Mean velocity and spectrum of dislocations moving along a Peierls relief

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The mean velocity and spectral intensity of the energy dissipated by a dislocation gliding along a Peierls relief is calculated. It is assumed that the main cause of energy loss by the moving dislocation is viscous friction. The nonlinear dependence of the Peierls force on the coordinates is taken into account exactly. A solution for the mean dislocation velocity is obtained for the case of high viscosity and of an arbitrary external force. From the expression for the spectral intensity of energy dissipation it follows that all elastic oscillation modes must be taken into account, if the external force is comparable with the amplitude of the Peierls force.

1. INTRODUCTION

When a linear dislocation glides in a crystal as a result of periodic variation of the atomic structure of the nucleus, the dislocation is acted upon by the lattice-resistance force—the Peierls force $F(x)$. Owing to the periodicity of the lattice we have

$$F(x)=F(x+a),$$

where a is the period of the lattice in the direction of the dislocation motion. It is customary to confine oneself to the first harmonic in the force $F(x)$, and to assume

$$F(x)=F_\pi \sin(2\pi x/a). \tag{1}$$

If an external constant force f acting in the slip plane is smaller than F_π, then the dislocation will be at rest inside one of the valleys of the Peierls relief. The equilibrium position is determined by solving the equation

$$F_\pi \sin(2\pi x/a)=f.$$

The quantum and thermal fluctuations cause the dislocation to go over, via production of double kinks, to the neighboring valley of the relief with exponentially small velocity[1].

When the stresses exceed the Peierls force, the potential relief has no valleys, and the dislocation will glide along the x axis. The average velocity is determined both by the forces f and $F(x)$ and by the forces of viscous and radiation friction. The nature of the force of the viscous friction differs in different intervals of velocity. At low velocities the main channel of dislocation energy dissipation are losses to overcome the energy barriers of various kinds. At high velocities, the principal role is played by transfer of energy to the phonons and to other elementary excitations from the moving dislocation[2]. The problem of dislocation motion with allowance for the reaction of an elastic field was considered in a self-consistent manner by Al'shitz, Indenbom, and Shtol'berg[5], who has shown that at a low value of the viscosity coefficient B there exists a critical average velocity, below which stationary motion of the dislocation is impossible. With increasing viscosity, the critical velocity decreases and vanishes if the viscosity coefficients exceeds the limiting value B_c.

At higher external stresses $f \gg F_\pi$, the dislocation moves practically uniformly and the shape of the Peierls relief is immaterial. However, to determine the dependence of the average velocity of motion v on the stress f/b at low velocities it is necessary to take into account exactly the Peierls relief. For a piecewise-parabolic relief, this problem was investigated in[5] with allowance for radiative dragging.

The average velocity in the spectral density of energy dissipation of a moving dislocation is obtained in the present paper, for a sinusoidal relief, in the case of large viscous friction, when the inertial force can be neglected against the background of the viscous losses.

2. FORMULATION OF PROBLEM

Equation of motion of the dislocation, with allowance for the reaction of the elastic field, is

$$-F_1(t)+B\dot{x}+F_\pi \sin(2\pi x/a)=f, \tag{2}$$

where $F_1(t)$ is the inertial force, which takes into account the effect of radiation dragging of the dislocation[5] and B is the coefficient of viscous friction. The dependence of $x(t)$ on the initial conditions vanishes within a time $t \sim 1/\omega_0$, after which $x(t)$ can be represented in the form

$$x=vt+\sum_{n=-\infty}^{\infty}\xi_n e^{i\omega_0 nt}, \qquad \omega_0=\frac{2\pi v}{a}. \tag{3}$$

If the inequality $f > F_\pi$ is not too strong, it is necessary to take into account exactly the Peierls relief. It is then necessary to distinguish between two cases: the case of large viscosity and the case of the existence of critical velocity. A criterion for the realized case was obtained in[5] for the case of a piecewise parabolic relief:

$$Q=\pi\mu b^2F_\pi/2ac_t^2B^2\ln\frac{\gamma c_t}{\omega_0 a}\sim 1,$$

where μ is the shear modulus, b is the value of the Burgers vector, c_t is the speed of sound, and $\gamma \sim 1$. For typical value $B \sim 10^{-4}$ poise, $c_t \sim 10^5$ cm/sec, $b \sim a \sim 10^{-8}$ cm, $\mu \sim 10^{11}$ dyn/cm^2, and $F \sim 10^{-1}$ dyn/cm we have $Q \sim 1/\ln(\gamma c_t/\omega_0 a) < 1$, from which it follows that the case of large viscosity can be easily realized under ordinary experimental conditions.

The characteristic times Ω_0^{-1} and Ω^{-1}, which are connected respectively with the external force f and with the Peierls force, are proportional to the viscosity coefficient

$$\Omega_0^{-1}=aB/2\pi f, \quad \Omega^{-1}=aB/2\pi F_\pi.$$

The characteristic times $\tau(n)$ that are connected with the inertial force depend on the number of the harmonic in the expansion of $x(t)$ in a Fourier series (3) and are inversely proportional to B. The largest value of $\tau(n)$ can be estimated by using for $F_1(t)$ the expression obtained in[5]:

$$\tau_1=\frac{\mu b^2}{4\pi c_t^2 B}\ln\frac{\gamma c_t}{\omega_0 a}.$$

This paper deals with the case of large viscosity, when the time τ_1 is the smallest characteristic time of the system:

$$\tau_1\ll\Omega_0^{-1},\quad \tau_1\ll\Omega^{-1}. \tag{4}$$

The inertia term in (2) can then be neglected. The obtained first-order equation can be easily solved. However, it is quite difficult to extract the quantities of interest to us from the explicit solution. We propose below a technical device which enables us to calculate the dependence of the average velocity and the spectral intensity of the energy dissipation on the external force.

