Quantum effects in an intense electromagnetic field

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The processes of photon emission by an electron and pair production in the field of an intense plane monochromatic wave of arbitrary elliptic polarization, as well as in the field of two linearly polarized waves with mutually perpendicular polarizations and propagating in the same direction, are considered. Expressions are obtained for the probabilities of these processes, and numerical computations are carried out for specific values of the parameters entering into the present problem. The characteristic features of the dependence of the total probabilities on the external-field frequency are discussed.

1. INTRODUCTION

Many papers have recently been published in which different quantum effects in the field of an intense electromagnetic wave are investigated[1-5]. The study of such effects is of great importance in connection with the production of high-power laser beams and the possibility of their use for different physical investigations. Interest in these investigations will evidently grow in future, and therefore it seems to us that it is necessary to thoroughly investigate the quantum processes that can occur in the field of a laser beam (γ-quantum emission by an electron, pair production, elementary-particle disintegration, etc.).

The enumerated effects have been considered in the papers[1-5] for the fields of linearly- and circularly-polarized monochromatic waves. Since, as it turned out, the course of the quantum processes have specific characteristics that depend on the polarization of the wave[5], it is of interest to consider the general case of elliptic polarization. Apparently, this circumstance has a more significant effect on the polarization characteristics of the particles (e.g., the emitted photons), although, as our calculations show, such integral characteristics as the magnitudes of the total probabilities of γ-quantum emission and pair production also depend essentially on the polarization of the wave.

Furthermore, it is also of interest to consider the quantum effects in the field of nonmonochromatic wave consisting of a collection of waves of different frequencies. The course of the quantum effects in this case can be accompanied by the absorption (or emission) of photons either from the various waves, or separately from each of them. In this paper we shall consider the simple model of two linearly polarized waves of different frequencies propagating in the same direction, the polarizations of these waves being assumed to be mutually perpendicular. In this case the exact solution of the Dirac equation for the electron has a particularly simple form, which significantly simplifies the numerical computations.

The probabilities of processes in a wave field depend on two invariants[1], to wit, $x = ea/m$ (a_μ is the amplitude of the potential) and $\chi = (kp)x/m^2$ (p_μ is the momentum and m the mass of the particle). For the case of several waves, the number of invariant parameters increases correspondingly. For $x \gg 1$, the analysis reduces to the study of processes in a constant crossed field[1]. The case $x \ll 1$ yields perturbation theory. We shall, in the present paper, assume that the values of the parameters x and χ entering into the problem are of the order of unity. This region requires the use of numerical integration, and has not been investigated in detail before (with the exception of the case of one circularly polarized wave[4]).

Let us note here another circumstance pertaining to the two-wave model. If for one monochromatic wave the effective values of the number s of quanta absorbed from the wave in the case when $x \gg 1$ are $\sim x^3$ and the quantum processes involve the absorption of a large number of the wave photons[5], then such a correspondence between the values of the parameters s_i and x_i cannot be established for the two-wave model. Here, for $x \gg 1$, processes involving the absorption of a large number of quanta from one of the waves and a small number of them from the other can also turn out to be important.

In the present paper we consider the effects of pair production and γ-quantum emission by an electron in the case of a monochromatic wave of arbitrary elliptic polarization, as well as for the above-indicated two-wave model. The decay of elementary particles will be considered later. We use the same metric and γ matrices used by Okun' in his book[6].

2. ELLIPTICALLY POLARIZED MONOCHROMATIC WAVE

As is well known, the exact solution to the Dirac equation with the potential $A_\mu(\varphi)$ of a plane electromagnetic wave has the form[7]

$$\psi_p(x) = e^{-ipx} F(\varphi) \frac{u(p)}{(2p_0)^{1/2}}, \tag{1}$$

where

$$F(\varphi) = \left[1 + \frac{e}{2(pk)} \hat{k}\hat{A}\right] e^{iS(\varphi)},$$

$$S(\varphi) = -\int_0^{kx} \left[\frac{e}{(kp)}(pA) - \frac{e^2}{2(kp)} A^2\right] d\varphi. \tag{2}$$

Here p_μ is a constant 4-vector, $p^2 = m^2$, the bispinor u(p) satisfies the free Dirac equation $(p - m)u(p) = 0$, $\varphi = (kx)$, and $k^2 = 0$.

