

# The possibilities of producing a $\gamma$ layer

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The possibility is discussed of creating an active medium for a  $\gamma$  laser by pumping by synchrotron radiation. A comparison is made of this scheme with the case of pumping by neutrons. The kinetics of stimulated  $\gamma$  radiation is discussed. The necessary preliminary experiments are discussed briefly.

## 1. INTRODUCTION

The promise of producing a  $\gamma$  laser involving nuclear transitions is extremely alluring. The difficulties which stand in the way of this achievement have been discussed by several authors.<sup>[1-3]</sup> There are two main problems:

- 1) creation of the necessary concentration of excited nuclei, i.e., the problem of sufficiently intense pumping;
- 2) creation of the necessary conditions for laser operation, the most important of which is the preparation of a crystal with a sufficiently narrow radiation line.

For development of a  $\gamma$  shower it is necessary that the increase in the number of  $\gamma$  rays as a result of the stimulated emission exceed the loss. The corresponding condition is of the form<sup>[1]</sup>

$$\frac{1}{l_+} = \left( n_2 - \frac{2I_2 + 1}{2I_1 + 1} n_1 \right) \sigma_{+} > n \sigma_{-} = \frac{1}{l_-}, \quad (1)$$

where  $n_2$  and  $n_1$  are the densities of excited and unexcited resonance nuclei,  $n$  is the density of the material,  $\sigma_{+}$  is the cross section for stimulated emission, and  $\sigma_{-}$  is the cross section for  $\gamma$ -ray absorption, which occurs mainly as the result of the photoeffect. The term corresponding to resonance absorption has been moved to the left side of the inequality. In Eq. (2) below we introduce the absorption length  $l_-$  and the stimulated emission length  $l_+$ . The quantity  $\sigma_{+}$  is given by<sup>[1]</sup>

$$\sigma_{+} = \frac{\lambda^2}{2\pi\Delta\omega\tau_\gamma} f. \quad (2)$$

Here  $\lambda$  is the wavelength of the  $\gamma$  ray,  $\Delta\omega$  is the real width of the crystal,  $\tau_\gamma$  is the radiation lifetime of the excited state, and  $f$  is the probability of the Mössbauer effect, which is necessary for achieving resonance interaction of  $\gamma$  rays with nuclei.

Two means of producing a  $\gamma$  laser have been discussed in the literature. Chirikov and Khokhlov<sup>[1,3]</sup> have suggested creation of a crystal of nuclei previously separated in an excited state. It is clear that growing a sufficiently good crystal will require a long lifetime of the nucleus. In this case according to Eqs. (1) and (2) it is necessary to have an extremely small width  $\Delta\omega$ , i.e., to remove every kind of inhomogeneity. Another means pointed out by Goldanskiĭ and Kagan<sup>[2]</sup> is to use relatively short-lived nuclei, so that  $\Delta\omega$  is determined by the natural line width. For pumping it is proposed to use a pulsed neutron flux. The difficulties of this approach are the need for ultrapowerful pumping and the problem of destruction of the crystal. The practical achievability of the parameters required for production of a  $\gamma$  laser is not clear at the present time.

In the present article we discuss a new possibility of producing a  $\gamma$  laser. For pumping it is proposed to use a powerful source of  $\gamma$  rays, the synchrotron radiation of high-energy electrons. A population inversion can be achieved by the usual three-level scheme. Specific eval-

uations are given in the second part of the article. Although the parameters required significantly exceed the characteristics of existing electron storage rings, this approach nevertheless appears more realistic than those proposed previously.

In the third part of the article we discuss the kinetics of a  $\gamma$  laser, which differ substantially from the known laser regimes. Our conclusions are in agreement with those of Chirikov,<sup>[1]</sup> who discusses the initial stage of the process on the basis of perturbation theory.

## 2. PUMPING OF A $\gamma$ LASER

We will discuss the possibility of creating a population inversion of nuclei on bombardment of a crystal by an intense  $\gamma$ -ray flux. As a source of such radiation we can use an electron storage ring of high energy.

Let us consider the case in which the nuclei have excited states suitable for pumping by the three-level scheme (see the figure). For example, the transitions  $1 \rightarrow 3$  and  $3 \rightarrow 2$  have multipolarity  $\Delta I = 1-2$ , and the transition  $2 \rightarrow 1$  has multipolarity  $\Delta I = 3-4$ . The latter transition gives a lifetime of the order of magnitude of interest here. Such levels occur in even-odd nuclei for  $A \sim 100$ .

