Stationary large-scale magnetic field in a gyrotropic turbulent medium

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An effective mechanism is discussed for the establishment of an equilibrium large-scale magnetic field in a weakly inhomogeneous, rotating, turbulent medium (the α effect) for large values of the hydrodynamic Reynolds number. It is shown that when $R_m \ll 1$ and $R_m \gg 1$, where R_m is the magnetic Reynolds number, the steady-state magnetic energy can be much less than the turbulence kinetic energy. Estimates are obtained for the ratio of these two energies. Conditions are found for the parameters of gyrotropic turbulence which ensure the generation of a regular magnetic field.

INTRODUCTION

The generation of large-scale magnetic fields in turbulent conducting fluids (the turbulent dynamo problem) has been discussed by many authors in connection with the origin and maintenance of magnetic fields in astrophysical objects. [1]

There is a well-known kinematic formulation of the problem where the statistical characteristics of the random velocity field are assumed given, and the reaction of the magnetic field on the motion of the conducting fluid is not taken into account. This analysis is valid, for example, for cosmic systems in which the energy associated with the magnetic field is much less than the kinetic energy of random motions. Existing estimates based on this approximation suggest that it is possible to amplify the large-scale magnetic field by small-scale turbulence when the latter exhibits helical symmetry. [2] This property of turbulence, called gyrotropy, is connected with the existence of helical motions with a special direction along the helix axis and is characterized by the quantity $\langle \mathbf{v}\cdot \operatorname{curl} \mathbf{v}\rangle$ which is averaged over the pulsations, where $\mathbf{v}$ is the velocity of turbulent motions. It is shown in [1,2] that the conditions prevailing in cosmic objects readily admit the appearance of turbulent plasma motions with the required type of symmetry. For example, the situation may arise in a rotating star with a developed convective envelope. The possibility that the magnetic field can be amplified by the turbulent dynamo is also confirmed by experimental data but these, unfortunately, are not as yet very extensive and have been obtained under conditions somewhat different from those for which the theory was developed. [3]

In the kinematic approximation, the equation describing the variation in the magnetic field $\mathbf{B} = \langle \mathbf{H}\rangle$, averaged over the velocity-field ensemble, may have solutions which increase exponentially with time. For sufficiently large times and, consequently, for large magnetic fields, this approximation will no longer be valid because the reaction of the electromagnetic forces on the motion of the fluid becomes important, restricting the unbounded increase in the field and leading to the establishment of a stationary state. The question then arises as to what are the conditions for the existence of the stationary regime and what is the value of the steady-state magnetic field.

In general, theoretical analyses of this problem are complicated by the fact that it is necessary to solve the self-consistent equations of hydrodynamics and electrodynamics. Estimates of the steady-state magnetic field are therefore frequently derived from the condition that the magnetic and kinetic energies are uniformly distributed, $B^2 \sim 4\pi\rho\langle v^2\rangle$ (where ρ is the fluid density). However, more careful analyses performed within the framework of various approximations have shown that it is possible to depart from this uniform distribution.

Thus, the authors of [4] have considered the case of homogeneous gyrotropic turbulence generated by a directed flux of random waves in a fluid rotating with high angular velocity Ω for large values of the hydrodynamic Reynolds number (Re $\gg 1$) and much smaller values of the magnetic Reynolds number ($R_m \ll$ Re). The angular velocity was assumed in [4] to be much greater than the characteristic frequencies of the conducting turbulent fluid, i.e., $\Omega \gg u_0 l_0^{-1}, \omega_0, \nu_m l_0^{-2}$, where l_0 is the turbulent energy scale, ω_0 is the frequency of the exciting force, u_0 is the root mean square velocity with scale l_0, and ν_m is the magnetic viscosity. It was assumed, moreover, that the large-scale magnetic field amplified in the gyrotropic medium was sufficiently large, i.e., $\omega_B \gg \nu_m l_0^{-2}$, where $\omega_B = B l_0^{-1}(4\pi\rho)^{-1/2}$. It was found that, under these conditions, the stationary large-scale magnetic field might reach values much greater than the kinetic energy associated with the turbulence.

