Theory of nonlinear interaction between monochromatic and noise waves in weakly dispersive media

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The problem of the interaction between monochromatic and noise waves in nondispersive media is solved. The results are applied to explain a number of phenomena, e.g., the cavitation spectrum, Fermi acceleration, and sound attenuation. General formulas are obtained for transforming the spectra of the interacting waves. The interaction between the sound waves and the noise, which consists of thermal elastic waves of the meidum, are considered in detail. The theory of sound attenuation developed in the paper takes into account effects due to the finite sound amplitude, the presence of external noise, etc. Under certain conditins, these effects lead, in particular, to the Landau–Rumer formula for the absorption coefficient in solids. Application of the theory to liquids and gases yields an estimate of the sound absorption due to the translational motion of the molecules with allowance for fluctuations in the media.

1. INTRODUCTION

The rigorous description of many physical phenomena requires the solution of nonlinear partial differential equations. In this connection, models based on exactly solved nonlinear equations have great value for the understanding of the basic regularities of nonlinear phenomena. The equation for simple waves, Burgers equation, the Korteweg-de Vries equation and others serve as standard equations of this type. The analysis of a number of phenomena in different areas of physics and mechanics reduces to these equations.[1, 2] The results obtained here are as a rule formulated in space-time language. Thus, if we are dealing with one-dimensional waves, then the corresponding solutions describe the distortion of the initial profiles of the waves with the coordinates or with time.

A deeper physical analysis of the phenoeman and interpretation of the experimental results require a knowledge of the behavior of the spectrum. The presence of a nonlinear medium, as is well known, prevents the obtaining of a closed equation for the spectrum in the general case; therefore, the exact solution of the dynamics of nonlinear processes is obtained only for regular waves. In media with strong dispersion, within the framework of transformation to the reduced equations, there exists a transformation to the "under-resolved" spectrum of the process; this facilitates the analysis of the interaction of waves with a finite spectrum width.[3,4] In media with weak dispersion, an infinite number of spectral components interact simultaneously, and the problem turns out to be extremely complicated.

In our recent researches we developed a general method[5] and obtained results[5, 6] which pertain to the dynamics of the spectra of noise waves as they propagate in nondispersive, nonlinear media. However, more interest attaches to the problem of the interaction of regular and noise waves which is due, in particular, to the presence of characteristic fluctuations of the nonlinear media.

The purpose of this work is the presentation of the results of the interaction of waves with regular and random modulations, described by the equation for simple waves. Exact formulas are obtained for the transformation of the spectra of the interacting waves. The results of the research are applicable for the explanation of the features of sound damping in nonlinear media, the dynamics of the cavitation spectrum and Fermi acceleration. It is shown that the developed statistical nonlinear wave theory of the damping of sound allows us to obtain more general results in comparison with the existing kinetic approach.[7, 8] Thus, it takes into account the generation of harmonics and the presence of external noise, which leads to excess absorption.

2. DERIVATION OF THE FUNDAMENTAL RELATIONS. THE SPECTRUM OF NONSTATIONARY NOISE DESCRIBED BY THE EQUATION FOR SIMPLE WAVES

We consider the evolution of a field u in a nondispersive medium, described by the equation

$$\frac{\partial u}{\partial z}-\beta u\frac{\partial u}{\partial \eta}=-\delta u. \tag{1}$$

Here z is the spatial coordinate along the direction of propagation of the wave, η is the time in a set of coordinates moving along with the wave; the coefficient β characterizes the nonlinearity of the medium, the parameter δ is responsible for the low-frequency dissipation ($\delta > 0$) or for the amplification of the wave ($\delta < 0$), when we are dealing with an active medium.

The solution of Eq. (1), which corresponds to the boundary condition $u = u_0(\eta)$ at $z = 0$ can be written in the form

$$v(\eta,x)=-\frac{i}{2\pi}\iint\limits_{-\infty}^{\infty}\omega^{-1}\dot{u}_0(\theta)\exp\{-i\omega[\theta-\beta x u_0(\theta)-\eta]\}d\omega\, d\theta, \tag{2}$$

where

$$v=ue^{\delta z},\quad x=\delta^{-1}[1-e^{-\delta z}]. \tag{3}$$

We analyze below the case in which $u_0(\eta)$ is a random function.

