A kinetic equation for a plasma in a high-frequency field

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The charged-particle collision integral has been obtained for a plasma placed in a high-frequency electric field with a frequency close to the plasma frequency. It is shown that the high-frequency field has a strong influence on the diffusion coefficient. An anomalous increase of the collision integral on account of the interaction of the particles with the ion sound is observed in a very narrow region near the threshold value of the intensity of the external field, and the interaction of the fast particles with Langmuir waves can lead to a considerable increase of the collision integral even far away from the threshold. The electron-diffusion coefficient of a weakly turbulent plasma with stationary level of the noise of the ion sound is calculated.

1. Theoretical investigations [1] carried out recently have indicated the possibility of a parametric instability developing in a plasma under the action of electromagnetic radiation on it. In order to exhibit the laws governing the relaxation processes and transport phenomena induced by the action of radiation on a plasma, it is necessary to construct a theory of the collision integral for the charged particles, which makes up the content of the present paper.

It is known [1-3] that the parametric instabilities which develop in a plasma lead to an anaomalous increase of the fluctuations of the internal fields. Starting from known results [2,3] corresponding to taking into account the Coulomb interaction, one can express the collision integral in terms of the fluctuations of the longitudinal field. The diffusion coefficient and the coefficient of systematic friction in momentum space have been calculated for a steady level of fluctuations for a completely ionized plasma placed in a spatially homogeneous monochromatic electric field

$$\mathbf{E}(t)=\mathbf{E}_0\sin\omega_0 t \tag{1.1}$$

with frequency ω_0 close to electron Langmuir frequency $\omega_{Le} = (4\pi e^2 n_e/m_e)^{1/2}$. We show that when the speed of oscillations of the electrons in the pumping field is small compared to their thermal velocity:

$$v_E=eE_0/m_e\omega_0\ll v_{Te}=(\varkappa T_e/m_e)^{1/2}, \tag{1.2}$$

then the coefficient of systematic friction does not change and the diffusion coefficient may increase considerably owing to the anomalous behavior of the fluctuation level of the field. Near the threshold the diffusion coefficient increases as $\ln|1 - E_0^2/E_{thr}^2|$ for nonresonant particles and like $|1 - E_0^2/E_{thr}^2|^{-1/2}$ for resonant particles. Accordingly, the anomalous increase of the collision integral in the first case is observed in a very narrow region near the threshold, in distinction from the resonant case, when such an increase can occur in a considerably wider region near the threshold. In this case the contribution of the interaction of the fast particles with high-frequency oscillations to the collision integral always exceeds the contributions of the interaction with the ion sound.

On the other hand it is known [1] that on account of the nonlinear interaction of the disturbances in a turbulent plasma state, occurring as a result of the parametric instability, a stationary level of plasma fluctuations can establish itself, by far exceeding the level of thermal fluctuations. In such a situation of developed fluctuations of the fields we have made explicit the dependence of the electron diffusion coefficient on the plasma parameters and the possibility that it increases compared to the usual diffusion coefficient which is determined by the thermal fluctuations.

2. In order to derive the collision integral we make use of the method of microscopic phase densities [3,4]

$$N_a(\mathbf{r}_a,\mathbf{p}_a,t)=\sum_i\delta(\mathbf{r}_a-\mathbf{r}_{ai}(t))\delta(\mathbf{p}_a-\mathbf{p}_{ai}(t)),$$

where the summation is over all particles of the kind a. Considering only the effects produced by the Coulomb interaction we consider as the starting equations the system of microscopic equations for the functions N_a:

$$\frac{\partial N_a}{\partial t}+\mathbf{v}_a\frac{\partial N_a}{\partial\mathbf{r}_a}+e_a[\mathbf{E}(t)+\mathbf{E}(\mathbf{r}_a,t)]\frac{\partial N_a}{\partial\mathbf{p}_a}=0. \tag{2.1}$$

