Gravitational waves that conserve the homogeneity of space

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A physical interpretation is proposed for the cosmological models of (Bianchi) type VII with homogeneous comoving space, in terms of circularly polarized gravitational waves with arbitrary wavelength λ, propagating respectively in a flat nonstationary space for models of type VII_0, or in a space of constant negative curvature in models of type VII_h. The models of type VII_0 describe standing gravitational waves, and those of type VII_h describe traveling gravitational waves. In particular, closed-form solutions are obtained for models of type VII_h having a single traveling gravitational wave.

INTRODUCTION

Below we propose a physical interpretation of cosmological models of Bianchi type VII with homogeneous comoving space. Such models can be considered as circularly polarized gravitational waves in a flat nonstationary space for the models of type VII_0, or in a space of constant negative curvature for the models of type $VIII_h$[1].

The propagation vector **k** of the gravitational wave is along the z axis and plays the role of arbitrary parameter in these models. For k = 0 (infinitely long waves) the metric of the VII_0 model goes over into a metric of type I (with flat comoving space), and the metric of the VII_h model goes over into a metric of Bianchi type V (with homogeneous comoving space of constant negative curvature). For $\mathbf{k} \neq 0$ the model of type VII_0 exhibits only standing waves, whereas in one synchronous comoving frame in the model of type VII_h there are only traveling (progressive) waves.

The possibility of describing homogeneous anisotropic cosmological models as isotropic models on which certain perturbations have been superimposed have been discussed repeatedly[1)] (cf., e.g., [2–4]). In its most general form this was done by Grishchuk, who has advanced the hypothesis that any homogeneous anisotropic model can be represented as a more symmetrical model with perturbations imposed on it (these perturbations are not necessarily small)[5].

A plane polarized gravitational wave of finite wavelength with nodes and antinodes clearly violates spatial homogeneity. The possibility of "inscribing" in a special way a gravitational wave of arbitrary wavelength λ into the homogeneous Friedmann model is related to the helical structure of the translation group (a combination of a displacement along the z axis with a rotation), thus leading to the homogeneous model of Bianchi type VII. (These questions are discussed in more detail in [6,7].) This property refers both to standing waves and to circularly polarized traveling waves. Thus, it is possible to construct anisotropic models which are either symmetric or not with respect to the reflections $z \to -z$.

As was noted before, these considerations are valid for any wavelength, the homogeneity is retained both at the stage $\lambda < t$, when a large number of wavelengths fits into the "horizon," and near a singularity, when $\lambda > t$ (we have set the speed of light and the Einstein constant equal to one). Since λ increases proportionally to the scale of the z axis ($\lambda = 2\pi k^{-1}c(t)$), one can follow the evolution through $\lambda \sim t$, i.e., the formation of the wave as such.

We consider below models filled with radiation (equation of state $p = \epsilon/3$) and nonrelativistic matter (p = 0) interacting with them gravitationally. (The observational data point to a diminishing role of radiation during the later stages of evolution, when the expansion is practically isotropic.) We assume that matter is at rest in a homogeneous synchronous coordinate system. A detailed description of the evolution of such models is given in [8–13]. The interpretation of cosmological models of type VII proposed in this paper allows one to understand better the physical processes which occur during the evolution of such models.

I. THE "FLAT" MODEL (TYPE VII_0)

1. Metric and Equations

The metric has the form (the Greek indices range over 1, 2, 3, Latin indices take on the values 0, 1, 2, 3)

$$ds^2=dt^2-g_{\alpha\beta}dx^\alpha dx^\beta, \quad x^1\equiv x, \quad x^2\equiv y, \quad x^3\equiv z, \tag{1}$$

$$g_{\alpha\beta}=\begin{pmatrix} a^2(\operatorname{ch}\mu+\operatorname{sh}\mu\cos kz) & a^2\operatorname{sh}\mu\sin kz & 0 \\ a^2\operatorname{sh}\mu\sin kz & a^2(\operatorname{ch}\mu-\operatorname{sh}\mu\cos kz) & 0 \\ 0 & 0 & c^2 \end{pmatrix}, \tag{2}$$

where a, c, and μ are functions of the time t (a and c are determined up to constant factors) $\gamma = \det g_{\alpha\beta} = a^4c^2$; the propagation vector **k** is directed along the z axis and is an arbitrary constant parameter, $k \geq 0$ (a change of sign of k in (2) corresponds to a change of polarization). We separate the metric (1), (2) into two parts:

$$g^{(0)}_{\alpha\beta}=\begin{pmatrix} a^2\operatorname{ch}\mu, & 0 & 0 \\ 0 & a^2\operatorname{ch}\mu & 0 \\ 0 & 0 & c^2 \end{pmatrix}, \quad h_{\alpha\beta}=a^2\operatorname{sh}\mu\begin{pmatrix} \cos kz & \sin kz & 0 \\ \sin kz & -\cos kz & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{3}$$

$$h_\alpha^\alpha=0, \quad h^\beta_{\alpha;\beta}=0, \quad h_{\alpha\beta;\gamma}^{\ \ \ ;\gamma}=-k^2h_{\alpha\beta}$$

(here the covariant differentiation is carried out by means of the metric tensor $g^{(0)}_{\alpha\beta}/c^2$).

For μ = 0 the three-dimensional cross section (slice) t = const is flat and the metric (1), (2) describes an axially symmetric model of type I (or type VII_0; $a \equiv c$ corresponds to the flat Friedmann model). In the case $\mu \ll 1$, $a \simeq c$ we may consider the tensor $h_{\alpha\beta}$ as a perturbation superimposed on the quasi-Euclidean Friedmann model (with metric $g^{(0)}_{\alpha\beta}$), the perturbation being a plane standing gravitational wave[14] [2)]. We refer to the tensor $h_{\alpha\beta}$ as a gravitational wave even in the case when $h_{\alpha\beta}$ is not small compared to $g^{(0)}_{\alpha\beta}$. The wavelength of the gravitational wave is $\lambda = 2\pi c/k$. The numerical value of k is not meaningful, because of the possibility of a scale transformation of the z axis. Only the ratio $k/c(t) = \omega = 2\pi/\lambda$, having the dimension $\sec^{-1}$, has a physical meaning. The quantity ω^{-1} should be compared with the characteristic time of changes in the metric ($a/\dot{a}$; cf. Eq. (5)), which in this case is the cosmological time t. The inequality

$$\lambda < a/\dot{a} \sim t \tag{4}$$

is obviously a necessary and sufficient condition for the validity of the adiabatic approximation.

