Galvanomagnetic phenomena in thin conductor layers

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A qualitative theory is developed for galvanogmagnetic phenomena in thin conducting layers for which the classical size-effect condition $\lambda \gg d$ is satisfied (λ is the mean free path of the carriers with respect to the volume scattering mechanisms and d is the sample thickness). The magnetic field is assumed to be oriented in the plane of the sample. Numerical calculations of the kinetic parameters for weak and intermediate fields such that $r_H \gtrsim d/2$ (r_H is the Larmor radius) are carried out for a degenerate carrier gas with a simple energy spectrum. The results of the calculation show that in the system under consideration the magnetoresistance ρ_H and Hall field E_H have a nonmonotonic dependence on the field H. For extremely weak fields corresponding to values $r_H \gtrsim \sqrt{\lambda d}$ and for diffuse scattering from the surface, satisfactory agreement is obtained with the experimental results obtained by Gaĭdukov and Danilova by measuring $\rho_H(H)$ of metallic whiskers. A detailed qualitative explanation of the anomalous $\rho_H(H)$ dependence in the systems under consideration in weak magnetic fields is presented.

INTRODUCTION

The study of galvanomagnetic phenomena in samples in which the conditions for the classical size effect are satisfied were intiated in the late 40's[1]. Nonetheless, interest in the study of these systems is growing apace, owing to the continuously developing possibilities of realizing the necessary conditions in the experiments.

Numerous investigations have shown that the appreciable influence of the sample boundaries on transport phenomena in size-effect systems leads to an abrupt change in the dependence of the kinetic coefficients on the magnetic field. It was found that there exist two principally different types of field orientation relative to the sample: a) the magnetic field is normal to the sample surface, b) the magnetic field lies in the sample surface. Systems with field orientation of the type (a) have been well investigated experimentally[2, 3] and theoretically[4–6]. They are characterized by an oscillatory dependence of the conductivity on H in fields for which $r_H \leq d/2$. Systems with orientation of type (b) have also been extensively investigated (e.g.,[7–12]). It was established that the magnetic field in such systems leads to a nonmonotonic dependence of the conductivity[7] and of the Hall field[9] on H even in weak fields[1)]. In the case of arbitrary field orientation, the dependence of the kinetic coefficients on H is a superposition of the relations noted above.

In this paper we consider systems with field orientation of the type (b). An interesting feature of these systems is that the field H in a size-effect sample sorts out the particles into groups with respect to their velocities v_z, where z is the thickness coordinate of the sample (see also[7]). As a result, each of the carrier groups is "localized" in a definite region of space between the scattering surfaces. Under this situation it turns out that the particles that collide with the surface have a larger mobility than the particles that are fully turned around by the field and do not collide with the surface. This phenomena, called the "static skin effect"[13] leads to different types of anomalies in the dependence of the magnetoresistance (MR) $\rho(H)$ and the Hall field $E_H(H)$ in these systems.

It is important to note that in the preceding studies they considered mainly the case of strong magnetic fields $r_H \ll d$. The case of weak fields was investigated qualitatively by Azbel'[7]. The results obtained in these studies in fields $r_H < \sqrt{\lambda d}$ are in satisfactory agreement with experiment. As to the region of weak fields, as indicated by the results of[10–12] on the measurements of the MR of metal whiskers, the existing theoretical papers do not explain satisfactorily the experimental MR dependences in fields $r_H \gtrsim \sqrt{\lambda d}$. In particular, there is no explanation for the dependence of the transverse MR on H, which is described in the region of weak fields, with accuracy ~20% (for the generalized curves), by the relation $\rho_H \sim H^n$ with n = 2/3[10].