Here $\mathbf{E}(t)$ is the external radiation field (1.1), $\mathbf{E}(\mathbf{r}, t)$ is the microscopic Coulomb field

$$\mathbf{E}(\mathbf{r},t)=-\frac{\partial}{\partial\mathbf{r}}\sum_b\int d\mathbf{p}_b d\mathbf{r}_b\frac{e_b}{|\mathbf{r}-\mathbf{r}_b|}N_b.$$

We introduce the deviations of the random functions N_a from their averages: $\delta N_a = N_a - \langle N_a\rangle$, where the brackets $\langle\ldots\rangle$ denote the averaging; the $\langle N_a\rangle$ are proportional to the first distribution functions $\langle N_a\rangle = n_a f_a$, where n_a is the average concentration of particles of type a. For a spatially homogeneous plasma the first distribution functions satisfy the equations

$$\frac{\partial f_a}{\partial t}+e_a\mathbf{E}(t)\frac{\partial f_a}{\partial\mathbf{p}_a}=-\frac{e_a}{n_a}\frac{\partial}{\partial\mathbf{p}_a}\langle\delta N_a\mathbf{E}\rangle\equiv J_a, \tag{2.2}$$

which follow directly from the averaging of (2.1). The right-hand side of (2.2), which takes into account the correlation of the Coulomb field and the particle distribution, is the collision integral denoted by J_a. To determine the deviations δN_a, we restrict our attention to the linearized equations

$$\frac{\partial\delta N_a}{\partial t}+\mathbf{v}_a\frac{\partial\delta N_a}{\partial\mathbf{r}_a}+e_a\mathbf{E}(t)\frac{\partial\delta N_a}{\partial\mathbf{p}_a}+e_a n_a\mathbf{E}\frac{\partial f_a}{\partial\mathbf{p}_a}=0. \tag{2.3}$$

In the sequel it will be convenient to replace the functions f_a by the distributions F_a defined by

$$F_a(\mathbf{p}_a,t)=f_a(\mathbf{p}_a+\mathbf{P}_a^E(t),t), \tag{2.4}$$

where $\mathbf{P}_a^E(t)$ are determined from the solution of the characteristic equations

$$\dot{\mathbf{P}}_a^E=e_a\mathbf{E}(t).$$

In searching for the solution of (2.3) we shall assume that the distributions $F_a(\mathbf{p}_a, t)$ are slowly varying functions of time (i.e., that they change little over the correlation time and over the period of oscillations of the external electric field). Taking this fact into account we will in the end obtain equations that express the space-time correlations in terms of the first distribution functions.

We effect a Fourier transformation with respect to the coordinates in Eq. (2.3):

$$\delta N_a(t,\mathbf{k},\mathbf{p}_a)=\int d\mathbf{r}_a\delta N_a(t,\mathbf{r}_a,\mathbf{p}_a)e^{-i\mathbf{k}\mathbf{r}_a}$$

and introduce the new functions:

$$\Psi_a(t,\mathbf{k},\mathbf{p}_a)=\delta N_a(t,\mathbf{k},\mathbf{p}_a+\mathbf{P}_a^E(t))\exp\{i\mathbf{k}\mathbf{R}_a^E(t)\},\quad \dot{\mathbf{R}}_a^E=m_a^{-1}\mathbf{P}_a^E(t).$$

For a monochromatic time dependence of the external field the functions Ψ_a and the longitudinal field E can be represented as expansions

$$\Psi_a(t,\mathbf{k},\mathbf{p}_a)=\sum_{n=-\infty}^{\infty}e^{-in\omega_0 t}\Psi_a^{(n)}(t,\mathbf{k},\mathbf{p}_a),\quad \mathbf{E}(t,\mathbf{k})=\sum_{n=-\infty}^{\infty}e^{-in\omega_0 t}\mathbf{E}^{(n)}(t,\mathbf{k}),$$

where the amplitudes $\Psi_a^{(n)}$ and $\mathbf{E}^{(n)}$ change little over the period of the external field. Using a one-sided Fourier transform with respect to time

