Anomalous skin effect in a plasma with a diffuse boundary in a magnetic field

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A theory is developed for the anomalous skin effect in a plasma having a diffuse boundary and located in a magnetic field parallel to the electron-concentration gradient. Expressions for the plasma surface resistance and for the electromagnetic-wave reflection coefficient of the plasma are obtained for the case when the electron concentration depends on the coordinates exponentially and the ratios of the electron mean free path, the penetration depth of the field in the plasma, and the size of the transition region at the boundary are arbitrary. The dependence of the surface resistance on the degree of anomaly of the skin effect and on the magnetic field intensity is investigated, as is the plasma diamagnetic-resonance line shape.

1. INTRODUCTION

The question of the anomalous skin effect in a plasma with diffuse boundary was raised by P. L. Kapitza[1] in connection with a study of a high-frequency discharge in a gas of high pressure. The theory of the anomalous skin effect in a plasma with diffuse boundary was developed by Liberman, Pitaevskiĭ and one of us[2]. In[2] an integral equation was obtained for the electromagnetic field in a plasma at an arbitrary ratio of the electron mean free path, the depth of penetration of the field into the plasma, and the dimension of the transition region at the boundary. This equation was solved for the case of an exponential decrease of the electron concentration outside the plasma under the conditions of the anomalous skin effect. The subject of the present study was the interaction of an electromagnetic wave with a plasma at an arbitrary anomaly of the skin effect, including the case when the magnetic field is parallel to the concentration gradient. The case when the magnetic field is perpendicular to the concentration gradient was considered in[3].

We shall assume that the dimension of the transition region at the boundary is small in comparison with the characteristic dimensions of the plasma, but large in comparison with the depth of penetration of the electromagnetic wave into the inhomogeneous plasma with the same electron concentration that is reached in the depth. Under these conditions, the electron concentration n_e can be regarded as a function of one coordinate x. In the absence of an electromagnetic wave, the plasma is assumed to be in equilibrium, and the specified electron concentration distribution is maintained by a static electric field $E_0(x)$ that acts on the electrons. The potential $\varphi(x)$ of this field is connected with $n_e(x)$ by the Boltzmann distribution formula

$$n_e(x)=n_0\exp(-e\varphi(x)/kT_e). \qquad (1.1)$$

In Sec. 2 we obtain an integral equation for the field in a plasma situated in the magnetic field H directed along the x axis, and find a solution of this equation for the case when the electron concentration outside the plasma decreases exponentially. In Sec. 3 we consider the dependence of the surface resistance of the plasma on the degree of anomaly of the skin effect without a magnetic field. It turns out that, in the low-frequency limit (3.3), the surface resistance does not depend on the collision frequency, and consequently on the degree of anomaly of the skin effect.

A study of the penetration of an electromagnetic wave into a plasma with a magnetic field parallel to the concentration gradient is conveniently carried out by expanding the incident wave into a sum of right- and left-circularly polarized waves. These waves propagate independently of one another, and the interaction of the wave that rotates in the same direction as the electrons in the magnetic field has a resonant character if the field frequency is close to the Larmor frequency of the electrons (diamagnetic resonance).

In Sec. 4 we obtain the dependence of the surface resistance of the plasma and of the reflection coefficient on the magnetic field.

2. DERIVATION AND SOLUTION OF THE INTEGRAL EQUATION

The system of equations for a plasma in a constant and homogeneous magnetic field H directed along the x axis consists of Maxwell's equations neglecting the displacement current

$$\frac{d^2E_\nu}{dx^2}=\frac{4\pi i\omega}{c^2}j_\nu \quad \nu=y,z \qquad (2.1)$$

and the kinetic equation for the electron distribution function. The kinetic equation linearized in terms of the electromagnetic field $E_\nu e^{i\omega t}$ takes the form

$$(i\omega+\nu_{\rm eff})f_1+v_x\frac{\partial f_1}{\partial x}-\frac{e}{m}\frac{d\varphi}{dx}\frac{\partial f_1}{\partial v_x}-\Omega\left(v_y\frac{\partial f_1}{\partial v_z}-v_z\frac{\partial f_1}{\partial v_y}\right)=\frac{eE_\nu(x)}{m}\frac{\partial f_0}{\partial v_\nu}. \qquad (2.2)$$

