,\alpha}+\frac{y_0}{w\sin\alpha} \tag{1}$$

in which w is the group velocity of the waves and α is the angle between the direction of the beam and the velocity of the source. An elementary examination of (1) for extrema shows that $t(\alpha)$ has a minimum at the value of α that satisfies the equation

$$\frac{1}{w}-\frac{1}{v\cos\alpha}=\mathrm{tg}\,\alpha\left(\frac{1}{w}\right)'_{\alpha}, \tag{2}$$

(we have taken into account the fact that in general the group velocity may depend on α). Now let us examine the consequences of Eq. (2) for a number of cases.

1. The source moves in an isotropic nondispersive medium, emitting a single frequency. Then $w = c/n$ and $\alpha = \theta$, where θ is the angle between the wave vector and the velocity of the source. In this case Eq. (2) becomes

$$\cos\theta=1/\beta n, \tag{3}$$

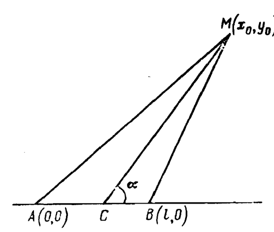

FIG. 1

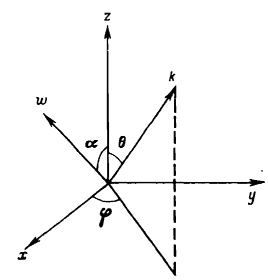

FIG. 2

where $\beta = v/c$. Thus, if the source moves with superlight velocity ($\beta n > 1$) and if its trajectory is long enough for the angle θ to fall within the field of view of the observer, the observer will first see the flash at the Vavilov-Cerenkov emission angle; but if the source moves with sublight velocity ($\beta n < 1$), Eq. (2) will have no solution and the entire radiation pattern will be trivial.

After the flash, the observer will see what appears to him to be two sources moving away from the locus of the flash in opposite directions with the apparent velocities

$$v_a=\pm\frac{v}{1-\beta n\cos\theta}. \tag{4}$$

One of these moves in the direction from A to B (the plus sign in (4)) and has the normal Doppler spectrum

$$\omega=\Omega/|1-\beta n\cos\theta|, \tag{5}$$

in which $\beta n\cos\theta < 1$ and Ω is the unshifted emission frequency, while the other moves in the opposite direction (the minus sign in (4)) and has an anomalous Doppler spectrum, which is given by (5) with $\beta n\cos\theta > 1$. Moreover, at the instant of the flash, the velocities of both sources seem to the observer to be infinite. Thus, the normal and anomalous parts of the Doppler emission seem to an observer in the medium to come from two different sources, and since the "anomalous" source moves away from the observer, the frequency of its radiation will be a monotonically decreasing function of the angle between the ray and the velocity of the apparent source. We note that if we understand the term "source with superlight velocity" in the spirit of the Bolotovsky-Ginzburg review[4], we can say that the apparent splitting of the source into two could be observed even in vacuo.

2. A similar situation obtains in the case of a source

moving in a crystal. Let the source move along the z axis and let the ray lie in the xz plane. In a crystal, the wave vector and the group velocity do not, in general, have the same direction. We shall assume that the direction of the ray is specified by the angle α, and that of the wave vector, by the angles θ and φ (Fig. 2). Then the refractive index for oscillations of type i in the crystal will be a function of θ and φ, i.e., $n_i = n_i(\theta, \varphi)$; moreover, θ and φ will be determined by α and the direction of the optical axis of the crystal. It is not difficult to show that in this case the group velocity will be determined by the formulas

$$\begin{gathered} w_x=\frac{c}{n^2}\left(-\cos\varphi\frac{\partial n\cos\theta}{\partial\theta}+\frac{\sin\varphi}{\sin\theta}\frac{\partial n}{\partial\varphi}\right), \\ w_y=\frac{c}{n^2}\left(-\sin\varphi\frac{\partial n\cos\theta}{\partial\theta}-\frac{\cos\varphi}{\sin\theta}\frac{\partial n}{\partial\varphi}\right), \\ w_z=\frac{c}{n^2}\frac{\partial n\sin\theta}{\partial\theta}, \\ w=\frac{c}{n^2}\left(n^2+n_\theta'^2+\frac{n_\varphi'^2}{\sin^2\theta}\right)^{1/2}. \end{gathered} \tag{6}$$

The condition $w_y = 0$ enables us to calculate n'_φ from the second of Eqs. (6), obtaining

$$n_\varphi'=-\operatorname{tg}\varphi\sin\theta\frac{\partial n\cos\theta}{\partial\theta}. \tag{7}$$

Now we calculate $(1/w)'_\alpha$ in terms of the derivatives with respect to θ and φ, and using Eqs. (6) and (7), we obtain

$$\left(\frac{1}{w}\right)'_\alpha=-\frac{n}{c}\frac{\cos\varphi[n_\varphi'+\operatorname{tg}^2\varphi(n\cos\theta)_\theta'\cos\theta]}{[n^2+n_\theta'^2+\operatorname{tg}^2\varphi(n\cos\theta)_\theta'^2]^{1/2}}. \tag{8}$$

On substituting (8) into (2) we obtain the relation

$$n_i\beta\cos\theta=1 \tag{9}$$

for the angle of the "first flash," which is again the Vavilov-Cerenkov emission angle. All that was said about case 1 proves also to be valid for a crystal, with the sole difference that a source moving in a crystal can emit several types of normal waves.

