Thermoelectric phenomena in the intermediate state of superconductors

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The thermoelectric-coefficient tensor α_{ij} is computed for the intermediate state of pure metallic type-I superconductors with excitation mean free paths $l \gg a$, where a is a characteristic layer dimension. The diagonal component of α_{ij} that is responsible for the electric current that flows along the layers and across the magnetic field is 1.5 times larger than the same coefficient for a dirty metal. The component for the direction transverse to the layers is determined by the angular dependence of the probability, w, of excitation passage across an n-s interface, and may be either positive or negative, depending on the properties of the metal and the orientation of the n-s interface. It is shown that in the presence of a heat flux the intermediate state tends, at low temperatures, to assume a structure in which the heat flows along the layers, in accord with the available observations. It is also shown that besides the excitations that undergo Andreev reflection at the interfaces, there is a small group of excitations that, sliding along the boundaries at an angle $\sim(T/\mu)^{1/2}$ (where μ is the chemical potential), undergo specular reflection.

Type-I superconductors can exist in an intermediate state that, under certain conditions, admits of a macroscopic description. In[1-4], it was shown (experimentally and theoretically) that the thermal phenomena could exert a significant influence on the dynamics of the intermediate state. In[4] a complete system of macroscopic equations describing the dynamics of the intermediate state with allowance for the thermal effects are derived. The kinetic coefficients are computed for dirty metals in the case when the excitation mean free path l in the normal and superconducting regions is less than the thickness $a_{n,s}$ of these layers, i.e., when $l \ll a_{n,s}$. In the present paper we consider the other limiting case of pure metals under conditions when $l \gg a_{n,s}$. The resistivity and thermal-conductivity tensors of pure metals in the intermediate state at low temperatures were computed in[5,6]. The question of the computation of the thermoelectric coefficients under these conditions was, however, left open.

The calculations are carried out for low temperatures $T \ll \Delta$, where Δ is the gap in the excitation spectrum of the superconductor. In this case practically all the excitations in the normal phase are reflected from the interface with the superconducting phase. For $T \sim \Delta \sim T_c$, it is necessary to know the probability, w, of excitation passage across an n–s interface. This problem cannot be solved in the general case, since for this purpose it is necessary to find the explicit form of the coordinate dependence $\Delta(z)$, which cannot be done even in the vicinity of the critical temperature T_c.

At finite $T \neq 0$, the Fermi distribution is blurred, which is the cause of the appearance of thermoelectricity. In order of magnitude, the thermoelectric coefficients $\alpha \sim T/\mu$, where μ is the chemical potential. Allowance for the smearing of the Fermi surface leads also to some new anomalies in the reflection of the excitations from the n–s interfaces. As is well known[6], the excitations in the vicinity of the Fermi surface are reflected from an interface via the Andreev mechanism. As is shown below, there also exist at finite T excitations that undergo specular reflection as they move almost parallel to the interfaces.

Notice that the computations are carried out for an isotropic model. The corresponding generalization to the case of an arbitrary energy spectrum $E(\mathbf{p})$ is obtained for the diagonal thermoelectric-tensor component α_{zz} (the z axis is perpendicular to the layers), which is responsible for the electric current j_z under the action of the temperature gradient $\partial T/\partial z$. This coefficient is determined by the angular dependence of the transit probability w and, as is demonstrated, may have a sign different from that of the thermoelectric coefficient of the normal metal, depending on the properties of the metal and the orientation of the interfaces between the n and s phases.

1. EXCITATION REFLECTION AT THE PHASE INTERFACES

Andreev[6] has explained the nature of the reflection of the excitations from the n–s phase interfaces. He has shown that upon reflection an electron above the Fermi surface goes over into a hole under the Fermi surface (and vice versa). This scattering practically does not change the excitation quasimomentum $\mathbf{p}$, but changes the sign of the distance–in terms of energy–from the Fermi surface E_0, i.e., of the quantity $\xi(\mathbf{p}) = E(\mathbf{p}) - E_0$. Thus, the Andreev reflection law can be written in the form

$$\xi' = -\xi, \quad \mathbf{n}' = \mathbf{n}, \tag{1}$$

where $\mathbf{n} = \partial\xi/\partial\mathbf{p}/|\partial\xi/\partial\mathbf{p}|$ and the prime indicates the reflected quantity. The relations (1) are suitable for the computation of the electrical and thermal conductivities, but in determining the thermoelectric coefficients (which are of the order of T/μ), we must take into account the blurring of the Fermi surface at $T \neq 0$, i.e., clarify the law of reflection of excitations of energy $\epsilon = |\xi| \sim T$.

We choose the coordinate system shown in Fig. 1. Let us first consider a stationary interface. The conditions for reflection reduce, obviously, to

$$\xi' = -\xi, \quad \mathbf{p}_t' = \mathbf{p}_t, \tag{2}$$

where $\mathbf{p}_t$ is the quasimomentum component tangential to the interface, a component which, in view of the homogeneity of the problem with respect to the x and y directions, is conserved[1)]. Each excitation is unambiguously characterized by the quantities ξ and $\mathbf{n} = \mathbf{p}/p$, both of which are changed in the first approximation in T/μ by

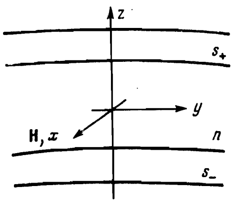

FIG. 1

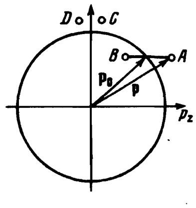

FIG. 2

the reflection. We can, however, characterize each excitation by the quantities ξ and $\mathbf{n}_0 = \mathbf{p}_0/p_0$, where $\mathbf{n}_0$ is the unit vector along the direction of the vector $\mathbf{p}_0$ drawn from the origin to a point on the Fermi surface such that $\mathbf{p}_{0t} = \mathbf{p}_t$. In this case the second condition in (2) is automatically satisfied. The unit vector $\mathbf{n}_0$ is then the same for both the incident and reflected excitations, and therefore the reflection amounts only to a change of the sign of ξ. For example, the reflection of an incident electron at the upper boundary of an n-layer ($z = a_n/2$) amounts to a transition from the point A to the point B in Fig. 2.