This disparity between theory and experiment calls for a detailed quantitative investigation of galvanomagnetic phenomena in weak fields[2)]. We have carried out such an investigation for systems with field orientation of type (b). In the case considered here, the inhomogeneity of the Hall fields plays practically no role[7, 9], and this greatly facilitates the quantitative aspect of the problem. In particular, the conditions on the surface can be written down in the usual Fuchs form. The results obtained in this manner make it possible to explain satisfactorily the $\rho_H(H)$ dependences obtained in[10–12] in the region of weak fields. In the region of fields $r_H < \sqrt{\lambda d}$, our results agree with the qualitative results of Azbel'[7]. In a preliminary communication[15], for the case of transverse MR, it was shown qualitatively that the dependence of the form of $\rho_H \sim H^n$ at $n < 1$ in the range of values of $r_H \gtrsim \lambda$ can be explained on the basis of the aforementioned sorting of the carriers with respect to the velocities v_z, which leads in final analysis to a decrease of the effective particle mean free path in the magnetic field.

In this paper we present a detailed exposition of the theory of galvanomagnetic phenomena in thin layers with field orientation of the type (b). We present the results of calculations for the longitudinal and transverse conductivity and of the Hall field at values $r_H \geq d/2$. Principal attention is paid to the limit of weak fields, when $r_H \gtrsim \sqrt{\lambda d}$. In this case it becomes possible to explain qualitatively the anomalous $\rho_H(H)$ dependence for both platelike and filamentary whiskers at arbitrary orientations of the field H relative to the sample.

2. DERIVATION OF FUNDAMENTAL EQUATIONS

We consider a conducting plate of thickness d, to which a drawing field E and a magnetic field $H_x \equiv H$ are applied (z is the coordinate in the direction of the sample thickness). The transport phenomena in this system are described by the equation

$$\mathrm{P}v+v_z\frac{\partial g}{\partial z}+\omega\left(v_z\frac{\partial g}{\partial v_y}-v_y\frac{\partial g}{\partial v_z}\right)=-\frac{g}{\tau}, \tag{1}$$

The solution of which, obtained by the method of characteristics, can be written in the form

$$g(z,v)=\exp\left(-\frac{t}{\tau}\right)\left[\int^{t}\exp\left(\frac{t'}{\tau}\right)\mathrm{P}v(t')\,dt'-C\right]. \tag{2}$$

Here g is the nonequilibrium part of the distribution function f, $\mathbf{P}\cdot\mathbf{v} = q\mathbf{E}\cdot\mathbf{v}\partial f_0/\partial\epsilon$ is the field term, v(t) is the dependence of the velocity component on the parameter t, $\omega = qH/mc$ is the cyclotron frequency, τ is the relaxation time, and C is a constant determined from the Fuchs condition on the surfaces z = 0, d (see the remark made above).

In this system there are two integrals of motion

$$\eta=v_z^2+v_y^2,\quad \vartheta=v_y-\omega z. \tag{3}$$

An analysis of the expressions for η and ϑ shows that the entire aggregate of the carriers breaks up into two groups: carriers with values $\eta \leq \eta_0 = (\omega d/2)^2$, and carriers with $\eta \geq \eta_0$. Among the carriers with $\eta < \eta_0$, there are particles that collide only with one surface (z = 0 or z = d) and carriers that are completely turned around by the field (see Fig. 1a). The carriers with $\eta \geq \eta_0$ also break up into two types:

a) carriers colliding with both surfaces (we call them carriers of type (a));

b) carriers colliding with only one of the surfaces (carriers of type (b)).

The range of variation of the parameter ϑ for each of the groups of carriers can be easily determined from Figs. 1a and b (the numbers designate the different regions of variation of ϑ). It will be convenient subsequently to find the nonequilibrium part of the distribution function g_k for each region of variation ϑ (k = 1, ..., 6).

We consider two cases of practical interest: transverse magnetic field ($E_x = 0$) and longitudinal field ($E \equiv E_x$).

A. Transverse Galvanomagnetic Phenomena

In this case the expression for g can be rewritten in the form

$$g=-q\frac{\partial f_0}{\partial\varepsilon}\frac{\tau\sqrt{\eta}}{1+\alpha^2}[E_z\,\mathrm{Re}\{(1-i\alpha)\tilde{g}\}+E_y\,\mathrm{Im}\{(1-i\alpha)\tilde{g}\}], \tag{4}$$

where

$$\tilde{g}=\exp\left(-\frac{t}{\tau}\right)\left[\exp\left\{(1+i\alpha)\frac{t}{\tau}\right\}-\tilde{C}\right]$$

$\alpha = \omega\tau$, and $\tilde{C}$ is a constant.