$$f_0=\left(\frac{m}{2\pi kT_e}\right)^{3/2}n_e(x)\exp\left(-\frac{mv^2}{2kT_e}\right) \qquad (2.3)$$

is the equilibrium distribution function of the electron; f_1 is an increment linear in the field; $\Omega = eH/mc$ is the Larmor frequency of the electron; ω is the frequency of the incident wave; $\nu_{\rm eff}$ is the effective collision frequency.

Since Eq. (2.2) is a first-order linear differential equation, its solution reduces to integration of the characteristic equations, which comprise Newton's equations for the motion of an electron in an electric field E_0 and a magnetic field H:

$$\frac{dx}{dt}=v_x, \quad m\frac{dv_x}{dt}=-e\frac{d\varphi}{dx}, \quad \frac{dv_y}{dt}=\Omega v_z, \quad \frac{dv_z}{dt}=-\Omega v_y. \qquad (2.4)$$

From the first two equations of (2.4) we obtain the in-

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tegral of motion, namely the law of conservation of the energy ϵ:

$$mv_x^2/2+e\varphi(x)=\varepsilon. \qquad (2.5)$$

To solve the last two equations of (2.4), we introduce $v_\pm = v_i \pm iv_z$; then their solutions take the form

$$v_\pm(t)=v_\pm(t_0)e^{\pm i\Omega(t-t_0)}. \qquad (2.6)$$

To solve the kinetic equation we introduce, as usual,

$$f_1=\begin{cases} f_+, & v_x>0,\\ f_-, & v_x<0.\end{cases} \qquad (2.7)$$

In the interior of the plasma, $x \to +\infty$, the wave attenuates and the plasma assumes the equilibrium state. From this we obtain the boundary condition

$$f_-=0, \quad x\to+\infty. \qquad (2.8)$$

On the other hand, the motion is bounded by the classical turning point $x = x^*$, which is determined from the equation

$$e\varphi(x^*)=\varepsilon. \qquad (2.9)$$

At the turning point, the electron reverses the direction of its motion. Therefore the boundary condition for the function f_+ is

$$f_+=f_-, \quad x=x^*. \qquad (2.10)$$

In accordance with the specifics of the interaction of the electrons with the wave, it is convenient to introduce right- and left-circularly polarized fields and current densities:

$$E_\pm=E_y\pm iE_z, \quad j_\pm=j_y\pm ij_z. \qquad (2.11)$$

Recognizing that $v_\nu E_\nu = (v_+E_- + v_-E_+)/2$, and also that

$$\frac{\partial f_0}{\partial v_\nu}(x')=-\frac{f_0(\varepsilon)}{kT_e}mv_\nu(x'), \qquad (2.12)$$

we obtain the solution of the kinetic equation (2.2) in the form

$$f_-=-\frac{ef_0}{2kT_e}\int_x^\infty e^{-\Phi(x',x)}\frac{v_+(x')E_-(x')+v_-(x')E_+(x')}{[v_x^2+2e(\varphi(x)-\varphi(x'))/m]^{1/2}}dx', \quad v_x<0; \qquad (2.13)$$

$$f_+=-\frac{ef_0}{2kT_e}\left\{\int_{x^*}^x \exp[\Phi(x',x)]+\int_{x^*}^\infty \exp[-\Phi(x',x^*)-\Phi(x,x^*)]\right\} \times\frac{v_+(x')E_-(x')+v_-(x')E_+(x')}{[v_x^2+2e(\varphi(x)-\varphi(x'))/m]^{1/2}}dx', \quad v_x>0, \qquad (2.14)$$

where

$$\Phi(x_1,x_2)=(i\omega+\nu_{\text{eff}})t(x_1,x_2)=\int_{x_2}^{x_1}\frac{(i\omega+\nu_{\text{eff}})dx}{[2m^{-1}(\varepsilon-e\varphi(x))]^{1/2}} \qquad (2.15)$$

is proportional to the time of flight of the electron $t(x_1, x_2)$ between the points x_1 and x_2.