On the other hand, we have the real possibility of a stationary state with a magnetic field much smaller than the value corresponding to the case of uniformly distributed energy, $\beta = B^2/4\pi\rho\langle v^2\rangle \ll 1$. This possibility was pointed out by Vaĭnshteĭn, [5] who noted that a growing magnetic field in a gyrotropic medium with Re $\gg 1$ led to the appearance of a force in the conducting fluid which acted as the source of additional (magnetic) gyrotropy. The sign of the helical symmetry of the excited gyrotropic motions is then opposite to the sign of the helical symmetry of unperturbed gyrotropy, and this leads to a reduction in the quantity $\langle \mathbf{v}\cdot \operatorname{curl} \mathbf{v}\rangle$ and, consequently, to a weakening in the growth of the large-scale field. In this case, the establishment of a stationary regular magnetic field satisfying the condition $\beta_{st} < 1$ is only achieved at the generation threshold when the growth rate γ_B is close to the growth rate $\gamma_B L_B^2 \gtrsim (\nu_T + \nu_m)$, where L_B is the characteristic size of the large-scale magnetic field and ν_T is the turbulent viscosity.

It is shown in the present paper that, for large values of the Reynolds number (Re $\gg 1$), one may have to take into account an effect of magnetic stresses on the gyroscopic media, which is analogous to the effect of dissipative forces with an effective viscosity proportional to B^2. This magnetic dissipation of the quantity $\langle \mathbf{v}\cdot \mathrm{curl}\, \mathbf{v}\rangle$ leads to a reduction in the growth rate and to the establishment of a stationary energy which is associated with the regular magnetic field generated in the gyrotropic field. The effectiveness of this stabilization mechanism is demonstrated below in two cases, namely, small values of the magnetic Reynolds number ($R_m < 1$) when $h < B$ (h is the random component of the magnetic field, and for $R_m \gg 1$ in the case of a Gaussian distribution of probabilities for the velocity field and a δ-function time correlation. In both cases, the characteristic frequency of turbulent pulsations in the conducting fluid $\tau_0^{-1} \sim u_0 l_0^{-1}$ is assumed to be much greater than Ω and ω_B and, therefore, the influence of resonance frequencies, which play the leading role in, [4] is not taken into account.

It is assumed that the amplification of the magnetic field occurs for $\beta \ll 1$ in a time much greater than all the hydrodynamic times. As the regular magnetic field increases under these conditions, its influence on the hydrodynamic quantities may be looked upon as a perturbation which does not affect the form of the generation equation obtained in the kinematic approximation. [1] We can derive from the generation equation the stationary equation for the magnetic energy averaged over the entire volume occupied by the turbulent fluid: [5]

$$\int d^3\mathbf{r}\, \alpha \mathbf{B}\, \mathrm{rot}\, \mathbf{B} = (\nu_T + \nu_m) \int d^3\mathbf{r}\, (\mathrm{rot}\, \mathbf{B})^2, \tag{1}$$

where

$$\alpha = -{}^1/_3 \tau_0 \langle \mathbf{v}\, \mathrm{rot}\, \mathbf{v}\rangle, \quad \nu_T = {}^1/_3 \tau_0 \langle v^2\rangle.$$

The right-hand side of (1) corresponds to a positive influx of magnetic energy from the kinetic energy of the pulsations for solutions which increase in time, when $\alpha \mathbf{B}\cdot \mathrm{curl}\, \mathbf{B} > 0$, and the left-hand side corresponds to the outflow of magnetic energy due to the turbulent viscosity.

Equation (1) can be used to estimate the stationary value β_{st}. All that is required is to establish the dependence of α and ν_T on the magnetic field.

1. ESTIMATES OF GYROTROPY IN MAGNETIC MEDIA

As a first step, let us estimate the gyrotropy in a turbulent medium in the absence of a magnetic field. Although this estimate has been obtained by Steenbeck and Krause, [2] it does not appear to be sufficiently accurate because the correct value of α must be obtained with allowance for the viscosity of the medium, and this was not taken into account in [2].