Let us choose the potential $A_\mu(\varphi)$ in the form

$$A_\mu(\varphi) = a_{1,\mu} \cos\varphi + \varepsilon a_{2,\mu} \sin\varphi. \tag{3}$$

The parameter ϵ varies within the limits $-1 \le \epsilon \le 1$, the 4-vectors a_1 and a_2 are mutually orthogonal: $a_1 a_2 = 0$ and, furthermore, $a_1 a_1 = a_2 a_2 = -a^2$.

Substituting (2) and (3) into (1) and integrating, we obtain

$$\psi_p(x) = \left[1 + \frac{e}{2(kp)} \hat{k}\hat{a}_1 \cos\varphi + \frac{e\varepsilon}{2(kp)} \hat{k}\hat{a}_2 \sin\varphi\right]$$

$$\times\frac{u(p)}{(2p_0)^{1/2}}\exp\left\{-i\left[\frac{e}{(kp)}(pa_1)\sin\varphi-e\varepsilon\frac{(pa_2)}{(kp)}\cos\varphi+\frac{e^2a^2}{8(kp)}(1-\varepsilon^2)\sin 2\varphi+(qx)\right]\right\}. \quad (4)$$

The quasimomentum

$$q_\mu=p_\mu+\frac{e^2a^2}{4(kp)}(1+\varepsilon^2)k_\mu$$

satisfies the relation

$$q^2=m_*^2,\quad m_*=m\left[1+\frac{1}{2}\left(\frac{ea}{m}\right)^2(1+\varepsilon^2)\right]^{1/2}.$$

The matrix element of photon emission by an electron has, as is well known, the form

$$M_{if}=-ie\int(\bar\psi_{p'}\hat e'\psi_p)\frac{e^{ik'x}}{(2k_0')^{1/2}}d^4x. \quad (5)$$

The notation here is the same as in[1]. The substitution of (4) into (5) gives rise to certain functions of φ, which we expand in Fourier series in a manner similar to what was done in[1]. Let us introduce the notation:

$$\cos^n\varphi\, e^{if(\varphi)}=\sum_{s=-\infty}^{\infty}A_n(s)e^{-is\varphi},$$
$$\sin\varphi\, e^{if(\varphi)}=\sum_{s=-\infty}^{\infty}A_1'(s)e^{-is\varphi}. \quad (6)$$

Here

$$f(\varphi)=\alpha_1\sin\varphi-\alpha_2\varepsilon\cos\varphi+\beta(1-\varepsilon^2)\sin 2\varphi,$$
$$\alpha_i=e\left[\frac{(a_ip')}{(kp')}-\frac{(a_ip)}{(kp)}\right],\quad \beta=\frac{e^2a^2}{8}\left[\frac{1}{(kp')}-\frac{1}{(kp)}\right]. \quad (7)$$

Let us represent with the aid of (6) and (7) the matrix element in the form

$$M_{if}=-ie\frac{1}{(2p_0'\cdot 2p_0\cdot 2k_0')^{1/2}}\sum_s\bar u(p)\left[\left(\hat e'+\frac{e^2a^2\varepsilon^2}{2(p'k)(pk)}(ke')\hat k\right)A_0+e\left(\frac{\hat a_1\hat k\hat e'}{2(kp')}+\frac{\hat e'\hat k\hat a_1}{(2kp)}\right)A_1+e\varepsilon\left(\frac{\hat a_2\hat k\hat e'}{2(kp')}+\frac{\hat e'\hat k\hat a_2}{2(kp)}\right)A_1'+\frac{e^2a^2(1-\varepsilon^2)}{2(kp')(kp)}(e'k)\hat kA_2\right]u(p)(2\pi)^4\delta(sk+q-q'-k'). \quad (8)$$

We see that the matrix element (8) for a wave of arbitrary polarization ϵ contains four complex functions A_i and A_i', in contrast to the cases of linear and circular polarizations analyzed in[1,4], when only three functions of the type (6) entered into the matrix element. Between these four functions exists a relation that can be established by a method similar to the one used in[1]:

$$[s-2\beta(1-\varepsilon^2)]A_0+\alpha_1A_1+\alpha_2\varepsilon A_1'+4\beta(1-\varepsilon^2)A_2=0. \quad (9)$$