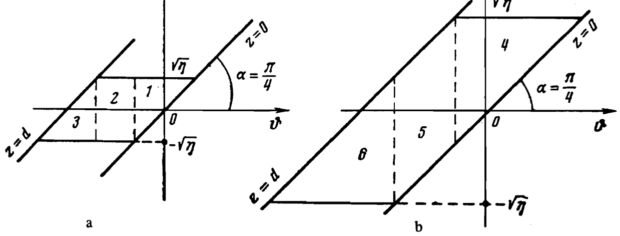

FIG. 1. Sorting of the carriers by velocities: a—groups of carriers with $\eta < \eta_0$, b—groups of carriers with $\eta \geq \eta_0$.

We call attention to the fact that the velocity components $v_z = \sqrt{\eta}\cos\omega t$ and $v_y = \sqrt{\eta}\sin\omega t$ have not been separated in g, i.e., g is not represented in the form $g(v) = g_z v_z + g_y v_y$. In this case the determination of $\tilde{C}$ from the boundary conditions, for example at z = 0, is carried out by introducing the values t_s^+ and t_s^- corresponding to the particle-motion states $\{z = 0, v_z > 0\}$ and $\{z = 0, v_z < 0\}$. We then separate v_z and v_y from the obtained expression for $\tilde{g}$.

It is easily seen that this approach to the determination of $\tilde{C}$ is due to the strict connection between the particle velocity components v_z and v_y, on the one hand, and the particle coordinate z on the other. This relation is given by Eqs. (3) and is typical of only the considered system.

The final expressions for g_k are given in the appendix.

Writing down the general expressions for the current, we can easily obtain final expressions for the Hall field E_H and for the conductivity $\sigma(H)$; these are too cumbersome to present here. A numerical check on the function $E_H(z)$ indicates that the Hall field is practically uniform over the sample thickness up to fields $r_H \sim d/2$.

B. Longitudinal Galvanomagnetic Phenomena

The expression for g has the form customary for size-effect systems

$$g=-q\frac{\partial f_0}{\partial\varepsilon}E_x v_x\left[1-C\exp\left(-\frac{t}{\tau}\right)\right]=-Pvg^{*}. \tag{5}$$

The constant C is obtained from the Fuchs conditions by introducing the values t_s^+ and t_s^- and then eliminating the parameter t_s^+ from the final expression for g. As a result we obtain for particles with $v_z > 0$

$$g_{1,4}^{*}=1-\frac{(1-p)\exp(K_0-K_z)}{1-p\exp(2K_0-\pi/\alpha)},\quad g_2^{*}=1,$$
$$g_{3,6}^{*}=1-\frac{(1-p)\exp(K_d-K_z)}{\exp(\pi/\alpha+2K_d)-p},\quad g_5^{*}=1-\frac{(1-p)\exp(K_0-K_z)}{1-p\exp(K_0-K_d)}. \tag{6}$$

Here p is the Fuchs specularity parameter, and the expression for K_z is

$$K_z=\frac{1}{\alpha}\arcsin\left(\frac{\vartheta+\omega z}{(\sqrt{\eta})}\right).$$

The final expression for the current density averaged over the thickness is

$$j_x=2E_x\int_0^\infty dv_x v_x^2 q\frac{\partial f_0}{\partial\varepsilon}\tau\left\{\int_0^{\eta_0}d\eta\sum_{k=1}^{k=3}\int d\vartheta_k\int dz_k g_k I\right.$$
$$\left.+\int_{\eta_0}^{\infty}d\eta\sum_{k=4}^{k=6}\int d\vartheta_k\int dz_k g_k I\right\}, \tag{7}$$

where $I = v_z^{-1}$ is the Jacobian of the transition from $dv_y dv_z$ to $d\eta d\vartheta$.