Thus, instead of (1) we have

$$\xi' = -\xi, \quad \mathbf{n}_0' = \mathbf{n}_0. \tag{3}$$

It is not difficult to derive the relation between $\mathbf{n}$ and $\mathbf{n}_0$ Let us write $\mathbf{p}$ in the form $\mathbf{p} = \mathbf{p}_0 + \boldsymbol{\nu}\delta p_z$, where $\boldsymbol{\nu}$ is the unit vector along the p_z axis. Then $\xi(\mathbf{p}) \approx \delta p_z \partial\xi/\partial p_z \approx v_0 n_{0z}\delta p_z$, where v_0 is the velocity at the Fermi surface. Taking also into account the fact that $p \approx p_0 + \xi/v_0$, we obtain

$$\mathbf{n} = \frac{\mathbf{p}}{p} \approx \frac{\mathbf{p}_0 + \mathbf{v}\xi/v_0 n_{0z}}{p_0 + \xi/v_0} \approx \mathbf{n}_0\left(1 - \frac{\xi}{2\mu}\right) + \frac{\mathbf{v}\xi}{2\mu n_{0z}}.$$

For the individual components we have

$$n_z \approx n_{0z} + \xi(1 - n_{0z}^2)/2\mu n_{0z},$$
$$n_t = p_t/p \approx \sqrt{1 - n_{0z}^2}\,(1 - \xi/2\mu). \tag{4}$$

Knowing n_t, we easily find

$$n_{x,y} = n_t \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix},$$

where φ is the azimuthal angle with respect to the axis p_z (which, of course, does not change in the reflection).

Let us now consider an n–s phase interface moving with velocity V along the z axis. To compute the corresponding thermoelectric coefficient (see below), we need to know only the change in the energy ϵ that occurs as a result of the reflection. In the moving reference system the energy does not change during the reflection, i.e., $\epsilon_0' = \epsilon_0$ (the index zero here indicates that the corresponding quantity is measured in the moving coordinate system). Applying the Galilean transformation

$$\varepsilon' - p_z'V = \varepsilon - p_z V,$$

we find[2)]

$$\Delta\varepsilon = \varepsilon' - \varepsilon = V(p_z' - p_z) \approx -Vp_0\xi/\mu n_{0z}. \tag{5}$$

Let us now consider the possibility of specular reflection of excitations from an interface. It is clear that if an electron with $p_t > p_0$ but with a small energy $\epsilon < \Delta$ is incident on an interface, then it must be specularly reflected, since for all the holes $p_t < p_0$ and the tangential component of the quasimomentum is preserved. Since only the excitations in the vicinity of the Fermi surface are important, in the isotropic case the electrons that undergo specular reflection are situated near the poles of the Fermi sphere at $p_z \approx 0$. For such electrons $v_z \approx 0$, i.e., they move almost parallel to the phase interfaces. In Fig. 2 the transition from the point C to the point D corresponds to a specular reflection of an electron at the interface $z = a_n/2$. Let us find the boundary in momentum space that separates the Andreev- and specular-reflection regions. Substituting the exact dependence $\xi(\mathbf{p})$, i.e.,

$$\xi(\mathbf{p}) = \frac{p^2 - p_0^2}{2m} = \frac{p_z^2 - p_{0z}^2}{2m}$$

in the isotropic case, into the first condition in (3), we find

$$p_z' = \sqrt{p_{0z}^2 - 2m\xi} = p_0\sqrt{n_{0z}^2 - \xi/\mu}.$$

Hence we obtain the boundary value n_{0zb} as a function of ξ:

$$n_{0zb} = \sqrt{\xi/\mu}. \tag{6}$$

An incident electron with a given ξ is reflected via the Andreev mechanism when $n_{0z} > n_{0zb}$ and specularly if $n_{0z} < n_{0zb}$ (for the incidence of holes in the present case there are no restrictions, since $\xi < 0$ in their case)[3)]. In the general case of an arbitrary spectrum $\epsilon(\mathbf{p})$ there can exist several extremal points at which $v_z = 0$. There exist near each such point of the Fermi surface excitations that undergo specular reflection. Notice, however, that the corresponding range of glide angles is small, i.e., $\sim(T/\mu)^{1/2} \sim 10^{-2}$, and therefore allowance for such excitations in the kinetic coefficients leads only to small corrections $\sim T^2/\mu^2$.

2. THE THERMOELECTRIC COEFFICIENTS OF THE LAMINAR STRUCTURE

In this case $l \gg a_{n,s}$ and the electric-current and thermal-flux densities $\mathbf{j}_n$ and $\mathbf{q}_n$ are determined with the aid of the excitation distribution function $n_n(\mathbf{r}, \mathbf{p})$ in the normal phase, which satisfies the following kinetic equation:

$$\mathbf{v}\frac{\partial n_n}{\partial \mathbf{r}} + e\mathbf{e}\frac{\partial n_n}{\partial \mathbf{p}} = -\frac{n_n - \bar{n}_n}{\tau}, \tag{7}$$

where $\mathbf{e}$ is the electric field in the n-layer (which field can be assumed to be independent of the coordinates[5]), $e = e_0 \operatorname{sgn}\xi$ is the charge of the excitation (e_0 is the electron charge), τ is the relaxation time that, at low temperatures, is due to scattering by impurities, lattice defects, etc., and the bar indicates averaging over the constant-energy surface. The magnetic field in the kinetic equation can be neglected, since usually $l \ll R_L$, where R_L is the Larmor radius in the critical magnetic field H_c. In Eq. (7) we have also dropped the small, nonlinear (in the perturbations $\mathbf{e}$ and V) term $\partial n_n/\partial t \approx -V\partial n_n/\partial z \approx 0$. As before[5, 4], from the boundary condition for the electric field follow the relations

$$e_x = 0, \quad V = -ce_y/H_c, \tag{8}$$

where the x axis has been chosen along the direction of the magnetic field in the n-layer (see Fig. 1).

