An investigation of a class of gravitational fields for a charged dustlike medium

I. S. Shikin

Moscow State University, Mechanics Institute
(Submitted February 20, 1974)
Zh. Eksp. Teor. Fiz. **67**, 433–440 (August 1974)

Gravitational fields with metrics admitting three-parameter groups of motion on two-dimensional manifolds of positive, negative, and zero curvature (spherical, pseudospherical, and flat symmetry) are discussed within the framework of general relativity. The source of the gravitational field is electrically charged dustlike matter. The elements of the medium have a 4 acceleration as a consequence of the electric ponderomotive force which is present. The consequences of the Einstein–Maxwell equations are analyzed qualitatively. Exact solutions in closed form involving arbitrary functions have been obtained for two cases. The asymptotic behavior near the singularities and for large times are discussed, as well as the conditions under which the solutions become asymptotically isotropic.

In the general theory of relativity (GTR) gravitational fields with spherical, pseudospherical and flat symmetry are of particular interest. Such gravitational fields, which have as their source electrically charged dustlike matter in a homogeneous magnetic field, admit of an analytic treatment. For uncharged dustlike matter in a magnetic field such an investigation was carried out before[1]. In the present paper a similar investigation is carried out for charged matter.

The solutions under consideration, which contain a generalization of the Tolman solution[2], are of interest as interior solutions in the study of the gravitational collapse of a charged dustlike sphere (or of a layer of the appropriate symmetry). On the other hand, these solutions can be treated as inhomogeneous cosmological models[1,3], which for a special choice of the arbitrary functions become asymptotically isotropic. As applied to inhomogeneous models the solutions under consideration with arbitrary charge distributions (in a shape with the appropriate symmetry) are models for a Universe which is electrically neutral on the average but has locally possible charge distributions. We note that Bondi and Lyttleton[4] have considered the possibility of a slight difference between the charges of the electron and of the proton, which manifests itself on a metagalactic scale.

An essential peculiarity of the gravitational fields under discussion is the presence of a ponderomotive force and of a 4-acceleration, which lead, in particular to a new asymptotic behavior near the singularity. In different special cases the dynamics of a charged dustlike medium in GTR has been considered by various authors[5-13]. Below we consider the general formulation of the problem and its analysis in the presence of a homogeneous magnetic field; analytic expressions have been obtained for a large class of cases.

1. Homogeneous anisotropic cosmological models, defined as gravitational fields which admit transitive groups of motions on three-dimensional manifods, are well known[2,14,15]. In addition to these, interest attaches also to gravitational fields of type D, which admit of multiply-transitive groups of motion on two-dimensional invariant manifolds V_2[1,3,14]. Such gravitational fields with three-parameter groups G_3 on spacelike V_2 are inhomogeneous models from the point of view of the mentioned group-theoretical criterion, and are the subject of this paper. The metric in this case has the form (we make use of the notations from[2])

$$ds^2=T^2(x^1,\tau)(cd\tau)^2-X^2(x^1,\tau)(dx^1)^2 -Y^2(x^1,\tau)[(dx^2)^2+f^2(x^2)(dx^3)^2] \quad (1.1)$$

with T = 1 [1,14]. The function $f(x^2)$ can take on the forms

$$f(x^2)=\operatorname{sh} x^2,\quad f(x^2)=\sin x^2,\quad f(x^2)=1 \quad (1.2)$$

respectively for V_2 of negative, positive, and zero curvature (pseudospheric, spheric and flat symmetry).

The source of the gravitational field in the Einstein equation

$$R_i^k-(R/2)\delta_i^k=(8\pi k/c^4)T_i^k \quad (1.3)$$

is electrically charge dustlike matter in an electromagnetic field, so that the right-hand side of (1.3) is formed by a superposition of components for dust and electromagnetic field:

$$T_{ik}=\mu_0c^2u_iu_k+T_{ik}^{(2)},\quad u^iu_i=1. \quad (1.4)$$

For the initial metric (1.1) with T = 1 the components u^0 and u^1 of the four-velocity are nonzero[1]; by means of a transformation of x^1 and τ the frame becomes comoving ($u^\alpha = 0$), with the metric (1.1). In the sequel we consider the reference frame comoving with the metric (1.1).