It is well known [6] that helical motions with a preferred direction of the helix axis may arise in an inhomogeneous rotating fluid. For the sake of simplicity, we consider a turbulent medium in which the density varies continuously and the root mean square velocity remains constant, $\nabla\rho/\rho = q \sim 1/L_f$, $L_f \gg l_0$ where ρ_0 is the density, l_0 the principal turbulence scale, and $\langle v^2\rangle = \mathrm{const}$. Starting with the Navier-Stokes equations, we have

$$\frac{d}{dt}\langle \mathbf{v}\, \mathrm{rot}\, \mathbf{v}\rangle = 4\langle \mathbf{v}\, \mathrm{rot}[\mathbf{v}\times\boldsymbol{\Omega}]\rangle - \left\langle \mathrm{div}\frac{1}{\rho}[\mathbf{v}\ \nabla p_\Omega]\right\rangle + \nu\{\langle \mathrm{rot}\, \mathbf{v}\Delta\mathbf{v}\rangle + \langle \mathbf{v}\, \mathrm{rot}\, \Delta\mathbf{v}\rangle\}. \tag{2*}$$

In this equation, $\boldsymbol{\Omega}$ is the angular velocity of the fluid, $\Omega\tau_0 < 1$, ν is the kinematic viscosity, p_Ω is the pressure connected with the effect of Coriolis forces. and $\Delta p_\Omega = 2\, \mathrm{div}\, \rho(\mathbf{v}\times\boldsymbol{\Omega})$.

The expressions

$$\langle \mathrm{div}\, [[\mathbf{v}\times \mathrm{rot}\, \mathbf{v}]\times\mathbf{v}]\rangle, \quad \langle \mathrm{div}\, [\mathbf{v}\times\nabla v^2]\rangle, \quad \langle \mathrm{div}\, [\mathbf{v}\times[\mathbf{v}\times\boldsymbol{\Omega}]]\rangle,$$

and $\langle \mathrm{div}\, \rho^{-1}(\mathbf{v}\times\nabla p_0)\rangle$, where p_0 is the isotropic part of the random pulsations, are removed in the course of averaging. Using the continuity equation $\mathrm{div}\, \rho\mathbf{v}_0 = 0$, and eliminating p_Ω from (2), we have, to within quantities of the order of $l_0^2 L_F^{-2}$

$$\frac{d}{dt}\langle \mathbf{v}\, \mathrm{rot}\, \mathbf{v}\rangle = 4Q - 4\nu\int_0^\infty dk\, k^2 F(k),$$
$$Q = \langle \mathbf{v}\, \mathrm{rot}\, [\mathbf{v}\boldsymbol{\Omega}]\rangle = -\langle \mathbf{v}\boldsymbol{\Omega}\, \mathrm{div}\, \mathbf{v}\rangle = {}^1/_3 q\Omega\langle v^2\rangle, \tag{3}$$
$$\left\langle \mathrm{div}\frac{1}{\rho}[\mathbf{v}, \nabla p_0]\right\rangle = 0,$$

where the spectral function F(k) corresponds to the gyrotropic part of the correlation tensor of the velocity field in k space $ik_f\epsilon_{ij}F(k)/4\pi k^4$ and defines the required quantity

$$\langle \mathbf{v}\, \mathrm{rot}\, \mathbf{v}\rangle = -2\int_0^\infty dk\, F(k).$$

In these expressions, Q is the source of the helical motions which determines, together with dissipation, the gyrotropy spectrum. The sign of Q corresponds to the predominance of the right-handed or left-handed spirals in the volume over which the average is evaluated. We note that, in the present case, Q is nonzero only when $\mathrm{div}\, \mathbf{v} \neq 0$, and this corresponds to the anisotropy of turbulent motions. It is clear that, in the case of weak inhomogeneity ($L_f \gg L_0$), the turbulent eddies will be anisotropic only for scales close to l_0 or greater, so that the main contribution to the source Q, and hence to the gyrotropy, will correspond to large scales. In the stationary state,

$$\langle \mathbf{v}\, \mathrm{rot}\, \mathbf{v}\rangle = Q\nu^{-1}\lambda_g^2,$$

where λ_g, which characterizes the equilibrium gyrotropy spectrum, is defined by

$$\lambda_g^2 = \int_0^\infty dk\, F(k)\left[\int_0^\infty dk\, k^2 F(k)\right]^{-1}.$$