It follows directly from Eq. (2) that for the calculation of the correlation function

$$B(\eta_1,\eta_2;\,x)=\langle v(\eta_1,x)v^*(\eta_2,x)\rangle \tag{4}$$

It is necessary to use the four-dimensional distribution law

$$W_4[u_0(\theta_1),\,u_0(\theta_2),\,\dot{u}_0(\theta_1),\,\dot{u}_0(\theta_2)],$$

which entails extraordinarily cumbersome calculations. In addition, the structure of Eqs. (2), (4) allow us to introduce the operator

$$L(\omega_1,\theta_1;\omega_2,\theta_2)=\frac{d^2}{d\theta_1\,d\theta_2}+i\omega_1\frac{d}{d\theta_2}-i\omega_2\frac{d}{d\theta_1}+\omega_1\omega_2. \quad (5)$$

Then

$$B(\eta_1,\eta_2;x)=\frac{1}{4\pi^2\beta^2x^2}\iint_{-\infty}^{\infty}e^{i\omega_1\eta_1-i\omega_2\eta_2}\frac{d\omega_1}{\omega_1^2}\frac{d\omega_2}{\omega_2^2}\cdot$$

$$\iint_{-\infty}^{\infty}L(\omega_1,\theta_1;\omega_2,\theta_2)\langle\exp\{i\omega_1\beta xu_0(\theta_1)-i\omega_2\beta xu_0(\theta_2)\}\rangle e^{-i\omega_1\theta_1+i\omega_2\theta_2}d\theta_1\,d\theta_2, \quad (6)$$

and for the averaging of (6) we can use the two-dimensional distribution $W_2[u_0(\theta_1), u_0(\theta_2)]$.

We assume that

$$u_0(\eta)=f(\eta)+\xi(\eta), \quad (7)$$

where $f(\eta)$ is a determinate function and $\xi(\eta)$ is a normal random process with mean value $\langle\xi\rangle = 0$ and correlation coefficient

$$R(\tau)=\langle\xi(\theta)\xi(\theta+\tau)\rangle\sigma^{-2},$$

$\sigma^2 = \langle\xi^2\rangle$ is the intensity of the random process. The statistical averaging in (6) gives

$$\langle\exp\{i\beta x[\omega_1\xi(\theta_1)-\omega_2\xi(\theta_2)]\}\rangle=$$
$$=\exp\{-{}^1/_2(\beta\sigma x)^2[\omega_1^2+\omega_2^2-2\omega_1\omega_2R(\theta_1-\theta_2)]\}. \quad (8)$$

Inasmuch as the random process $u_0(\eta)$ (7) is nonstationary, Eq. (6) must still be averaged over the time η_1, $(\eta_2 = \eta_1 + \tau)$; as a result, we obtain

$$B(\tau,x)=\int_{-\infty}^{\infty}\frac{e^{-i\omega\tau}}{2\pi\omega^2}e^{-(\beta\sigma\omega x)^2}d\omega\iint_{-\infty}^{\infty}(\Phi_1+\Phi_2+\Phi_3)$$
$$\times\exp\{i\omega(\theta_2-\theta_1)+(\beta\sigma\omega x)^2R(\theta_1-\theta_2)+i\beta\omega x[f(\theta_1)-f(\theta_2)]\}d\theta_1\,d\theta_2; \quad (9)$$

$$\Phi_1=-\sigma^2\left[\frac{\partial^2R}{\partial\theta_1^2}+(\beta\sigma\omega x)^2\left(\frac{\partial R}{\partial\theta_1}\right)^2\right],$$

$$\Phi_2=-i\sigma\beta\sigma\omega x\left[\frac{\partial R}{\partial\theta_1}\frac{\partial f}{\partial\theta_2}-\frac{\partial R}{\partial\theta_2}\frac{\partial f}{\partial\theta_1}\right],\quad \Phi_3=\frac{\partial f}{\partial\theta_1}\frac{\partial f}{\partial\theta_2}. \quad (10)$$