The current density $j_\pm$ is expressed in terms of the distribution function (2.13) and (2.14) by the formula

$$j_\pm=-e\int v_\pm f_1 d^3v. \qquad (2.16)$$

Recognizing that $v_\pm^2$ averaged over the equilibrium state vanishes, and that the mean value of the product v_+v_- is $\overline{v}^2$, where $\overline{v} = (2kT_e/m)^{1/2}$, we can obtain expressions for the current $j_\pm$ in terms of the field $E_\pm$. Changing the order of integration with respect to v_x and x', and also integrating with respect to v_y and v_z, we obtain

$$j_\pm=\frac{e^2n_0}{\pi^{1/2}m\overline{v}}\left[\int_{-\infty}^x E_\pm(x')G_\pm(x,x')dx'+\int_x^\infty E_\pm(x')G_\pm(x',x)dx'\right], \qquad (2.17)$$

where the kernel is

$$G_\pm(x',x)=\exp\left(-\frac{e\varphi(x)}{kT_e}\right)\int_0^\infty dv_x\exp\left(-\frac{mv_x^2}{2kT_e}\right) \cdot\frac{\exp[-\Phi_\pm(x',x)]+\exp[-\Phi_\pm(x',x^*)-\Phi_\pm(x,x^*)]}{[v_x^2+2e(\varphi(x)-\varphi(x'))/m]^{1/2}}. \qquad (2.18)$$

The function $\Phi_\pm(x_1, x_2)$ is proportional to $t(x_1, x_2)$:

$$\Phi_\pm(x_1,x_2)=[i(\omega\mp\Omega)+\nu_{\text{eff}}]t(x_1,x_2). \qquad (2.19)$$

In the absence of a magnetic field, $\Omega = 0$, expression (2.18) goes over into expression (14) of[2]. Substituting (2.17) in (2.1), we obtain the integro-differential equation

$$\frac{d^2E_\pm(x)}{dx^2}=\frac{i\omega\omega_0^2}{\pi^{1/2}c^2\overline{v}}\left\{\int_{-\infty}^x E_\pm(x')G_\pm(x,x')dx'+\int_x^\infty E_\pm(x')G_\pm(x',x)dx'\right\} \qquad (2.20)$$

where $\omega_0^2 = 4\pi e^2n_0/m$, and ω_0 is the plasma frequency of the electrons in the interior of the plasma. We note that the waves of the right-hand and left-hand polarizations propagate in the plasma independently of one another.

From the solution of (2.20) we can obtain for each polarization a macroscopic characteristic (the reflection coefficient $r_\pm$) by carrying out the same calculations as in[2]. Outside the plasma at $a \ll -x \ll c/\omega$, the field has the asymptotic value $E_\pm = A_\pm(x + B_\pm)$, where a is a quantity on the order of the width of the transition layer at the plasma boundary. The quantities $r_\pm$ are connected with $B_\pm$ in the following manner:

$$r_\pm=1+2i\omega B_\pm/c, \quad \omega B_\pm/c\ll1. \qquad (2.21)$$

In the case of a small mean free path, we can neglect the variation of the field over the free path

$$l=\overline{v}/|i(\omega\mp\Omega)+\nu_{\text{eff}}|,$$

i.e., we can take $E_\pm(x') = E_\pm(x)$ outside the integral sign in (2.20), after which the integro-differential equation (2.20) goes over into the known ordinary differential equation for the normal skin effect:

$$\frac{d^2E_\pm}{dx^2}=\frac{i\omega\omega_0^2(x)E_\pm}{c^2(i(\omega\mp\Omega)+\nu_{\text{eff}})}, \quad l\ll\delta, \qquad (2.22)$$

where $\omega_0^2(x) = 4\pi e^2n_e(x)/m$, and δ is the depth of the skin layer.

It is impossible to solve (2.20) without making assumptions concerning the form of the potential $\varphi(x)$.