Figure 1 shows schematically the relative motion of the particles (on which the coordinate system (1) is constructed) in the z = 0 plane ($\dot{\mu} > 0$; the dot denotes differentiation with respect to t). The same motion of the particles occurs in any plane z = const at a given instant, but the whole picture is turned through an angle ±kz (the signs correspond to the two polarizations). Such a configuration does not create inhomogeneous perturbations of the density, deformation (strain) velocities, etc. and corresponds to circular polarization. For such a wave we know the complete solution (cf. [10,12,13]). The Einstein equations are of the form[3)]

$$\ddot{\mu}+\dot{\mu}\left(2\frac{\dot{a}}{a}+\frac{\dot{c}}{c}\right)+\omega^2\frac{\operatorname{sh}2\mu}{2}=0, \tag{5}$$

$$\frac{\ddot{a}}{a}+\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a}+\frac{\dot{c}}{c}\right)=\frac{\varepsilon-p}{2},$$
$$\frac{\ddot{c}}{c}+2\frac{\dot{c}}{c}\frac{\dot{a}}{a}=\frac{\varepsilon-p}{2}+\frac{1}{2}\omega^2\operatorname{sh}^2\mu, \tag{6}$$

$$\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a}+2\frac{\dot{c}}{c}\right)=\varepsilon+\frac{1}{4}(\dot{\mu}^2+\omega^2\operatorname{sh}^2\mu). \tag{7}$$

The amplitude μ of the gravitational wave, determined by the wave equation (5) depends on the expansion of the Universe in the x, y plane, which is perpendicular to the propagation vector **k**; when transcribed in terms of the time ζ, the wave equation does not contain the function c(t):

$$\mu''+2\frac{a'}{a}\mu'+\frac{1}{2}\operatorname{sh}2\mu=0,$$
$$(')=\frac{d}{d\zeta}=\omega^{-1}\frac{d}{dt}$$

We denote by $\mu_{max}(t) > 0$ the amplitude of the oscillations of the function $\mu(t)$; $|\mu| \le \mu_{max}$. The following assertions are true about the function $\mu_{max}(t)$ (cf., e.g.,[13]): a) $\mu_{max}(t)$ decreases monotonically as t increases, going to zero as $t \to \infty$; b) $\mu_{max} \lesssim 1$ if $\lambda < t$ and $\lambda \gtrsim t$ if $\mu_{max} > 1$. Equations (6) and (7) describe the evolution of the scale a(t) in the x, y plane and of the scale c(t) in the z direction; (7) is a first integral of the (5) and (6).

2. Properties of the Solution

Figure 2 illustrates the most characteristic case of isotropization of the cosmological model with a gravitational wave during the expansion from the singularity

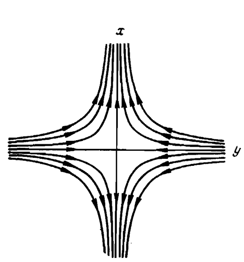

FIG. 1

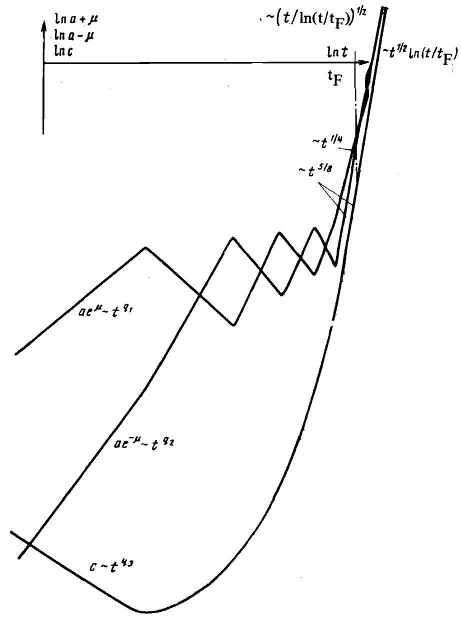

FIG. 2

at t = 0. Considering a reversed flow of time, i.e., a contraction of a Friedmann Universe with a weak ($\mu \ll 1$) standing gravitational wave (in the z direction), one can give the following physical interpretation of the properties of the solution. The "energy" of the wave (cf. (7), (11)) increases faster during the contraction than the energy of ordinary matter ($p \le \epsilon/3$), leading to the "vacuum" stage; the "pressure" of the gravitational wave in the z direction (cf. (6), (11)) becomes so large that the compression in the z direction is replaced by an expansion and the model goes over into the Kasner asymptotic behavior. (The comoving volume $\gamma^{1/2}$ tends monotonically to zero as $t \to 0$.) One can, however, select the phase of the gravitational wave in a special way so that the influence of matter gravitation will be essential up to t = 0 (there is no "vacuum" stage). In this case two types of asymptotic behaviors are realized as $t \to 0$:

$$\varepsilon=\frac{3}{4t^2},\quad \mu=\text{const},\quad a\sim c\sim\sqrt{t} \tag{8}$$

a quasiisotropic start of the expansion, and

$$\varepsilon=\frac{21}{32t^2},\quad \mu=\ln\frac{c\sqrt{3}}{4t},\quad a\sim t^{7/16},\quad c\sim t^{5/8}. \tag{9}$$

We again return to the expanding cosmological model. In the most general case matter starts affecting the evolution already during the sharply anisotropic stage of expansion, when the wave amplitude is large ($\mu \gg 1$)[10]. In this case the instant of isotropization t_F is preceded by a special stage of expansion [the stage of "damping" of the anisotropy of strain (deformation) (9)], during which both the gravitation of the isotropically distributed matter and the curvature terms ("energy and anisotropic pressure" of the gravitational wave) are important[10]. For arbitrary initial conditions the length of this intermediate stage turns out automatically to be such that at the instant of isotropization ($t \sim t_F$) the wavelength of the gravitational wave is of the order of the horizon $\lambda \sim t$, and the amplitude is $\mu \sim 1$. (At the time t_F the anisotropy of the strain tensor is of the order of one. The horizon is defined as the distance traveled by light from the genuine singularity to the time t, viz., $c(t)\int_0^t dt/c(t)$.) This case of isotropization is the most general one[4)]. Thus, within the framework of models with homogeneous comoving space it is possible to solve the program of "chaotic cosmology"[15].

A strongly anisotropic beginning of the expansion ("vacuum" stage) is related to the presence of large-amplitude gravitational waves $\mu_{max} \gg 1$.