Further simplification in the general case has not been accomplished. At this stage, it is necessary to specify the function $f(\theta)$. Let $f(\theta)$ be the harmonic function

$$f(\theta)=A\sin\omega_0\theta, \quad (11)$$

where ω_0 is the frequency of the monochromatic wave. In the case considered, the spectral density $S(\omega,x)$ of the process v in an arbitrary section z of a nonlinear medium can be represented in the form

$$S(\omega,\ x)=S_1+S_2+S_3; \quad (12)$$

$$S_1(\omega,x)=-\frac{\sigma^2}{2\pi}\int_{-\infty}^{\infty}\left[\frac{d^2R}{d\theta^2}+(\beta\sigma\omega x)^2\left(\frac{dR}{d\theta}\right)^2\right]e^{\chi(\theta)}J_0(\psi(\theta))\,d\theta, \quad (13a)$$

$$S_2(\omega,x)=\frac{\sigma A}{\pi\omega}\int_{-\infty}^{\infty}J_1(\psi(\theta))e^{\chi(\theta)}\frac{dR}{d\theta}\cos\frac{\omega_0\theta}{2}d\theta, \quad (13b)$$

$$S_3(\omega,x)=\frac{A^2\omega_0^2}{4\pi\omega^2}\int_{-\infty}^{\infty}[J_0(\psi(\theta))\cos\omega_0\theta-J_2(\psi(\theta))]e^{\chi(\theta)}d\theta, \quad (13c)$$

$$\psi(\theta)=2\beta A\omega\sin(\omega_0\theta/2),\quad \chi(\theta)=-i\omega\theta-(\beta\sigma\omega x)[1-R(\theta)]. \quad (14)$$

Each of the terms of (13) has a clear meaning. The expression $S_1(\omega,x)$ describes the process of nonlinear distortion of the spectral density of the noise $\xi(\eta)$, "corrected" under the integral (13a) by the factor $J_0(\psi)$, which takes into account the presence of the monochromatic wave (11); as the amplitude $A\to 0$, we obtain the result for the noise wave.[5] The formula (13c) describes the nonlinear distortion of the monochromatic wave with correction for the noise wave; in the limit as $\sigma\to 0$, we can obtain a result which corresponds to the well-known Bessel-Fubini solution (see, for example,[9]). The term $S_2(\omega,x)$ (13b) gives information on the cross interaction of the monochromatic wave with noise.

To separate the spectra at the frequencies of interest, we can use the addition theorem for cylindrical functions, which allows us to expand the Bessel function $J_n(\psi)$ of argument ψ (14) in a series of paired products of the functions $J_n(\beta A\omega x)$. We then obtain an expression for the distortion of the spectrum of the noise wave "in pure form":

$$S_\xi(\omega,x)=-\frac{\sigma^2}{\pi}J_0^2(\beta A\omega x)\int_0^{\infty}\frac{dR}{d\theta}\frac{\sin\omega\theta}{\omega}\exp\{(\beta\sigma\omega x)[R(\theta)-1]\}d\theta. \quad (15)$$

The factor $J_0^2(\beta A\omega x)$, as in (13a), takes into account the interaction with the monochromatic wave.

More complicated calculations are connected with the separation of the terms which describe the creation of new sections of the spectrum in the process of interaction of monochromatic waves and noise waves. Keeping the terms in $\sin\omega_0\theta$ or $\cos\omega_0\theta$ in Eqs. (13), and performing a number of transformations, we arrive at the following result:

$$S_{\xi,f}(\omega,x)=-\frac{\sigma^2}{\pi}J_1^2(\beta A\omega x)\int_0^{\infty}\frac{dR}{d\theta}\exp\{(\beta\sigma\omega x)^2[R(\theta)-1]\}\frac{\sin(\omega-\omega_0)\theta}{\omega-\omega_0}d\theta. \quad (16)$$

The structure of the integrals in (15) and (16) is the same; this circumstance simplifies the analysis of the resultant expressions.

Expansion of (13c) leads to formulas which describe the dynamics of the spectrum of the initial monochromatic wave (11); as a result, we have, at the fundamental frequency ω,

$$S_{f,1}(\omega,\ x)=A^2\exp\{-(\beta\sigma\omega x)^2\}[J_1(\beta A\omega x)/\beta A\omega x]^2\delta(\omega-\omega_0) \quad (17)$$

and, for example, at its second harmonic

$$S_{f,2}(\omega,\ x)=A^2\exp\{-(2\beta\sigma\omega x)^2\}[J_2(2\beta A\omega x)/2\beta A\omega x]^2\delta(\omega-2\omega_0). \quad (18)$$

It is seen from (17) that, because of the nonlinear interaction with the noise wave and the generation of the characteristic harmonics, the intensity of the initial monochromatic wave decreases.