If we introduce the vector $e''=e'-k'(ke')/(kk')$, then we can, with the aid of (9), reduce the matrix element (8) to the form

$$M_{if}=-ie\frac{1}{(2p_0'\cdot 2p_0\cdot 2k_0')^{1/2}}\sum_s\bar u(p')\left[\hat e''A_0+e\left(\frac{\hat a_1\hat k\hat e''}{2(kp')}+\frac{\hat e''\hat k\hat a_1}{2(kp)}\right)A_1+e\varepsilon\left(\frac{\hat a_2\hat k\hat e''}{2(kp')}+\frac{\hat e''\hat k\hat a_2}{2(kp)}\right)A_1'\right]u(p)(2\pi)^4\delta(sk+q-q'-k'). \quad (10)$$

Squaring (10), and summing over the polarizations of the electrons, we obtain

$$\sum_{r,r'}\frac{|M_{if}|^2}{VT}=\frac{e^2}{2q_0'q_0k_0'}\sum_s K(s)(2\pi)^4\delta(sk+q-q'-k'). \quad (11)$$

Here

$$K(s)=|A_0|^2[2(pe'')^2+(pp')-m^2]+\mathrm{Re}\,A_0A_1^*[\alpha_1(kk')-4e(pe'')(a_1e'')]+\mathrm{Re}\,A_0A_1'^*\varepsilon[\alpha_2(kk')-4e(pe'')(a_2e'')]+|A_1|^2e^2\left[2(a_1e'')^2+\frac{a^2(kk')^2}{2(kp)(kp')}\right]+4\,\mathrm{Re}\,A_1A_1'^*e^2\varepsilon(a_1e'')(a_2e'')+|A_1'|^2e^2\varepsilon^2\left[2(a_2e'')^2+\frac{a^2(kk')^2}{2(kp)(kp')}\right]. \quad (12)$$

For ϵ = 0 this expression goes over into the corresponding formula obtained by Nikishov and Ritus in[1].

The expression (12) gives the square of the matrix element with allowance for the polarization of the γ quantum. However, we shall not in this paper consider the polarization effects: we shall consider only the corresponding total probabilities.

Summing (12) over the polarizations of the γ quanta, we obtain

$$K(s)=-2m^2|A_0|^2-a^2e^2\varepsilon^2\left[2+\frac{(kk')^2}{(kp)(kp')}\right]|A_0|^2+a^2e^2\left[2+\frac{(kk')^2}{(kp)(kp')}\right][|A_1|^2+|A_1'|^2\varepsilon^2-\mathrm{Re}\,A_0A_2^*(1-\varepsilon^2)]. \quad (13)$$

As was to be expected, only quantities quadratic in ϵ enter into this expression. The effects linear in ϵ can be observed by studying, for example, the circular polarization of the γ quantum.

For pair production by a polarized photon of momentum l, we obtain a quantity similar to (12) if we make the substitutions $k'\to -l$ and $p\to -p$ in this expression (i.e., in (12)) and change the overall sign in front of K(s).

After averaging over the polarizations of the incident photon, the quantity, which is similar to (13) and which we denote by $\bar K(s)$, has the form

$$\bar K(s)=2m^2|A_0|^2+a^2e^2\varepsilon^2\left[2-\frac{(kl)^2}{(kp)(kp')}\right]|A_0|^2-a^2e^2\left[2-\frac{(kl)^2}{(kp)(kp')}\right][|A_1|^2+|A_1'|^2\varepsilon^2-\mathrm{Re}\,A_0A_2^*(1-\varepsilon^2)]. \quad (14)$$

The total probability of emission of a photon from a unit volume in a unit time is equal to[1]

$$W_\gamma=W_q\sum_{s=1}^{\infty}\int_0^{2\pi}d\varphi'\int_0^{u_s}\frac{du}{(1+u)^2}\frac{1}{m^2}K(s). \quad (15)$$

Here n is the mean density of the incident particles,

$$W_q=\frac{e^2m^2n}{16\pi^2q_0},\quad u_s=\frac{2s(kq)}{m_*^2},\quad u=\frac{(kk')}{(kq')}. \quad (16)$$

For the probability of pair production we have

$$W_p=W_l\sum_{s>s_0}^{\infty}\int_0^{2\pi}d\varphi'\int_1^{s/s_0}\frac{du}{u[u(u-1)]^{1/2}}\frac{1}{m^2}\bar K(s), \quad (17)$$

where

$$W_l=\frac{e^2m^2n}{32\pi^2l_0},\quad s_0=\frac{2m_*^2}{(kl)},\quad u=\frac{(kl)^2}{4(kp)(kp')}. \quad (18)$$

In (15) and (17), φ' is the azimuthal angle[1]. In the general form, the expressions (15) and (17) contain three integrations, which cannot be performed analytically. The only exception is the case, considered in[4], of circular polarization of the wave, when the dependence of the functions A_i on the azimuthal angle is trivial and these functions can themselves be computed analytically in terms of the Bessel functions. The number of integrations thus decreases to one. In the general case, however, we can obtain concrete values for the probabilities only through a numerical integration over the angle φ' and the phase φ of the wave.