We consider the case of an exponential decrease of the electron concentration outside the plasma:

$$n_e(x)=n_0\exp(x/a), \quad a=kT_e/eE_0, \quad x\to-\infty.$$

The electron density in all of space is of course not described by this formula. We assume that $n_e(x)$ tends to a constant value n as $x \to +\infty$. Under the condition

$$L\equiv\ln(a/\delta)^3\gg1, \quad \delta^3=\frac{c^2\overline{v}m}{4\pi^{1/2}e^2\tilde{n}\omega} \qquad (2.23)$$

the electromagnetic wave attenuates strongly and in practice does not reach the regions in which the electron concentration begins to differ from exponential. To solve the integral equation in this case it sufficies to know only the asymptotic form of the potential

$$\varphi(x)=-E_0x, \quad x\to-\infty. \qquad (2.24)$$

We now proceed to dimensionless coordinates in accordance with the formulas

$$x=a(\xi-L),\quad f_\pm(\xi)=E_\pm(x(\xi)),$$
$$\gamma_\pm=\frac{2a}{\bar v}(i(\omega\mp\Omega)+\nu_{\rm eff}),\quad u=\xi'-\xi. \tag{2.25}$$

The difference between the right- and left-polarized waves lies only in the parameters $\gamma_\pm$. For simplicity, we shall not write out the subscripts $\pm$. Using (2.24) and (2.25), we can represent expression (2.18) for the conductivity kernel in the form

$$G(\xi',\xi)=e^{\xi-L}\int_0^\infty dx\frac{\exp\{-x^2-\gamma\sqrt{x^2+u}\}}{\sqrt{x^2+u}}(e^{\gamma x}+e^{-\gamma x}). \tag{2.26}$$

Making the substitutions $e^w=-x+\sqrt{x^2+u}$ in the integral (2.26) with the first term and the substitution $e^w=x+\sqrt{x^2+u}$ in the second term, we obtain

$$G(\xi',\xi)=\exp\left\{\frac{\xi+\xi'}{2}-L\right\}\int_{-\infty}^\infty dw\exp\left(-\frac{e^{2w}}{4}-\gamma e^w-\frac{(\xi'-\xi)^2e^{-2w}}{4}\right)\equiv\exp((\xi+\xi')/2-L)R(\xi'-\xi). \tag{2.27}$$

The equation for the field

$$\frac{d^2f(\xi)}{d\xi^2}=i\int_{-\infty}^\infty d\xi' f(\xi')\exp[(\xi+\xi')/2]R(\xi'-\xi) \tag{2.28}$$

is solved by the method first proposed by Hartmann and Luttinger[4] and then used many times in[2,3,5]. We apply to this equation the bilateral Laplace transformation

$$F(k)=\int_{-\infty}^\infty f(\xi)e^{-k\xi}d\xi,\quad f(\xi)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}F(k)e^{k\xi}dk. \tag{2.29}$$

Recognizing that the integral

$$\int_0^\infty du[e^{u(k+1/2)}+e^{-u(k+1/2)}]R(u)=2\pi^{1/2}\int_0^\infty dx\, e^{-k(k+1)x^2-\gamma x}\equiv P(k) \tag{2.30}$$

converges in the band $-1<\mathrm{Re}\,k<0$, we obtain

$$\int_{c-i\infty}^{c+i\infty}k^2F(k)e^{k\xi}dk=\int_{c-i\infty}^{c+i\infty}iP(k)F(k)e^{(k+1)\xi}dk,\quad -1<c<0. \tag{2.31}$$