Let us assume that the temperature T is constant and seek the solution to Eq. (7) in the form

$$n_n = n_0 + \frac{\partial n_0}{\partial \varepsilon}\chi, \tag{9}$$

where $n_0 = [\exp(\epsilon/T) + 1]^{-1}$ and χ is the new unknown function. Substitution of (9) into (7) yields

$$\partial\chi/\partial\zeta+u(\chi-\chi_0)=0. \tag{10}$$

Here we have introduced the following notation:

$$\zeta=\frac{2z}{a_n},\quad u=\frac{a_n}{2\tau v_z}=\frac{k\,\mathrm{sgn}\,\xi}{n_z},\quad k=\frac{a_n}{2l}\ll 1,$$
$$l=\tau v,\quad \chi_0=-\tau e\mathbf{e}\mathbf{v}=-e_0 l n_i e_i. \tag{11}$$

We have also dropped the term containing $\bar{\chi}$, a term which, for $l \gg a_n$, yields a small contribution to the kinetic coefficients. The solution to Eq. (10) is of the form

$$\chi=\chi_0+e^{-\zeta u}C(\mathbf{n},\xi). \tag{12}$$

The arbitrary function $C(\mathbf{n}, \xi)$ is determined from the boundary conditions, which, for $T \ll \Delta$, reduce to the equality of the incident- and reflected-excitation fluxes in the reference system moving together with the interfaces. This leads to the equations

$$n_0(\varepsilon)+\frac{\partial n_0}{\partial\varepsilon}\chi(\zeta=\pm1;\,\mathbf{n},\xi)=n_0(\varepsilon')+\frac{\partial n_0}{\partial\varepsilon}\chi(\zeta=\pm1;\,\mathbf{n}',-\xi). \tag{13}$$

Substituting (12) into these equations, we obtain

$$C(\mathbf{n},\xi)\begin{bmatrix}e^{-u}\\ e^{u}\end{bmatrix}-C(\mathbf{n}',-\xi)\begin{bmatrix}e^{-u'}\\ e^{u'}\end{bmatrix}=[\Delta\varepsilon+\chi_0(\mathbf{n}')-\chi_0(\mathbf{n})]\begin{bmatrix}1\\ 1\end{bmatrix}. \tag{14}$$

Expressing the reflected quantities (characterized by the prime) in terms of the incident quantities with the aid of (3)–(5), and using (11), we find from (14) the sought quantity $C(\mathbf{n}, \xi)$. The nonequilibrium part χ of the distribution function can be completely determined at the same time from (12) in the form

$$\chi=-\frac{Vp_0\xi e^{-\zeta u}}{2\mu n_z\,\mathrm{ch}\,u}-e_0le_yn_y\left(1+\frac{\xi e^{-\zeta u}}{2\mu\,\mathrm{ch}\,u}\right)-e_0le_z\left[n_z-\frac{\xi e^{-\zeta u}(1-n_z^2)}{2\mu n_z\,\mathrm{ch}\,u}\right]$$
$$\approx -Vp_0\xi/2\mu n_z-e_0le_yn_y(1+\xi/2\mu)-e_0le_z[n_z-\xi(1-n_z^2)/2\mu n_z]. \tag{15}$$

In the last equality we have retained only the leading terms in the expansion of the function χ in powers of $k \ll 1$.

The macroscopic thermal flux $\mathbf{q}$ is the volume average of the thermal fluxes in the n- and s-layers. In the s-phase $\mathbf{q}_s = 0$, since all the excitations are reflected from the boundaries back into the normal phase. Since χ does not depend on ζ in pure metals (see (15)), the macroscopic thermal flux is simply

$$\mathbf{q}=x_n\mathbf{q}_n=\frac{2x_n}{(2\pi\hbar)^3}\int d^3p\,\varepsilon\mathbf{v}\frac{\partial n_0}{\partial\varepsilon}\chi, \tag{16}$$

where $x_n = a_n/(a_n + a_s)$ is the concentration of the normal phase. It can be seen from (15) that $q_x = 0$, since the integral (16) vanishes upon integration over the angle φ. For the same reason, only the second term in (15) makes a contribution to q_y. We have

$$q_y=-\frac{2x_ne_0le_y}{(2\pi\hbar)^3}\int d^3p\,\varepsilon v_y\frac{\partial n_0}{\partial\varepsilon}n_y\left(1+\frac{\xi}{2\mu}\right)$$
$$=-\frac{2e_0lE_y}{(2\pi\hbar)^3}\int_0^{2\pi}\sin^2\varphi\,d\varphi\int_{-1}^{1}(1-n_z^2)\,dn_z\int_{-\infty}^{+\infty}d\xi\frac{p^2}{v}\xi v\frac{\partial n_0}{\partial\varepsilon}\left(1+\frac{\xi}{2\mu}\right)$$
$$=\frac{e_0lp_0^2T^2}{6\hbar^3\mu}E_y=\gamma_{yy}E_y.$$

Here we have introduced the macroscopic electric field $\mathbf{E} = x_n\mathbf{e}$ (see[4]). Notice that γ_{yy} is exactly 1.5 times larger than the corresponding thermoelectric coefficient γ_n of the normal metal:

$$\gamma_{yy}=\frac{e_0lp_0^2T^2}{6\hbar^3\mu}=\frac{3}{2}\gamma_n. \tag{17}$$

This can be easily verified by computing q_y, neglecting beforehand the terms $\sim\xi/\mu$ in (15) (also cf., for example,[7]).