The derived expressions (15)–(18) have a rather general character: they were obtained in the absence of any restrictions on the intensity σ^2, on the form of the noise spectrum, and on its location relative to the spectrum of the monochromatic wave. Formulas (15)–(18) allow us to consider various problems which, in the final analysis, reduce to the problem of the interaction of noise with a monochromatic wave (Fermi acceleration, damping of sound, cavitation spectrum, and so forth).

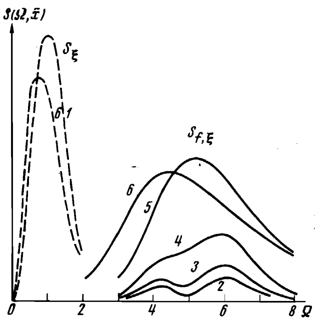
3. INTERACTION OF BROADBAND NOISE WITH A MONOCHROMATIC WAVE

As an example, we consider here the case in which the noise correlation coefficient at the input of the nonlinear medium is of the form

$$R(\theta)=(1-2\gamma^2\theta^2)\exp(-\gamma^2\theta^2) \quad (19)$$

The results of the calculation of the spectra (15) and (16) for this $R(\theta)$ are shown in the figure. The initial noise spectrum undergoes in the nonlinear medium relatively weak distortions that have a tendency to energy redistribution both in the lower and in the upper frequency ranges.

Special interest attaches to the consideration of the dynamics of the new spectral region created in the vicinity of the frequency ω_0. The width of this pedestal at small distances $\tilde{x}$ is equal, approximately, to twice the width of the noise spectrum and increases with increase in $\tilde{x}$. It is important in principle to note the nonsymmetrical form of the pedestal. Its high-frequency wing has a large amplitude and grows rapidly as a result of the nonlinear energy pumping. The dip between the two wings gradually disappears (curve 4) and both wings of the pedestal fuse into a single broad line, the energy center of which is shifted in the direction of frequencies $\omega > \omega_0$. Subsequently, this line increases still more in amplitude (curve 5) and then begins the stage of "spreading" over the spectrum 'the solid curve 6) and, again, a small fraction of the energy is pumped over into the high frequencies. The motion of the energy center upwards in the frequency spectrum in the process of interaction with a low-frequency noise is similar to the well known Fermi acceleration effect.[10]



Noise spectrum $S_\xi(\omega,x)$ (dashed curve) and the spectrum $S_{f,\xi}(\omega,x)$ that develops in the region of the fundamental frequency ω_0 of a monochromatic wave as a result of interaction with noise (solid curves) at various reduced distances $\tilde{x} = 48\beta\gamma\sigma x$: 1) $\tilde{x} = 0$; 2) $\tilde{x} = 1$; 3) $\tilde{x} = 4/3$; 4) $\tilde{x} = 2$; 5) $\tilde{x} = 4$; 6) $\tilde{x} = 8$. In construction of the graphs it has been assumed that $\Omega = \omega/2\gamma$, $\omega_0/2\gamma = 5$, $A = \sigma/2$.

If the spectral line of the initial wave has a sufficiently small finite width, then sooner or later it joins with the pedestal formed at its foot into a single complex, and it is not possible to speak of the location of the line; at this stage it is necessary to follow the motion of the energy center of the entire complex as a whole. The complete picture is, however, more complicated: Together with the generation of the harmonics $n\omega_0$ ($n = 2, 3, \ldots$) of the initial frequency ω_0, new spectral complexes appear in the vicinity of these harmonics.

It must be emphasized that in the analysis of the nonlinear dynamics of the superposition of regular and noise waves we have, for simplicity, considered such a location of the spectral line of the signal that it does not intersect with the spectrum of the random perturbation. The results obtained allow us to consider the general case also. However, if the initial spectral distributions of the wave and the noise intersect, then the resolution of the spectrum $S(\omega,x)$ into the components S_ξ, $S_{f\xi}$, S_f for $x \neq 0$ becomes a formal procedure.

Just this latter case is often encountered in practice. The spectra of cavitation,[11] of the noise of jet engines and certain other sources of intense noise consists of discrete lines located against a background of broadband noise. Inasmuch as a wide spectrum is reproduced at the foot of each discrete component, in accord with the theory presented above, it is not difficult to conclude that the continuous part of the spectrum grows rapidly as the sound propagates in the medium.

