Narrowing and spreading of a radiation beam incident to induced Compton scattering

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Induced Compton scattering by plasma electrons may lead to narrowing or spreading of a radiation beam traversing the plasma, depending on the spectrum of the radiation. Angular effects become apparent earlier than spectral effects. Results of numerical calculations of the evolution of the angular distribution and spectrum of a radiation beam traversing a low-density plasma are presented. The possibility that one radiation beam may be partially turned by another beam is discussed. The effect under consideration may cause the low-frequency radiation from certain astronomical objects to become anisotropic.

1. INTRODUCTION

Detailed studies of induced Compton scattering of high-intensity isotropic radiation by free electrons have been reported in a number of papers. Induced scattering leads to heating of a low-density plasma [1-3] and is accompanied by a downward movement of the photons on the frequency axis—a dynamical Bose condensation of photons [4-6].

In this paper we shall show that induced scattering by electrons also leads to a change in the angular distribution of a directed beam of radiation. We note that induced Compton scattering, unlike spontaneous scattering, does not alter the angular distribution of the radiation in the zeroth approximation. The change in the angular distribution of the beam, like the change in the spectrum of the radiation, appears in the first order in an expansion of the kinetic equation in powers of $h\nu/m_ec^2$.

The nature of the change in the angular distribution of the radiation depends on the behavior of its spectrum. In the part of the spectrum in which $d(n\nu^2)/d\nu \sim d(I_\nu/\nu)/d\nu < 0$ the angular distribution of the radiation becomes narrower (Fig. 1, c), whereas in the part of the spectrum in which $d(n\nu^2)/d\nu > 0$, the angular distribution broadens (Fig. 1,b). When $d(n\nu^2)/d\nu \sim 0$, the angular distribution does not change. All these characteristics involve certain angular averages. Here and below, ν is the frequency of the radiation, I_ν[erg/cm^2 · sec · sr · Hz] is the intensity of the radiation, and $n = c^2I_\nu/2h\nu^3$ is the occupation number in the photon phase space. The change in the angular distribution of the beam is accompanied by a change in its spectrum (Fig. 1,a). We note that the rate at which the spectrum changes within the limits of the beam depends on the direction (Fig. 2).

The effect under discussion is a consequence of the nonlinearity of the process of induced scattering by free electrons. It has nothing to do with the well known self-focusing effect, but depends on different characteristics of the scattering medium.

The characteristic distance x_0 in which a significant change in the angular distribution of the radiation beam takes place as a result of induced scattering by free electrons is given in order of magnitude by the expression

$$x_0 \approx \frac{1}{\sigma_T N_e}\frac{m_ec^2}{kT_b}\frac{1}{\theta_0^4}. \quad (1)$$

However, numerical calculations have shown that appreciable narrowing and broadening of the angular distribution as well as spectral changes (especially at large angles) can take place in distances x of the order of $(10^{-2}-10^{-1})x_0$. Here and below $\Delta\nu$ is the spectral width of the radiation beam, θ_0 is its angular spread, $kT_b = nh\nu$ is the brightness temperature of the intense radiation in the beam, N_e is the electron density, and $\sigma_T = (8/3)\pi r_0^2$ is the Thompson cross section. In the conditions under discussion, $kT_b \gg m_ec^2$ (we recall that the brightness temperatures of the radio emission from pulsars may be as high as 10^{30} K, while for lasers, for which $kT_b \approx Wc^2/4\pi\theta_0^2 s\nu^2\Delta\nu$, brightness temperatures of $10^{22}-10^{25}$ K are not unusual) and the effect may still appear even when the optical thickness τ_T of the plasma