The electric 4-current j^i is the convective current of the charged particles of the medium with the invariant charge density ρ_0^e:

$$j_i=\rho_0^ecu_i.$$

The conservation laws according to (1.3) and (1.4) yield the momentum equations

$$\mu_0c^2u^ku_{i;k}=-T_{i;k}^{k\,(2)}=\mathscr{F}_{ik}j^k/c \quad (1.5)$$

and the continuity equation for the matter 4-current

$$u_iT_{i;k}^{k}\equiv c^2(\mu_0u^k)_{;k}=0,$$

from which we obtain for the matter density

$$\mu_0=\Phi_0(x^1)/XY^2. \quad (1.6)$$

For the description of the electromagnetic field we make use of the 4-vectors e^i and h^i defined according to[16,1]

$$e_i=u^k\mathscr{F}_{ki},\quad h_i=E_{kilm}u^k\mathscr{F}^{lm}/2,\quad e^iu_i=h^iu_i=0,$$
$$e^ie_i=-E^2,\quad h^ih_i=-H^2 \quad (1.7)$$

(here $\mathscr{F}_{ik}$ is the electromagnetic field tensor and

E_{iklm} is the antisymmetric unit tensor[2]). In the case under consideration with the comoving frame (1.1) we have, taking (1.7) into account[1]:

$$e_i=EX\delta_i^1,\quad h_i=HX\delta_i^1,\quad u_i=T\delta_i^0,\quad j_i=c\rho_0^e T\delta_i^0. \tag{1.8}$$

The Maxwell equations[16,1]

$$\mathcal{F}^{ik}{}_{;k}\equiv(u^ie^k-u^ke^i-E^{iklm}u_lh_m)_{;k}=-4\pi j^i/c,\quad j^i{}_{;i}=0,$$
$$(u^ih^k-u^kh^i+E^{iklm}u_le_m)_{;k}=0$$

together with (1.8) yield (K_1 = const, $\Psi(x^1)$ is an arbitrary function)

$$E=\Psi(x^1)/Y^2,\quad 4\pi\rho_0^e XY^2=\Psi'(x^1),\quad H=K_1/Y^2. \tag{1.9}$$

According to (1.7) we have for the ponderomotive force in (1.5)

$$-T^{k\,(2)}_{i;\,k}=-\rho_0^e e_i=-\rho_0^e EX\delta_i^1. \tag{1.5a}$$

The energy-momentum tensor of the electromagnetic field in (1.4)[16,1]

$$T_{ik}{}^{(2)}=\frac{1}{4\pi}\left[\left(u_iu_k-\frac{1}{2}g_{ik}\right)(E^2+H^2)-e_ie_k-h_ih_k\right]$$

has then the following nonvanishing components

$$T_0^{0(2)}=T_1^{1(2)}=-T_2^{2(2)}=-T_3^{3(2)}=c^4\Psi_1{}^2(x^1)/8\pi kY^4,$$
$$\Psi_1{}^2(x^1)\equiv k(K_1{}^2+\Psi^2)/c^4. \tag{1.10}$$

In the kinematic decomposition[17]

$$u_{i;k}=\Theta_{ik}+\omega_{ik}+(u^lu_{i;l})u_k,\quad \Theta_{ik}=\Theta_{ki},\quad \omega_{ik}=-\omega_{ki},\quad \omega_{ik}u^k=0$$

for (1.1), (1.8) the rotation tensor ω_{ik} = 0, and the nonvanishing components are

$$\Theta_1{}^1=h_1/cT,\quad \Theta_2{}^2=\Theta_3{}^3=h_2/cT;\quad u^lu_{1;\,l}=-\lambda_0,$$
$$h_1=\dot X/X,\quad h_2=\dot Y/Y,\quad \lambda_0=T'/T \tag{1.11}$$

(differentiation with respect to τ is denoted by a dot, differentiation with respect to x^1, by a prime) so that there is an isotropic expansion, a displacement, and a 4-acceleration.