$$\Psi_a^{(n)}(\omega,\mathbf{k},\mathbf{p}_a)=\int_0^\infty dt\Psi_a^{(n)}(t,\mathbf{k},\mathbf{p}_a)e^{i\omega t-\Delta t}$$

we obtain the following solution of equation (2.3):

$$\Psi_a^{(n)}(\omega,\mathbf{k},\mathbf{p}_a)=\frac{i}{\omega+n\omega_0+i\Delta-\mathbf{k}\mathbf{v}_a}\Big[\Psi_a^{(n)}(\mathbf{k},\mathbf{p}_a,t=0)+i\frac{4\pi}{k^2}e_a n_a\mathbf{k}\frac{\partial F_a}{\partial\mathbf{p}_a}\sum_{b,m}J_{n-m}(A_{ab})u_b^{(m)}(\omega,\mathbf{k})\Big],\tag{2.5}$$

$$A_{ab}\equiv A_a-A_b=\frac{\mathbf{k}\mathbf{E}_0}{\omega_0^2}\left(\frac{e_a}{m_a}-\frac{e_b}{m_b}\right).$$

Here J_n is the Bessel function of the first kind of integer index n and the functions

$$u_a^{(n)}(\omega,\mathbf{k})=e_a\int d\mathbf{p}_a\Psi_a^{(n)}(\omega,\mathbf{k},\mathbf{p}_a)$$

satisfy the linear system of algebraic equations

$$u_a^{(n)}(\omega,\mathbf{k})+\delta\varepsilon_a(\omega+n\omega_0,\mathbf{k})\sum_{m,b}J_{n-m}(A_{ab})u_b^{(m)}(\omega,\mathbf{k})=ie_a\int\frac{d\mathbf{p}_a}{\omega+n\omega_0+i\Delta-\mathbf{k}\mathbf{v}_a}\Psi_a^{(n)}(\mathbf{k},\mathbf{p}_a,t=0),\tag{2.6}$$

where $\delta\epsilon_a$ is the partial longitudinal dielectric permittivity for particles of the type a:

$$\delta\varepsilon_a(\omega,\mathbf{k})=\frac{4\pi e_a^2n_a}{k^2}\int d\mathbf{p}_a\frac{1}{\omega+i\Delta-\mathbf{k}\mathbf{v}_a}\mathbf{k}\frac{\partial F_a}{\partial\mathbf{p}_a}.$$

The system of equations (2.6) obtained in this manner differs from the one studied in the theory of parametric resonance by the presence of a right-hand side. The Poisson equation allows us to write down the following relations between the Fourier components of the amplitudes of the harmonics of the longitudinal field, $\mathbf{E}^{(n)}(\omega,\mathbf{k})$ and of the charge densities, $u_a^{(n)}(\omega,\mathbf{k})$:

$$i\mathbf{k}\mathbf{E}^{(n)}(\omega,\mathbf{k})=4\pi\sum_{a,m}J_{m-n}(A_a)u_a^{(m)}(\omega,\mathbf{k}).$$

Assuming a slow time dependence of the functions $F_a(\mathbf{p}_a,t)$, we obtain from (2.2)

$$\frac{\partial F_a}{\partial t}=-i\frac{4\pi e_a}{n_a}\sum_{n,m,l,b}\frac{\partial}{\partial\mathbf{p}_a}\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{\mathbf{k}}{k^2}J_{n-l}(A_a)J_{m-l}(A_b)\times\int\frac{d\omega}{2\pi}(\Psi_a^{(n)}u_b^{(m)})_{\omega,\mathbf{k}}\equiv J_a.\tag{2.7}$$

The possibility of defining the collision integral (2.7) in terms of space-time spectral functions $(\Psi_a^{(n)}u_b^{(m)})_{\omega,\mathbf{k}}$ follows from the invariance of the two-time correlation functions:

$$\langle\Psi_a(\mathbf{k},\mathbf{p}_a,t)u_b^*(\mathbf{k}',t')\rangle=(2\pi)^3\delta(\mathbf{k}-\mathbf{k}')(\Psi_a u_b)_{\mathbf{k},t,t'},$$

with respect the transformations

$$t\to t+2\pi n/\omega_0,\quad t'\to t'+2\pi n/\omega_0,\quad n=0,\pm1,\pm2,\dots$$