Since $f(\xi)\to\alpha(\xi+\beta)$ as $\xi\to-\infty$, it is necessary that $F(k)$ have at k a second-order pole:

$$\lim_{k\to0}k^2F(k)=\alpha. \tag{2.32}$$

We seek a function $F(k)$ which is analytic in the band $-1<\mathrm{Re}\,k<1$, with the exception of the origin. Then $k^2F(k)$ is analytic in this entire band and we can shift the integration contour in the left integral of (2.31) by unity to the right, and then redesignate k as $k+1$. Since (2.31) holds at any value of ξ, we can change over from equality of the integrals to the functional equation

$$F(k+1)/F(k)=iP(k)/(k+1)^2. \tag{2.33}$$

A solution of (2.33) having the required analytic properties is unique apart from a normalization constant, which is arbitrary by virtue of the linearity and homogeneity of (2.28). In[2-5], the corresponding equations had right-hands side in the form of a relatively simple function, so that the solution could be found by trial and error. In our case thus is not possible, and we therefore use a regular method that yields the solution of (2.33) for arbitrary $P(k)$ in the form of an integral.

We take the logarithm of Eq. (2.33). We obtain the functional difference equation

$$Q(k+1)-Q(k)=M(k),$$
$$Q(k)=\ln F(k),\quad M(k)=\ln(iP(k)/(k+1)^2). \tag{2.34}$$

The solution of (2.34) is obtained with the aid of the known property of the Laplace transformation. Namely, if $Q(k)$ is the Laplace transform of $q(t)$, then $Q(t+1)$ is the transform of $e^{-t}q(t)$. We thus obtain for $q(t)$ the simple algebraic equation

$$(e^{-t}-1)q(t)=m(t). \tag{2.35}$$

The function $Q(k)$, which vanishes at $k=0$, is given by

$$Q(k)=\frac{1}{2\pi i}\int_{-\infty}^\infty dt\frac{e^{-kt}-1}{e^{-t}-1}\int_{c-i\infty}^{c+i\infty}e^{tz}M(z)dz. \tag{2.36}$$

Choosing c in (2.36) such that

$$-1<c<\mathrm{Re}\,k\ \text{ for }\ \mathrm{Re}\,k\leqslant0\ \text{ and}$$
$$\mathrm{Re}\,k-1<c<0\ \text{ for }\ \mathrm{Re}\,k\geqslant0, \tag{2.37}$$

we can interchange the order of integration with respect to t and z. Calculating the integral with respect to t ([6], 3.311), we obtain for $Q(k)$ the expression

$$Q(k)=\frac{1}{2i}\int_{c-i\infty}^{c+i\infty}dz\frac{M(z)\sin k\pi}{\sin z\pi\sin(z-k)\pi} \tag{2.38}$$

The solution of Eq. (2.33) is now expressed in the form

$$F(k)=g(k)\exp Q(k), \tag{2.39}$$

where $g(k)$ is an arbitrary periodic function with unity period. The necessary analytic properties of $F(k)$ can be obtained by assuming

$$g(k)=\alpha\frac{2\pi^2}{1-\cos2\pi k}. \tag{2.40}$$

Substituting the solution (2.39) in (2.29), we obtain a formula for determining the field $f(\xi)$. As $\xi\to-\infty$, the main contribution to the integral was made by the residue at the pole $k=0$. Expanding $F(k)$ in a Laurent series, we obtain in accordance with the residue theorem

$$f(\xi)=\alpha(\xi+Q'(0)), \tag{2.41}$$

whence

$$\beta=Q'(0)=\frac{\pi}{2}\int_{-\infty}^\infty\frac{M(iw-1/2)dw}{\mathrm{ch}^2\pi w}. \tag{2.42}$$

Comparing formulas (2.42), (2.34), and (2.30), we obtain

$$B=a(\beta+L),\quad \beta=\ln\pi+3C+\frac{i\pi}{2}+p(\gamma), \tag{2.43}$$

where $C=0.577\ldots$ is the Euler constant and

$$p(\gamma)=\frac{\pi}{2}\int_0^\infty\frac{dx}{\mathrm{ch}^2(\pi x/2)}\left\{\frac{\gamma^2}{x^2+1}+\ln[1-\Phi(\gamma/(x^2+1)^{1/2})]\right\},$$
$$\Phi(x)=\frac{2}{\sqrt\pi}\int_0^x e^{-\alpha^2}d\alpha \tag{2.44}$$

is the error integral

In the case $|\gamma|\gg1$, the expression for the field coincides with the solution of Eq. (2.22) for the normal skin effect:

$$f_\pm(\xi)=AK_0\left(2^{3/2}\pi^{1/4}|\gamma_\pm|^{-1/2}\exp\left(\frac{\xi}{2}+i\left(\frac{\pi}{4}-\frac{1}{2}\operatorname{arc\,tg}\frac{\omega\mp\Omega}{\nu_{\rm eff}}\right)\right)\right), \tag{2.45}$$

and the expression for β goes over into

$$\beta_\pm=\frac{1}{2}\ln\pi+2C+\ln2-\ln\gamma_\pm+i\frac{\pi}{2}. \tag{2.46}$$

3. DEPENDENCE OF THE SURFACE RESISTANCE ON THE DEGREE OF ANOMALY OF THE SKIN EFFECT

The derived formulas (2.43) and (2.44) enable us to determine the dependence of the surface resistance and

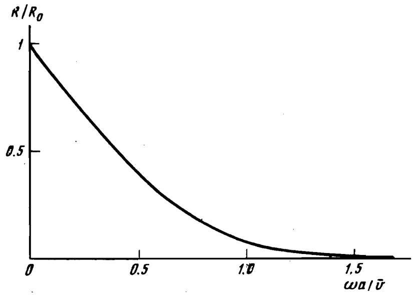

FIG. 1. Dependence of the surface resistance R on the parameter $\omega a/\bar{v}$ for a collisionless plasma.

of the reflection coefficient on the degree of the anomaly in the absence of a magnetic field. In this case the surface resistance and the reflection coefficient are connected with β (2.43) in accordance with the formulas[2]

$$R=\frac{4\pi\omega a}{c^2}\operatorname{Im}\beta=\frac{4\pi\omega a}{c^2}\left(\frac{\pi}{2}+\operatorname{Im}p(\gamma)\right), \tag{3.1}$$

$$r=1+2i\frac{\omega a}{c}(\beta+L). \tag{3.2}$$

We note that at $\omega a/\bar{v} \ll 1$ the imaginary part of $p(\gamma)$ is small (on the order of $\omega a/\bar{v}$), so that in the low-frequency limit the surface resistance of a plasma having an exponential $n_e(x)$ dependence does not depend at all on the degree of anomaly of the skin effect, and is determined by the structure of the transition layer at the plasma boundary:

$$R_0=2\pi^2\omega a/c^2=2\pi^2\cdot 10^{-9}\ \omega a\ [\Omega], \qquad \omega\ll\bar{v}/a, \quad \nu_{eff}a/\bar{v} - \text{arbitrary} \tag{3.3}$$

For a collisionless plasma ($\nu_{eff} = 0$), the surface resistance as a function of the parameter $\omega a/\bar{v}$ is given by

$$R=R_0\,\rho(\omega a/\bar{v}),$$
$$\rho(x)=1-\int_0^\infty\frac{dy}{\operatorname{ch}^2(\pi y/2)}\operatorname{arctg}\left(2\pi^{-1/2}\int_0^{2x/\sqrt{y^2+1}}e^{z^2}dz\right). \tag{3.4}$$

The function $\rho(x)$ is plotted in Fig. 1. At low frequencies (extremely anomalous skin effects) we have

$$\rho(x)=1-2I_1x, \quad x=\omega a/\bar{v}\ll 1, \tag{3.5}$$

where

$$I_1=2\pi^{-1/2}\int_0^\infty dx\Big/\sqrt{x^2+1}\operatorname{ch}^2\frac{\pi x}{2}=0.6461. \tag{3.6}$$

Using the saddle-point method, we find that in the inverse limiting case $\omega a/\bar{v} \gg 1$ the surface resistance of the plasma decreases exponentially with increasing frequency:

$$\rho(x)\approx\frac{2}{\sqrt{3}}(\pi x)^{2/3}\exp(-3(\pi x)^{2/3}), \quad x\gg 1. \tag{3.7}$$

At $\nu_{eff} = 0$, the surface resistance is due to collisionless dissipation of the Landau-damping type, the mechanism of which consists in the fact that the electrons interacting with the field in the skin layer transport energy into the interior of the plasma. With increasing field frequency, the effectiveness of the interaction of the electron field in the skin layer decreases, as a result of which the collisionless dissipation also decreases. In the limiting case of the ordinary skin effect $\omega \gg \bar{v}/a$ the collisionless dissipation is exponentially small. This result agrees with the fact that in the usual skin effect the surface resistance of the plasma tends to zero as the number of the collisions decreases.