4. STATISTICAL NONLINEAR WAVE THEORY OF SOUND DAMPING

The examples of the preceding section to not cover all the applications of the general results of the present paper. By using these results, we can, from one point of view, consider the problem of sound damping in various media, due to the translational thermal motion of the molecules. As has already been pointed out, the theory of sound damping existing at the present time for solids uses the kinetic approach;[7, 8] the foundations of this theory were developed in the works of Landau and Rumer[12] and Akhiezer.[13] So far as the theory of sound attenuation in liquids and gases is concerned, it is based on the equations of hydrodynamics[14] and does not take into account the presence of fluctuations in the media.

Equation (1) can serve as a simple model of the interaction of a sound wave with internal thermal noises of the medium; it can be derived from the complete set of equations of hydrodynamics or the theory of elasticity under the assumption that the distortion of the wave profile is slow at distances of the order of a wavelength. Here the variable u in (1) has the meaning of the oscillating velocity of the particles, $\beta = \epsilon/c_0^2$, where c_0 is the velocity of linear sound. For solids, ϵ is expressed in terms of the elastic constants of second and third orders; for liquids and gases, $\beta = (\gamma + 1)/2c_0^2$, γ is the adiabatic coefficient in the isentropic equation of state or the corresponding constant in the Tait equation.

We return to Eq. (17); it describes the decrease in the intensity of the fundamertal wave

$$I=2A^2[J_1(\beta A\omega_0 x)/\beta A\omega_0 x]^2\exp\{-(\beta\sigma\omega_0 x)^2\} \quad (20)$$

for two reasons. First, this is nonlinear self-action of the wave, taken into account by the factor $F(\kappa) = 4J_1^2(\kappa)\kappa^{-2}$ and connected with the transfer of energy into its harmonics; as $\kappa \to 0$, this effect is small, $F \approx 1$. The second reason for the decrease in intensity is the process of interaction of the wave with noise; this noise is specified on the boundary and its effect is described by the exponential factor.

In the general case, the noise intensity σ^2 can be represented in the form of the superposition of the thermal-noise intensity σ^2_{ther} produced by thermal elastic waves of the medium, and the intensity of outside (or external) noise σ^2_{ext}, which owes its presence to other sources. It is evident that the external noise leads to an excess attenuation of the sound wave. When account is taken of only the thermal noise of the medium, we obtained the known results for the sound attenuation coefficient. According to (20), we get for the attenuation coefficient α $(2\alpha I = -\partial I/\partial x)$ of the fundamental wave due to interaction with the noise

$$\alpha=\varepsilon^2(\rho_0\sigma^2)\omega_0^2 x(c_0^4\rho_0)^{-1}. \quad (21)$$

The quantity $\Delta\xi = \rho_0\sigma^2$ is the volume energy density of the noise which interacts in synchronism with the signal wave.

We proceed from one-dimensional to three-dimensional noise, taking into account the spatial isotropy of the latter. Only the part of the total noise energy formed by those Fourier components which propagate relative to the wave at an angle less than the width of the angle of synchronous interaction θ_c interact most effectively with the initial wave. In the medium considered without dispersion, only waves traveling strictly in one direction are in exact synchronism. However, the finite distance z to the point of observation permits an x-dependent detuning of the wave vectors, at which the interaction of the waves still takes place effectively. This leads to a definite solid angle of so-called parametric capture $\Delta\Omega = \pi\theta_c^2(x)$. It is not difficult to show that at low effectiveness of the interaction (see, for example,[15]),

$$\theta_c^2 = 2\pi c_0 |\omega \pm \omega_0| (x\omega\omega_0)^{-1}. \tag{22}$$

The fundamental contribution to the sound absorption is made by the high-frequency thermal phonons, for which $\omega \gg \omega_0$. With account of this, $\theta_c^2 = 2\pi c_0/\omega_0 x$ and the fraction of the noise energy in "synchronism," $\Delta\xi/\xi$, is equal to

$$\Delta\mathscr{E} = \mathscr{E}\Delta\Omega/4\pi = \mathscr{E}\pi c_0/2\omega_0 x. \tag{23}$$

The expressions (21) and (23) determine the value of the absorption coefficient

$$\alpha' = \pi\varepsilon^2\mathscr{E}\omega_0/2c_0^3\rho_0. \tag{24}$$

Inasmuch as we have assumed, in the derivation of (24), that the momentum and the energy of a thermal phonon are determined exactly, the condition of applicability of (24) is: the path length of the thermal phonon is greater than its wavelength or $\omega_0\tau > 1$ (τ is the relaxation time of the phonon).