Indeed, since we consider a correlation process which is stationary in time, a simultaneous shift of the times t and t′ by an integral number of periods of the external field does not change the physical picture, i.e.,

$$(\Psi_a u_b)_{\mathbf{k},t,t'}=(\Psi_a u_b)_{\mathbf{k},t+2\pi n/\omega_0,t'+2\pi n/\omega_0}.\tag{2.8}$$

Thus, from the expansion

$$(\Psi_a u_b)_{\mathbf{k},t,t'}=\sum_{n,m}\exp(-in\omega_0t+im\omega_0t')(\Psi_a^{(n)}u_b^{(m)})_{\mathbf{k},t,t'}$$

and the conditions (2.8) we obtain that the functions $(\Psi_a^{(n)}u_b^{(m)})_{\omega,\mathbf{k}}$ depend only on the time difference $t-t'$. Hence, in particular, follows the applicability of the formulas of the stationary theory[3,4]

$$(2\pi)^3\delta(\mathbf{k}-\mathbf{k}')(\Psi_a^{(n)}u_b^{(m)})_{\omega,\mathbf{k}}=\lim_{\Delta\to0}2\Delta\langle\Psi_a^{(n)}(\omega,\mathbf{k},\mathbf{p}_a)u_b^{*(m)}(\omega,\mathbf{k}')\rangle,\tag{2.9}$$

where the spectral functions can be expressed in terms of the equal-time functions

$$\langle\Psi_a^{(n)}(\mathbf{k},\mathbf{p}_a,t=0)\Psi_b^{*(m)}(\mathbf{k}',\mathbf{p}_b,t=0)\rangle=(2\pi)^3\delta(\mathbf{k}-\mathbf{k}')[J_n(A_a)J_m(A_b)\times n_a\delta_{ab}\delta(\mathbf{p}_a-\mathbf{p}_b)F_a+n_an_bG_{ab}^{(n,m)}].\tag{2.10}$$

For the slowly varying function F_a, expression (2.10) is a consequence of the equation

$$\langle\delta N_a(\mathbf{k},\mathbf{p}_a,t=0)\delta N_b^*(\mathbf{k}',\mathbf{p}_b,t=0)\rangle=(2\pi)^3\delta(\mathbf{k}-\mathbf{k}')[\delta_{ab}\delta(\mathbf{p}_a-\mathbf{p}_b)n_af_a+n_an_bg_{ab}],$$

where $g_{ab}(\mathbf{k},\mathbf{p}_a,\mathbf{p}_b)$ are the pair correlation functions. The correlation functions G_{ab} are introduced in analogy with the functions Ψ_a

$$G_{ab}(t,\mathbf{k},\mathbf{p}_a,\mathbf{p}_b)=g_{ab}(t,\mathbf{k},\mathbf{p}_a+\mathbf{P}_a^E(t),\mathbf{p}_b+\mathbf{P}_b^E(t))\exp\{i\mathbf{k}(\mathbf{R}_a^E(t)-\mathbf{R}_b^E(t))\}=\sum_{n,m}\exp(-in\omega_0t+im\omega_0t)G_{ab}^{(n,m)}.$$

The usefulness of Eq. (2.9) is due to the important simplifying circumstance that in the limit as $\Delta\to0$ the terms which contain $G_{ab}^{(n,m)}$ do not contribute to the spectral function.