For the thermal-flux density component perpendicular to the layer boundaries we find

$$q_z=\frac{2x_n}{(2\pi\hbar)^3}\int d^3p\,\varepsilon v_z\frac{\partial n_0}{\partial\varepsilon}\chi=-\frac{p_0^2x_n}{\pi^2\hbar^3}\int_0^1 dn_z\int_{-\infty}^{+\infty}d\xi\cdot\frac{\partial n_0}{\partial\varepsilon}\xi$$
$$\times\left(1+\frac{\xi}{\mu}\right)\left\{Vp_0\frac{\xi}{2\mu}+e_0le_z\left[n_z^2-\frac{\xi(1-n_z^2)}{2\mu}\right]\right\}$$
$$=-\frac{p_0^2x_n}{\pi^2\hbar^3}\int_0^\infty d\xi\frac{\partial n_0}{\partial\varepsilon}\xi^2\left[Vp_0/\mu+\frac{2e_0le_z}{\mu}\int_0^1 dn_z(3n_z^2-1)\right]=\frac{x_np_0^2T^2V}{3\hbar^3v_0}=x_nQV.$$

Here we have used the temperature dependence of the critical magnetic field $H_c(T)$ that follows from the BCS theory at low temperatures, and we have introduced the heat Q of formation of the n-phase:

$$Q=T(S_n-S_s)=-\frac{TH_c}{4\pi}\frac{dH_c}{dT}\approx\frac{p_0^2T^2}{3\hbar^3v_0}. \tag{18}$$

Taking (8) into account, we have

$$q_z=x_nQV=-\frac{cQ}{H_c}E_y=\gamma_{zy}E_y,\quad \gamma_{zy}=-\frac{cQ}{H_c}. \tag{19}$$

The formula (19) describes the transport of the excess entropy of the n-phase with velocity V (see[4, 2]). Notice that, as is to be expected under conditions of total reflection of the excitations from the boundaries, the diagonal thermoelectric coefficient $\gamma_{zz} = 0$.

Let us now take into account the rare (for $T \ll \Delta$) event of the passage, with probability w, of an excitation across an n–s interface in the same way as was done by Andreev[8] in the computation of the transverse thermal conductivity κ_{iz}. It is easier to find the electric current $j_{n,z}$ due to $\partial T/\partial z$. In the geometry under consideration (Fig. 1), let us set e = 0 and V = 0, and let us assume that the temperature in the n- and $s_\pm$-layers differ by δT, which is connected with the observable quantity $\partial T/\partial z$ by the relation

$$\delta T=\frac{a_n+a_s}{2}\frac{\partial T}{\partial z}=\left(\frac{1}{x_n}\right)\frac{\partial T}{\partial\zeta}. \tag{20}$$

Let us seek the solution to Eq. (7) in a n-layer in the form

$$n_n=n_0(T)+\frac{\partial n_0}{\partial T}\chi.$$

With the aid of (12) we find

$$n_n=n_0(T)+\frac{\partial n_0}{\partial T}e^{-\zeta u}C_n(n_z,\xi). \tag{21}$$

Assume that $C_n(n_z, \xi)$ is of the form

$$C_n=\theta(v_z)C_{n1}+\theta(-v_z)C_{n2}=\frac{C_{n1}+C_{n2}}{2}+\frac{C_{n1}-C_{n2}}{2}\mathrm{sgn}\,v_z. \tag{22}$$

We write the boundary conditions for the distribution functions n_n and $n_{s\pm}$ in the n- and $s_\pm$-layers with allowance for the passage of the quasiparticles across the n–s interfaces. We have in terms of the variables n_{0z} and ξ, instead of (13), the expressions

$$\zeta=1{:}\ n_0(T)+\frac{\partial n_0}{\partial T}\exp[-u(n_{0z},\xi)]C_{n2}(n_{0z},\xi)=w(n_{0z},\xi)n_{s+}+[1-w(n_{0z},-\xi)]$$
$$\times\left\{n_0(T)+\frac{\partial n_0}{\partial T}\exp[u(n_{0z},-\xi)]C_{n1}(n_{0z},-\xi)\right\},$$
$$\zeta=-1{:}\ n_0(T)+\frac{\partial n_0}{\partial T}\exp[u(n_{0z},\xi)]C_{n1}(n_{0z},\xi)=w(n_{0z},\xi)n_{s-} \tag{23}$$

$$+[1-w(n_{0z},-\xi)]\cdot\left\{n_0(T)+\frac{\partial n_0}{\partial T}\exp[-u(n_{0z},-\xi)]C_{n2}(n_{0z},-\xi)\right\},$$

where for $T \ll \Delta$ the transit probability w is[6]

$$w=f(n_z)\sqrt{\frac{\varepsilon-\Delta}{\Delta}}. \qquad (24)$$

Taking the smallness of w into account, we retain in (23) only the equilibrium parts of the distribution functions $n_{s\pm}$:

$$wn_{s\pm}\approx wn_0(T)\pm w\frac{\partial n_0}{\partial T}\delta T.$$

Making the substitution $\xi\to-\xi$ in the first equality in (23), and expanding in powers of k, we can rewrite the boundary conditions as follows:

$$\begin{gathered}[1-u(n_{0z},\xi)]C_{n1}(n_{0z},\xi)-[1+u(n_{0z},-\xi)]C_{n2}(n_{0z},-\xi)\\ =\frac{\delta T}{2}\Delta w-w\delta T+\frac{n_0\Delta w}{\partial n_0/\partial T},\\ [1+u(n_{0z},\xi)]C_{n1}(n_{0z},\xi)-[1-u(n_{0z},-\xi)]C_{n2}(n_{0z},-\xi)\\ =-\frac{\delta T}{2}\Delta w-w\delta T+\frac{n_0\Delta w}{\partial n_0/\partial T}.\end{gathered} \qquad (25)$$

where we have introduced the notation:

$$\begin{gathered}\Delta w=w(n_{0z},\xi)-w(n_{0z},-\xi),\\ w={}^1/{}_2[w(n_{0z},\xi)+w(n_{0z},-\xi)].\end{gathered} \qquad (26)$$

From (25) we find

$$\begin{gathered}C_{n1}(n_{0z},\xi)=-\frac{\delta T}{4u}\Delta w-\frac{w\delta T}{2}+\frac{w\delta T}{4}\frac{\Delta u}{u}+\frac{n_0\Delta w}{2\partial n_0/\partial T},\\ C_{n2}(n_{0z},\xi)=-\frac{\delta T}{4u}\Delta w+\frac{w\delta T}{2}-\frac{w\delta T}{4}\frac{\Delta u}{u}+\frac{n_0\Delta w}{2\partial n_0/\partial T}\end{gathered} \qquad (27)$$

Here, in analogy to (26),

$$\begin{gathered}\Delta u=u(n_{0z},\xi)-u(n_{0z},-\xi),\\ u={}^1/{}_2[u(n_{0z},\xi)+u(n_{0z},-\xi)].\end{gathered}$$