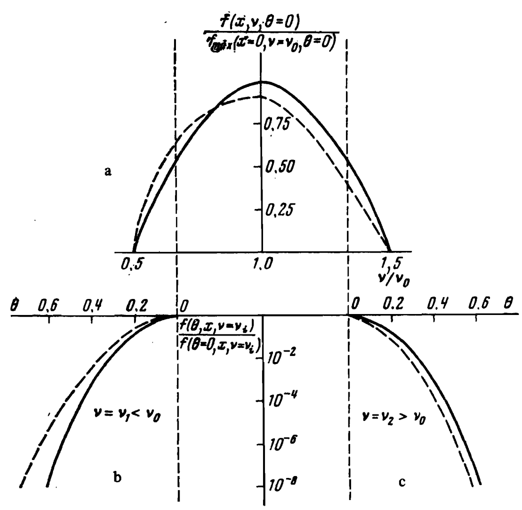
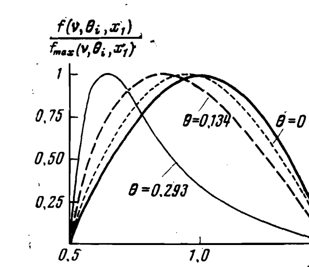


FIG. 1. Changes in the spectrum (a) and angular directionality (b, c) of radiation incident to induced scattering. The full curves represent the initial distributions (x = 0), and the dashed curves, the distributions at $x = x_1 = 0.04x_0$.

FIG. 2. Spectra of the radiation at $x_1 = 0.04x_0$ for three angles θ. The initial spectrum at x = 0, the same for all angles, is shown by the heavy curve.



with respect to spontaneous scattering may be very small: $\tau_T = \sigma_T N_e x \ll 1$. Here W[erg/sec] is the peak power of the laser, and $s = \pi r^2$ is the area into which the beam is focused.

Induced scattering is a classical process—not a quantum one. The effect is independent of the electron temperature (the electrons are assumed to be nonrelativistic). The basic equation is derived in the random phase approximation under the assumption that the radiation is spectrally broad:

$$\Delta\nu > \Delta\nu_D = \nu\left[\frac{2kT_e}{m_e c^2}(1-\cos\alpha)\right]^{1/2} \approx \nu\theta_0\left(\frac{2kT_e}{m_e c^2}\right)^{1/2}.$$

At the same time it is evident that the effect should also be observed for spectrally narrow radiation for which $\Delta\nu < \Delta\nu_D$ (in this case the characteristic length x_0 is greater by a factor of $(\Delta\nu_D/\Delta\nu)^3$[7]).

Plasma effects were neglected in deriving Eqs. (1)–(4), the electrons being assumed to be free and isotropically distributed. This is the case when $\Delta\nu > \nu_{pl} = \sqrt{e^2 N_e/\pi m_e}$. The effect must obviously also take place in the case of a high-density plasma with $\Delta\nu < \nu_{pl} < \nu$, when plasma effects cannot be neglected. The characteristic length x_0 is also greater in this case (see [8, 9]).

Moreover, the change in angular distribution as result of nonlinear interaction appears to be characteristic not only of induced Compton scattering of radiation, but also of a wide range of plasma processes that involve the interaction of waves and are described by similar equations (in particular, see [10]).

2. BASIC EQUATION

An integro-differential kinetic equation for the changes in the spectrum and angular distribution of the radiation incident to induced scattering in a homogeneous isotropic medium was derived in [7]:

$$\frac{\partial n(\nu,\theta,\varphi,t)}{\partial t} = n(\nu,\theta,\varphi,t)\int K(\nu',\nu,\alpha)\,n(\nu',\theta',\varphi',t)\,d\nu'\,d\cos\theta'\,d\varphi' \tag{2}$$

with the antisymmetric kernel

$$K(\nu',\nu,\alpha) = \frac{3}{8\pi(2\pi)^{1/2}}\frac{\sigma_T N_e h}{m_e c}\frac{1+\cos^2\alpha}{(1-\cos\alpha)^{1/2}}\frac{\nu'^2(\nu-\nu')}{\nu^3} \times\left(\frac{m_e c^2}{2kT_e}\right)^{3/2}\exp\left[-\frac{m_e c^2(\nu-\nu')}{4kT_e\nu^2(1-\cos\alpha)}\right],$$

in which the scattering angle α is defined by the formula

$$\cos\alpha = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\varphi-\varphi').$$

The kernel $K(\nu', \nu, \alpha)$ is a derivative of the Gaussian function that describes the broadening of the spectrum of the radiation incident to spontaneous scattering by electrons having a Maxwellian distribution.