The cross section for resonance photoabsorption by nuclei with the transition  $1 \rightarrow 3 \rightarrow 2$  is

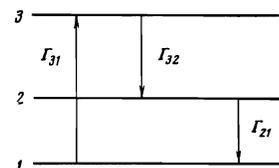
$$\sigma = \frac{\lambda_{13}^2}{8\pi} \frac{\Gamma_{13}\Gamma_{32}}{(\omega - \omega_{13})^2 + \Gamma_3^2/4} \frac{2I_3 + 1}{2I_1 + 1}, \quad (3)$$

where  $\lambda_{13} = 2\pi c/\omega_{13}$  is the wavelength of the  $\gamma$  rays absorbed,  $\Gamma_{13}$  and  $\Gamma_{32}$  are the partial widths of the transitions,  $\Gamma_3$  is the total width of the third level, and  $I_3$  and  $I_1$  are the spins of the corresponding states.

The  $\gamma$ -ray spectrum of synchrotron radiation  $dn/d\omega$  (in photons/sec-Hz) has the form<sup>[4]</sup>:

$$\frac{dn}{d\omega} = \frac{\sqrt{3}}{2\pi} \frac{e^2}{\hbar c} \frac{\gamma\omega_H}{\omega} F\left(\frac{\omega}{\omega_H}\right) N_e, \quad (4)$$

where  $\gamma = E/mc^2$ ,  $E$  is the electron energy,  $\omega_H = eH/mc\gamma$  is the Larmor frequency,  $\omega_k = (3/2)\gamma^3\omega_H$  is the location of the maximum of the spectrum,  $F(x)$  is a known function (for  $x \sim 1$ ,  $F(x) \sim 1$ ), and  $N_e$  is the number of electrons. At distances from the beam  $r \lesssim d/\theta$  (where  $d$  is the vertical beam dimension and  $\theta$  is the angular divergence of the radiation  $\theta \sim (\omega/\omega_H)^{-1/3}$ ) the dimensions of the illuminated area are determined by the beam height  $d$ . In this case the flux per unit area is  $(1/2\pi R d) dn/d\omega$ ,



where R is the distance from the crystal to the center of the orbit.

It is evident that the pumping time is of the order of the lifetime  $\tau_2$  of the isomer, and for large lifetimes the ratio of the number of excited nuclei  $N_2$  to the number of nuclei in the ground state  $N_1$  remains constant:

$$\frac{N_2}{N_1} \approx \frac{\tau_2}{2\pi R d} \int \frac{dn}{d\omega} \sigma(\omega) d\omega. \quad (5)$$

According to the condition for achieving laser operation (1) we obtain

$$\frac{\tau_2}{2\pi R d} \int \frac{dn}{d\omega} \sigma(\omega) d\omega > \frac{\sigma_+(2I_2+1)/(2I_1+1)+\sigma_-}{\sigma_+-\sigma_-}. \quad (6)$$

This inequality determines the requirements on the pumping.

Let us turn now to estimation of the values of the parameters occurring in Eq. (6). According to refs. 1-3 the optimum  $\gamma$ -ray energies are  $\hbar\omega_{21} \sim 100$  keV. It is not desirable to decrease this value, since in this case the cross section for the photoeffect rises more rapidly than  $\lambda^2$ . In addition, for the multipolarity  $\Delta I = 3-4$  of interest to us the conversion probability increases rapidly.<sup>[5]</sup> An increase of  $\hbar\omega_{21}$  reduces the probability of the Mössbauer effect. For these energies the greatest transition widths are  $\Gamma_{13}/\hbar \sim \Gamma_{32}/\hbar \sim 10^{12}$  Hz.