3. RESULTS OF CALCULATION OF THE KINETIC COEFFICIENTS

The dependences of the conductivity $\sigma(H)$ and of the Hall field $E_H(H)$, obtained by numerical computer calculation for different mechanisms of scattering from the surface and for two types of orientation of H relative to E, are shown in Figs. 2a, b, and c. In the case of small ratios $d/\lambda = \delta \ll 1$ of interest to us, these dependences are identical for different δ, and we therefore present here the results only for $\delta = 0.02$.

As expected, even in the considered case of spherical equal-energy surfaces the dependence of the kinetic parameters on the field H is quite appreciable (with the exception of $\sigma(H)$ at $p = 1$, which is nonmonotonic). We turn to the analysis of the results.

We call attention first to the strong dependence of the conductivity on the field H in the considered system in the case of weak fields under the condition $p = 0$. This result differs significantly from the known theoretical result of[7] and is close to the results of[14]. A simple recalculation shows that the dependence of the relative change of the resistance $\rho_H(H) = [\rho(H) - \rho(0)]/\rho(0)$ in fields when $\lambda^2/d > r_H \gtrsim \sqrt{\lambda d}$, for both longitudinal and transverse fields, follow the law $\rho_H \sim H^n$ with $n < 1$. At values $r_H \sim \sqrt{\lambda d}$ we have $\rho_H \sim H$ in the case of a transverse field. This behavior of $\rho_H(H)$ is in satisfactory agreement with the experimental results of[10-12] obtained in metal whiskers[3)]. A physical explanation of the anomalous dependence of the MR on the magnetic field in the considered systems is given below.

As to the field region $r_H < \sqrt{\lambda d}$, the results, for example for the case of transverse magnetoresistance, agree with the results of Azbel'[7]. Understandably, the qualitative character of the results is due to the fact that it is impossible to take into account the inhomogeneity of the field E_H over the sample thickness (see[9]).

We note in addition that the course of the function $\sigma(H)$ under the condition $p = 1$ does not depend in practice on the value of δ at $\delta \ll 1$. Starting with values $r_H \sim d/2$, the $\sigma(H)$ dependence is determined by the relation $\sigma(H) \sim 1/H$. This behavior of $\sigma(H)$ can be used to determine v_F (from the relation $r_H = v_F/\omega$).

Let us dwell on some details of the function $E_H(H)$. In weak fields, when $r_H \geq \sqrt{\lambda d}$, the Hall field at arbitrary values of p is determined by the relation $E_H \approx \omega\tau E_y$. With further increase of the field H, a sharp decrease takes place in the Hall field. Under conditions of specular reflection from the surface, starting with values $r_H \sim d$, the function $E_H(H)$ takes the form $E_H \sim 1/H$. If $p = 0$, the function $E_H(H)$ has a weakly pronounced minimum at values $r_H \sim d$; this minimum gives way to an abrupt growth of the field E_H at values $r_H \gtrsim d/2$. In this case E_H reaches values $\sim 2\omega\tau E_y/3$, i.e., it increases by several orders of magnitude in comparison with $E_{H\,min}$; this agrees with the abrupt growth of the conductivity in this magnetic-field region.

It is important to note that the results obtained for $E_H(H)$ in medium fields corresponding to $r_H < \sqrt{\lambda d}$ are only qualitative. In this connection, the experimental investigations of the Hall effect in the considered systems become of fundamental interest. We add that the measurements of $E_H(H)$ are of interest also from the point of view of determining the parameters τ, v_F, and λ.

4. DISCUSSION

As already noted, our results for the conductivity differ from the known results only in the region of weak fields, when $r_H > \sqrt{\lambda d}$. Therefore principal attention will be paid precisely to this limit of the values of the field H. The quantitative agreement between the theoretical results for plates with the results of Gaĭdukov and Danilova for whiskers in weak fields makes it most interesting to explain physically the anomalous behavior of the $\rho_H(H)$ dependence in the considered case of field orientation of type (b). This explanation can be obtained by taking into account that distinguishing feature of the system, whereby the carriers are sorted out into groups with respective velocities v_z.