We shall be interested in the case of a weak external field, when the inequality (1.2) holds. Such an approximation allows one to retain in the collision integral only three amplitudes of the charge densities, $u_a^{(0)}$ and $u_a^{(\pm1)}$:

$$J_a=\frac{\partial}{\partial p_{ai}}D_{ij}^a\frac{\partial F_a}{\partial p_{aj}}+\frac{\partial}{\partial p_{ai}}A_i^aF_a,\tag{2.11}$$

where the diffusion coefficients D_{ij}^a and the systematic friction coefficients A_i^a in momentum space are defined by

$$D_{ij}^a=\frac{e_a^2}{16\pi^3}\int d\mathbf{k}\frac{k_ik_j}{k^2}\int d\omega\,\delta(\omega-\mathbf{k}\mathbf{v}_a)(\mathbf{E}\mathbf{E})^a_{\omega,\mathbf{k}},\tag{2.12}$$

$$A_i^aF_a=\frac{e_a}{4\pi^3n}\mathrm{Re}\sum_b\int d\mathbf{k}\frac{k_i}{k^2}\int d\omega\Big[\sum_{n=0,\pm1}\left(\frac{\Psi_a^{(n)}(\mathbf{k},\mathbf{p}_a,t=0)}{\omega+n\omega_0+i\Delta-\mathbf{k}\mathbf{v}_a}u_b^{(n)}\right)_{\omega,\mathbf{k}}+\frac{1}{2}A_{ab}\sum_{|n-m|=1}(n-m)\left(\frac{\Psi_a^{(n)}(\mathbf{k},\mathbf{p}_a,t=0)}{\omega+n\omega_0+i\Delta-\mathbf{k}\mathbf{v}_a}u_b^{(m)}\right)_{\omega,\mathbf{k}}.\tag{2.13}$$

In Eq. (2.12) we have used the notation

$$(\mathbf{E}\mathbf{E})^a_{\omega,\mathbf{k}}=\frac{16\pi^2}{k^2}\sum_{b,b'}\Big[\sum_{n=0,\pm1}(u_b^{(n)}u_{b'}^{(n)})_{\omega-n\omega_0,\mathbf{k}}\tag{2.14}$$

$$+A_{ab}\,\mathrm{Re}\sum_{|n-m|=1}(m-n)\,(u_b^{(n)}u_{b'}^{(m)})_{\omega-m\omega_0,\,\mathbf{k}}\Big].$$

We restrict our attention to a plasma consisting of electrons and a single kind of ions, neglecting the effect of the high-frequency field on the ions. Solving the system of equations (2.6) for $A_e^2 \equiv (\mathbf{k}\cdot\mathbf{r}_E)^2 \ll 1$ and using an equation similar to (2.9), we obtain the spectral correlation functions of the harmonics of the charge densities and, consequently, the correlation functions of the Coulomb field (2.14). If the inequality

$$(\mathbf{kr}_E)^2\,\mathrm{Re}\frac{\delta\varepsilon_e(\omega,\mathbf{k})}{1+\delta\varepsilon_e(\omega\pm\omega_0,\mathbf{k})}\ll 1$$

holds, we obtain that the spectral functions (2.14) for the electrons and ions coincide

$$(\mathbf{EE})^e_{\omega,\mathbf{k}}=(\mathbf{EE})^i_{\omega,\mathbf{k}}\equiv(\mathbf{EE})_{\omega,\mathbf{k}},$$

and in the frequency region ω and $\omega \pm \omega_0$ ($|\omega| \ll \omega_0$) they take the following form:

$$(\mathbf{EE})_{\omega,\mathbf{k}}=(\mathbf{E}^{(0)}\mathbf{E}^{(0)})_{\omega,\mathbf{k}}+(\mathbf{E}^{(-1)}\mathbf{E}^{(-1)})_{\omega+\omega_0,\mathbf{k}}+(\mathbf{E}^{(1)}\mathbf{E}^{(1)})_{\omega-\omega_0,\mathbf{k}}, \tag{2.15}$$

$$(\mathbf{EE})_{\omega\pm\omega_0,\mathbf{k}}=(\mathbf{E}^{(0)}\mathbf{E}^{(0)})_{\omega\pm\omega_0,\mathbf{k}}+(\mathbf{E}^{(\pm1)}\mathbf{E}^{(\pm1)})_{\omega,\mathbf{k}}, \tag{2.16}$$