The i = 1, k = 0 component of (1.3), taking into account (1.8), (1.10) has the form

$$cR_{01}=2[\lambda_2(h_1-h_2)+\lambda_0h_2-\dot\lambda_2]=0,$$
$$\lambda_1=X'/X,\quad \lambda_2=Y'/Y,\quad h_0=\dot T/T. \tag{1.12}$$

The i = k = 1 component of the equations (1.3) can be written in the form

$$\frac{1}{c^2T^2}(2\dot h_2+3h_2{}^2-2h_2h_0)-\frac{1}{X^2}(\lambda_2{}^2+2\lambda_0\lambda_2)-\frac{1}{Y^2}\frac{d^2f}{f(dx^2)^2}=\frac{8\pi k}{c^4}T_1{}^1. \tag{1.13}$$

The i = k = 0 component of (1.3) has the form

$$\frac{1}{c^2T^2}h_2(2h_1+h_2)+\frac{1}{X^2}(-2\lambda_2'+2\lambda_1\lambda_2-3\lambda_2{}^2)-\frac{1}{Y^2}\frac{d^2f}{f(dx^2)^2}=\frac{8\pi k}{c^4}T_0{}^0. \tag{1.14}$$

The i = k = 2, 3 components of (1.3) are satisfied if (1.3) and (1.5) are satisfied for i = 1[1]. Taking into account (1.5a), (1.6)—(1.9) we obtain from (1.5) (cf. (1.11))

$$T'/T=\chi(x^1)X/Y^2,\quad \chi(x^1)\equiv\Psi'(x^1)\Psi(x^1)/4\pi\Phi_0(x^1)c^2. \tag{1.15}$$

Integrating (1.12), taking (1.15) into account, leads to

$$X=Y\dot Y'/[\psi(x^1)Y-\chi(x^1)];\quad \psi(x^1)\equiv[\varphi(x^1)-d^2f/f(dx^2)^2]^{1/2}. \tag{1.16}$$

Integrating (1.13) once and taking (1.16) and (1.10) into account yields

$$(Y\dot Y/c)^2=T^2\{\varphi(x^1)Y^2+F_1(x^1)Y+[\chi^2(x^1)-\Psi_1{}^2(x^1)]\} \tag{1.17}$$

with an arbitrary function $F_1(x^1)$.

Substitution of (1.15)—(1.17) into (1.14), account being taken of (1.10), yields in (1.6)

$$\Phi_0(x^1)=c^2[F_1(x^1)+2\chi(x^1)\psi(x^1)]'/8\pi k\psi(x^1). \tag{1.18}$$

2. The possible regions of variation of Y are determined by the condition that the right-hand side of (1.17) be nonnegative. Introducing the determinant

$$D(x^1)\equiv F_1{}^2(x^1)+4\varphi(x^1)[\Psi_1{}^2(x^1)-\chi^2(x^1)], \tag{2.1}$$

we obtain then five possible cases:

$$D(x^1)<0,\quad \varphi(x^1)>0, \tag{2.2a}$$
$$D(x^1)>0,\quad \varphi(x^1)>0, \tag{2.2b}$$
$$D(x^1)=0,\quad \varphi(x^1)>0, \tag{2.2c}$$
$$D(x^1)>0,\quad \varphi(x^1)<0, \tag{2.2d}$$
$$\varphi(x^1)=0. \tag{2.2e}$$

Taking into account (1.6), (1.15), (1.9) and (1.10) we have in (2.1) and (1.17)

$$c^4(\Psi_1{}^2-\chi^2)=kK_1{}^2+\Psi^2[k-(\rho_0{}^e/\mu_0)^2]. \tag{2.3}$$

In the absence of a magnetic field (K_1 = 0) the quantity (2.3) is positive for $(\rho_0^e/\mu_0)^2 < k$ (in this case there exist two horizon surfaces for the Reissner-Nordstrøm metric[18]) and is negative for $(\rho_0^e/\mu_0)^2 > k$ (then there are no horizon surfaces for the Reissner-Nordstrøm metric). In the Newtonian case for a spherically symmetric uniform charge distribution, for $(\rho_0^e/\mu_0)^2 < k$, the gravitational force dominates over the electrostatic repulsion force.

In the case when there is no charge (χ = 0), (2.2a) and (2.2c) are not realized[1].