3. Now let us consider the case of a point charge moving uniformly in a straight line through an isotropic dispersive medium that has just one resonance frequency. The spectrum of the Vavilov-Cerenkov emission will be determined by the well-known formula

$$\cos\theta=1/\beta n, \tag{10}$$

in which the refractive index of the medium is given by

$$n(\omega)=\left(1-\frac{\omega_0^2}{\omega^2-\omega_s^2}\right)^{1/2}. \tag{11}$$

Relation (2) for the Vavilov-Cerenkov emission in a dispersive medium can be rewritten with the aid of (10) in the form

$$\omega\left(\frac{dn}{d\omega}\right)^2=(\beta^2n^2-1)n\frac{d^2n\omega}{d\omega}. \tag{12}$$

Some explanation is required in connection with the application of (2) to the emission in dispersive media. This relation is obviously valid in the wave zone of the emitter for frequencies that are not too close to the absorption frequency of the medium. Then by the instant of emission we must understand the instant at which the corresponding wave packet is formed.

Equation (12) has roots only when $d^2n\omega/d\omega^2 > 0$. On substituting (10) and (11) into (12) we obtain the following equations for the frequency:

$$\omega=\left\{\omega_s\left[4\omega_s^2-\frac{3\beta^2\omega_0^2}{1-\beta^2}\right]^{1/2}-\omega_s^2\right\}^{1/2} \tag{13}$$

and the angle of the "first flash":

$$\cos\theta^0=\sqrt{3}\left\{2+\beta^2+(1-\beta^2)\left[4-\frac{3\beta^2}{1-\beta^2}\frac{\omega_0^2}{\omega_s^2}\right]^{1/2}\right\}^{-1/2}. \tag{14}$$

Neither of these equations is meaningful unless

$$\beta<\omega_s/(\omega_s^2+\omega_0^2)^{1/2}. \tag{15}$$

As in the preceeding case, the observer will see sources "running away from one another" on opposite sides of the angle θ^0 with the apparent velocity

$$v_a=v\left[1-\frac{v}{w}\cos\theta+v\sin\theta(w^{-1})_\theta'\right]^{-1}, \tag{16}$$

Moreover, the frequency of the source moving away from the observer will decrease with time, and that of the one moving toward the observer will increase with time.

4. Finally, let us consider the case of a point charge moving along a strong magnetic field in a cold collisionless electronic plasma. In the frequency region $\omega_{Hi} \ll \omega \ll \omega_{He}$ (ω_{Hi} and ω_{He} are the gyromagnetic frequencies of the plasma ions and electrons) the plasma has the properties of a uniaxial anisotropic crystal whose refractive index for the extraordinary wave is given by

$$n(\omega,\theta)=\omega/(\omega^2-\omega_0^2\sin^2\theta)^{1/2}. \tag{17}$$

Equations (6) can be used to obtain the following relation between the angles θ and α, which were defined in connection with case 2:

$$\operatorname{tg}\theta=-(\omega_0^2/\omega^2-1)\operatorname{tg}\alpha. \tag{18}$$

We note that since $\omega < \omega_0$, the projections of the wave vector and the group velocity onto the direction of the velocity of the charge have opposite signs, and that here the Vavilov-Cerenkov emission cone is reversed, its generator making an obtuse angle with the velocity of the charge (see, e.g.,[5]). The spectrum of the radiation can be put in the following form with the aid of Eqs. (10), (17), and (18):

$$\omega=\omega_0\sin\alpha/[1-\beta^2\cos^2\alpha]^{1/2}. \tag{19}$$

Then from Eqs. (6), (17), (18), and (19), we obtain the following equation for the group velocity:

$$w(\alpha)=v|\cos\alpha|. \tag{20}$$

Substitution of (20) into (2) leads to the following expression for the angle at which the "first flash" is seen:

$$\operatorname{tg}\alpha=-\sqrt{2}\quad\text{or}\quad\operatorname{tg}\theta=(1-\beta^2)\sqrt{2}, \tag{21}$$

the frequency of the light in this flash being

$$\omega=\left(\frac{2}{3-\beta^2}\right)^{1/2}\omega_0. \tag{22}$$

In this case the observer sees the "first flash" at an obtuse angle to the velocity of the source—not at an acute angle as in the preceeding cases. As before, the observer sees two sources moving apart, the frequency of the receding source decreasing with time, and that from the approaching source, increasing.

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