The presence of isotropically distributed matter leads to an interaction (via gravitation) with the gravitational waves (the intermediate "damping" stage), as a result of which there occurs isotropization[10] (at the instant $t \sim t_F$ we have $\lambda \sim t$ and $\mu \sim 1$).

If the wavelength is shorter than the horizon, $\lambda < t$, the amplitude of the gravitational wave is small $\mu_{max} < 1$, and for the determination of the function $\mu(t)$ it suffices to make use of the adiabatic WKB approximation[10,12]:

$$\mu \approx \mu_{max} \sin \int \omega\, dt, \quad \mu_{max}(t) \approx \frac{\text{const}}{a(t)} < 1. \tag{10}$$

We stress the fact that this equation is valid during any stage of expansion (including the "vacuum" stage) and assumes only the condition (4).

In the model of Bianchi type VII_0 (cf. (1)) a standing gravitational wave can be represented as a sum of two traveling waves propagating in opposite directions and having the same amplitude. In the case $\mu < 1$ one may neglect the mutual interaction of these waves: in the leading approximation the waves propagate freely with the speed of light (cf. (10)). Moreover, in the determination of the functions a(t) and c(t) in the curvature terms of the Einstein equations one may average all oscillating terms over a period, since the condition $\lambda < t$ means that the period of oscillations is smaller than the cosmological time (cf. Eq. (4)). We thus arrive at the conclusion that on the cosmological level such a plane gravitational wave ($\lambda < t$) is physically equivalent to two opposite fluxes of free particles of zero rest mass in the axially symmetric model of Bianchi type I (VII_0) (cf.[16-20]) and is described by an energy-momentum tensor

$$T_0^0 = -T_3^3 = \varepsilon_w = p_{zw} = \frac{\omega^2}{2}\overline{\mu^2} = \left(\frac{\omega \mu_{max}}{2}\right)^2, \quad -T_1^1 = -T_2^2 = p_x = p_y = 0, \tag{11}$$

where $\mu_{max}(t)$ is determined from (10).

The corresponding cosmological solution for the model of type VII_0 was determined in [12] (independently, a solution was obtained in [19] for two opposite fluxes of free particles in a model of type I filled with radiation, $p = \epsilon/3$). The most important distinction of this solution is the fact that during the isotropic expansion stage ($t > t_F$), which is determined by the gravitation of the isotropically distributed matter, an extremely important role is played by the terms of second order in the wave amplitude μ, i.e., by the anisotropic pressure of the gravitational wave (cf. (11)). This leads to the result that as time increases the cosmological expansion very slowly approaches an isotropic one (compared, in particular, with models which only take into account the first-order perturbation[14,21,22]). Thus, for the $p = \epsilon/3$ model filled with radiation the solution has the form (cf. [12,19])

$$\varepsilon = 3p = \frac{6A^2}{\gamma}\nu\eta e^{\nu^2}, \quad \varepsilon_w = p_{zw} = \frac{6A^2}{\gamma}\nu^2\eta^2,$$
$$\eta = \int_1 \frac{e^{\nu^2}}{\nu^2}\, d\nu + \eta_0, \quad \dot\nu = \frac{A}{\sqrt{\gamma}}\nu\eta, \tag{12}$$

where A and η_0 are constants. During the isotropic expansion stage (here one can neglect the constant η_0 compared to the integral in (12)), the strain anisotropy decays logarithmically:

$$\gamma^{1/3} \sim t\left(1 + \frac{\alpha_1}{2} + \alpha_2\right), \quad \frac{\varepsilon_w}{\varepsilon} \approx \alpha_1(1 + 3\alpha_1 + 2\alpha_2) \sim \left(\frac{a}{c}\right)^{2/6}, \tag{13}$$

where $\alpha_1 \simeq 1/\ln(t/t_\lambda)$ and $\alpha_2 \simeq (t_F/t)^{1/2}$; here t_λ ($\lesssim t_F$) denotes the time when $\lambda \sim t$ (in estimating the function $\alpha_1(t)$ it was assumed that for $t - t_\lambda$ the quantity μ is of the order $\mu \sim 1$; more accurate estimates can be found in [10,12]). The correction $\alpha_2(t)$ can be calculated in first order of perturbation theory. The logarithmic contribution to the volume $\alpha_1(t)$ which appears in the isotropic stage in the second order in the amplitude $\mu \ll 1$ takes into account the self-energy of the gravitational waves, averaged with respect to the scale inside the horizon. In the most general case $t_\lambda \sim t_F$ and

$$t/t_\lambda \sim t/t_F \sim (t/\lambda)^2, \tag{14}$$

i.e., the logarithm manifests itself only under the condition $\lambda \ll t$. In the special case when $t_\lambda \ll t_F$ (at the instant $t \sim t_F$ one has $\lambda \ll t$, cf. footnote 4), then $\ln(t_F/t_\lambda) \gg 1$ and one may neglect the variation of the logarithm during the isotropic stage (up to the instant of recombination). In this case the anisotropy decays according to a power law and the contribution of the gravitational wave reduces to the contribution to the general density of matter.

II. THE "OPEN" MODEL (TYPE VII_h)

1. Metric and Equations

In the "open" model of type VII in addition to the scale there is another characteristic scale, the magnitude of the general (Milne) curvature c(t), z = 1:

$$g_{\alpha\beta} = \begin{pmatrix} a^2e^{2z}[\operatorname{ch}\mu + \operatorname{sh}\mu\cos(kz+\tilde\varphi)], & a^2e^{2z}\operatorname{sh}\mu\sin(kz+\tilde\varphi), & 0 \\ a^2e^{2z}\operatorname{sh}\mu\sin(kz+\tilde\varphi), & a^2e^{2z}[\operatorname{ch}\mu - \operatorname{sh}\mu\cos(kz+\tilde\varphi)], & 0 \\ 0 & 0 & c^2 \end{pmatrix}; \tag{15}$$

a, c, μ, $\tilde\phi$ are functions of the time t (the function a is defined up to a constant multiple, cf. (1)). As in the "flat" model, in the "open" model of type VII the metric tensor splits into two parts, one of which, $g_{ik}^{(0)}$, describes on the slice t = const ($g_{\alpha\beta}^{(0)}$) a space of constant negative curvature (cf. (16)), and the other part h_{ik} ($h_{\alpha;\beta}^{\beta} = 0$ on the pseudosphere $g_{\alpha\beta}^{(0)}$) satisfies in the linear approximation the wave equation in the open nonstationary Friedmann model ($g_{ik}^{(0)}$). Indeed, in the case when $\mu = 0$, the three-dimensional slice t = const is a space of constant negative curvature and the metric (1), (15), describes an open Friedmann model ($a \equiv c$, cf. (21))[13]:

$$dl^{(0)2} = dz^2 + e^{2z}(dr^2 + r^2 d\varphi^2) = d\chi^2 + \operatorname{sh}^2\chi(d\theta^2 + \sin^2\theta\, d\varphi^2), \tag{16}$$

here $r^2 = (x^2 + y^2)(a/c)_{t = \text{const}}$, and χ, θ, and ϕ are the Friedmann coordinates. The transformation formulas have the form

$$e^z = \operatorname{ch}\chi + \operatorname{sh}\chi\cos\theta, \quad r = \operatorname{sh}\chi\sin\theta/(\operatorname{ch}\chi + \operatorname{sh}\chi\cos\theta) \tag{17}$$