According to (1.17) in the case (2.2a) the quantity Y can vary from $-\infty$ to $+\infty$ passing through the value Y = 0. In the cases (2.2b) and (2.2d) there are two roots $Y_1(x^1)$ and $Y_2(x^1)$ in the right-hand side of (1.17); for (2.2b) there are two regimes: $Y \le Y_1$ and $Y \ge Y_2$, whereas for (2.2d) we have $Y_1 \le Y \le Y_2$. The quantity Y does not vanish for (2.2d) for positive values of (2.3). In the case (2.2b) Y does not vanish in both regimes for $\Psi_1^2 - \chi^2 > 0$ and in one of the regimes for $\Psi_1^2 - \chi^2 \le 0$. In the case (2.2c) we have $Y_1 = Y_2$. Then (for $F_1 \ne 0$) there are two regimes with $Y < Y_1 = Y_2$ and $Y > Y_1 = Y_2$, and according to (1.17) for $Y \to Y_1 = Y_2$ we have $|\tau| \to \infty$ (for T → const); in one of these regimes Y does not vanish for positive (2.3). We note that for all cases (2.2) and positive (2.3), Y does not vanish.

In the cases (2.2a)—(2.2e) according to the admissible regions of variation of Y one can introduce a parameter according to

$$Y=[-F_1(x^1)+|D(x^1)|^{1/2}\operatorname{sh}\eta]/2\varphi(x^1), \tag{2.4a}$$
$$Y=[-F_1(x^1)\pm D^{1/2}(x^1)\operatorname{ch}\eta]/2\varphi(x^1), \tag{2.4b}$$
$$Y=[-F_1(x^1)\pm e^\eta]/2\varphi(x^1), \tag{2.4c}$$
$$Y=[-F_1(x^1)+D^{1/2}(x^1)\cos\eta]/2\varphi(x^1), \tag{2.4d}$$
$$Y=F_2(x^1)+{}^1/_4F_1\lambda^2\,(F_1\ne0),\quad F_2(x^1)\equiv(\Psi_1{}^2-\chi^2)/F_1. \tag{2.4e}$$

In each case the solution contains three "physically distinct" arbitrary functions of x^1 corresponding to the arbitrariness in the velocity distributions (along x^1) and the distributions of the matter and charge densities. In general one cannot integrate (1.15)—(1.17) further. Below we consider two special cases.

3. We consider the special case in (1.16)[1)]:

$$\psi(x^1)=0,\quad \varphi(x^1)=d^2f(x^2)/f(dx^2)^2. \tag{3.1}$$

According to the meaning of (1.8) in the case (3.1) we should have $F_1(x^1)$ = const = F_0. From (1.15)—(1.16) with the condition (3.1) we have up to a normalization constant

$$T=1/Y,\quad X=-Y\dot Y'/\chi(x^1). \tag{3.2}$$

According to (3.1). (1.2) and (2.2) to the value $f(x^2)$

$= \sinh x^2$ in (1.1) correspond the cases (2.4a), (2.4b), (2.4c), to $f(x^2) = \sin x^2$ corresponds the case (2.4d) and to $f(x^2) = 1$ corresponds the case (2.4d).

In all cases (2.4) we have from (1.17)

$$\pm c\,d\tau = Y^2 d\eta, \quad \pm c\,d\tau = Y^2 d\lambda. \tag{3.3}$$

The dependence of τ on x^1 and a parameter, calculated from (3.3) and (3.2), are given in the Appendix.

We consider the asymptotic formulas near the singular state. Near the singularity $\tau = \tau^*(x^1)$ with $Y = 0$, according to (1.17) we have[2)]

$$\begin{gathered} Y \approx [\pm 3c\Phi(x^1)(\tau-\tau^*)]^{1/3}, \quad XY^2 \approx (\tau-\tau^*)^{1/3}\Delta \\ \text{for } \Psi_1^2-\chi^2 \equiv -\Phi^2(x^1) < 0; \\ Y \approx [5cF_1^{1/2}(\tau-\tau^*)/2]^{2/5}, \quad XY^2 \approx (\tau-\tau^*)^{3/5}\Delta \\ \text{for } \Psi_1^2-\chi^2 = 0, \quad F_1 \neq 0; \\ Y \approx [\pm 2c\varphi^{1/2}(\tau-\tau^*)]^{1/2}, \quad XY^2 \approx (\tau-\tau^*)\Delta \\ \text{for } \Psi_1^2-\chi^2 = 0, \quad F_1 = 0, \quad \varphi > 0 \end{gathered} \tag{3.4}$$