The optimal lifetime  $\tau_2$  is determined, on the one hand, by the pumping possibilities given by (6) and, on the other hand, by the possibilities of narrowing the radiation line  $\Delta\omega$ . For guidance we will give an estimate of  $\tau_2$  for certain arbitrary storage-ring parameters.<sup>1)</sup> It is assumed that to obtain the synchrotron radiation a small deflecting magnet is used (beam deflection angle  $\sim 1^\circ$ ) with a field  $H \sim 150$  kOe. For an electron energy  $E \sim 5$  GeV the peak intensity lies in the region  $\hbar\omega = 100$  keV. It is assumed also that the beam in the region of the field is compressed in the vertical direction to a dimension  $d \sim 10^{-4}$  cm determined by the quantum fluctuations of the radiation. For a circulating current of  $\sim 10$  amperes we find that the characteristic time of pumping to  $n_2 \sim n$  is of the order of one hour.

In the evaluation it was assumed that the right-hand side of Eq. (6) is of the order of unity, i.e.,  $\sigma_+ > \sigma_-$ . This condition imposes a severe restriction on the required line width  $\Delta\omega$ . Specifically, on setting the conversion coefficient  $\alpha \sim 1$  and  $f \sim 1$  we have  $\Delta\omega \lesssim \sigma_+/\tau_2 \sim 1$  Hz. The minimum width achieved experimentally for a Mössbauer line is<sup>[6]</sup>  $\Delta\omega \sim 10^5$  Hz. This value of  $\Delta\omega$  can be explained by the magnetic dipole interaction of the nuclei.<sup>[6]</sup> However, this form of interaction can be removed by the technique of radio-frequency pulsing of the magnetic field,<sup>[7]</sup> developed in NMR spectroscopy.<sup>[8]</sup> The line width due to defects has been discussed in refs. 2 and 3. According to ref. 3,  $\Delta\omega$  can be reduced<sup>2)</sup> to  $\Delta\omega \sim 10^{-2}$  sec<sup>-1</sup>.

Let us compare the proposed scheme for creating a  $\gamma$ -laser active medium with the possibilities of pumping by neutrons. The intensity of pumping can be conveniently characterized by the increase in the concentration of active atoms per unit time,  $\dot{n}_2 = j\sigma n$ , where  $j$  is the flux of photons or neutrons,  $\sigma$  is the absorption cross section, and  $n$  is the density of the material. For the synchrotron-radiation parameters described above,  $\dot{n}_2 \approx 2 \times 10^{19}$  nuclei/sec-cm<sup>3</sup>, and for neutrons from the best reactors  $j \sim 10^{15}$  neutrons/sec-cm<sup>2</sup>,  $\sigma \sim 10^{-20}$  cm<sup>2</sup>, and  $\dot{n}_2 \sim 10^{18}$  nuclei/sec-cm<sup>3</sup>. A definite advantage of

neutron pumping is the fact that it is carried out from a neighboring isotope so that, for a favorable isomeric ratio, the number of nuclei in the ground state is small. For the parameters given, conditions (3) and (6) give  $\Delta\omega \leq 0.1$  sec<sup>-1</sup>, and a lifetime  $\tau_2 \geq 10$  sec. Thus, with respect to pumping intensity the two methods are comparable.

An important advantage of pumping by  $\gamma$  rays appears when we consider the effect of the intense bombardment on the crystal structure of the material. The recoil energy necessary to remove a nucleus from a lattice site amounts<sup>[9]</sup> to 10-20 eV. It is easy to see that this quantity  $E_{\text{sep}} = (\hbar\omega)^2/2Mc^2$  ( $\omega$  is the  $\gamma$ -ray frequency and  $M$  is the mass of the nucleus) for  $\hbar\omega \approx 200$  keV is 0.2 eV for  $A = 100$ , and in the case  $\hbar\omega = 2$  MeV (a typical  $\gamma$ -ray energy in radiative neutron capture)  $E_{\text{sep}} = 20$  eV. Thus, in pumping by  $\gamma$  rays the recoil energy is less than the threshold for production of a dislocation, while in pumping by neutrons the crystal lattice is already destroyed. It is clear that for this reason it is difficult to hope for success in use of pulsed neutron pumping and nuclear transitions with short half-lives.

The remaining possibility is recrystallization of the sample,<sup>[3]</sup> which obviously requires a large lifetime of the isomer:  $\tau_2 \gtrsim 10^4-10^6$  sec. The crystal must be highly perfect, since for this value of  $\tau_2$  the limitations on  $\Delta\omega$  are very great. Radiation pumping is more promising also for this scheme, since (at least in principle) it requires a shorter lifetime of the isomer and provides the possibility of preparing a crystal with the required properties beforehand.