For the case of anisotropic spectrally broad radiation with $\Delta\nu \gg \Delta\nu_D$ in which we are interested, the frequency integration in (2), together with allowance for the spatial dependence, leads to an integro-differential transport equation for the distribution function

$$f(\mathbf{r},\nu,\mathbf{\Omega},t) = \nu^2 n(\mathbf{r},\nu,\mathbf{\Omega},t) = \nu^2 n(\mathbf{r},\nu,\theta,\varphi,t),$$

$$\frac{1}{c}\frac{\partial f}{\partial t} + \mathbf{\Omega}\nabla f = Af\int\frac{\partial f}{\partial\nu}(1-\cos\alpha)(1+\cos^2\alpha)\,d\cos\theta'\,d\varphi', \tag{3}$$

where $A = 3\sigma_T N_e h/8\pi m_e c^2$.

The equation simplifies for the case of an isotropic distribution $f = f(\nu, t)$ in homogeneous space:

$$\frac{1}{c}\frac{\partial f}{\partial t} = \frac{16\pi}{3}Af\frac{\partial f}{\partial\nu}. \tag{4}$$

This equation can easily be obtained from Kompaneets' equation[11] by neglecting the spontaneous processes in the latter.

Equation (3) enables us to investigate the change in the angular characteristics of a radiation beam incident to induced scattering. We note that this equation conserves the total number of photons while leaving open the possibility that the photons may be redistributed with respect to frequency and direction. This equation makes it possible to study the stability of isotropic radiation with respect to small angular perturbations, as was done in [12].

In investigating the evolution of the angular distribution of a narrow beam of radiation we may consider the steady-state problem. For a narrow beam, the changes in time and in a spatial coordinate are correlated. When $r \gg \theta_0 x$, we may assume that the intensity of the radiation depends only on the single Cartesian coordinate x, the angle θ, and the frequency ν. Then we obtain the following simple equation for f:

$$\mu\frac{\partial f}{\partial x} = Af\int\frac{\partial f}{\partial\nu}P(\mu',\mu)\,d\mu', \tag{5}$$

where

$$\mu = \cos\theta, \quad f = n\nu^2 \text{ and } P(\mu',\mu) = \int_0^{2\pi}(1-\cos\alpha)(1+\cos^2\alpha)\,d\varphi'$$
$$= 2\pi(1-\mu\mu')(1+\mu^2\mu'^2) + \pi(1-3\mu\mu')(1-\mu^2)(1-\mu'^2).$$

3. APPROXIMATE ANALYSIS

Let us consider the motion of a plane parallel beam in the direction $\mathbf{\Omega}_1(\theta_1, \varphi_1)$. For this purpose we write the distribution function in the form

$$f(\mathbf{r},\nu,\theta,\varphi,t) = \varphi(\mathbf{r},\nu,t)\,\delta(\cos\theta-\cos\theta_1)\,\delta(\varphi-\varphi_1).$$

On substituting this expression into Eq. (3), the right-hand side vanishes since $\cos\alpha$ becomes unity. Consequently, induced Compton scattering cannot alter a plane parallel beam. As is evident from Eq. (2), no changes can take place in a monoenergetic beam with $f(\mathbf{r}, \nu, \theta, \varphi, t) = \delta(\nu - \nu_1)\psi(\mathbf{r}, \theta, \varphi, t)$ either. The effect under discussion is due to the change incident to photon scattering in the direction and frequency of the photon by the amount of the quantum correction $\delta\nu \sim h\nu^2(1-\cos\alpha)/m_e c^2$ and is manifest only in the case of photons having angular and spectral distributions of finite width.