It is well known that in size-effect samples, in which the condition $\lambda/d \gg 1$ is satisfied, the main contribution to the conductivity in the absence of a magnetic field is made by carriers that are emitted at small angles to the surface, i.e., carriers with small v_z. Obviously, it suffices to consider carriers for which the emission angles α exceed a certain critical angle $\alpha_{cr} = \alpha_0 \approx \delta$. We now take into account the fact that in the presence of a magnetic field lying in the plane of the sample, particles with small v_z are turned around by the field and do not reach the opposite surface (these are type (b) particles). It is easy to ascertain from Fig. 1b that the maximum emission angle α_1 of particles of type (b) is determined by the ratio $v_{z\,max}/v_{y\,min}$ and is of the order of $\sqrt{d/r_H}$.

Thus, in a field H particles of type (b) are emitted at angles smaller than $\alpha_1 \approx \sqrt{d/r_H}$ to the surface, and cover a path on the order of αr_H. Particles of (a), which are emitted at angles $\alpha\,(\pi/2 \geq \alpha \geq \alpha_1)$, cover a type on the order of $d/\sin\alpha$, and are only slightly turned around by the field. Therefore, in very weak fields, when $r_H > \lambda^2/d$ and $\alpha_1 < \alpha_0$, the conductivity of the system is determined completely by the carriers of type (a), and $\sigma(H)$ is close to the typical dependence for bulky samples. This section of the dependence is extremely small and is not reflected in Figs. 2a and 2c; the experimental curves do reveal such a section (see[10]).

With increasing field H, when r_H lies in the range $\lambda^2/d > r_H \gtrsim \lambda$, the mean free path of particles of type (a) begins to change significantly with changing field:

$$\lambda_1 \sim d \ln \frac{1}{\alpha_1} \approx d \ln \sqrt{\frac{r_H}{d}}.$$

In the same field, the free path of the particles (b) is

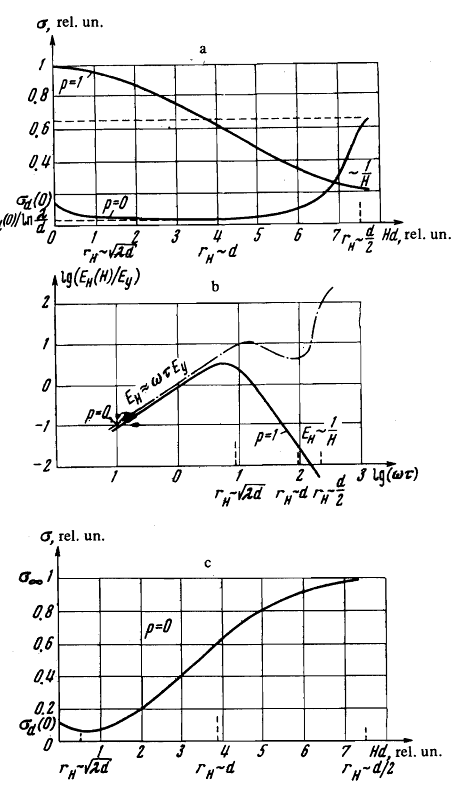

FIG. 2. a—Dependence of the transverse conductivity on the magnetic field H; b—dependence of the Hall field on H; c—dependence of the longitudinal conductivity on H; $\delta = 0.02$ and

$$\sigma_\infty \sim \frac{\lambda}{d}\sigma_d(0) \Big/ \ln\frac{\lambda}{d}.$$

small as before: $\lambda_2 \sim \alpha_1^2 r_H \approx d$. If it is now recognized that the relative number of carriers (b) is of the order of (d/r_H), and the number of carriers with $\eta < \eta_0$ is of the order of $(d/r_H)^2$, then we readily obtain

$$\sigma(H) \approx \sigma_d(0)\left[1 - \ln\frac{\lambda}{\sqrt{r_H d}} \Big/ \ln\frac{\lambda}{d}\right], \tag{8}$$

where $\sigma_d(0) \sim a\, ln(\lambda/d)$ is the conductivity of the sample in the absence of a field.