We will estimate the effect of the pumping on the quality of the crystal, resulting from the heating during irradiation. The depth of the working zone of the crystal is one of the order of the mean free path of the resonance  $\gamma$  rays, which amounts to  $1/n\lambda_{13}^2 \sim 10^{-4}$  cm. If the sample is chosen in the form of a film of this thickness, nonresonance  $\gamma$  rays pass through it almost without absorption. For the parameters described above the heat dissipation is given as  $\sim 10$  watts in 1 cm of length in a narrow band whose width is of the order of the vertical beam dimension  $d$ . The heat dissipation can be reduced by many orders of magnitude by cutting out a narrow band of frequencies by means of Bragg scattering. Removal of the heat can be accomplished by a gas or liquid with a small nuclear charge, which is transparent for  $\gamma$  rays. The temperature must be maintained well below the melting point. Since the lifetime of the isomer is large (of the order of hours), the conditions necessary for stimulated radiation can be produced in a separate apparatus at low temperatures in a magnetic field. The radio-frequency pulses, which lead to an artificial narrowing of the line, serve as a unique triggering mechanism of the  $\gamma$  shower.

### 3. KINETICS OF A $\gamma$ LASER

The problem of radiation of electromagnetic waves by a medium consisting of excited atoms has been discussed by many authors (a bibliography may be found, for example, in ref. 10). These studies differ greatly both in their approaches and in the approximations used. In fact, the existence in the problem of several important parameters leads, for various relations between them, to various laser regimes. Various levels of discussion are also possible: from the strictest quantum

approach, utilizing the equation for the density matrix of the system of atoms and the electromagnetic field,<sup>[11]</sup> to the semiclassical approach utilizing Maxwell's equations together with the equations for polarization of the medium.<sup>[12, 13]</sup> In order to make the relation between the kinetics of a  $\gamma$  laser and other laser regimes clearer, we will begin with simple estimates.

The most important phenomenon which can take place in the radiation of an excited medium is the cooperative radiation of many atoms or nuclei. Here the intensity of the radiation is significantly increased and the duration  $\tau$  of the radiation decreases so that  $\tau \ll \tau_\gamma$ .

Let the excited medium be placed in tube of length  $L$  and cross section  $S$ ; for the time being we will neglect absorption. Let us estimate the probability of radiation per unit time  $1/\tau$ , neglecting the probability of spontaneous radiation:

$$\tau^{-1} \sim \tau_\gamma^{-1} N_{\text{coh}} \Delta\Omega/4\pi, \quad (7)$$

where  $\Delta\Omega/4\pi$  is the fraction of the solid angle in which stimulated radiation is possible (in the case of a sufficiently great length of the system,  $\Delta\Omega$  is the diffraction solid angle:  $\Delta\Omega/4\pi \approx \lambda^2/S$ ), and  $N_{\text{coh}}$  is the number of nuclei radiating coherently. We will write it in the form  $N_{\text{coh}} = nS l_{\text{coh}}$ , where  $l_{\text{coh}}$  is the length of the region of coherence. Substituting this expression and estimate of  $\Delta\Omega$  into Eq. (7), we have

$$\tau \approx \tau_\gamma l_+ / l_{\text{coh}} \quad (8)$$

Depending on the parameters of the system,  $l_{\text{coh}}$  is determined by various factors. For large system dimensions,  $l_{\text{coh}}$  is the length of the wave train of the emitted radiation,  $l_{\text{coh}} = c\tau$  (ref. 13). We find from this and from Eq. (8) that  $\tau \approx (\tau_\gamma l_+ / c)^{1/2}$ . Here the radiation is emitted in individual pulses of duration  $\sim \tau \ll \tau_\gamma$ . If the system dimensions are less than the length of the wave train, then  $l_{\text{coh}} = L$ —the dimensions of the system. Finally, in the case of a  $\gamma$  laser the factor determining  $l_{\text{coh}}$  is the absorption length  $l$ . Accordingly, the characteristic time of the radiation is

$$\tau \sim \tau_\gamma l_- / l_+ \quad (9)$$

If the threshold condition (1) is satisfied, we have  $\tau < \tau_\gamma$ . The stimulated emission is radiated along the system into an angle  $\Delta\Omega$ . If the threshold condition is not satisfied,  $l_- < l_+$ , then the radiation along the tube is absorbed and the background from spontaneous radiation dominates.