(here $\Delta \equiv \Phi_1(x^1)d\tau^*/dx^1 + \Phi_2(x^1)(\tau - \tau^*)$). Near the singularity with $X = 0$, $Y \neq 0$, taking into account (3.3) we have

$$X \approx \Phi_3(x^1)[\tau-\tau^*(x^1)]. \tag{3.5}$$

In the cases (2.2a), (2.2b), (2.2d), (2.2e), the quantity X vanishes according to (A.1), (A.2), (A.4), (A.5). In the case (2.2c), according to (A.3), the quantities Y and X do not vanish for finite values of τ (and η) for the solution (2.4c) with the upper sign for $F_0 > 0$, and for the solution (2.4c) with the lower sign for $F_0 < 0$. In these cases for large $|\tau|$ (and $|\eta|$) X tends to a finite value for values of τ (and η) of one sign and $X \to 0$ for values of τ of the opposite sign (correspondingly $Y \to \infty$ and $Y \to Y_1 = Y_2 = -F_0/2$). We note that similar solutions are realized for $\Psi_1^2 - \chi^2 < 0$.

Consider X for $Y \to \infty$. According to (A.1)–(A.3) in the cases (2.2a), (2.2b), (2.2c) for $Y \to \infty$ the quantity X grows like η or remains finite. In the case (2.2e), however, for $Y \to \infty$ and $F_0 \neq 0$, $F_2' \neq 0$, taking into account the transformation $dt = T\,d\tau$ we have

$$Y \propto t^{2/3}, \quad X \approx YF_2'/3\chi. \tag{3.6}$$

Making use of the admissible transformations of x^1 one can assume that in (3.6) $X \simeq Y \propto t^{2/3}$. Thus, in the case $\Psi = 0$ the solutions (2.4e), (A.5) with $f(x^2) = 1$ for $F_0 \neq 0$, $F_a' \neq 0$ tend for $|\tau| \to \infty$ to the flat Friedmann model.

Comparing with the case of absence of charge[23], we note that for $\chi = 0$ if $\psi = 0$ we have $X = X(\tau)$ and $Y = Y(\tau)$[1]. In this situation only the cases (2.2b). (2.2d), (2.2e) can be realized[23], with $\chi(\tau)$ determined by (1.14). The quantity Y is positive and does not vanish.

4. For $\psi \neq 0$ it is possible to carry out the integration explicitly if one selects the function $\psi(x^1)$ in the form

$$\psi(x^1) = \chi(x^1)/k_1, \quad k_1 = \text{const}.$$

For $k_1 = 0$ this reduces to the charge-free case[1]. Taking into account the admissible transformation of τ it then follows from (1.15) that

$$T = (Y-k_1)/Y. \tag{4.1}$$

Substitution of (2.4) and (4.1) into (1.17) leads to the relations

$$\pm c\frac{\partial\tau}{\partial\eta} = \frac{Y^2}{(Y-k_1)|\varphi|^{1/2}}, \quad \pm c\frac{\partial\tau}{\partial\lambda} = \frac{Y^2}{Y-k_1},$$

the integration of which leads to the functions $\tau(\eta, x^1)$ and $\tau(\lambda, x^1)$.

5. For the original groups G_3 with (1.1–1.2) the algebras of their generators can be extended to G_4 and G_6, as shown in[1,3], corresponding to a specialization of the functions in (1.1). The extension to G_6 corresponds to a transition from (1.1), (1.2) to the Friedmann models[1,3].

In the presence of a charge of the medium the extension to G_4 is possible only through a central extension, corresponding to the functions in (1.1) depending on only one variable. Then, on account of (1.15), in the regions with $\chi \neq 0$ the added fourth Killing vector must be timelike and the functions in (1.1) must depend only on x^1, leading to static solutions. According to (1.17) for static solutions the conditions $\dot{Y} = \ddot{Y} = 0$ imply that $D(x^1) = 0$; then for $\Psi_1^2 - \chi^2 \neq 0$ we have $Y = Y_1 = Y_2 = -F_1/2\varphi$, and for $\Psi_1^2 - \chi^2 = 0$ one must set $F_1 = 0$, $\varphi = 0$.