In accordance with [7], we introduce a scale which characterizes the energy spectrum E(k) of the turbulence:

$$\lambda^2 = \int_0^\infty dk\, E(k)\left[\int_0^\infty dk\, k^2 E(k)\right]^{-1},$$

where $\lambda^2\nu^{-1} \sim \tau_0$. We then have

$$\langle \mathbf{v}\, \mathrm{rot}\, \mathbf{v}\rangle = Q\tau_0\xi \sim \Omega L_f^{-1} l_0^2 \tau_0^{-1}\xi,$$

where $\xi = \lambda_g^2\lambda^{-2}$. If the spectral function F(k) is substantially nonzero in a narrower interval of values of k near l_0^{-1} than the spectral density E(k), we have the condition $\xi > 1$ (or $\xi \gg 1$), and the resulting estimate for the stationary value of the gyrotropy differs from the corresponding estimate in [2] by the factor ξ.

The above example shows that the special direction of the helix axis in the medium is closely connected with the existence of anisotropic pulsations. There is experimental evidence that the anisotropy is exhibited only by eddies with the maximum scales, the wave numbers of which lie in the above energy interval and are shifted toward lower values. On the other hand, experiment indicates the existence of a high degree of isotropy of turbulent motion [7,8] for scales much smaller than l_0 (large k), which is the basis of the proposed existence of the universal equilibrium scales. We assume that the gyrotropic part of the turbulent motions must also undergo isotropization in the course of transition to smaller scales (large k). Its spectrum must therefore fall more rapidly toward larger k than the spectrum of the main isotropic part, so that for Re $\gg 1$ we have the condition $\xi > 1$. For example, if we suppose that $E(k) \sim k^{-5/3}$ and $F(k) \sim k^{-2}$ right up to the dissipation scales, then $\xi = \ln \mathrm{Re}$. Of course, this assumption requires experimental verification.

The conclusions of this section do not depend on the adopted model of turbulence. In particular, they remain the same for inhomogeneous velocity fields $\langle \mathbf{v}^2 \rangle = f(\mathbf{r})$ with characteristic inhomogeneity scale $L_f \gg l_0$. When this is so, expressions of the form $\langle \mathrm{div} \ldots \rangle$ in (2) will be nonzero (these expressions correspond to the flow of gyrotropy through the boundary of the volume in which the average is taken). It can be shown that, if we neglect terms of the order of $(l_0 L_f^{-1})^3$, the contribution of these terms is of the order of the gyrotropy source Q, i.e., $\mathbf{\Omega}(v^2\mathbf{q} + \nabla f)$, and that allowance for these terms does not essentially affect the estimated level of gyrotropy. This is readily verified by substituting a pseudoscalar of the form div $\mathbf{A}$, where $\mathbf{A}$ is a pseudovector determined by a combination of $\mathbf{q}$ and the pseudovector $\mathbf{\Omega}$, or one of these vectors. We note that, in general, the existence of gyrotropy is not necessarily connected with the compressibility of the fluid and, moreover, any anisotropy produced by the presence of pseudovector and vector quantities in the turbulent medium may lead to the formation of the pseudoscalar quantity $\langle \mathbf{v} \cdot \mathrm{curl}\, \mathbf{v} \rangle$.

2. EFFECT OF A WEAK REGULAR MAGNETIC FIELD ON TURBULENCE

We shall assume that the condition $\beta \ll 1$ is satisfied for a regular magnetic field frozen into turbulent plasma. We shall show that, for large hydromagnetic Reynolds numbers (Re $\gg 1$), the effect of a weak large-scale magnetic field on the balance of the kinetic energy $\langle v^2 \rangle$ of stationary turbulent motions in a conducting fluid can be much less than the effect on the balance of the quantity $\langle \mathbf{v} \cdot \mathrm{curl}\, \mathbf{v} \rangle$ which characterizes the turbulence gyrotropy.