Let us turn now to discussion of the equations describing the kinetics of a  $\gamma$  laser. Although these equations have been obtained previously,<sup>[11, 12]</sup> we will nevertheless mention their derivation, in order to make clear the nature of the approximations used. Taking into account only the resonance part of the interaction of the nuclei with the electromagnetic field, we will consider the nuclei to be two-level systems with coordinates  $\mathbf{R}_a$ . We introduce the matrix density operator

$$\hat{\rho}_{ik}'(\mathbf{x}) = \sum_a \delta(\mathbf{x} - \mathbf{R}_a) \hat{\Psi}_i(a) \hat{\Psi}_k^+(a), \quad (10)$$

where the subscripts  $i$  and  $k$  take on values 1 and 2;  $\hat{\Psi}_1(a)$  is the operator for annihilation of the  $a$ -th nucleus in state  $i$ . The Hamiltonian of the interaction we will write in the form

$$\hat{H}_{\text{int}} = \int d^3x \text{Sp} [\hat{\rho}'(\mathbf{x}) \hat{A}(\mathbf{x})], \quad (11)$$

where  $\mathbf{j}$  is the nuclear electromagnetic transition operator and  $\hat{A}(\mathbf{x})$  is the field operator. The trace is taken over the subscripts  $i$  and  $k$ . We will write the coordinates of the nuclei as  $\mathbf{R}_a = \mathbf{R}_a^{(0)} + \delta\mathbf{R}_a$ , where  $\mathbf{R}_a^{(0)}$  are the crystal lattice sites and  $\delta\mathbf{R}_a$  are the displacements, i.e., the operators acting on the oscillator degrees of freedom of the lattice. Since we are interested in Mössbauer transitions, i.e., transitions without radiation of phonons, it is necessary to average Eq. (11) over the states of the crystal:

$$\langle \hat{\rho}_{ik}'(\mathbf{x}) \rangle = \sum_a \langle \exp i\mathbf{p}_a \delta\mathbf{R}_a \rangle \delta(\mathbf{x} - \mathbf{R}_a^{(0)}) \hat{\Psi}_i(a) \hat{\Psi}_k^+(a) = f^h \rho_{ik}(\mathbf{x}),$$

where  $f^{1/2} = \langle \exp i\mathbf{p} \cdot \delta\mathbf{R} \rangle$  and  $\rho_{ik}(\mathbf{x})$  differs from Eq. (10) in replacement of  $\mathbf{R}_a$  by  $\mathbf{R}_a^{(0)}$ . The possibility of separating the Mössbauer effect probability  $f$  as a factor is due to the fact that  $\langle \exp(i\mathbf{p} \cdot \delta\mathbf{R}_a) \rangle$  does not depend on  $a$ .

The operator equations of motion obtained with use of the intrinsic Hamiltonian of the nuclei in the fields  $H_0$  and  $H_{\text{int}}$  of Eq. (11) have the form

$$\begin{aligned} \dot{\hat{\rho}}_i &= \left(1 + \frac{2I_2 + 1}{2I_1 + 1}\right) \frac{f^h}{\hbar\omega_{21}} \hat{\mathbf{p}} \hat{A}, \\ \dot{\hat{\mathbf{p}}} + \omega_{21} \hat{\mathbf{p}} &= -(c^2 \Gamma_{21} / 2\hbar) f^h \hat{A} \hat{\rho}_i, \\ \square \hat{A} &= (4\pi/c) f^h \hat{\mathbf{p}} - \hat{A} / c l_-, \end{aligned} \quad (12)$$

where  $\hat{\rho}_i$  and  $\hat{\mathbf{p}}$  are defined as

$$\begin{aligned} \hat{\rho}_i &= -(2I_2 + 1) \text{Sp} [\sigma_i \hat{\rho}(\mathbf{x})] = (2I_2 + 1) [\rho_{22}(\mathbf{x}) - \rho_{11}(\mathbf{x})] = n_2 - \frac{2I_2 + 1}{2I_1 + 1} n_1, \\ \hat{\mathbf{p}} &= \text{Sp} [j\mathbf{p}(\mathbf{x})], \end{aligned} \quad (13)$$

and  $\hbar\omega_{21}$  is the energy of the transition. The last term in the lower equation (12) is introduced to take into account in addition the volume absorption of  $\gamma$  rays. The square of the matrix element of the electromagnetic transition operator in the second equation is expressed in terms of the  $\gamma$ -transition probability<sup>3)</sup>  $\Gamma_{21}$ . In the case in which  $\gamma$ -ray conversion is important, it is necessary in addition to add damping in Eqs. (12) for  $\hat{\rho}_i$  and  $\hat{\mathbf{p}}$ .