Expression (8) for $\sigma(H)$ cannot be represented in the form of a series in powers of H. Elementary numerical estimates show that relation (8) corresponds approximately to the relation $\rho_H \sim H^n$ with $n < 1$. By way of example we indicate that at $\ln(\lambda/d) = 4$ the relation $\sigma(H)$ given by (8) satisfies with ~10% accuracy the relation $\rho_H \sim H^{0.55}$. Thus, in weak fields a strong dependence of the magneto-resistance on the magnetic field should indeed be observed, and the foregoing estimates explain the anomalous behavior of $\rho_H(H)$ in thin layers in the case of a magnetic-field orientation of type (b).

According to the experimental results[10-12], the magneto-resistance in whisker metals, for platelike samples in weak fields, is given with ~20% accuracy by $\rho_H \sim H^{2/3}$. The qualitative estimates given above agree well within the limits of the accuracy with the experimental $\rho_H(H)$ dependence. It is interesting to note in this connection that for the transverse magnetoresistance, the experimental $\rho_H(H)$ relations for platelike and filamentary samples were identical. On the other hand, for the longitudinal magnetoresistance of filamentary samples, a weaker dependence of $\rho_H(H)$ was obtained, with $\rho_H \sim H^{2/3}$ as before. These details of the dependence of the magnetoresistance of whiskers on H can also be explained on the basis of the foregoing qualitative analysis.

Indeed, it follows from the statements made above that in weak fields the change of conductivity with changing fields is determined by the decrease of the mean free path of the particles of type (a), owing to the fact that the carriers with small v_z are knocked out of the play. It is clear that in weak fields for platelike samples the difference between the cases of longitudinal and transverse orientation of the fields plays practically no role in the approach to the estimate of the critical emission of angle α_{cr}. For filamentary samples, the transverse field is also, roughly speaking, parallel to one of the surfaces, and the approach to the estimate α_{cr} for particles of type (a) remains unchanged.

On the other hand, in the case of a longitudinal magnetic field for filamentary samples it is necessary to change the approach to the estimate of α_{cr}. In this case it is necessary to take into account the dependence of the critical emission angle on the coordinate of the particle on the surface, and to average over the coordinate in order to determine the average path of the particles of type (a). Approximate estimates given in the Appendix explain the reason for the weak linear dependence of the longitudinal conductivity of the filaments on the field H.

The results make it possible to establish a satisfactory qualitative correspondence between theory and experiment in weak fields with respect to the basic factors (the character of the $\rho_H(H)$ dependence and the scaling law) without specifying concretely the shape of the Fermi surface of the crystal or the orientation of the crystal axes relative to the field H. One can hardly expect, however, a qualitative agreement between theory and experiment even in weak fields (see also footnote 3). In particular, within the framework of our calculations it is impossible to establish the orders of magnitude of the characteristic fields H_1 and H_2 which are involved in[10-12]. With respect to the field H_{inv}, it can be assumed that H_{inv} corresponds to the value $r_H \sim \sqrt{\lambda d}$.

In the case of medium and strong fields, starting with values when $r_H < \sqrt{\lambda d}$, the character of the function $\rho_H(H)$ is significantly influenced by the individual features of the Fermi surface and by other characteristics of the crystal. Favoring the effectiveness of taking into account the nonsphericity of the equal-energy surfaces is the fact that a qualitative explanation of the behavior of $\rho_H(H)$ in strong fields, given in[10] from the point of view of the static skin effect, turns out to be quite satisfactory.

In conclusion, the authors thank Yu. P. Gaĭdukov and N. P. Danilova for interest in the work, for a discussion of the results, and for many useful remarks.