The approach developed in this paper allows us to describe N-processes that lead to stimulated redistribution of the acoustic energy over the frequencies of the Debye spectrum. The processes which bring the spectrum to equilibrium, on the other hand, can only be taken into account phenomenologically by changing the model of the medium (1) in such a way as to introduce in it a relaxation term in terms of the noise component. However, there is no need of complicating the problem. Allowance for the finite value of τ leads to a decrease in the interaction length and reduces (see[8]) to the appearance of an additional factor $\omega_0\tau/\pi$ in Eq. (24). This gives the result

$$\alpha'' = \varepsilon^2\mathscr{E}\omega_0^2\tau/2c_0^3\rho_0, \tag{25}$$

which is valid for $\omega_0\tau \ll 1$.

An important aspect of the sound damping theory that we have developed is the use of the fact that the thermal motion of the molecules in a medium can be expanded in Debye elastic waves. Just this assumption was used in the kinetic approach for the derivation of sound attenuation in solids. For solids, $\xi \sim C(T)T$, where C(T) is the heat capacity, T the absolute temperature; Eqs. (24), (25) give well-known results: (24) gives the Landau-Rumer result[12] for high-frequency sound, and (25) gives that of Akhiezer[13] for low-frequency sound.

Along with this, experiments on molecular scattering of light show[16] that the expansion of the thermal motions of a medium into elastic waves is applicable also for liquids and gases. Consequently, Eqs. (24) and (25) also apply in principle to these media. Then the problem of calculation of the absorption coefficient reduces to finding the kinetic energy of the translational motion of the molecules in the media concerned. For an ideal gas, for example, $E = \rho v^2/2 = 3kTN_L/2$, where k is Boltzmann's constant and N_L is Loschmidt's number. In this case, the damping coefficient (25) is large in comparison with that calculated by the classical theory of Stokes,[14] which determines the damping in terms of the shear viscosity and the thermal conductivity. This difference is due to the fact that the nonlinearity parameter $\epsilon > 1$ and the mean-squared velocity of the molecules $\overline{v^2}$ enter in (25), while $(\bar{v})^2$, $(\overline{v^2} > (\bar{v})^2)$ appears in the Stokes formula.

CONCLUSION

In the present work we have obtained exact expressions for the evolution of the spectra of monochromatic and noise waves which interact in nonlinear, nondispersive media. The results of the work are applicable to the analysis of phenomena described by the equation for simple waves. As is shown, in particular, the effect of Fermi acceleration can be explained with the help of formula (16), and the behavior of the cavitation spectrum can be explained with the help of formula (16) and (17).

The equations (24) and (25) for the sound attenuation coefficient are similar to those obtained earlier with the help of the kinetic approach. Along with this, our results permit us to take into account a number of factors which have value in principle for the propagation of elastic waves. Above all, Eq. (20) rigorously describes the losses due to the finiteness of the sound amplitude A. The possibility of taking into account the nonlinear losses broadens the range of intensities used for the experimental measurement of the sound absorption coefficient. The presence of the external noise σ_{ext}^2 leads to an excess sound attenuation which can also be taken into account on the basis of the developed theory; it is possible that the excess sound absorption in experiments at low temperatures can be attributed to this external noise.

The formulas in the paper contain a variable x that is connected with the length z of the nonlinear medium by the expression (3), which allows us to take into consideration the features of the interaction of acoustic waves in the presence of excitations of any nature ($\delta \neq 0$). In the approximation of the nondispersive medium, the decrease in the intensity of the signal is determined by the total energy $\Delta\xi$ of the noise "in synchronism" and does not depend on the shape of the noise spectrum and the location of it relative to the signal. The effect of the dispersion reduces to a change $\Delta\xi$ which is determined by the competition of two factors: on the one hand, the dispersion limits the frequency width of synchronism, and on the other—it increases the effective solid angle of capture $\Delta\Omega$. The reasons that have been indicated account for the difference in the expressions for the absorption coefficients from (24) and (25).

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