We have carried out such computations of the expressions (15) and (17), and the results are shown in Fig. 1.

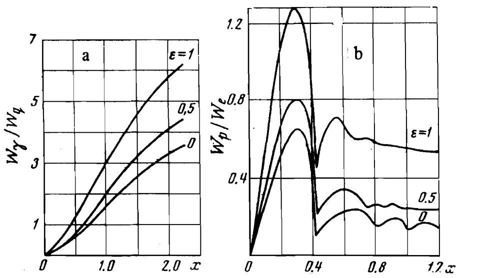

FIG. 1. The probability W_γ of emission of a photon, (a), and the probability W_p of pair production, (b); $\chi = 1$.

The values of the parameters χ and ϵ used in the computations are indicated in the figure. For the circular polarization $\epsilon = 1$, the corresponding curves were obtained also in[4] by Narozhnyĭ, Nikishov, and Ritus. For comparison with their results, the values of the probabilities given in Fig. 1 should be divided respectively by π and 2π. The curves computed by us for $\epsilon = 1$ agree with the curves given in[4]. As can be seen from Fig. 1, the magnitudes of the probabilities W_γ and W_p essentially depend on the polarization of the wave, this dependence becoming stronger and stronger as ϵ approaches unity. The nonmonotony of the x dependence of W_p is, as has already been indicated in[4], due to the existence of a threshold for pair production. It can be seen from Fig. 1b that the nearer the polarization of the wave is to being linear, the more pronounced this nonmonotony is in the region $x > 0.4$.

3. TWO LINEARLY POLARIZED WAVES WITH MUTUALLY PERPENDICULAR POLARIZATIONS

Let us choose the wave potentials in the form

$$A_1=a_1\cos\varphi_1,\quad A_2=a_2\cos(\varphi_2+\varphi_0). \tag{19}$$

Here $\varphi_i = k_ix$, $a_1a_2 = 0$, and φ_0 is the phase shift.

Taking into account the fact that $k_1k_2 = 0$, we find the exact solution of the Dirac equation with the potential (19) to have the form

$$\psi_p(x)=e^{-ipx}F_1(\varphi_1)F_2(\varphi_2)\frac{u(p)}{(2p_0)^{1/2}}. \tag{20}$$

Here $F_i(\varphi_i)$ has the form of (2) with k and A replaced respectively by k_i and A_i. We shall henceforth assume that k_1 and k_2 are incommensurate. There will arise in the computation of the matrix element (5) in the present case two types of functions A_n and B_n defined by:

$$\cos^n\varphi_1 e^{if_1(\varphi_1)}=\sum_{s_1}A_n(s_1;\alpha_1,\beta_1)e^{-is_1\varphi_1},$$
$$\cos^n(\varphi_2+\varphi_0)e^{if_2(\varphi_2)}=\sum_{s_2}B_n(s_2;\alpha_2,\beta_2)e^{-is_2\varphi_2}. \tag{21}$$

We have introduced the following notation:

$$\alpha_i=e\left[\frac{(a_ip')}{(k_ip')}-\frac{(a_ip)}{(k_ip)}\right],\quad \beta_i=\frac{e^2a_i^2}{8}\left[\frac{1}{(k_ip')}-\frac{1}{(k_ip)}\right],$$
$$f_i(\varphi_i)=\alpha_i\sin(\varphi_i+\delta_i)+\beta_i\sin 2(\varphi_i+\delta_i),\quad \delta_1=0,\quad \delta_2=\varphi_0. \tag{22}$$

Notice also that in the present case the quasimomentum

$$q_\mu=p_\mu+\frac{e^2a_1^2}{4(k_1p)}k_{1,\mu}+\frac{e^2a_2^2}{4(k_2p)}k_{2,\mu}$$

and the effective mass

$$m_*=m\left[1+\frac{1}{2}\left(\frac{ea_1}{m}\right)^2+\frac{1}{2}\left(\frac{ea_2}{m}\right)^2\right]^{1/2}.$$

The functions A_n and B_n are connected by relations of the type

$$(s_i-2\beta_i)K_0^i+\alpha_iK_1^i+4\beta_iK_2^i=0. \tag{23}$$

Here $K_j^1 = A_j$ and $K_j^2 = B_j$.