With the aid of (24) and (4) we find from (26) that

$$\Delta w=\frac{\partial w}{\partial n_z}[n_z(n_{0z},\xi)-n_z(n_{0z},-\xi)]=\frac{\partial f}{\partial n_z}\sqrt{\frac{\varepsilon-\Delta}{\Delta}}\frac{\xi(1-n_z^2)}{\mu n_z}. \qquad (28)$$

Retaining in (27) only the leading terms $\sim l$, and using (21), (11), and (28), we obtain

$$n_n=n_0(T)+\frac{\partial n_0}{\partial T}\left(-\frac{\delta T}{4k\mu}\right)\varepsilon\sqrt{\frac{\varepsilon-\Delta}{\Delta}}(1-n_z^2)\frac{df}{dn_z}. \qquad (29)$$

Let us now compute the transverse–to the interfaces–component of the electric-current density in the normal phase:

$$j_{nz}=\frac{2}{(2\pi\hbar)^3}\int d^3p\,ev_zn_n=-\frac{e_0p_0^2\delta T}{2\pi^2\hbar^3k\mu}\int_0^1 dn_zn_z(1-n_z^2)\frac{df}{dn_z}\int_\Delta^\infty\varepsilon\frac{\partial n_0}{\partial T}\sqrt{\frac{\varepsilon-\Delta}{\Delta}}d\varepsilon.$$

Here we have taken into account the obvious evenness of the function $f(n_z)$. Let us evaluate, under the assumption that this function has no singularities, the first integral by parts and denote it by f_1–a number of the order of unity:

$$\int_0^1 dn_zn_z(1-n_z^2)\frac{df}{dn_z}=\int_0^1 dn_z(3n_z^2-1)f(n_z)=f_1. \qquad (30)$$

The energy integral is easy to evaluate when $T \ll \Delta$, since in this case $n_0 \approx e^{-\epsilon/T}$:

$$\begin{gathered}\int_\Delta^\infty\varepsilon\frac{\partial n_0}{\partial T}\sqrt{\frac{\varepsilon-\Delta}{\Delta}}d\varepsilon=\frac{\partial}{\partial T}\int_\Delta^\infty\varepsilon n_0\sqrt{\frac{\varepsilon-\Delta}{\Delta}}d\varepsilon\\ \approx\frac{\partial}{\partial T}\left(\frac{\sqrt{\pi}}{2}T^{3/2}\Delta^{1/2}e^{-\Delta/T}\right)\approx\frac{\sqrt{\pi}}{2}\Delta^{3/2}T^{-1/2}e^{-\Delta/T}.\end{gathered}$$

Using (20) and (11), we finally find

$$j_{nz}=-\frac{f_1e_0lp_0^2\Delta^{3/2}T^{-1/2}e^{-\Delta/T}}{4\pi^{3/2}x_n\mu\hbar^3}\frac{\partial T}{\partial z}=\frac{9f_1}{4\pi^{3/2}x_n}\left(\frac{\Delta}{T}\right)^{3/2}e^{-\Delta/T}\beta_n\frac{\partial T}{\partial z}=\beta_{zz}\frac{\partial T}{\partial z}, \qquad (31)$$

where β_n is the corresponding thermoelectric coefficient of the normal metal[7]

$$\beta_n=-\frac{e_0lp_0^2T}{9\hbar^3\mu}. \qquad (32)$$

As can be seen from (30) and (31), the sign of β_{zz} may not coincide with that of β_n. It all depends on the sign of f_1, which is determined by the behavior of the function $f(n_z)$. It is therefore of interest to find the coefficient β_{zz} in the case of an arbitrary spectrum $\epsilon(\mathbf{p})$. From (27) we have, as above

$$C_{n1}\approx C_{n2}\approx-\frac{\delta T}{4u}\Delta w.$$

Instead of (28), we find for Δw the expression

$$\Delta w=2\delta\mathbf{n}\frac{\partial w}{\partial\mathbf{n}}=\frac{2\xi}{\partial\xi/\partial p_z}\frac{\partial w}{\partial\mathbf{n}}\frac{\partial\mathbf{n}}{\partial p_z}$$

where now

$$\mathbf{n}=\frac{\partial\xi/\partial\mathbf{p}}{|\partial\xi/\partial\mathbf{p}|},\qquad w=f(\mathbf{n})\sqrt{\frac{\varepsilon-\Delta(\mathbf{n})}{\Delta(\mathbf{n})}}.$$

The distribution function assumes the form

$$n_n=n_0(T)+\frac{\partial n_0}{\partial T}\left(-\frac{\delta T}{4k}\right)\frac{2\varepsilon}{|\partial\xi/\partial\mathbf{p}|}\frac{\partial w}{\partial\mathbf{n}}\frac{\partial\mathbf{n}}{\partial p_z}.$$

Let us find the electric current density

$$\begin{gathered}j_{nz}=-\frac{\delta T}{k(2\pi\hbar)^3}\int d^3p\,ev_z\frac{\partial n_0}{\partial T}\frac{\varepsilon}{v}\frac{\partial w}{\partial\mathbf{n}}\frac{\partial\mathbf{n}}{\partial p_z}=-\frac{e_0\delta T}{\pi^3\hbar^3a_n}\int_{n_z>0}\frac{dS}{v}n_zl(\mathbf{n})\\ \times\int_{\Delta(\mathbf{n})}^\infty\frac{\partial n_0}{\partial T}\varepsilon\,d\varepsilon\frac{\partial w}{\partial\mathbf{n}}\frac{\partial\mathbf{n}}{\partial p_z}=-\frac{e_0\delta T}{\pi^3\hbar^3a_n}\int_{n_z>0}\frac{dS}{v}n_zl\frac{\partial n_k}{\partial p_z}\\ \times\frac{\partial}{\partial T}\int_\Delta^\infty\varepsilon n_0\,d\varepsilon\left(\frac{\partial f}{\partial n_k}\sqrt{\frac{\varepsilon-\Delta}{\Delta}}-\frac{\varepsilon f}{2\Delta^{3/2}\sqrt{\varepsilon-\Delta}}\frac{\partial\Delta}{\partial n_k}\right).\end{gathered}$$