Equation (4) describes the downward motion along the frequency axis of isotropically distributed photons. The motion of the photons takes place along the characteristic $c^{-1}d\nu/dt = -(16/3)\pi Af$ with maintenance of f[5].

We note that if the initial spectrum contains flex points, then in time shock waves will form in the spectrum of the radiation from the low-frequency side[5], and this will lead to the characteristic oscillatory frequency dependence of the intensity[6].

The downward motion of the photons along the frequency axis should obviously also take place in a spatially directed beam of radiation. We shall show in a qualitative way that a change in the photon spectrum must be accompanied by a change in the angular distribution of the photons. Equation (5) can be rewritten in the form

$$f(\nu,\mu,x)=f(\nu,\mu,x=0)\exp\left[\frac{A}{\mu}\int_0^x d\xi\int_0^1\frac{\partial f(\nu,\mu',\xi)}{\partial\nu}P(\mu',\mu)\,d\mu'\right].$$

At small distances $x \ll x_0$, the deviations of $f(\nu, \mu, x)$ from its initial value are small, so that we can write $\partial f(\nu, \mu, 0)/\partial \nu$ in the integrand in the argument of the exponential. Then the solution to Eq. (5) assumes the following form, in which it is simple to analyze:

$$f(\nu,\mu,x)=f(\nu,\mu,x=0)\exp\left[\frac{Ax}{\mu}\frac{\partial}{\partial\nu}\int_0^1 f(\nu,\mu',0)P(\mu',\mu)\,d\mu'\right]. \tag{6}$$

Equations (3) and (5) do not factor. However, it is advantageous to analyze Eq. (6) for the simplest case, in which the initial distribution is factored: $f(\nu, \mu, 0) = \varphi(\nu)\psi(\mu)$; for this case we obtain

$$f(\nu,\mu,x)=\varphi(\nu)\psi(\mu)\exp\left[\frac{Ax}{\mu}\frac{\partial\varphi(\nu)}{\partial\nu}\int_0^1\psi(\mu')P(\mu',\mu)\,d\mu'\right]. \tag{7}$$

It is evident from Eq. (7) that when $\partial\varphi(\nu)/\partial\nu < 0$, the distribution function f decreases—the photons are moved downward on the frequency axis. However, this decrease does not take place at the same rate for all angles. It is easily shown that

$$\frac{1}{\mu}\int_0^1\psi(\mu')P(\mu',\mu)\,d\mu'$$

is a monotonically decreasing function of μ for any function $\psi(\mu)$; hence the rate at which the beam weakens is minimal in the region $\mu \sim 1$ (near the beam axis) and increases as θ increases (as μ decreases). This differential effect should lead to narrowing of the beam, i.e., to concentration of photons toward the direction of the beam axis.

On the other hand, when $\partial\varphi(\nu)/\partial\nu > 0$, the downward motion of the photons on the frequency axis tends to increase $f(\nu, \mu, x)$. As before, this increase is small near the axis of the beam where $\theta \sim 0$ and $\mu \sim 1$, and increases with increasing θ. The beam should accordingly broaden, and indeed to such an extent that the radiation at sufficiently low frequencies may even become isotropic. If $\psi(\mu)$ vanishes somewhere, however, the radiation cannot become isotropic by virtue of the nonlinearity of the effect under discussion.

When $\partial\varphi/\partial\nu = 0$ there are no changes in the directionality of the beam. From Eq. (7) we can easily find the characteristic distance x_0 in which the effects of the change in the angular directionality of the radiation should appear in full force. When the angular spread θ_0 of the beam is small, the transfer function $P(\mu', \mu)$ will be approximately equal to $\pi\theta_0^2$ and $d\mu'$ can be replaced by $\theta_0^2/2$. Then on replacing $\partial f/\partial\nu$ by $f/\Delta\nu$, we obtain the expression for x_0 given above as Eq. (1).