(accurate to the transformation $\theta \to \pi - \theta$, which changes the sign of $\cos\theta$).

The general metric (15) can be interpreted as two monochromatic circularly polarizes gravitational waves propagating in opposite directions (in the directions $\pm z$) in an open nonstationary Friedmann Universe (the propagation vector $\pm\mathbf{k}$ determines the wavelength $\lambda = 2\pi c(t)/k$ in terms of the curvature "radius" c(t)).

However, one may no longer regard these waves as plane waves on a scale larger than the curvature radius, $\chi \gtrsim 1$. The constant-phase surfaces (wave front, z = const.) form nested rotation paraboloids (in the Friedmann space with radial coordinate sinh χ; cf. Fig. 3: the hyperbolas are the lines r = const which are tangent to the vector **k**). Therefore a wave which propagates as time increases in the positive +z direction ("outgoing" wave) will propagate along a series of diverging paraboloids and will be damped owing to dilution (in addition to the damping in the expanding Universe; cf. Sec. I.2); the wave which propagates in the opposite direction −z ("incoming" wave), is amplified (compared to the adiabatic damping law obtained by E. M. Lifshitz for weak waves[14])[5)].

During the expansion stage $t < t_M$, when the horizon is smaller than the radius of curvature ($t < c, \chi, z < 1$), the wave front is weakly curved on the scale of the horizon (i.e., practically coincides with a plane wave front) and therefore one may neglect the dilution factor so that the amplitudes of both waves are damped in the same manner.

During the Milne stage of the expansion, $t > t_M$ ($c \sim t$), when the horizon becomes larger than the radius of curvature, the curving of the wave front within the horizon becomes essential. Under these conditions the "incoming" wave is amplified (in comparison with the "outgoing" wave), i.e., in the comoving space there appears a flux of gravitational energy in the z direction. One must stress the fact that the amplitudes of the two traveling waves change in a different manner just on account of the curvature of the three-dimensional space t = const. (cf. (16), (17)) in the VII_h model. In the "flat" model (of type VII_0) the surfaces of constant phase are the surfaces z = const. and the amplitudes of both waves are damped identically. Since in the model of type VII_0 the amplitudes of both waves are equal ($\tilde{\varphi} \equiv 0$, cf. Eq. (2)), the two waves propagating in opposite directions are equivalent to one standing wave and no directed fluxes of energy appear in the comoving space. As was already stressed, in a model of type VII_h during the interval of evolution during which the gravitational wave (which propagates with the speed of light, cf. (22), (26)) passes through a distance smaller than the curvature radius c(t), one may neglect the curvature of the wave front and the amplitudes of both waves are damped identically (as in the model of type VII_0). However, in an "open" model (in distinction from the "flat" model) the amplitudes (μ_1 and μ_2) of the two waves can be in fact prescribed independently of each other (for more detail, cf. (28), (30), (31)) and therefore in this model traveling gravitational waves ($\mu_1 \lessgtr \mu_2$) are possible during the Friedmann portion of the expansion with the critical density $\epsilon \simeq \epsilon_{cr} = \frac{1}{3}(\ln\sqrt{\gamma})^2$.

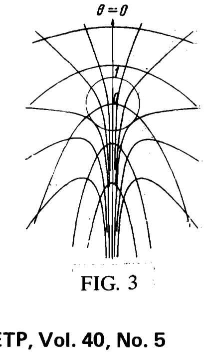

FIG. 3

The Einstein equations take the form

$$\ddot{\mu}+\dot{\mu}\left(2\frac{\dot{a}}{a}+\frac{\dot{c}}{c}\right)+(\omega^2-\dot{\tilde{\varphi}}^2)\frac{\operatorname{sh}2\mu}{2}=0,$$
$$\frac{1}{\sqrt{\gamma}}(\dot{\tilde{\varphi}}\sqrt{\gamma}\operatorname{sh}^2\mu)^{\cdot}=2\frac{\omega}{c}\operatorname{sh}^2\mu; \quad (18)$$

$$\frac{\ddot{a}}{a}+\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a}+\frac{\dot{c}}{c}\right)-\frac{2}{c^2}=\frac{\varepsilon-p}{2},$$
$$\frac{\ddot{c}}{c}+2\frac{\dot{c}}{c}\frac{\dot{a}}{a}-\frac{2}{c^2}=\frac{\varepsilon-p}{2}+\frac{1}{2}\omega^2\operatorname{sh}^2\mu; \quad (19)$$

$$\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a}+2\frac{\dot{c}}{c}\right)-\frac{3}{c^2}=\varepsilon+\frac{1}{4}[\dot{\mu}^2+(\dot{\tilde{\varphi}}+\omega^2)\operatorname{sh}^2\mu]; \quad (20)$$

$$\left(\ln\frac{c}{a}\right)^{\cdot}=\frac{k}{4}\dot{\tilde{\varphi}}\operatorname{sh}^2\mu. \quad (21)$$

(Formally, Eqs. (19)−(21) describe the evolution of an axially symmetric model of type V in a synchronous homogeneous coordinate system (1)

$$ds^2=dt^2-c^2dz^2-a^2e^{2z}(dx^2+dy^2)$$

with an energy-momentum tensor T_i^k:

$$T_0^{\ 0}=-T_3^{\ 3}={}^1/_4[\dot{\mu}^2+(\dot{\tilde{\varphi}}^2+\omega^2)\operatorname{sh}^2\mu],$$
$$-T_1^{\ 1}=-T_2^{\ 2}={}^1/_4[\dot{\mu}^2+(\dot{\tilde{\varphi}}^2-\omega^2)\operatorname{sh}^2\mu], \quad T_3^{\ 0}=\frac{k}{2}e^z\dot{\tilde{\varphi}}\operatorname{sh}^2\mu,$$

where μ and $\dot{\tilde{\phi}}$ are functions of time and satisfy the equations of "motion" $T^k_{i;k} = 0$, cf. (18); T_i^k goes over into (29) at $\lambda < t$.)