If ν_{eff} differs from zero, then at $|\gamma| \gg 1$ it follows from (2.43) that

$$R=R_0\varphi(\nu_{eff}/\omega)=R_0\frac{2}{\pi}\operatorname{arctg}(\nu_{eff}/\omega). \tag{3.8}$$

If the collision frequency is low, $\nu_{eff} \ll \bar{v}/a$, then the surface resistance is determined by the sum

$$R=R_0(\rho(\omega a/\bar{v})+\varphi(\nu_{eff}/\omega)), \tag{3.9}$$

and, depending on the value of the parameter

$$\varkappa=\frac{\omega}{\nu_{eff}}\left(\frac{a\omega}{\bar{v}}\right)^{1/3}\exp\left(-3\left(\frac{\pi a\omega}{\bar{v}}\right)^{2/3}\right)$$

we have either collisionless dissipation (3.4) if $\kappa \gg 1$, or collision-dominated dissipation (3.8) if $\kappa \ll 1$.

In the region of the extremely anomalous skin effect, at an arbitrary ratio of the frequencies ν_{eff} and ω, we have

$$p(\gamma)=\frac{\pi}{2}(-I_1\gamma+I_2\gamma^2+I_3\gamma^3+\ldots), \quad |\gamma|\ll 1, \tag{3.10}$$

where I_1 is defined in (3.6), and

$$I_2=\left(1-\frac{2}{\pi}\right)\int_0^\infty\frac{dx}{(x^2+1)\operatorname{ch}^2(\pi x/2)}=\frac{\pi}{6}-\frac{1}{3}=0.1903,$$
$$I_3=\frac{2}{3\pi^{3/2}}(4-\pi)\int_0^\infty\frac{dx}{(x^2+1)^{3/2}\operatorname{ch}^2(\pi x/2)}=0.0498. \tag{3.11}$$

4. INFLUENCE OF MAGNETIC FIELD. DIAMAGNETIC RESONANCE

The reflection coefficient $r_\pm$ of a circularly-polarized wave is determined from the general solution (2.43) of Eq. (2.28):

$$r_\pm=1+2i\frac{\omega a}{c}(\beta_\pm+L). \tag{4.1}$$

We recall that the subscripts $\pm$ designate right and left circularly polarized waves. The absorption coefficient $\eta_\pm$ of a circularly polarized wave, defined by the formula

$$S_{x\pm}=\eta_\pm\frac{c|E_{0\pm}|^2}{8\pi} \tag{4.2}$$

(S is the flux density of the electromagnetic energy, $E_{0\pm}$ is the amplitude of the field of the incident wave) is connected with β in the following manner:

$$\eta_\pm=\frac{2\omega a}{c}\operatorname{Im}\beta_\pm. \tag{4.3}$$

When the magnetic field tends to zero, the absorption coefficient agrees, apart from a constant factor, with the surface resistance, $\eta = cR/2\pi$.

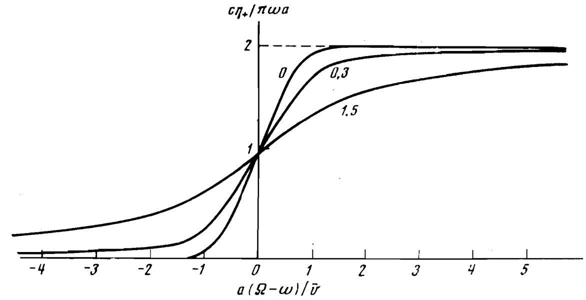

FIG. 2. Depedence of the absorption coefficient η_+ of a circularly polarized wave on the parameter $a(\Omega-\omega)/\bar{v}$. The figures on the curves indicate the dimensionless collision frequencies $a\nu_{eff}/\bar{v}$.

