Instability in smectic *A*

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Instability induced in smectic *A* by a stress is considered. Formulas are presented for the threshold and period of the instability.

Recent papers[1,2] have dealt with the instability that appears in smectic A liquid crystals under the influence of tension. This instability appears when the smectic A is stretched (but not compressed), when the distance between the plates bounding the smectic A increases by a certain critical amount δ. The smectic layers are then set in a motion that is characterized by a spatial period, after which the liquid crystal relaxes to its initial, i.e., unstretched state. This phenomenon, and in particular the presence of an instability period, can be observed by optical methods (see, e.g., [2]). In this paper we describe instability in the smectic; we shall also show that when the smectic is stretched no unstable periodic structure is produced, so that the instability is of the relaxation type.

The smectic A liquid crystal in its unperturbed state is a layered one-dimensional system consisting of parallel equidistant planes on which are located the molecule centers, with the long axes of the molecules perpendicular to the planes. The smectic A has, with respect to tension and compression in a direction perpendicular to the planes, elastic properties that are analogous to crystalline elasticity, while inside the layers its properties are analogous to those of a nematic liquid crystal. Thus, the smectic A is a one-dimensional crystal in the sense indicated above. The free energy of such a system in the lowest order in the gradients of the displacements is described in[3,4]. To investigate the considered effect it is necessary to consider terms of higher order than are customarily included (see[2,4]).

We denote by (xy) the plane parallel the smectic layers, and by u(x, z) the local displacement of the smectic layers perpendicular to their unperturbed position. Then the procedure for separating the corresponding terms leads to the following expression for the free energy[2]:

$$F=\frac{1}{2}B\left[\frac{\partial u}{\partial z}-\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2\right]^2+\frac{1}{2}K\left(\frac{\partial^2 u}{\partial x^2}\right)^2; \qquad (1)$$

we are considering a situation that is one-dimensional in the (xy) plane, so that there are no derivatives with respect to y.

It is seen from (1) that for a stretched smectic, when $\partial u/\partial z = \delta/d > 0$ (d is the distance between planes and δ is the change of this distance), the corresponding term in the free energy can be decreased if $\theta = \partial u/\partial x \neq 0$, corresponding to deformation of the smectic layers. At the same time, however, the second term of (1) increases and it is therefore necessary to find the external function θ corresponding to the minimum value of the free energy. We note here that deformation is not favored when the smectic is compressed ($\partial u/\partial z < 0$), i.e., instability can occur only in tension.

The test of the functional

$$\int F\,dx \qquad (2)$$

for a minimum is relatively simple; it leads to the following results: the solutions of the Euler equation that can be easily obtained from (1) and satisfy the boundary conditions of our problem can be subdivided into three types.

Solutions for which the boundary conditions call for introduction of disclinations. Such solutions were investigated in detail by Parodi[5] in a study of the rotation of the layers of smectic A under the influence of a magnetic field; the energy of these structures is quite high (for details see[5], and are of no interest to us here.

When considering solutions in which θ is not bounded, we go outside the framework of the selected approximation of small gradients; in addition, difficulties arise with the boundary conditions, analogous to the difficulties considered above, i.e., attempts to investigate a structure of this kind lead inevitably to the appearance of disclinations, and consequently the statements made above concerning structures with disclinations remain equally in force for them.

Finally, we have solutions with bounded θ. An investigations of these solutions, with allowance for the boundary conditions, leads to the following result. Their energy exceeds the energy of a homogeneously stretched smectic, i.e., states with the displacement field

$$u=u_0=\alpha z, \qquad (3)$$

where $\alpha = \delta/d$. Consequently, such structures are not energywise favored. On the other hand, it is easy to note that the structure (3) is dynamically unstable (see below).

In fact, let us consider the Lagrangian

$$\mathscr{L}=F-{}^1/{}_2\rho(\partial u/\partial t)^2, \qquad (4)$$

for which the Euler equations are the equations of motion of the smectic A, and investigate the stability of the state (3), i.e., seek solutions in the form

$$u=u_0+u_1e^{i(\omega t+\mathbf{kr})}, \quad \mathbf{k}=(k_x, k_z), \qquad (5)$$

where **k** and ω, as usual, are the wave vector and the perturbation frequency.

We linearize further the Euler equations obtained from (4) with respect to u_1 and consider a situation in which ω becomes complex for real k, which indeed represents stability. Elementary calculations lead to the following equation for ω:

$$\rho\omega^2=B[(\lambda_c k_x^2-\alpha/\lambda_c)^2+k_z^2-(\alpha/\lambda_c)^2], \qquad (6)$$

where $\lambda_c = (K/B)^{1/2}$ is the penetration depth introduced by de Gennes see[6]. The boundary conditions yield $k_z \sim \pi d$. Using this circumstance, we can easily obtain from (6) the threshold conditions for the onset of the instability

$$\alpha_{cr}=k_z\lambda_c\sim\pi\lambda_c/d \quad \text{or} \quad \delta_{cr}=\pi\lambda_c. \tag{7}$$

The wave vector k_x, which describes the periodicity of the instability in the direction of x, is determined from the expression

$$k_x^2\lambda_c d=\pi. \tag{8}$$

Thus, the described instability leads to an increase in the amplitude of the perturbation, which in turn involves production of disclinations. A relaxing disclination increases the number of layers of smectic A until the tension δ/d becomes less than critical.

We note in conclusion that the described instability arises only when the smectic A is stretched, and does not occur under compression. The instability begins to develop when the change in the distance between the plates exceeds δ_{cr} (see (7)), the spatial periodicity of the instability is then described by formula (8). We emphasize further that the instability has a dynamic character and does not lead to the formation of the stable structure, i.e., the growth of the perturbation amplitude leads to the production of the disclinations with the aid of which the system relaxes to the state $\delta < \delta_{cr}$.

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[1]C. Gerritsma, P. Van Zenten, Phys. Lett. **37A**, 47, 1971.
[2]M. Delaje, R. Ribotta, G. Durand, Phys. Lett. **44A**, 139, 1973.
[3]L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika (Statistical Physics), Nauka (1964), p. 474.
[4]P. G. de Gennes, J. de Physique, **30**, C4, 65, 1969.
[5]O. Parodi, Solid State Commun, 10, 753, 1972.
[6]P. G. de Gennes, Solid State Commun., **11**, 1503, 1972.

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