We shall consider stationary uniform turbulence, produced by uniformly distributed sources, and denote by ϵ the amount of energy entering the fluid per unit time per unit mass in the energy interval with characteristic scale l_0. Let us write down the equation for kinetic-energy dissipation with allowance for the magnetic force in the Navier-Stokes equation for

$$\langle v^2 \rangle = 2\int_0^\infty dk\, E(k), \tag{4}$$

$$\varepsilon = 4\nu \int_0^\infty dk\, k^2 E(k) + (2\pi\rho)^{-1}\{-\mathrm{rot}\,\mathbf{B}\langle[\mathbf{vh}]\rangle + \mathbf{B}\langle[\mathbf{v}\,\mathrm{rot}\,\mathbf{h}]\rangle + \langle[\mathrm{rot}\,\mathbf{h}, \mathbf{h}]\mathbf{v}\rangle\},$$

and also the balance equation for the energy associated with the random component of the magnetic field $\mathbf{h}$, when

$$\langle h^2 \rangle = 2\int_0^\infty H(k)dk,$$

$$0 = \nu_m (\pi\rho)^{-1} \int_0^\infty dk\, k^2 H(k) - (2\pi\rho)^{-1}\{\mathbf{B}\langle[\mathbf{v}\,\mathrm{rot}\,\mathbf{h}]\rangle + \langle[\mathrm{rot}\,\mathbf{h}, \mathbf{h}]\mathbf{v}\rangle\}. \tag{5}$$

Combining (4) and (5), we obtain a generalization of the corresponding expression obtained by Chandrasekhar [9] for the case when there is a regular component of the magnetic field:

$$\varepsilon = 4\nu \int_0^\infty dk\, k^2 E(k) + \nu_m (\pi\rho)^{-1} \int_0^\infty dk\, k^2 H(k) - (2\pi\rho)^{-1}\,\mathrm{rot}\,\mathbf{B}\langle[\mathbf{v}\times\mathbf{h}]\rangle. \tag{6}$$

For small values $R_m \ll 1$, the random component of the magnetic field is small in comparison with the regular component ($h \ll B$), and the second term on the right-hand side of (6) can be neglected in comparison with the first.

When $R_m \ll 1$ we then have

$$\langle[\mathbf{vh}]\rangle = -\frac{2}{3}R_m\left\{\mathbf{B}\tau_0\int_0^\infty dk\, F(k) - \mathrm{rot}\,\mathbf{B}\tau_0\int_0^\infty dk\, E(k)\right\};$$
$$\langle \mathbf{v}\,\mathrm{rot}\,\mathbf{v}\rangle = -2\int_0^\infty dk\, F(k), \tag{7}$$

if the inhomogeneity is neglected.

The last term on the right-hand side of (6) is of the order of $R_m \beta(l_0 L_B^{-1})\tau_0^{-1}\langle v^2\rangle$, i.e., it is also much less than the first term which is equal to $\nu\lambda^{-2}\langle v^2\rangle$ to within an order of magnitude. The effect of a weak ($\beta \ll 1$) regular magnetic field on the quantity $\langle v^2\rangle$, i.e., the stationary turbulence level, can therefore be neglected for $R_m \ll 1$.

For large values of R_m, the random component of the magnetic field, h, can reach values much greater than B and may have an important effect on the equilibrium turbulence spectrum. One cannot then neglect the second term on the right-hand side of (6). To estimate the contribution of the regular magnetic field B to the energy balance equation (6), we shall use the well-known result for $\langle \mathbf{v}\times\mathbf{h}\rangle$ [1] for the Gaussian distribution of velocity-field probabilities and δ-function correlations in time. This expression differs from (7) by the factor R_m^{-1}. The last term on the right-hand side of (6) is therefore of the order of $\beta(l_0 L_B^{-1})^2\tau_0^{-1}\langle v^2\rangle$. This is negligible in comparison with the first term on the right-hand side of (6) if the characteristic time of the large-scale magnetic field $T_b \sim \tau_0(L_B l_0^{-1})^2$ is much greater than the characteristic dissipation time $\lambda^2\nu^{-1}$ which, in general, is no longer a quantity of the order of τ_0.

When $R_m \gg 1$ and $\nu_m > \nu$, we can follow [10] and take $\lambda \lesssim l_0 R_m^{-1/2}$ as the upper estimate for the scale λ of the spectral function E(k) for uniformly distributed turbulent kinetic energy and magnetic pulsation energy. This is done by analogy with the fact that, in the absence of the magnetic field, $\lambda \lesssim l_0 \mathrm{Re}^{-1/2}$. The requirement that the mean magnetic field must be regular, $T_B \gg \lambda^2\nu^{-1}$, can then take the form $R_m(L_B l_0^{-1})^2 \gg \mathrm{Re}$, where R_m may be less than Re. We note that for the steady-state regular magnetic field generated in the gyrotropic fluid, the last term on the right-hand side of (6) is $\nu_m(2\pi\rho)^{-1} \times (\mathrm{curl}\,\mathbf{B})^2$.