Performing the simple integration over the energy, and differentiating with respect to T with allowance for the fact that $T \ll \Delta$, we arrive at the integral over the Fermi surface:

$$j_{nz}=-\frac{e_0\delta T}{2\pi^{5/2}a_n\hbar^3}\int_{n_z>0}\frac{dS}{v}ln_z\left(\frac{\Delta}{T}\right)^{3/2}e^{-\Delta/T}\frac{\partial n_k}{\partial p_z}\left(T\frac{\partial f}{\partial n_k}-f\frac{\partial\Delta}{\partial n_k}\right).$$

In view of the strong dependence of the integrand on the gap Δ, the integration can be performed near those points of the Fermi surface where the gap is minimum[8]. The expansion of $\Delta(\mathbf{n}(\theta,\varphi))$ in the vicinity of Δ_{min}, where θ and φ are the spherical coordinates of the points of the surface, is of the form

$$\begin{gathered}\Delta(\theta,\varphi)=\Delta_{min}+\frac{1}{2}\left\{\frac{\partial^2\Delta}{\partial(\cos\theta)^2}(\cos\theta-\cos\theta_{min})^2\right.\\ \left.+2\frac{\partial^2\Delta}{\partial\varphi\,\partial\cos\theta}(\cos\theta-\cos\theta_{min})(\varphi-\varphi_{min})+\frac{\partial^2\Delta}{\partial\varphi^2}(\varphi-\varphi_{min})^2\right\}.\end{gathered}$$

In that case

$$\begin{gathered}j_{nz}=-\frac{e_0\delta T}{2\pi^{5/2}a_n\hbar^3T^{1/2}}\left[\frac{n_zl\Delta^{5/2}}{vK(\theta,\varphi)}e^{-\Delta/T}\frac{\partial f}{\partial\mathbf{n}}\frac{\partial\mathbf{n}}{\partial n_z}\right]_{min}\\ \times\int d(\cos\theta-\cos\theta_{min})\int d(\varphi-\varphi_{min})\exp\left\{-\frac{1}{2T}\left[\frac{\partial^2\Delta}{\partial(\cos\theta)^2}(\cos\theta-\cos\theta_{min})^2\right.\right.\\ \left.\left.+2\frac{\partial^2\Delta}{\partial\varphi\,\partial\cos\theta}(\cos\theta-\cos\theta_{min})(\varphi-\varphi_{min})+\frac{\partial^2\Delta}{\partial\varphi^2}(\varphi-\varphi_{min})^2\right]\right\},\end{gathered}$$

where $K(\theta,\varphi)$ is the Gaussian curvature of the surface and the coefficient in front of the double integral is evaluated at the minimum point of the gap. The double integral entering into this expression is equal to $2\pi T/\sqrt{|\Delta''|}$[8], where

$$|\Delta''| = \frac{\partial^2\Delta}{\partial(\cos\theta)^2}\frac{\partial^2\Delta}{\partial\varphi^2} - \left(\frac{\partial^2\Delta}{\partial\varphi\,\partial\cos\theta}\right)^2.$$

Thus,

$$j_{nz} = -\frac{e_0\delta T T^{1/2}}{\pi^{1/2}a_n\hbar^3\sqrt{|\Delta''|}}\left[\frac{n_z l\Delta^{1/2}}{vK}e^{-\Delta/T}\frac{\partial f}{\partial \mathbf{n}}\frac{\partial \mathbf{n}}{\partial p_z}\right]_{min} = \beta_{zz}\frac{\partial T}{\partial z}.$$

Allowing for (20), we finally have in the anisotropic case

$$\beta_{zz} = -\frac{e_0 T^{1/2}}{2\pi^{1/2}x_n\hbar^3\sqrt{|\Delta''|}}\left[\frac{n_z l\Delta^{1/2}}{vK}e^{-\Delta/T}\frac{\partial f}{\partial \mathbf{n}}\frac{\partial \mathbf{n}}{\partial p_z}\right]_{min}. \quad (33)$$

3. DISCUSSION AND COMPARISON WITH THE AVAILABLE DATA

Let us write out the components of the density of the thermal flux arising in the general case under the action of E and ∇T. Using the formulas (17), (19), and (31) and the expression found for κ_{ik} by Andreev[6, 8], we obtain

$$\begin{aligned} q_x &= -\varkappa_\parallel\frac{\partial T}{\partial x} = -\frac{21\zeta(3)}{4\pi^2}\frac{a_n^2}{dl}\varkappa_n\frac{\partial T}{\partial x}, \\ q_y &= -\varkappa_\parallel\frac{\partial T}{\partial y} + \gamma_{yy}E_y = -\frac{21\zeta(3)}{4\pi^2}\frac{a_n^2}{dl}\varkappa_n\frac{\partial T}{\partial y} + \frac{3}{2}\gamma_n E_y, \\ q_z &= -\varkappa_\perp\frac{\partial T}{\partial x} + \gamma_{zy}E_y + \gamma_{zz}E_z = -\frac{9f_0}{4\pi^{1/2}}\frac{d}{l}\left(\frac{\Delta}{T}\right)^{1/2}e^{-\Delta/T}\varkappa_n\frac{\partial T}{\partial z} \\ &\quad -\frac{cQ}{H_c}E_y + \frac{9f_1}{4\pi^{1/2}x_n}\left(\frac{\Delta}{T}\right)^{1/2}e^{-\Delta/T}\gamma_n E_z, \end{aligned} \quad (34)$$

where κ_n is the thermal conductivity of the normal metal, $d = a_n + a_s$ is the structure period of the intermediate state, $\zeta(x)$ is the Riemann zeta function, and

$$f_0 = \int_0^1 dn_z f(n_z) n_z$$

is a number of the order of unity. We have also used the Onsager relations, according to which

$$\gamma_{ik}(\mathbf{H}) = -T\beta_{ki}(-\mathbf{H}). \quad (35)$$

The equalities (34) are equivalent to the single vector relation

$$\begin{aligned} \mathbf{q} &= -\varkappa_\parallel\nabla T - (\varkappa_\perp - \varkappa_\parallel)\mathbf{k}(\mathbf{k}\nabla T) + \gamma_{yy}\left\{\mathbf{E} - \mathbf{k}(\mathbf{kE}) - \frac{\mathbf{H}}{H_c^2}(\mathbf{HE})\right\} \\ &\quad + \frac{\gamma_{zy}}{H_c}\mathbf{k}(\mathbf{E}[\mathbf{k}\times\mathbf{H}]) + \gamma_{zz}\mathbf{k}(\mathbf{kE}). \end{aligned} \quad (36)$$