On account of the transversality of the gravitational waves, their amplitudes (determined by the functions μ and $\tilde{\phi}$) decrease owing to the expansion of the Universe in the plane x, y, of the wave front, cf. (18). The increase of the amplitude owing to dilution in the "incoming" wave does not exceed the corresponding decrease in the expanding Universe, so that the amplitude of the "incoming" wave does not increase. Regarding the function $\mu_{max}(t)$ the same assertions are valid as in the model of type VII_0, with the exception of the limiting asymptotic behavior[6)]: $\sinh\mu \to \text{const} < 2k^{-1}$ as $t \to \infty$, cf. (36). Equations (19) describe the evolution of the functions a(t) and c(t); Eqs. (20) and (21) are first integrals of (18) and (19). We call the stage of evolution in which the expansion of the Universe in the x, y plane (scale a(t)) is determined by the general (Milne) curvature $2/c^2$ the Milne stage (cf. the first equation of (19)): $a \sim e^{\xi}$, $d\xi = dt/c$, $(') = d/d\xi$. From the inequality $(a'/a)' + 2(a'/a)^2 \geq 2$ one obtains easily an estimate for the curvature radius c(t):

$$c \geqslant a/\dot{a} \sim t,$$

from where it follows that in the wave zone one can only find waves of large wave number (which is physically obvious): if $\lambda < t$ then $k > 1$; if $k < 1$, then $\lambda > t$.

2. Properties of the Solution

First of all there arises the question: are there solutions in the open model of type VII for one "incoming" or for one "outgoing" wave? We look for solutions for which

$$\tilde{\varphi}=\pm\int\omega\,dt, \quad \omega=\frac{2\pi}{\lambda}=\frac{k}{c(t)}. \quad (22)$$

It is easy to show that such solutions are possible only in the empty model. The only possible solutions for solitary "incoming" or "outgoing" waves can be integrated in closed form.

1) A solitary "incoming" traveling gravitational wave:

$$a = At^{1/(1+\varkappa^2)}, \quad c = (1+\varkappa^2)t,$$
$$\tilde{\varphi} = \frac{k \ln t}{1+\varkappa^2} + \tilde{\varphi}_0, \quad \varkappa = \frac{k}{2} \operatorname{sh} \mu = \text{const}; \tag{23}$$

where A and $\tilde{\phi}_0$ are constants.

2) A solitary "outgoing" traveling gravitational wave:

$$a^2 = A \operatorname{sh} 2\xi, \quad c^2 = Be^{11/4\xi}/\operatorname{sh}^{3/4} 2\xi; \tag{24}$$

$$\tilde{\varphi} = -\sqrt{11}\,\xi + \tilde{\varphi}_0, \quad \mu = \frac{1}{2} \ln \operatorname{cth} \xi, \quad \xi = \int_0^t \frac{dt}{c}; \tag{25}$$

where A, B, $\tilde{\phi}_0$ are constants and ξ is the horizon (in the Lagrangian coordinate z).

A solitary "outgoing" wave is possible only with the wave number $k = \sqrt{11}$. For $\xi = 1$ the solution (24), (25) describes a Kasner asymptotic behavior with exponents (12/13, 4/13, −3/13). At $\xi > 1$ the solution (24), (25) goes over into the Milne asymptotic behavior: $a = c = t$ (the constant A has no physical meaning and is related to the choice of scale in the x, y plane).

In the case of a solitary "incoming" wave the Milne solution is realized up to t = 0. The wave traverses an infinite distance as $t \to 0$ (in the Milne solution there is no horizon in the z direction), and an increase of the wave amplitude in the contracting Universe ($t \to 0$) is exactly compensated by the dilution factor (the divergence of the front as $t \to 0$). It is easy to show that the converse also holds: in the case of a Milne singularity (or in the absence of a horizon as $t \to 0$), only an "incoming" wave is possible.

In all other situations (in particular, in models with matter) both waves are present (except, of course, the trivial case of a Friedmann Universe, $\mu \equiv 0$, when there are no waves). As $t \to 0$ the solution goes into its Kasner asymptotic behavior, since the amplitude of the "outgoing" wave increases to infinity in a Universe which undergoes unlimited contraction (exceptions are cases of special focusing of the gravitational waves as $t \to 0$, when the asymptotic behaviors (8) and (9) are realized and the initial amplitudes of both waves are equal, $0 < \dot{\tilde{\phi}}/\omega \to 0$). It is just the presence of the "outgoing" wave which is responsible for the Kasner asymptotic behavior at $t \to 0$. Conversely, in the case of a Kasner asymptotic behavior the "initial amplitude" of the "outgoing" wave is always larger than the amplitude of the "incoming" wave ($\dot{\tilde{\phi}} < 0$ as $t \to 0$). The Kasner parameters (and the "number" k of the harmonic) relate the "initial amplitudes" of the waves (both waves traverse a finite path for $t \to 0$, the phase $\tilde{\phi}$ is finite); cf. (20), (21). As $t \to \infty$, only one "incoming" wave remains (since the "outgoing wave" is damped out), so that Eqs. (23) describe the asymptotic behavior of any solution of Eqs. (18)–(21) for the cosmological expansion (cf. also (36)).

A prolonged isotropic Friedmann stage of expansion with $\epsilon \simeq \epsilon_{cr}$ is possible only if $\lambda \ll t$, i.e., $k \gg 1$ (for $k \lesssim 1$ the isotropization is possible only for a special choice of phases and amplitudes for the gravitational waves, when a quasiisotropic expansion is realized: $\sinh \mu \simeq \text{const} \ll k^{-1}$). The solution of Eqs. (18) in the wave zone (we recall that in this case $\mu < 1$) can be determined in the WKB approximation[7]:

$$\sqrt{\gamma}\,\omega\mu^2 \approx k\left(\alpha^2 e^{2\xi} + \beta^2 e^{-2\xi} + 2\alpha\beta \cos 2\int \omega\, dt\right),$$

$$\sqrt{\gamma}\,\dot{\tilde{\varphi}}\mu^2 \approx k\alpha^2 e^{2\xi} - k\beta^2 e^{-2\xi} + 2\alpha\beta \sin 2\int \omega\, dt, \tag{26}$$

where α and β are constants.