Using (21)–(23), and introducing the notation $e'' = e' - k'(k_ie')/(k_ik')$, we can write the matrix element (5) for our case in the form

$$M_{if}=-ie\frac{1}{(2p_0'\cdot 2p_0\cdot 2k_0')^{1/2}}\sum_{s_1s_2}\bar{u}(p')\left\{\hat{e}''C_{00}+e\left[\frac{\hat{a}_1\hat{k}_1\hat{e}''}{2(k_1p')}+\frac{\hat{e}''\hat{k}_1\hat{a}_1}{2(k_1p)}\right]C_{10}\right.$$
$$\left.+e\left[\frac{\hat{a}_2\hat{k}_2\hat{e}''}{2(k_2p')}+\frac{\hat{e}''\hat{k}_2\hat{a}_2}{2(k_2p)}\right]C_{01}\right\}u(p)(2\pi)^4\delta(s_1k_1+s_2k_2+q-q'-k'). \tag{24}$$

We have introduced the notation $C_{ij} = A_iB_j$. The sum over s_1 and s_2 extends to both negative and positive whole numbers.

Upon squaring (24) there arises, besides the double sum over s_1 and s_2, another double sum over s_1' and s_2'. However, since we assume that k_1 and k_2 are incommensurable, we obtain on the basis of the conservation laws that $s_1' = s_1$ and $s_2' = s_2$, and the summation remains a double summation. As a result of the definition (21), the functions B_n depend on φ_0 only through the exponential factor $e^{is_2\varphi_2}$; therefore, on account of the equality $s_2 = s_2'$, the dependence of the probabilities of the processes on φ_0 vanishes, and we shall assume that $\varphi_0 = 0$. The functions B_i are in this case real. Squaring (24) and summing over the polarizations of the electrons, we obtain

$$\sum_{r,r'}\frac{|M_{if}|^2}{VT}=\frac{e^2}{2q_0'q_0k_0'}\sum_{s_1s_2}N(s_1s_2)(2\pi)^4\delta(s_1k_1+s_2k_2+q-q'-k'). \tag{25}$$

Here

$$N(s_1s_2)=C_{00}^2[2(pe'')^2+(pp')-m^2]+C_{10}C_{00}[\alpha_1(k_1k')-4e(pe'')(a_1e'')]$$
$$+C_{01}C_{00}[\alpha_2(k_2k')-4e(pe'')(a_2e'')]+C_{10}^2e^2\left[2(a_1e'')^2+\frac{a_1^2(k_1k')^2}{2(pk_1)(p'k_1)}\right]$$
$$+C_{01}^2e^2\left[2(a_2e'')^2+\frac{a_2^2(k_2k')^2}{2(pk_2)(p'k_2)}\right]+4C_{01}C_{10}e^2(a_1e'')(a_2e''). \tag{26}$$

If we are not interested in the polarization of the emitted γ quantum, then, summing over the polarizations, we obtain

$$N(s_1s_2)=-2m^2C_{00}^2+a_1^2e^2\left[2+\frac{(k_1k')^2}{(k_1p)(k_1p')}\right](C_{10}^2-C_{00}C_{20})$$
$$+a_2^2e^2\left[2+\frac{(k_2k')^2}{(k_2p)(k_2p')}\right](C_{01}^2-C_{00}C_{02}). \tag{27}$$

For the photon-emission probability we have

$$\overline{W}_\gamma=W_q\sum_{s_1s_2}\int_0^{2\pi}d\varphi'\int_0^{u_s}\frac{du}{(1+u)^2}m^{-2}N(s_1s_2). \tag{28}$$

Here

$$u=\frac{(k_ik')}{(k_iq')},\quad u_s=\frac{2s_1\chi_1m^2}{x_1m_*^2}+\frac{2s_2\chi_2m^2}{x_2m_*^2},$$
$$\chi_i=\frac{(k_ip)}{m^2}x_i,\quad x_i=\frac{ea_i}{m}. \tag{29}$$

The summation over s_1 and s_2 at fixed values of the remaining invariants is carried out over the region admissible on the basis of the condition $u_s > 0$. In contrast to the case of one monochromatic wave, the processes for which s_1 and s_2 have different signs—which corresponds to absorption from one wave, and emission into the other, of a definite number of quanta—also contribute to the probability (28).