Here k is the unit vector normal to the layer boundaries. To write down a similar expression for the macroscopic electric current density j, it is necessary to know its relation with the "microscopic" current $\mathbf{j}_n$ in the region occupied by the normal phase. It is not difficult to verify that the relation between the current components $\mathbf{j}_\perp$ and $\mathbf{j}_{n\perp}$ perpendicular to the magnetic field H takes, as in[4], the form

$$\mathbf{j}_\perp = \mathbf{j}_{n\perp} + \frac{cQ}{TH_c^2}[\mathbf{H}, \times\nabla T - \nabla\tau_n] = \mathbf{j}_{n\perp} - \frac{cQ}{TH_c^2}[\mathbf{k}\times\mathbf{H}](\mathbf{k}\nabla T), \quad (37)$$

since for $l \gg a_{n,s}$ the gradients of the "microscopic" and macroscopic temperatures $\nabla\tau_n$ and ∇T are connected by the relations

$$\frac{\partial\tau}{\partial x, y} = \frac{\partial T}{\partial x, y}, \quad \frac{\partial\tau}{\partial z} = 0.$$

Using (34), (35), and the expression given in[5] for σ_{ik}, we find from (37) that

$$\begin{aligned} \mathbf{j}_\perp &= \frac{\sigma_n}{x_n}\mathbf{E} + \frac{3\beta_n}{2}\left\{\nabla T - \mathbf{k}(\mathbf{k}\nabla T) - \frac{\mathbf{H}}{H_c^2}(\mathbf{H}\nabla T)\right\} \\ &\quad + \frac{9f_1}{4\pi^{1/2}x_n}\left(\frac{\Delta}{T}\right)^{1/2}e^{-\Delta/T}\beta_n\mathbf{k}(\mathbf{k}\nabla T) - \frac{cQ}{TH_c^2}[\mathbf{k}\times\mathbf{H}](\mathbf{k}\nabla T), \end{aligned} \quad (38)$$

where σ_n is the electrical conductivity of the normal metal. It is also easy to find the formulas expressing E and q in terms of the total current j and the temperature gradient ∇T:

$$E_i = \rho_{ij}j_j + \alpha_{ij}\frac{\partial T}{\partial x_j}, \quad q_i = T\alpha_{ji}(-\mathbf{H})j_j - \varkappa_{ij}\frac{\partial T}{\partial x_j}. \quad (39)$$

These formulas completely determine the resistivity, thermoelectricity, and thermal-conductivity tensors, ρ_{ij}, α_{ij}, and κ_{ij} respectively, under the conditions under consideration.

Let us first find κ_{ij}. With the aid of (36) and (38) we obtain

$$\begin{aligned} \varkappa_{ij} &= \varkappa_\parallel\delta_{ij} + \left(\varkappa_\perp + \frac{x_n c^2Q^2}{\sigma_n TH_c^2} - \varkappa_\parallel\right)k_ik_j - \frac{3x_n\alpha_n cQ}{2H_c^2}\varepsilon_{ijl}H_l \\ &= \varkappa_\parallel\delta_{ij} + (\varkappa_\perp' - \varkappa_\parallel)k_ik_j + \frac{\varkappa_{yz}}{H_c}\varepsilon_{ijl}H_l. \end{aligned} \quad (40)$$

The ratio of the second term in $\kappa_\perp'$ to the first is, in order of magnitude, equal to

$$\frac{c^2Q^2}{\varkappa_\perp\sigma_n TH_c^2} \sim \left(\frac{T}{\Delta}\right)^{1/2}e^{\Delta/T}\frac{l}{d}\frac{c^2Q^2}{\sigma_n\varkappa_n TH_c^2} \sim \left(\frac{T}{\Delta}\right)^{1/2}e^{\Delta/T}\frac{l}{d}\left(\frac{e_0cH_c}{\sigma_nT_c}\right)^2 \sim \left(\frac{T}{\Delta}\right)^{1/2}e^{\Delta/T}\varkappa_{GL}{}^2\xi_0{}^2/ld,$$

where κ_{GL} is the parameter of the Ginzburg-Landau theory and ξ_0 is the coherence length. As $T \to 0$, this relation grows without limit; therefore, $\kappa_\perp' = x_n c^2Q^2/\sigma_n TH_c^2$. However, at not too low temperatures, at which experiments are usually performed, this relation is small, since $l \gg \xi_0$ and $d \gg \xi_0$. Let us estimate the order of the off-diagonal component κ_{yz} of the thermal-conductivity tensor. We compare it with the same component for a real metal, a component which was computed in[8], and which is of the same order of magnitude as $\kappa_\perp$:

$$\frac{\varkappa_{zy}}{\varkappa_\perp} \sim \left(\frac{T}{\Delta}\right)^{1/2}e^{\Delta/T}\frac{l}{d}\frac{\alpha_n cQ}{\varkappa_n H_c} \sim \left(\frac{T}{\Delta}\right)^{1/2}e^{\Delta/T}\frac{ecH_cT}{\sigma_nT_c\mu} \sim \left(\frac{T}{\Delta}\right)^{1/2}e^{\Delta/T}\varkappa_{GL}\frac{\xi_0 T}{d\mu}.$$

This relation is also small at not too low temperatures ($t \ll \mu$).