Substituting (26) in (15) we obtain

$$\mu \exp\{i(kz+\tilde{\varphi})\} = \mu_1 \exp\left\{i\left(kz + \int \omega\, dt\right)\right\} + \mu_2 \exp\left\{i\left(kz - \int \omega\, dt\right)\right\}, \tag{27}$$

where

$$\mu_1 = \frac{\alpha}{a} e^{\xi}, \quad \mu_2 = \frac{\beta}{a} e^{-\xi}. \tag{28}$$

Thus, in the leading approximation the small-amplitude waves do not interact with one another and propagate freely with the speed of light. The quantities α and β are adiabatic invariants and determine the amplitudes of the "incoming" (μ_1) and "outgoing" (μ_2) waves ($\alpha < \beta$). The instant $\xi \sim 1$ corresponds to the transition to the Milne stage of evolution; before this time ($\xi < 1$) the amplitudes of both waves are equally damped; in the Milne stage ($\xi > 1$, $a \sim e^{\xi}$) the amplitude of the "incoming" wave becomes constant: $\mu_1 \sim$ const, and $\mu_2 \sim 1/t^2$, according to (23)–(25). On the cosmological level the influence of such gravitational waves on the evolution ($\lambda < t$) reduces to an equivalent action of the energy-momentum tensor in an axially-symmetric model of type V (VII_h) cf. (16)):

$$T_0^0 = -T_3^3 = \varepsilon_w = p_{zw} = \frac{\omega^2}{2}\overline{\mu^2} = \frac{k\omega}{2\sqrt{\gamma}}(\alpha^2 e^{2\xi} + \beta^2 e^{-2\xi}), \quad p_x = p_y = 0,$$
$$T_3^0 = -cF_w = \frac{k}{2} e^z \overline{\dot{\tilde{\varphi}}\mu^2} = \frac{k^2}{2\sqrt{\gamma}}(\alpha^2 e^{2\xi} - \beta^2 e^{-2\xi})\, e^z. \tag{29}$$

It is easy to verify that the energy-momentum tensor (29) corresponds exactly to two colliding fluxes of free particles in an axially symmetric model of type V (or VII_h). The corresponding cosmological solutions with the energy-momentum tensor (29) have the following forms:

A) A solution with prevalent "outgoing" wave, $\mu_2 \gg \mu_1$, $\beta \gg \alpha$ (the upper sign in Eqs. (30)–(31)). If $p = \epsilon/3$, then

$$a^2 = A\nu^2 e^{\mp 2\xi}, \quad c^2 = B\frac{\eta^3}{\nu} e^{\mp 8\xi},$$
$$\varepsilon = \frac{12A^2\nu\eta}{\gamma} e^{\mp 4\xi}, \quad \varepsilon_w = p_{zw} = \pm F_w = \frac{6A^2\xi_F\nu^2}{\gamma} e^{\mp 4\xi}, \tag{30}$$

where

$$\eta = \nu \pm \nu^2 \pm \xi_F, \quad \frac{d\nu}{d\xi} = \frac{2\eta}{\nu}, \quad A = \frac{k^2}{12\xi_F}\beta^2(\alpha^2);$$

if, on the other hand, p = 0, then

$$a^2 = A\nu^4 e^{\mp 2\xi}, \quad c^2 = B\frac{\eta^2}{\nu^2} e^{\mp 4\xi},$$
$$\varepsilon = \frac{12A^2\nu^3\eta}{\gamma} e^{\mp 4\xi}, \quad \varepsilon_w = p_{zw} = \pm F_w = \frac{6A^2\xi_F^3\nu^4}{\gamma} e^{\mp 4\xi}, \tag{31}$$

where

$$\eta = \nu^3 \pm \nu^4 \pm \xi_F^3, \quad \frac{d\nu}{d\xi} = \frac{\eta}{\nu^3}, \quad A = \frac{k^2}{12\xi_F^3}\beta^2(\alpha^2);$$

A, B, $\xi_F > 0$ are constants.

A sufficiently lengthy Friedmann stage exists in the case when $\xi_F \ll 1$; in this case ξ_F has the meaning of a horizon at the instant when the Friedmann stage begins. The solution (30), (31) is applicable starting from the instant $\mu \sim 1$ (for this for $\mu > 1$ there exists a Kasner asymptotic behavior with the exponents near the solution (12/13, 4/13, −3/13)), and is valid as long as $\mu_1 < \mu_2$ (cf. (26), (28)). This condition is violated during the Milne stage, after which the evolution is described by the solution with prevalently "incoming" wave.

B) A solution with prevalent "incoming" wave (the lower sign in Eqs. (30), (31)). As in the preceding case the requirement of a sufficiently lengthy isotropic stage leads to the condition $\xi_F \ll 1$; in this case ξ_F is the horizon ξ at the instant when the Friedmann stage begins. Here and in the solutions (30), (31) the normalization of the function ψ differs from (25):

$$\xi=\xi_F+\int_{\xi_F} d\nu \Big/ \frac{d\nu}{d\xi}. \tag{32}$$

For $|\xi| > 1$ ($\eta \to 0$) the solutions (30), (31) describe the Milne asymptotic behavior:

$$c=(1+\varkappa_{\pm\infty}^2)(t+t_{\pm\infty}), \quad a=A_{\pm\infty}(t+t_{\pm\infty})^{1/(1+\varkappa_{\pm\infty}^2)}, \tag{33}$$

where $A_{\pm\infty}$ and $t_{\pm\infty}$ are constants,

$$\varkappa_{\pm\infty}^2=\lim_{\xi\to\pm\infty}\frac{k^2}{4}\operatorname{sh}^2\mu=\lim_{\xi\to\pm\infty}\begin{cases} 3\xi_F/\nu^2, & p=\varepsilon/3. \\ 3\xi_F^3/\nu^4, & p=0 \end{cases} . \tag{34}$$

From (30), (31) follows the obvious restriction on the constant ξ_F:

$$\xi_F<1/4, \quad p=\varepsilon/3, \quad \xi_F<3/2^{2/3}, \quad p=0, \tag{35}$$

hence

$$\begin{aligned} &\varkappa_{-\infty}^2>3, \quad \varkappa_{+\infty}^2<3, \quad p=\varepsilon/3, \\ &\varkappa_{-\infty}^2>1, \quad \varkappa_{+\infty}^2<1, \quad p=0. \end{aligned} \tag{36}$$