For a charged medium the specialization of (1.1), (1.2) to a form corresponding to the Friedmann models (extension to G_6) is not possible in the whole region of the parameters as a consequence of the anisotropy of T_{ik}, but can be achieved asymptotically, corresponding to asymptotic isotropization. In regions with $\varphi \neq 0$ we have as $Y \to \infty$, according to (1.15)–(1.16), $T \to T(\tau) \times \exp(-\chi/\psi Y)$ and taking into account the admissible transformation of τ we get $T \to 1$. According to (1.17), (1.17) the behavior of Y and X as $Y \to \infty$ is determined by $\varphi(x^1)$ for $\varphi \neq 0$ and $F_1(x^1)$ for $\varphi = 0$, so that the presence of charge does not influence the leading terms of the asymptotic behavior for $Y \to \infty$ (with the exception of the case $\varphi = 0$, $F_1 = 0$, which does not lead to isotropization). The choice of the arbitrary function which leads to asymptotic isotropization, to the open and flat Friedmann models for $\psi \neq 0$, not related to the value of is determined by Eqs. (3.27)–(3.30) of ref.[1] (the case $\psi = 0$ is considered above in Section 3).

APPENDIX

The dependence of τ and $X = -YY'/\chi(x^1)$ on x^1 and a parameter determined by (2.1), (2.2) and (3.1)–(3.3) for the case $\psi = 0$ is of the form ($\tau_0(x^1)$ is an arbitrary function)

a) $f(x^2) = \operatorname{sh} x^2$, case (2.4a), $\varphi(x^1) = 1$:

$$\begin{gathered} \pm c[\tau-\tau_0(x^1)] = [2(2F_0^2+D)\eta - 8F_0|D|^{1/2}\operatorname{ch}\eta - D\operatorname{sh}2\eta]/16, \\ 16\chi(x^1)X = \{2F_0D' - 2[|D|^{1/2}]'(F_0^2+D)\operatorname{sh}\eta + |D|^{1/2}(\pm 8c\tau_0' + D'\eta)\operatorname{ch}\eta\}/Y; \end{gathered} \tag{A.1}$$

b) $f(x^2) = \operatorname{sh} x^2$, case (2.4b), $\varphi(x^1) = 1$:

$$\begin{gathered} c[\tau-\tau_0(x^1)] = [2(2F_0^2+D)\eta \mp 8F_0D^{1/2}\operatorname{sh}\eta + D\operatorname{sh}2\eta]/16, \\ 16\chi(x^1)X = [2F_0D' \mp 2(D^{1/2})'(F_0^2+D)\operatorname{ch}\eta \pm D^{1/2}(8c\tau_0' + D'\eta)\operatorname{sh}\eta]/Y; \end{gathered} \tag{A.2}$$

c) $f(x^2) = \operatorname{sh} x^2$, case (2.4c), $\varphi(x^1) = 1$:

$$\begin{gathered} \pm c[\tau-\tau_0(x^1)] = (e^{2\eta} \mp 4F_0e^{\eta} + 2F_0^2\eta)/8, \\ X = \pm c\tau_0'(2Y+F_0)/2\chi(x^1)Y; \end{gathered} \tag{A.3}$$

d) $f(x^2) = \sin x^2$, case (2.4d), $\varphi(x^1) = -1$:

$$\begin{gathered} c[\tau-\tau_0(x^1)] = [2(2F_0^2+D)\eta - 8F_0D^{1/2}\sin\eta + D\sin 2\eta]/16, \\ 16\chi(x^1)X = [-2F_0D' + 2(D^{1/2})'(F_0^2+D)\cos\eta + D^{1/2}(8c\tau_0' + D'\eta)\sin\eta]/Y; \end{gathered} \tag{A.4}$$

e) $f(x^2) = 1$, case (2.4e), $\varphi = 0$:

$$\begin{gathered} c[\tau-\tau_0(x^1)] = F_2^2\lambda + (F_0F_2/6)\lambda^3 + (F_0^2/80)\lambda^5, \\ 3\chi(x^1)X = [F_0^2F_2'\lambda^4 + 24F_0F_2F_2'\lambda^2 + 24F_0c\tau_0'\lambda - 48F_2^2F_2']/16Y \\ = F_2'Y + 4F_2F_2' + Y^{-1}[(3/2)F_0c\tau_0'\lambda - 8F_2^2F_2']. \end{gathered} \tag{A.5}$$