To take into account the effect of the weak regular magnetic field on $\langle \mathbf{v}\cdot\mathrm{curl}\,\mathbf{v}\rangle$, which determines the gyrotropic properties of the turbulence, let us consider the uniform gyrotropic fluid approximation, when the medium contains a gyrotropy source Q which is determined by the influence of external conditions on the maximum turbulence scale. It is well known [11] that, in the case of a uniform fluid, the quantity $\langle \mathbf{v}\cdot\mathrm{curl}\,\mathbf{v}\rangle$ is an invariant as $\nu \to 0$, since the equation for this quantity, which is given by (2), does not contain contributions due to nonlinear terms, just as in the case of the equation for $\langle v^2\rangle$. Therefore, the gyrotropy balance equation obtained from the Navier-Stokes equation with allowance for the magnetic field can be written in the form

$$Q=4\nu\int_0^\infty dk\,k^2F(k)+(2\pi\rho)^{-1}\{\mathbf{B}\langle[\mathrm{rot}\,\mathbf{v},\times\,\mathrm{rot}\,\mathbf{h}]\rangle+\mathrm{rot}\,\mathbf{B}\langle[\mathbf{h}\,\mathrm{rot}\,\mathbf{v}]\rangle+\langle[\mathrm{rot}\,\mathbf{v},\mathrm{rot}\,\mathbf{h}]\mathbf{h}\rangle\}. \quad (8)$$

When $R_m \ll 1$ and $h \ll B$, and if we neglect terms of the order of βR_m^2, $\beta(l_0 L_B^{-1})^3$, and $\beta^2 l_0 L_B^{-1}$, we obtain

$$Q=4\nu\int_0^\infty dk\,k^2F(k)+\frac{4}{3}\beta R_m\tau_0^{-1}\int_0^\infty dk\,F(k). \quad (9)$$

The second expression on the right-hand side of (9) describes the additional (magnetic) dissipation of gyrotropic motions, and the first term has the form

$$4\nu\lambda_g^{-2}\int_0^\infty dk\,F(k).$$

In accordance with (9), when the weak regular magnetic field grows slowly, its influence on $\langle\mathbf{v}\ \mathrm{curl}\ \mathbf{v}\rangle$ is described by

$$|\langle\mathbf{v}\ \mathrm{rot}\ \mathbf{v}\rangle|\approx|Q|\tau_0\xi(1+{}^1/_3\beta R_m\xi)^{-1},\quad \xi>1. \quad (10)$$

Vaĭnshteĭn [5] has calculated the contribution of the magnetic force in (8) within the framework of the Gaussian distribution of velocity-field probabilities and δ-function correlations in time in the case $R_m \gg 1$. If terms of the order of $\beta(l_0 L_B^{-1})^3$ and $\beta^2 l_0 L_B^{-1}$ are neglected, the second term on the right-hand side of (9) is, in this case, replaced by

$$\frac{4}{3}\beta T_1\nu\int_0^\infty dk\,k^2F(k),$$

and if this is taken into account, we obtain

$$|\langle\mathbf{v}\ \mathrm{rot}\ \mathbf{v}\rangle|\approx|Q|\tau_0\xi(1+{}^1/_3\beta T_1)^{-1},\quad \mathrm{Re}\geqslant T_1>1. \quad (11)$$

Therefore, the influence of a weak regular magnetic field on $\langle\mathbf{v}\cdot\mathrm{curl}\ \mathbf{v}\rangle$ may turn out to be important, whereas its influence on the kinetic energy of the turbulence is still negligible.