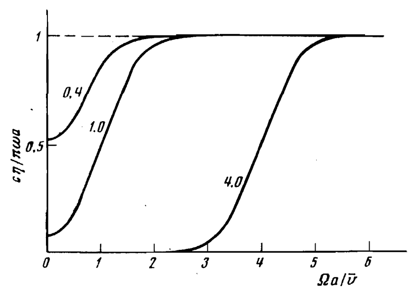

FIG. 3. Dependence of the absorption coefficient η of a linearly polarized wave on the parameter $a\Omega/\bar{v}$ for a collisionless plasma. The numbers on the curves indicate the values of the parameter $a\omega/\bar{v}$.

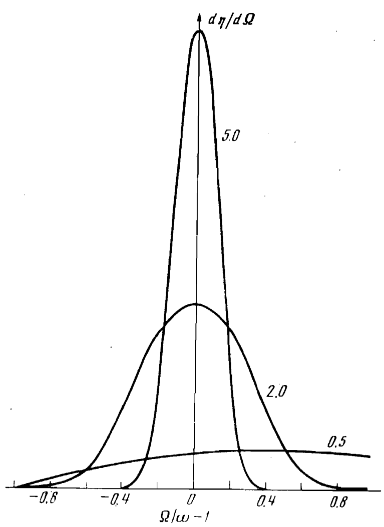

FIG. 4. Diamagnetic-resonance line shape at different values of ω/v (indicated by the figures on the curves) at $\nu_{\text{eff}} = 0$.

The dependence of $\eta_\pm$ on the magnetic field is described by the same formulas as the dependence of the surface resistance on the frequency in Sec. 3, except that the frequency ω is replaced in the formulas by the combination $\omega \mp \Omega$. Thus, for example, for a collisionless plasma ($\nu_{\text{eff}} = 0$), the dependence of $\eta_\pm$ on the magnetic field is determined, apart from a factor $c/2\pi$, by formula (3.4), in which now $x = a(\omega \mp \Omega)/\bar{v}$. Figure 2 shows the corresponding plot for different values of the parameter $\nu_{\text{eff}}a/\bar{v}$.

By regarding a linearly polarized wave as a sum of left and right polarized waves, we obtain the connection between the components of the reflection-coefficient tensor $r_{\alpha\beta}$ of a linearly-polarized wave and reflection coefficients $r_\pm$ of circularly-polarized waves:

$$r_{yy}=r_{zz}=(r_+ + r_-)/2, \quad r_{yz}=-r_{zy}=(r_- - r_+)/2i. \tag{4.4}$$

The off-diagonal elements $r_{\alpha\beta}$, $\alpha \neq \beta$, describe the transformation of the wave from one linear polarization to the other upon reflection from the plasma. The absorption coefficient of a linearly polarized wave is equal to

$$\eta=(\eta_+ + \eta_-)/2. \tag{4.5}$$

Figure 3 shows plots of η against the magnetic field for a collisionless plasma at different values of the parameter $\omega a/\bar{v}$.

At high frequencies, $\omega \gg \bar{v}/a$, the absorption coefficient increases sharply when the normal frequency of the electrons reaches the frequency ω of the external field. In this case the electrons, rotating in phase with the circularly polarized field component, interact effectively with this component. In accordance with the standard terminology, the corresponding resonance is called diamagnetic. Figure 4 shows the dependence of $d\eta/d\Omega$ on $\Omega/\omega - 1$ at different values of the parameter $\omega a/\bar{v}$. When the parameter $\omega a/\bar{v}$ decreases, and also when the number of collisions increases, the width of the resonance line increases. Thus, the diamagnetic resonance is most strongly pronounced in the region $\nu_{\text{eff}} \ll \bar{v}/a$, $\omega \gg \bar{v}/a$. Its experimental observation could offer evidence that the number of collisions is small, and consequently that the plasma temperature is high.

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