Taking the foregoing into account, we finally find the sought tensors:

$$\begin{aligned} \rho_{ij} &= \frac{x_n}{\sigma_n}\left(\delta_{ij} - \frac{H_iH_j}{H_c^2}\right), \quad \varkappa_{ij} = \varkappa_\parallel\delta_{ij} + (\varkappa_\perp - \varkappa_\parallel)k_ik_j, \\ \alpha_{ij} &= \frac{3x_n\alpha_n}{2H_c^2}[\mathbf{k}\times\mathbf{H}]_i[\mathbf{k}\times\mathbf{H}]_j + \frac{x_ncQ}{\sigma_nTH_c^2}[\mathbf{k}\times\mathbf{H}]_ik_j + \frac{9f_1\alpha_n}{4\pi^{1/2}}\left(\frac{\Delta}{T}\right)^{1/2}e^{-\Delta/T}k_ik_j. \end{aligned} \quad (41)$$

As to the macroscopic-electrodynamics equations (including the heat equations, equations which determine the shape of the layers and their traveling speed) themselves, they clearly coincide completely with the equations derived in[4], since no assumptions about the mean free path of the excitations were made in their derivation.

Let us make a few observations about the found thermoelectric coefficients of the intermediate state. The off-diagonal component of the tensor α_{ij} (the second term in the last formula in (41)) is due to the heat release Q at the boundaries of the n- and s-regions. This quantity naturally does not depend on l and coincides therefore with the same component for a dirty metal[4]. The diagonal component of α_{ij} that is responsible for the electric current along the layers (and across the magnetic field) is greater by a factor of $1.5x_n$ (in the isotropic case) than the thermoelectric coefficient of the normal bulk metal. This occurs because of the influence of the n–s phase interfaces, at which the excitations undergo Andreev reflection[4)]. The diagonal transverse component α_{zz}, which is $\sim\beta_{zz}$ defined by (31) and (33), is an exponentially small quantity $\sim\exp(-\Delta/T)$ at low temperatures. It is determined by the angle derivatives of the probability, w, of passage of an excitation across an interface. Unlike w itself, which is positive (or zero), these derivatives can be positive or negative. In the isotropic case, (30), the sign of β_{zz} does not depend on the arrangement of the interfaces; it is determined only by the behavior of the function $f(n_z)$. Any possible decrease of w that occurs with increasing angle of incidence of the excitations at an interface as a result of the increase of the "effective" thickness of the transition layer should change into growth, since for a large thickness (large angles of incidence) the above-the-barrier reflection becomes quasiclassical (cf.[6]), and in this case w is exponentially close to unity[10]. It can be seen, therefore, that, depending on the properties of the metal, α_{zz} may be either negative, which corresponds to the predominance of the electronic mechanism, or positive (the hole mechanism)[5)] In an anisotropic metal β_{zz}, (33), also depends on the orientation of the n–s interface. It may be inferred therefore that, by smoothly turning the layers of the intermediate state, we can observe the reversal of the sign of this thermoelectric coefficient.

In spite of the absolute smallness of the thermoelectric coefficients, their existence leads to entirely observable phenomena. This is because the various components of the tensor α_{ij} strongly differ from each other. The ratio of the second term in (41) to the first is, in order of magnitude, equal to

$$\frac{cQ}{\sigma_n\alpha_n TH_c} \sim \varkappa_{GL}\frac{\xi_0}{l}\frac{\mu}{T}$$

This quantity significantly exceeds unity in not too pure metals. The ratio of the first term to the third is of the order of $(T/\Delta)^{3/2}\exp(\Delta/T)$, and is also large at low temperatures. A consequence of this anisotropy (as well as of the anisotropy of the thermal-conductivity tensor) is the recently observed[11] alignment of the structure of the intermediate state arising in a plate of pure Pb located in an external transverse magnetic field in the presence in the plate of a thermal flux q (the plate itself was electrically insulated). The angle φ_q between the direction q and the normal **k** to the layers increased monotonically from zero to $\pi/2$ as the temperature decreased from T_c to $\approx T_c/3$. The structure as a whole was stationary, but the resulting irregularity of the n–s interfaces moved with some velocity V along the layers. As T decreased, this velocity first decreased, attaining a minimum at $T \approx T_c/2$, and then began to increase. In[9], formulas for $\tan\varphi_q$ and V in the case of a dirty metal are derived that qualitatively explain the observed alignment for all temperatures. Such an alignment should also occur in pure metals. With the aid of (36) and (38) in the geometry of Fig. 1 (the plane of the plate coincides with the y-z plane), we find (see[9])

$$\operatorname{tg}\varphi_q = \frac{q_y}{q_z} = \frac{14\zeta(3)\dot{x}_n^2 cQ}{9\sqrt{\pi} f_0\sigma_n\alpha_n TH_c}\left(\frac{T}{\Delta}\right)^{1/2} e^{\Delta/T},$$

$$V = -\frac{c\alpha_n l}{\varkappa_n x_n d}\cos\varphi_q\frac{f_1 q}{f_0 H_c}.$$

It can be seen that even in an extremely pure metal, where $cQ/\sigma_n\alpha_n TH_c \ll 1$, at low $T \ll \Delta$, the quantity $\operatorname{tg}\varphi_q \to \infty$, i.e., the structure tends to assume a form in which the heat flows along the layers (i.e., in which $\mathbf{k}\perp\mathbf{q}$). As to the velocity, it should tend to zero as $T \to 0$. Notice that the velocity did not change its sign in the temperature range considered in[11]. By the same token, the thermoelectric coefficient α_{zz}, which determines this velocity, did not change its sign. In this instance this is natural, since the sample was not a single crystal, in which such a change of the sign of the velocity could have been observed.

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1)Since the aim of our investigation is to obtain macroscopic equations, and since a macroscopic description is possible only when the radius R of curvature of the layers significantly exceeds the structure period d (i.e., only when $R \gg d$), the curvature of the layers can be neglected in the present case.

2)Since we are interested only in the terms linear in V, we evaluate δp_z at the stationary interface.

3)The bounding surface separating the Andreev- and specular-reflection regions is an ellipsoid inscribed near the Fermi surface and touching it at the poles: $p_z^2/2 + p_t^2 = p_0^2$.

4)In dirty metals (in which $l \ll a_{n,s}$) such growth naturally does not occur. At low T in a dirty metal $\alpha = x_n\alpha_n$ [9].

5)The monotonic decrease of f with increasing angle of incidence corresponds to $\alpha_{zz} < 0$.

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