APPENDIX

1. Expression for g_k in the case of a Transverse Magnetic Field

To complete the exposition, we present the final expression for the nonequilibrium part of the distribution function in each region of variation of the parameter ϑ (see Figs. 1a and 1b):

$$\tilde{g}_{1,4} = \tilde{v}\left\{1 - \frac{[1 + p\exp(-2i\alpha K_0)]\exp[\tilde{\alpha}(K_0 - K_z)]}{1 - p\exp(2K_0 - \pi/\alpha)}\right\},$$

$$\tilde{g}_2 = \tilde{v},$$

$$\tilde{g}_{3,6} = \tilde{v}\left\{1 + \frac{[1 + p\exp(2i\alpha K_d)]\exp[-\tilde{\alpha}(K_z + K_d)]}{1 - p\exp(-2K_d - \pi/\alpha)}\right\},$$

$$\tilde{g}_5 = \tilde{v}\left\{1 - \frac{[1 - p\exp[i\alpha(K_d - K_0)]]\exp[\tilde{\alpha}(K_0 - K_z)]}{1 - p\exp(K_0 - K_d)}\right\}.$$

Here $\tilde{v} = v_z + iv_y$, $\tilde{\alpha} = 1 + i\alpha$, p is the Fuchs parameter, and the expression for K_z is of the same form as in the main text. The expressions given here for g_k are valid for particles with velocities $v_z > 0$ provided that both surfaces scatter in equal fashion.

2. The $\rho_H(H)$ Dependence of Filamentary Whiskers in a Weak Longitudinal Field

In the case of platelike samples, considered above, it was possible to obtain clear estimates of the $\sigma(H)$ dependence because the system was quasi-one-dimensional (the system was bounded only with respect to one coordinates). In the case of filamentary samples and longitudinal orientation of the field, the system is essentially two-dimensional, and therefore quantitative estimates of the effective carrier range become quite cumbersome. We confine ourselves here therefore only to general considerations.

It is physically clear that in filamentary samples the effective mean free path λ_{eff} depends much less on the

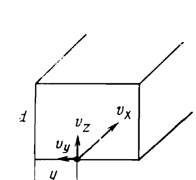

FIG. 3

value of the critical angle $\alpha_{cr}(H)$ in comparison with the analogous dependence in platelike samples (if the field H is longitudinal)[4]. Indeed, for a particle that starts at the surface of the sample (for example, z = 0) and is located at a distance y from the surface y = 0 (see Fig. 3), we can write

$$\lambda_{\mathrm{eff}} \sim \frac{1}{d}\int_0^d \sqrt{\left(d \ln \frac{\lambda}{d}\right)\left(y \ln \frac{\lambda}{y}\right)}\, dy$$

$$\approx \sqrt{d \ln \frac{\lambda}{d}} \int_0^d \sqrt{y}\, \frac{dy}{d} \approx d \sqrt{\ln \frac{\lambda}{d}} \sim d.$$

Thus, in fields such that $d/r_H \lesssim d/\lambda$, the $\rho_H(H)$ dependence is close to the typical one for bulky samples. With further increase of the field, $\rho_H(H)$ is determined mainly by the decrease of the number of type (a) carriers with changing field H. This yields $\sigma(H) \approx \sigma(0)(1 - d/r_H)$, and $\rho_H(H)$ increases linearly with H. In the range of values $r_H \sim d$, the principal role in the transport is assumed by particles that are completely turned around by the field, and this leads to a change in the character of the $\rho_H(H)$ dependence in these fields.

1) An exception is the case of specular scattering by a surface, when $\rho_H(H)$ increases monotonically with H.

2) Mention should be made here of the work by Ditlefsen and Lothe [14], who investigated quantitatively the $\rho_H(H)$ dependence at a field orientation of the type (b). They did not analyze, however, the results in weak fields and did not cite the $E_H(H)$ dependence.

3) A more complete quantitative comparison of the theoretical and experimental results, even in the region of weak fields, is made difficult by the lack of quantitative values of λ and $r_{H\,eff}$ for whiskers. In addition, it must be remembered that even platelike whiskers are physically not plates.

4) This is indicated also by the higher values of the magnetic fields (in the case of longitudinal orientation of H), starting with which an anomalous $\rho_H(H)$ dependence appears in filamentary samples.

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