For sufficiently intense fields in which stimulated emission is dominant over spontaneous emission, the quantities occurring in the system (12) can be considered classical. Just this situation exists for radiation in the laser regime, while the initiating stage of the shower must be discussed quantum-mechanically.

The system (12) has the following integral of motion:

$$|\dot{\mathbf{p}}|^2 + \omega_{21}^2 |\mathbf{p}|^2 = -\rho_i^2 \Gamma_{21} \omega_{21} c^3 / 4 \left(1 + \frac{2I_2 + 1}{2I_1 + 1}\right) + \text{const.} \quad (14)$$

In the quantities  $\mathbf{A}$  and  $\mathbf{p}$  we will separate the rapid dependence on the coordinates and time in the form

$$\mathbf{A} = \alpha_k \exp(-i\omega_{21}t + ikx), \quad \mathbf{p} = \pi_k \exp(-i\omega_{21}t + ikx), \quad (15)$$

assuming that the wave is propagated along the  $x$  axis. Then Eq. (14) is converted to direct coupling. The latter can be taken into account explicitly by transforming to the variable  $u_k$ :

$$\begin{aligned} \rho_i &= \rho_i(0) \cos u_k; \\ \pi_k &= \nu_k \left[ \frac{c^3}{8[1 + (2I_2 + 1)/(2I_1 + 1)]} \frac{\Gamma_{21}}{\omega_{21}} \rho_i^2(0) \right]^{1/2} \sin u_k, \end{aligned} \quad (16)$$

where  $\nu_k$  is the unit polarization vector and  $\rho_i^{(0)}$  is the initial population inversion. Substituting (16) into Eqs. (12), we have<sup>[14]</sup>

$$\frac{1}{c} \ddot{u}_k + \frac{\partial \dot{u}_k}{\partial x} = \frac{1}{2c\tau^2} \sin u_k - \frac{\dot{u}_k}{2L_-}, \quad (17)$$

where

$$1/c\tau^2 = (\pi c^2 \Gamma_{21} / \hbar \omega_{21}^2) f \rho_1(0).$$

Equation (17) is the basic equation describing the interaction of an intense electromagnetic field with a resonant medium. Its general solution is unknown, and therefore we will discuss possible approximations and the qualitative nature of the solutions.

The simplest case is an infinite uniform medium in which the quantity  $u_k$  depends only on time. Here Eq. (17) is identical to the equation of motion of a physical pendulum with friction proportional to the velocity. Two modes of motion are possible: If the friction is small,  $L \gg c\tau$ , it can be neglected, and then the equation is easily integrated.<sup>[14]</sup> In the opposite case of strong friction,  $L \ll c\tau$ , it is possible to drop the term with "acceleration"  $\ddot{u}_k$ . The solution takes the form:

$$\sin u_k = 2 \operatorname{tg} \frac{u_0}{2} \exp\left(\frac{L}{c\tau^2} t\right) / \left[1 + \operatorname{tg}^2 \frac{u_0}{2} \exp\left(\frac{2L}{c\tau^2} t\right)\right]. \quad (18)$$

Just this regime of "slipping" of the pendulum, an equilibrium between radiation and absorption, corresponds to a  $\gamma$  laser. The quantity  $u_0$  in Eq. (18) is the initial value of  $u_k$ . For  $u_0 = 0$  the pendulum is in a position of unstable equilibrium. Near the turning point the classical approach is not valid and instead of (17) we must use the quantum equations (12). However, the classical equation (17) can be used if we take as initial the level of the field from spontaneous radiation.