For $F_0 = 0$ we have

$$\begin{gathered} Y = \pm(9c^2F_3)^{1/6}[\tau-\tau_0(x^1)]^{1/3}, \\ X = \mp c[F_3'\tau - (F_3'\tau_0 + 2F_3\tau_0')]/2F_3^{1/2}\chi Y, \end{gathered}$$

where $F_3(x^1) \equiv \chi^2(x^1) - \Psi_1^2(x^1) > 0$.

[1)]In the absence of charge ($\chi = 0$) solutions of the type (3.1) have been considered in [19-24].

[2)]The asymptotic forms (3.4) and (3.5) near the singularity are, according to (1.15)–(1.17) valid for $\chi \neq 0$ and in the general case of an arbitrary $\psi(x^1)$.

[1] I. S. Shikin, Commun. Math. Phys. 26, 24 (1972).

[2] L. D. Landau and E. M. Lifshitz, Teoriya polya (Classical Theory of Fields), 6-th ed., Nauka, 1973 [Engl. Transl. of earlier editions, Addison-Wesley] Pergamon, various years).

[3] I. S. Shikin, Abstracts of talks at the 3-rd Soviet Gravitation Conference, Erevan, 1972, p. 178.

[4] H. Bondi and R. A. Lyttleton, Proc. Roy. Soc. A252, 313 (1959).

[5] W. Bonnor, Z. Physik 160, 59 (1960); Mon. Not. Roy. Astr. Soc. 129, 443 (1965).

[6] I. D. Novikov, Astron. Zh. 43, 911 (1966) [Sov. Astron. 10, 731 (1967)] Zh. Eksp. Teor. Fiz. 59, 262 (1970) [Sov. Phys.-JETP 32, 142 (1971)].

[7] Y. P. Shah and P. C. Vaidya, Ann. Inst. H. Poincaré A6, 219 (1967).

[8] A. Hamoui, Ann. Inst. H. Poincaré A10, 195 (1969).

[9] M. C. Faulkes, Canad. J. Phys. 47, 1969 (1969).

[10] A. K. Raychaudhuri and U. K. De, Proc. Roy. Soc. A303, 97 (1968).

[11] U. K. De, Prog. Theor. Phys. 49, 1546 (1973).

[12] M. A. Markov and V. P. Frolov, Teor. Matem Fiz. 3, 3 (1970).

[13] P. A. Vickers, Ann. Inst. H. Poincaré A18, 137 (1973).

[14] A. H. Taub, Ann. Math. 53, 472 (1951).

[15] G. F. R. Ellis and M. A. H. MacCallum, Commun. Math. Phys. 12, 108 (1969). A. R. King and G. F. R. Ellis, Commun. Math. Phys. 31, 209 (1973).

[16] A. Lichnerowicz, Relativistic Hydrodynamics anf Magnetohydrodynamics, Benjamin, N. Y. 1967.

[17] Gravitation: an Introduction to Current Research, ed. by L. Witten Wiley, N. Y., 1962, Ch. 2.

[18] R. Penrose, The Structure of Space-Time, in Batelle Rencontres 1967, edit. by C. De Witt and J. A. Wheeler, Benjamin, N. Y. 1968 [Russ. Transl., Mir, 1972].

[19] A. S. Kompaneets and A. S. Chernov, Zh. Eksp. Teor. Fiz. 47, 1939 (1964) [Sov. Phys.-JETP 20, 1303 (1965)].

[20] R. Kantowski and R. K. Sachs, J. Maht. Phys. 7, 443 (1966).

[21] V. A. Ruban, Zh. Eksp. Teor. Fiz. 56, 1914 (1969) [Sov. Phys.-JETP 29, 1027 (1969)].

[22] A. G. Doroshkevich, Astrofizika 1, 255 (1965).

[23] I. S. Shikin, Dokl. Akad. Nauk SSSR 171, 73 (1966) [Sov. Phys.-Doklady 11, 944 (1967)].

[24] K. S. Thorne, Astrophys. J. 148, 51 (1967).

Translated by Meinhard E. Mayer
50