Numerical calculations confirm the conclusions reached via the above approximate analysis.

We can also analyze the more general equation (3) in a similar approximate manner. Integrating this equation along a characteristic of the differential operator forming the left-hand side of it, we obtain

$$f(\mathbf{r},\nu,\mathbf{\Omega},t)=f(\mathbf{r}^*,\nu,\mathbf{\Omega},t^*)\exp\left[A\int_{\xi^*}^{\xi}d\eta\int\frac{\partial f(\eta)}{\partial\nu}(1-\cos\alpha)(1+\cos^2\alpha)\,d\Omega'\right], \tag{8}$$

where ξ is a variable measured along the characteristic and corresponding to the point $\mathbf{r}$, t, and symbols bearing asterisks represent initial or boundary values. An approximate solution to (8) valid for short times can be obtained by expanding the value of f at the point ξ', which occurs in the argument of the exponential, in powers of $(\xi' - \xi^*)$ and retaining only the zeroth-order term; this gives

$$f=f^*\exp\left[A(\xi-\xi^*)\int\frac{\partial f^*}{\partial\nu}(1-\cos\alpha)(1+\cos^2\alpha)\,d\Omega'\right]. \tag{9}$$

For the case of an initial distribution f^* that can be factored, the approximate analysis of (9) leads to conclusions similar to those drawn above from the analysis of (7).

The problem of a radiation source that is not spherically symmetric in shape and is filled with scattering plasma is of astrophysical interest (see the Conclusion). For this case the steady-state transport equation has the form

$$\mathbf{\Omega}\nabla f=A(\mathbf{r})f\int\frac{\partial f}{\partial\nu}(1-\cos\alpha)(1+\cos^2\alpha)\,d\Omega'+q(\mathbf{r},\nu,\mathbf{\Omega}),$$

where q represents the distributed radiation source and the coefficient A depends on the coordinates by virtue of the coordinate-dependence of N_e. On integrating along a characteristic of the operator $\mathbf{\Omega}\nabla$, we obtain

$$f(\mathbf{r},\nu,\mathbf{\Omega})=\int_{\xi^*}^{\xi}d\eta\,q(\eta)\exp\left[\int_{\eta}^{\xi}d\zeta A(\zeta)\int\frac{\partial f(\zeta)}{\partial\nu}(1-\cos\alpha)(1+\cos^2\alpha)\,d\Omega'\right].$$

When the effects of induced scattering are small we have

$$f\approx q(\xi-\xi^*),\quad \partial f/\partial\nu\approx(\xi-\xi^*)\,\partial q/\partial\nu.$$

at each point. For the astrophysical objects of interest $\partial q/\partial\nu$ is negative, and this means that we may expect narrowing of the angular distribution, depending on the path traversed by the photons. The radiation leaving the source should be anisotropic even if the radiation so sources are locally isotropic. This is associated with differences in the effective length $\xi - \xi^*$ for interaction of the radiation with the scattering plasma. In this case rotation of the source may lead to characteristic variability of the intensity and spectrum of the radiation.

4. NUMERICAL CALCULATIONS

For the numerical calculations we chose the following initial angular and spectral distribution for the radiation in factored form:

$$f(\nu,\mu,x=0)=\sin\pi\left(\frac{\nu}{\nu_0}-0{,}5\right)\frac{1}{\theta_0^2}\exp\left[-\frac{1-\mu}{\theta_0^2}\right],$$

Calculations were made for $\theta_0 = 0.1$ and for frequencies in the range $0.5 \le \nu/\nu_0 \le 1.5$. The results of the calculations are presented in Figs. 1–3. Figure 3 shows angular distributions $f(\mu, x)$ for several values of x and two fixed frequencies $\nu = \nu_1$ and $\nu = \nu_2$ so chosen that

$$\frac{\partial f}{\partial\nu}(x=0)\Big|_{\nu=\nu_1}>0 \text{ and } \frac{\partial f}{\partial\nu}(x=0)\Big|_{\nu=\nu_2}<0.$$

It will be seen that the effects of narrowing and spreading of the beam are especially marked in the region of large angles θ where the radiation has a relatively low intensity. It is evident from Fig. 2, which illustrates the evolution of the spectrum $f(\nu)$ for three values of θ, so that the spectrum of the radiation also changes most rapidly in the large-angle region.