If the radiating system has the form of an infinitely long and thin rod, the principal radiation occurs in the longitudinal direction. It is clear that the discussion given above is valid for longitudinal modes. For a finite rod length  $L$  it is necessary to take into account the dependence of  $u_k$  on the lateral coordinates. The mode of radiation depends on the relations between the parameters  $c\tau$ ,  $L$ , and  $L$ . In contrast to ordinary lasers, where  $L \gg L \gg c\tau$ , in a  $\gamma$  laser the relation is the reverse:  $c\tau \gg L \gtrsim L$ . If we estimate the term with the spatial derivative in (17) as  $\dot{u}_k/L$ , we see that in the case  $L \gg L$  both terms in the left-hand side of (17) can be dropped, so that the radiation is similar to the case of an infinite medium. Therefore a  $\gamma$  laser does not require a mirror, as has been noted by Chirikov.<sup>[1]</sup> Furthermore, for the same reason we should not expect an amplification of the radiation intensity with increase of the length  $L$  for  $L > L$ , in contrast to the conclusions of Khokhlov and Il'inskiĭ.<sup>[3]</sup>

In the initial stage of development of the process,  $u_0 \ll u_k \ll 1$ , we neglect the nonlinearity of Eq. (17). This corresponds to neglecting the change in population. As was pointed out above, for  $L \ll c\tau$  it is possible to drop also the term  $\ddot{u}_k$ . The equation obtained is identical to Chirikov's equation obtained from perturbation theory. The solution found in ref. 1 of this equation with special initial conditions is consistent with the estimates given above.

Our discussion has been carried out for a continuous medium without taking into account the crystal structure of the radiator. Since the wavelength  $\lambda \lesssim a$  ( $a$  is the lattice constant), we can expect in principle that the process will depend on the orientation of the radiation relative to the crystal axes. If the Bragg condition is satisfied it is possible to create a waveguide, i.e., a traveling wave squeezed between crystal planes. How-

ever, its development is no different from the case of a longitudinal wave discussed above if  $L$  is replaced by  $L/\cos \theta$ , where  $\theta$  is the Bragg angle.

#### 4. NECESSARY EXPERIMENTS

As follows from the estimates presented, creation of a  $\gamma$  laser is possible in principle. It is necessary first of all to check the possibility of practical realization of the parameters used in the estimates made. At the present time the following experiments are the most important:

- 1) Observation of the Mössbauer effect with a long-lived isomer (of the type of the experiment of Bizina et al.<sup>[6]</sup>) with use of artificial line narrowing. Achievement of the maximum possible line narrowing of the radiation is fundamentally important not only for the problem of the  $\gamma$  laser, but also for increasing the accuracy of the Mössbauer technique.
- 2) Experiments on the population of working levels of nuclei through higher-lying levels in  $\gamma$ -ray bombardment. Note that in ordinary Mössbauer sources the working level is populated in the decay of some long-lived state, so that the intensity of these sources is low. In the proposed three-level scheme, on the contrary, the population occurs through the state with the largest possible width. Therefore the drop in intensity of a source irradiated by a pulse of synchrotron radiation (characteristic duration of a burst  $10^{-9}$  sec) is determined by the working transition itself. This arrangement is a frequency transformer which transforms the incident radiation in the band of frequencies of the upper level to a narrower frequency band determined by the width of the working level. It should be noted that synchrotron radiation is conveniently different in its background conditions from the pulsed bombardment by protons or neutrons which excite a large number of nuclear levels. These experiments can be begun in accelerators which already exist. For example, the radiation of the storage ring VEPP-3 permits obtaining the isotopes <sup>161</sup>Dy and <sup>145</sup>Nd, which have a suitable three-level scheme with very low energies. Increasing the energy of the accelerators will permit extending the "nuclear lumino-phor" technique described to a wide class of isotopes.

- 3) Experiments on observation of the stimulated radiation effect for active-nucleus concentrations much less than the critical concentration necessary for the generation regime. This can be done, for example, by measuring a small change in the  $\gamma$ -ray absorption cross section as a function of the concentration of excited nuclei.

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<sup>1</sup>There are no storage rings with these parameters at the present time. The operating storage rings closest to meeting these requirements at the present time are VEPP-4, DORIS (West Germany), and SPEAR (USA), the latter now working at a lower energy. The achievement of such a small beam size is extremely problematical.

<sup>2</sup>The estimate given in ref. 2 of the role of screw dislocations appears to us to be incorrect: The local energy shifts are proportional not to the lattice shifts but to their gradients.

<sup>3</sup>For unpolarized nuclei. If the radiation occurs in a magnetic field, the coefficients are somewhat changed.

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