3. SOLUTION OF THE EQUATION OF MOTION

We use the fact that the quantities $e^{imy(t)}$, where $y = 2\pi x/a$ and m is an integer, can be represented in the form

$$e^{imy(t)}=\sum_{\nu=-\infty}^{\infty} C_m^\nu e^{i\omega_0\nu t}.$$

The coefficients C_m^ν should satisfy the conditions

$$C_0^\nu=\delta_{\nu,0},\qquad C_{m+n}^{\nu_1}=\sum_{\nu=-\infty}^{\infty} C_m^\nu C_n^{\nu_1-\nu}. \tag{5}$$

Multiplying the reduced equation (2) by $e^{imy(t)}$, we obtain the following finite-difference equation for C_m^ν:

$$m\left[\Omega_0-\frac{\Omega}{2i}(e^{\partial/\partial m}-e^{-\partial/\partial m})\right]C_m^\nu=\nu\omega_0 C_m^\nu. \tag{6}$$

Here $e^{\alpha\,\partial/\partial m}$ is the operator for shifting the index m by α:

$$e^{\alpha\partial/\partial m}C_m^\nu=C_{m+\alpha}^\nu.$$

To solve the recurrence relation (2), we use the method of generating functions. We introduce the notation

$$\Phi_z^\nu=\sum_{m=-\infty}^{\infty} e^{imz}C_m^\nu. \tag{7}$$

In expression (7), the variable z ranges from $-\pi$ to π. Equation (6) is transformed into

$$-i\frac{\partial}{\partial z}[(\Omega_0+\Omega\sin z)\Phi_z^\nu]=\nu\omega_0\Phi_z^\nu. \tag{8}$$

Equation (8) can be easily solved.

$$C_m^\nu=\frac{C}{2\pi}\int_{-\pi}^{\pi}\frac{dz}{d+\sin z}e^{-imz}\exp\left\{2i\nu\frac{\alpha}{\sqrt{d^2-1}}\operatorname{arctg}\frac{d\,\operatorname{tg}(z/2)-1}{\sqrt{d^2-1}}\right\}, \tag{9}$$

where $d = \Omega_0/\Omega$ and $\alpha = \omega_0/\Omega$. The constant C should be obtained from the normalization condition (5). The first equation of (5) can be fulfilled if the following relations hold between the average velocity of motion of the dislocation v and the external force f:

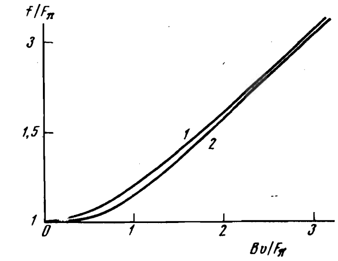
$$f=F_\pi\sqrt{1+(Bv/F_\pi)^2}. \tag{10}$$

Here $C = \sqrt{d^2-1}$. It is easy to verify that relation (10) with allowance for formula (9) satisfies the time-averaged reduced equation (2).

The integral (9) can be taken at arbitrary values of m and ν. We shall need subsequently an expression for $C_1^\nu - C_{-1}^\nu$:

$$C_1^\nu-C_{-1}^\nu=-2i\alpha\left[\frac{(\omega_0-\Omega_0)(\Omega-i\omega_0)}{\Omega\Omega_0}\right]^\nu,\quad \nu>0. \tag{11}$$

The analytic relation (10) for $f(v)$ in the case of a sinusoidal barrier differs from the corresponding expression for a piecewise-parabolic barrier, obtained in[5]:



$$f=F_\pi\operatorname{cth}(F_\pi/Bv). \tag{12}$$

However, as seen from the figure, curve 1, calculated by formula (10), lies quite close to curve 2, calculated by formula (12). As expected, the largest deviation between the curves is obtained for comparable values of f and F_π. When $f \gg F_\pi$ we obtain in both cases the same asymptotic relation $f \sim Bv$.

The oscillations of the Peierls force in time give rise to a field of elastic oscillations, the excitation of which consumes part of the dislocation energy. If we disregard the broadening of the spectral line due to the interaction between the moving dislocation and the lattice defects of other types of energy barriers, then the oscillation field consists of monochromatic lines with frequencies that are multiples of ω_0. It is convenient in the calculation to introduce a finite line width, which is then made to approach zero in the final expression. To this end we add to the frequency of the oscillations the term $i\delta$, $\delta = +0$.

The spectral density of the energy dissertation is proportional to the correlator of the Peierls forces

$$R(\omega)=\operatorname{Re}\int_0^\infty dt\,e^{i\omega t}R(t), \tag{13}$$

where

$$R(t)=\lim_{t_1\to\infty}F_\pi^2\sin y(t+t_1)\sin y(t_1).$$

The correlator $R(\omega)$ is expressed in terms of C_m^ν as follows:

$$R(\omega)=\frac{F_\pi^2}{4}\sum_\nu|C_1^\nu-C_{-1}^\nu|^2\frac{1}{-i(\omega+\omega_0\nu+i\delta)}, \tag{14}$$

where it is necessary to take in the final expression the real part of $-1/i(\omega + \omega_0 + i\delta)$, i.e., $\pi\delta(\omega + \omega_0\nu)$. Using expressions (14) and (11), we can find the relation between the intensity of the energy dissipation by the oscillations with various harmonics:

$$\frac{R(\omega=\omega_0 i)}{R(\omega=\omega_0 j)}=\left(\frac{f}{F_\pi}-\sqrt{\frac{f^2}{F_\pi^2}-1}\right)^{2(i-j)}. \tag{15}$$

It is seen from (15) that if the constant external force is comparable with the amplitude of the Peierls force, then the motion of the dislocation has an essentially non-single-mode character.

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