The estimates obtained here[8)] have a simple interpretation and follow from the condition of decrease (increase) of the matter density compared to the general curvature for $\xi \to +\infty$ ($\xi \to -\infty$):

$$\frac{d\Omega}{dt}<0, \quad \xi\to\infty\left(\frac{d\Omega}{dt}>0, \quad \xi\to-\infty\right), \quad \Omega=\frac{\varepsilon}{\varepsilon_{cr}}. \tag{37}$$

The solutions (30), (31) are obviously valid during the stage ($\mu > 1$) when $\mu_1 > \mu_2$. The latter condition is violated as $\xi \to -\infty$. Since near the singularity we always have $\mu_2 > \mu_1$ ($\alpha < \beta$), in order that an "incoming" wave should be obtained during the Friedmann stage it is necessary that the gravitation of matter should start influencing the evolution during the Milne stage, when the "incoming" wave separates (in addition it is necessary that the condition $\mu_1 > (1-\sqrt{3})2/k$ be satisfied, cf. (36)).

C) A standing gravitational wave during the Friedmann stage. Since the amplitudes of both waves vary identically during the isotropic Friedmann stage of expansion with $\epsilon \simeq \epsilon_{cr}$ ($\xi < 1$, cf. (28)), one can always choose the initial amplitudes of the waves close to each other, so that during the Friedmann stage a standing wave is formed: $0 < \beta - \alpha \ll \alpha$.

In this case, up to the beginning of the Milne stage of expansion ($\xi \sim 1$), the evolution in the "open" model of type VII_h occurs in the same manner as in the "flat" model of type VII_0 (cf. (11), (14))[10]. (At $\mu_1 \simeq \mu_2$ (26) goes over into (10); the phase accumulation $\Delta\tilde{\varphi} \simeq \pi/2$ near the point $\mu \sim \mu_{min} \ll \mu_{max}$ is equivalent to a change of the sign of the function μ.) To the instant $\xi \sim 1$ we will have $(\mu_1 - \mu_2) \sim \mu_2$ and in the sequel the solution is approximated by the equations (30), (31) for the "incoming" wave.

Thus, during the Milne stage one traveling wave separates from the standing wave, i.e., there appears a flow of energy. Thus, the dipole component of the anisotropy of the microwave background radiation (cf. [13]) could be explained by a Doppler shifting of the frequency owing to the presence of the energy flow in the traveling gravitational wave.

We list some estimates for the energy flux F_W. If the instant of isotropization is of the order of the Planck time, $t_F \sim t_{Pl} \sim 10^{-43}$ sec, then

$$\mu\sim10^{-31}, \quad \lambda\sim1 \text{ cm} \quad \omega\sim10^5 \text{ MHz}, \quad F_\nu\approx0.01-1 \text{ erg/cm}^2\text{/sec}, \tag{38}$$

i.e., the gravitational waves end up in the same waveband as the background radiation. In principle, experiments aiming at the detection of such "relic" gravitational waves can be performed under laboratory conditions[24].

CONCLUSION

We enumerate the main results. In the "flat" and "open" Friedmann models gravitational waves of arbitrary wavelength λ are possible without violation of the spatial homogeneity, the waves having amplitudes constant over the space, (1). (In the "closed" Friedmann model such waves can only have a maximal wavelength compatible with the closed character of the space[2].)

The homogeneity condition determines uniquely the profile of the wavefront (surface of constant phase). In the "flat" Friedmann model standing plane waves are present. In the "open" model the wavefront is curved over a scale of the order of the curvature radius (cf. (16), (17)), therefore the amplitudes of the waves which travel in opposite directions vary differently owing to dilution.

If a standing wave is given near a singularity, a special choice of phase is possible for which the amplitude of the wave remains finite for $t \to 0$, $\mu \to$ const, i.e., there is a quasiisotropic start of the expansion. Only in this case there occurs an isotropization of models with long gravitational waves $k \lesssim 1$. For $k \gg 1$ the models under consideration always become isotropic[9)].

During the period $\lambda < t$ the equations coincide with those obtained for the average values which characterize the coarse-grained metric, if one considers the gravitational waves on the same footing as other types of waves (e.g., electromagnetic waves), described by an energy-momentum tensor[16,17]—cf. (11), (29).

A strongly anisotropic oscillatory expansion regime during the vacuum stage near the singularity is realized in models with standing gravitational waves. During the isotropic Friedmann expansion stage ($\lambda < t$) the second order of perturbations with respect to the wave amplitude $\mu < 1$ is quite important in these models (at the instant of isotropization we have $t \sim t_F \sim \lambda$, $\mu \sim 1$; at the instant when the Milne stage of expansion starts we have $t \sim t_M \sim k^3(t_F^{3/2}/t_c^{1/2})\ln(t_c/t_F)$, $\mu \sim (t_F^{1/2}t_c^{1/6}/t_M^{2/3}) \times \ln^{1/2}(t_c/t_F)$, where t_c is the instant when the equation of state "changes"), leading to a slower isotropization (compared to models in which only the first order of perturbation theory is taken into account[14,21,22]), and this can cause a considerable anisotropy of the background radiation if the isotropization does not occur sufficiently early[10,13]. A slow logarithmic isotropization at the stage $p = \epsilon/3$ (cf. (13)) is related to the fact that homogeneity is compatible only with the presence of one-two gravitational waves in the Friedmann model (the distribution function is anisotropic). If the gravitational waves are present in all directions (such a model will be nonhomogeneous), then at the stage when the distribution function is close to the isotropic one, the anisotropy of expansion decays according to a power law (for $\lambda < t$ the system of gravitational waves is equivalent to a set of free particles of zero rest mass[16-20] (11), (29), and the distribution function of the free particles tends, as is well known[18], to an isotropic one).

The appearance of a flux of gravitational energy, $F_W = \pm\epsilon_W$ in the "open" Friedmann models is due to the curvature of the wavefront at the scale of the horizon. Therefore, if on the quasieuclidean expansion stage ($\epsilon \simeq \epsilon_{cr}$) there is only one traveling wave, it will not affect the cosmological expansion and its amplitude will be small: for $t \sim t_F$, $\epsilon_W/\epsilon \sim \xi_F \ll 1$, at the instant $t \sim t_M$ ($\xi \sim 1$) we have $\mu \sim t_F^{1/4} t_c^{1/4}/k t_M^{1/2}$, cf. (28)–(31).