The remaining invariants entering into the expression for $N(s_1 s_2)$ have the form

$$\beta_i = u x_i^3/8\chi_i, \quad \alpha_1 = -Z_1 \cos\varphi',$$
$$\alpha_2 = -Z_2 \sin\varphi', \quad Z_i = \frac{x_i^2}{\chi_i}\sqrt{u}\left[\frac{2s_1\chi_1}{x_1} + \frac{2s_2\chi_2}{x_2} - u\frac{m_*^2}{m^2}\right]^{1/2}. \quad (30)$$

We have chosen the x axis in the direction of the vector $\mathbf{a}_1$, the y axis in the direction of $\mathbf{a}_2$.

Similarly, for pair production, after averaging over the polarizations of the incident photon, we have

$$\bar{N}(s_1 s_2) = 2m^2 C_{00}^2 - a_1^2 e^2\left[2 - \frac{(k_1 l)^2}{(k_1 p)(k_1 p')}\right](C_{10}^2 - C_{00}C_{20})$$
$$- a_2^2 e^2\left[2 - \frac{(k_2 l)^2}{(k_2 p)(k_2 p')}\right](C_{01}^2 - C_{00}C_{02}). \quad (31)$$

For the total probability we have

$$\bar{W}_p = W_l \sum_{s_1 s_2} \int_0^{2\pi} d\varphi' \int_1^{u_{max}} \frac{du}{u[u(u-1)]^{1/2}} \frac{\bar{N}(s_1 s_2)}{m^2}, \quad (32)$$

where

$$u = \frac{(k_i l)^2}{4(k_i p)(k_i p')},$$
$$u_{max} = \frac{s_1\chi_1/x_1 + s_2\chi_2/x_2}{2(1 + x_1^2/2 + x_2^2/2)}. \quad (33)$$

The values of α_i are also formally given in this case by the formula (30), but for β_i and Z_i we have

$$\beta_i = u x_i^3/2\chi_i,$$
$$Z_i = \frac{4x_i^2 u}{\chi_i}\left\{\left(1 + \frac{x_1^2}{2} + \frac{x_2^2}{2}\right)\left[\frac{u_{max}}{u} - 1\right]\right\}^{1/2}. \quad (34)$$

The s_i summation in (32) is performed in the following fashion. For a fixed value of one of the s_i's, say $s_1 = \bar{s}_1$, we find y from the equality

$$\frac{\bar{s}_1\chi_1}{x_1} + \frac{y\chi_2}{x_2} = 2\left(1 + \frac{x_1^2}{2} + \frac{x_2^2}{2}\right). \quad (35)$$

Let s_2^{min} be the whole number nearest to, and greater than, y. Then the s_2 summation (for a fixed $s_1 = \bar{s}_1$) is carried out over the values $s_2 \geq \bar{s}_2^{min}$ and the $\bar{s}_1$ summation is performed over all whole numbers.

Using (28)–(32), we computed the probabilities $\bar{W}_\gamma$ and $\bar{W}_p$ as functions of x_1 for some concrete values of the other invariants x_2 and χ_i. The results for $\bar{W}_\gamma$ and $\bar{W}_p$ are respectively shown in Fig. 2, a) and b). As in the single-wave case, $\bar{W}_\gamma$ is a smooth function of x_i, although it grows more slowly with increasing x_i than in the case of one monochromatic wave.

The dependence of $\bar{W}_p$ on x_1 is nonmonotonic, the

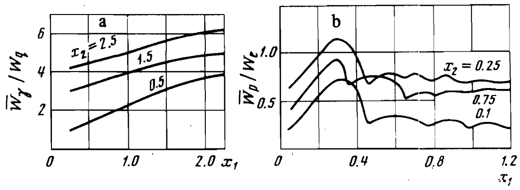

FIG. 2. The probability $\bar{W}_\gamma$ of emission of a photon, (a), and the probability $\bar{W}_p$ for pair production, (b); $\chi_1 = 1$ and $\chi_2 = 1.2$.

reason for this nonmonotony being the same as in the single-wave case, i.e., the existence of a threshold for the pair-production effect. However, the principal maximum of each of the curves in the present case is more smoothed out than in the case of one monochromatic wave.

Let us note another circumstance concerning the nature of the behavior of the curves in Fig. 2 as x_2 is varied. For γ-quantum emission the corresponding probability in the x_1-value interval under consideration increases with increasing x_2. In contrast, the x_2 dependence of the probability for pair production is also nonmonotonic.

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