Calculations made for other initial angular and spectral distributions for the radiation beam confirm the presence and importance of the effect under discussion. We note that the effect is appreciable even at $x \sim (10^{-2}–10^{-1})x_0$. No numerical calculations were made for $x \ge 0.1x_0$ since the method used for the numerical calculations is not accurate enough for that case.

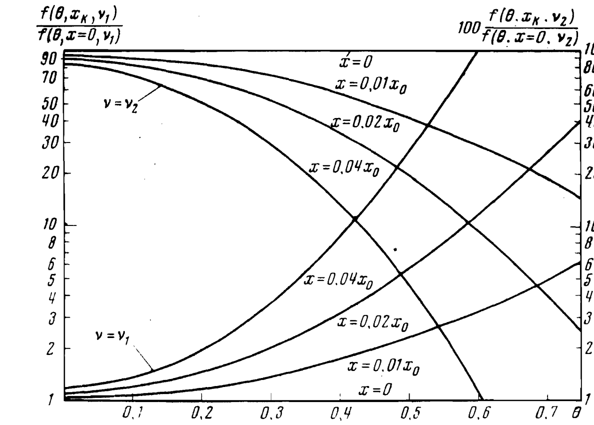


FIG. 3. Angular directionality of the radiation for two frequencies ($\nu_1 = 0.65\nu_0$ and $\nu_2 = 1.35\nu_0$) and several values of the optical thickness (proportional to z) of the plasma. For convenience, each of the two series of curves is separately normalized to the ordinate axis.

5. CONCLUSION

The effect under consideration may be important under astrophysical conditions (see the discussion in [13]). It may affect the widths of the directional patterns for radio pulsars. In the case of compact nonspherical cosmic sources of rf synchrotron radiation—clouds of relativistic electrons and thermal plasma with frozen-in magnetic fields—induced scattering of the radiation by the thermal electrons may cause the radiation to become directional at low frequencies, even though the cloud is optically thin to the synchrotron radiation and the synchrotron radiation is isotropic.

It may be possible to observe the effect in the laboratory. The problem of induced Compton interaction of two spectrally broad laser beams propagating in opposite directions [3] or at an angle to one another is of special interest.

When two intersecting beams interact, the change in the intensity in one direction is determined by the value of $\partial f/\partial\nu$ for the other direction, and various effects of partially weakening or strengthening of one of the beams by the other are obviously possible, depending on the signs of $\partial f/\partial\nu$ for the two beams. The equations for the interaction of two plane parallel beams of which one has the direction $\mathbf{\Omega}_1(\theta_1, \varphi_1)$ and the distribution function $f_1(\mathbf{r}, \nu, t)$, and the other, the direction $\mathbf{\Omega}_2(\theta_2, \varphi_2)$ and the distribution function $f_2(\mathbf{r}, \nu, t)$, are

$$\frac{1}{c}\frac{\partial f_1}{\partial t}+\Omega_1\nabla f_1=Af_1\frac{\partial f_2}{\partial\nu}(1-\cos\alpha)(1+\cos^2\alpha),$$
$$\frac{1}{c}\frac{\partial f_2}{\partial t}+\Omega_2\nabla f_2=Af_2\frac{\partial f_1}{\partial\nu}(1-\cos\alpha)(1+\cos^2\alpha).$$

The weakening of one beam is due to the rotation of some of its photons into the direction of the other beam as a result of induced scattering (we emphasize again that no effects are obtained unless allowance is made for the change in energy incident to scattering). We note that in this situation one can speak only of a preferential downward motion of the photons along the frequency axis.

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