It is important to stress that a traveling gravitational wave propagates strictly with the speed of light (cf. (23)–(25)). If there are two gravitational waves present, the phase velocities of the waves are equal to the speed of light only in the case when their amplitudes are small, $\mu < 1$, since in this case one may neglect in the leading approximation the gravitational interactions between the waves. Only in this case does the quantity $\omega = 2\pi/\lambda$ defined in (4) coincide with the frequency of the wave. If one takes into account the interaction, the phase velocity is larger than the speed of light and formally tends to infinity[10)] as $\mu_{max} \to \infty$. However, in the nonlinear region, for $\mu_{max} > 1$ ($\mu_{max} \gg \mu_{min}$) the concept of phase velocity has a purely formal character, since the interaction between the waves is of the same order as the "energy" of each of them, so that the increase of the "phase velocity" refers to standing waves. The physical meaning of the exact cosmological solutions is, of course, established on the basis of an analysis of the linear region ($\mu < 1$) when the different modes of the "perturbation" are independent and admit a unique interpretation.

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1)In [2], the metric of the closed model of type IX was represented as a sum of the isotropic part and a superposition of gravitational waves of maximally possible wavelength (standing waves in the comoving homogeneous space). In our case we are dealing with gravitational waves in a flat (or open, Sec. II) Universe with arbitrary wavelength λ.

2)In this connection we note that if within the framework of the Lifshitz theory [14] one selects perturbations of the metric (in a flat Friedmann model) in the form of a plane monochromatic circularly-polarized standing gravitational wave with arbitrary wavelength, then such a model coincides exactly with the metric of the model of type VII_0.

3)It is interesting to note that formally the equations (6), (7) describe the evolution of an axially symmetric model of type I in a synchronous homogeneous coordinate system (1) with the energy-momentum tensor T_i^k

$$T_0^0 = -T_3^3 = \tfrac{1}{4}(\dot\mu^2 + \omega^2 \operatorname{sh}^2 \mu), \qquad -T_1^1 = -T_2^2 = \tfrac{1}{4}(\dot\mu^2 - \omega^2 \operatorname{sh}^2 \mu),$$

where μ is a function of the time t. The equations of "motion" $T^k_{i;k} = 0$ have the form (5). (For $\lambda < t$, T_i^k goes over into (11).) Taking into account what was said above, it is more natural to use the following decomposition of the metric tensor g_{ik} (1)–(3)

$$ds^{(0)2} = dt^2 - c^2 dz^2 - a^2(dx^2 + dy^2)$$

plus the perturbation (gravitational waves)

$$-h_{ik} = a^2 \operatorname{sh}\mu \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos kz & \sin kz & 0 \\ 0 & \sin kz & -\cos kz & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + 2a^2 \operatorname{sh}^2\frac{\mu}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$h^{\beta}_{\alpha;\beta} = 0.$$

4)All other cases of isotropization are realized only for special choices of the initial conditions. Indeed, the case of isotropization when at the time $t \sim t_F$ we have $\lambda \gg t$ is degenerate, since it requires a special choice of the phase of the gravitational wave (the gravity of matter starts affecting the expansion at the instant when $|\mu| \ll 1$—quasi-isotropic expansion). On the contrary, if for $t \sim t_F$ we have $\lambda \ll t$, this means that the amplitude of the gravitational wave must be small already during the vacuum stage, $\mu_{max} \ll 1$. This case also requires a special choice of initial conditions. In all "degenerate" cases ($\lambda \gtrsim t$ for $t \sim t_F$) the isotropic Friedmann expansion stage starts practically right after the end of the vacuum stage, i.e., the interaction via gravity of the gravitational wave with matter becomes unimportant, and one may neglect the second order terms (in the amplitude μ) during the isotropic stage ($t > t_F$) during a sufficiently lengthy time of evolution (the logarithm in the solution (13) is large at the time $t \sim t_F$). In this case the solution is well approximated by the Lifshitz approximation [14].

5)E. M. Lifshitz has not obtained explicitly the damping effect of the amplitude of the gravitational waves on account of dilution, since he considered the solution in the form of a series in standing spherical tensorial waves. In this case the amplitude of each individual harmonic is not constant in space and is chosen in such a way that it compensates the decrease (or increase of the amplitude in the wave traveling away from (toward) the center owing to the curvedness of the wavefront. In principle, since the set of harmonics is complete, a gravitational wave which does not violate the homogeneity (with an amplitude independent of the coordinates) in an open Friedmann model (type VII_h) can be represented as a Fourier integral with respect to the spherical harmonics. (Such a wave has a nondegenerate spectrum.)

6)If the condition $\dot{\tilde\varphi} \neq 0$ holds, then μ does not vanish ($\mu > 0$), so that in the case of an oscillatory regine of the function $\mu(t)$ one can determine the minimal amplitude of oscillations $\mu_{min}(t) > 0$, $\mu_{min} \leqslant \mu_{max}$ [10,13]. As $t \to \infty$ we have $\mu_{max} - \mu_{min} \to 0$.

7)We note that if $a = Ae^{\xi}$, then Eqs. (26) (as solutions of (18) for weak waves, $\mu \ll 1$) are valid for arbitrary $k > 0$; the corrections to (26) are related to the fact that the expansion law $a = Ae^{\xi}$ is only asymptotic and can be determined by means of perturbation theory methods.

8)Such estimates for the parameter $\kappa_{+\infty}$ for the equation of state $p = n\epsilon$ ($n < 1$) have been obtained independently by Novikov and Bogoyavlenskiĭ [23] in investigations of the stability of the asymptotic solution (for $t \to \infty$) of the model of type VII_h (cf. (23)). It is important to note that the equations (30–(36) describe the exact limiting law of evolution of the system of equations (18)–(21) for $\xi \to \infty$ ($t \to \infty$; $\epsilon_W = p_{zW} = -F_W = \omega^2 \sinh^2 \mu/2$, $p_x = p_y = 0$), therefore the estimates (34)–(36) for κ_+ are valid for any solution.

9)By isotropization we mean the isotropization of the strain (deformation) tensor; at the same time, the isotropization of the curvature tensor is in general not necessary [10,13]. The anisotropy of the expansion during the stage $\epsilon < \epsilon_{cr}$ is in reality unobservable in the asymptotic solution (33), $\kappa \ll 1$, cf. [10].

10)Physically, it is precisely the increase of the phase velocity that can explain why the space derivatives of the metric tensor (curvature terms in (5)–(7) and (18)–(21)) are important during the vacuum oscillation stage in the epoch when the horizon is smaller than the wavelength, $\lambda > t$.

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172