The term in (8), which corresponds to the magnetic gyrotropy source, [5] $M_\alpha B^2\mathbf{B}\cdot\mathrm{curl}\ \mathbf{B}$ is of the order of $T_2\beta^2 L_B^{-1}\tau_0^{-1}\langle v^2\rangle$, where $\mathrm{Re} \ge T_2 > 1$, when $R_m \gg 1$ and $T_2 \sim R_m$ when $R_m \ll 1$. If we substitute this expression into the numerator in (10) and (11), then, using (1), we can verify that when $\beta_{st} < 1$, allowance for the magnetic gyrotropy source gives small corrections (of the order of β_{st}^2) to the stationary value β_{st}.

3. ESTIMATE OF THE STATIONARY LARGE-SCALE MAGNETIC FIELD

The stationary value of the magnetic energy averaged over all space when $R_m \ll 1$ can be obtained, using (1) and (10), from the expression

$$R_m\alpha_0 L_B\sim\nu_m(1+{}^1/_3\beta_{s.}R_m\xi).$$

Here, $R_m\alpha_0$ is the coefficient in (1) when the effect of the magnetic field on the motion of the fluid is not taken into account, $\alpha_0 \sim {}^1/_3\Omega l_0^2 L_f^{-1}\xi$. For the steady value of the ratio of the magnetic energy to the kinetic energy of the turbulence, we have

$$\beta_{st}\sim 3R_m\{\Omega\tau_0 L_B L_f^{-1}-3\xi^{-1}R_m^{-2}\}.$$

Positive β_{st} is in agreement with the condition for magnetic-field generation in the kinematic formulation of the problem, $R_m\alpha_0 L_B > \nu_m$, which has the form $\Omega\tau_0 L_B L_f^{-1} > 3\xi^{-1}R_m^{-2}$. Since $\Omega\tau_0 < 1$, the generation condition for the regular magnetic field in the gyrotropic medium with $R_m \ll 1$ can be satisfied when $3R_m^{-2}\xi^{-1} < 1$, i.e., for $\xi \gg 1$. The maximum estimate for $\xi = \lambda_g^2\lambda^{-2} \sim \mathrm{Re}$ yields $\xi \le l_0^2\lambda^{-2} \sim \mathrm{Re}$, and hence we must satisfy the condition $\mathrm{Re} > 3R_m^{-2} \gg 1$.

When $R_m \gg 1$, we have from (1) and (11)

$$\beta_{st}\sim 3T_1^{-1}\{\Omega\tau_0 L_B L_f^{-1}-\xi^{-1}\}.$$

The condition that the regular magnetic field will grow $\alpha_0 L_B > \nu_T$ then assumes the form $\Omega\tau_0 L_B L_f^{-1} > \xi^{-1}$. It is clear that, in the above cases, β_{st} can be much greater than unity, and this is in agreement with the assumption made in Sec. 2. If we substitute $L_B \sim L_f$, we can take $\beta_{st} \lesssim \Omega\tau_0$ as the upper limit for the regular magnetic field.

Let us now estimate the stationary magnetic field which can be maintained by turbulent motion in the convective envelope of a star, without taking into account differential rotation which tends to decrease with increasing turbulent viscosity. Consider, for example, a red giant of radius $R \approx 10\,R_\odot$ (where $R_\odot$ is the solar radius) with a broad convective zone $\sim 0.9R$, rotational velocity $V_\varphi \sim 5\times10^5$ cm/sec, and height of homogeneous atmosphere $H \sim 10^9$ cm. If we take the characteristic turbulence scale to be $l_0 \sim 0.1H$, and root mean square velocity $v \sim 5\times10^3$ cm/sec, we find that the upper limit for the steady-state magnetic energy is

$$B^2\lesssim 4\pi\rho v^2\Omega\tau\sim{}^1/_3 4\pi\rho v^2.$$

Hence it follows that $B \le 4v\sqrt{\rho/3}$, i.e., when the density in the envelope is $\rho \sim 10^7$ g/cm^3, we have $B \lesssim 1$ G.

We note, in conclusion, that the generation of magnetic fields in a gyrotropic turbulent medium is determined by the magnitude of external factors which lead to the anisotropy of the eddies. The small ratio of the magnetic energy to the turbulent kinetic energy in this case is, therefore, due to the fact that the parameters characterizing the effects of inhomogeneity and rotation on turbulence are small ($l_0 L_f^{-1} \ll 1$, $\Omega\tau_0 < 1$).

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