$$\frac{1}{8\pi}(\mathbf{E}^{(0)}\mathbf{E}^{(0)})_{\omega,\mathbf{k}}=\frac{4\pi^2}{k^2}\frac{1}{|\tilde\varepsilon|^2}\sum_a e_a^2 n_a\int d\mathbf{p}_a\delta(\omega-\mathbf{kv}_a)F_a, \tag{2.17}$$

$$\frac{1}{8\pi}(\mathbf{E}^{(0)}\mathbf{E}^{(0)})_{\omega\pm\omega_0,\mathbf{k}}=\frac{4\pi^2}{k^2}\frac{|\varepsilon|^2}{|\tilde\varepsilon|^2|\varepsilon_{\pm1}|^2}\sum_a e_a^2 n_a\int d\mathbf{p}_a\,\delta(\omega\pm\omega_0-\mathbf{kv}_a)F_a, \tag{2.18}$$

$$(\mathbf{E}^{(\pm1)}\mathbf{E}^{(\pm1)})_{\omega,\mathbf{k}}=\frac{1}{4}(\mathbf{kr}_E)^2\frac{|1+\delta\varepsilon_e|^2}{|\varepsilon_{\pm1}|^2}(\mathbf{E}^{(0)}\mathbf{E}^{(0)})_{\omega,\mathbf{k}}, \tag{2.19}$$

$$(\mathbf{E}^{(\pm1)}\mathbf{E}^{(\pm1)})_{\omega\mp\omega_0,\mathbf{k}}=\frac{1}{4}(\mathbf{kr}_E)^2\frac{|1+\delta\varepsilon_e|^2}{|\varepsilon|^2}(\mathbf{E}^{(0)}\mathbf{E}^{(0)})_{\omega\mp\omega_0,\mathbf{k}}. \tag{2.20}$$

Here $\tilde\epsilon(\omega, \mathbf{k})$ is the nonlinear longitudinal dielectric permittivity:

$$\tilde\varepsilon=\varepsilon+{}^1\!/\!_4(\mathbf{kr}_E)^2\delta\varepsilon_i(1+\delta\varepsilon_e)(1/\varepsilon_1+1/\varepsilon_{-1}),$$

$\epsilon(\omega, \mathbf{k})$ is the usual linear permittivity $\epsilon = 1 + \delta\epsilon_e + \delta\epsilon_i$, $\epsilon_{\pm1}$ is the linear dielectric permittivity for the frequency $\omega \pm \omega_0$. In the derivation of Eqs. (2.15)—(2.20) we recognize that the main contribution to the collision integral comes from the zeros of the functions $\mathrm{Re}\,\tilde\epsilon(\omega, \mathbf{k}) \simeq \mathrm{Re}\,\epsilon(\omega, \mathbf{k})$.

A calculation of the correlation functions that enter in (2.13) leads to the following pair of equations for the coefficient of systematic friction respectively in the region of frequencies ω and $\omega \pm \omega_0$ ($|\omega| \ll \omega_0$):

$$A_i^a=\frac{e_a^2}{2\pi^2}\int dk\frac{k_i}{k^2}\int d\omega\,\delta(\omega-\mathbf{kv}_a)\,\mathrm{Im}\frac{1}{\tilde\varepsilon^*}, \tag{2.21}$$

$$A_i^a=\frac{e_a^2}{2\pi^2}\int d\mathbf{k}\frac{k_i}{k^2}\int d\omega\,\delta(\omega\pm\omega_0-\mathbf{kv}_a)\,\mathrm{Im}\frac{\varepsilon^*}{\tilde\varepsilon^*\varepsilon^*_{\pm1}}. \tag{2.22}$$

It follows from (2.21) and (2.22) that a weak high-frequency field does not change the systematic friction coefficient. To the contrary, the diffusion coefficient can increase considerably as a result of the anomalous behavior of the level of electromagnetic fluctuations (2.17)—(2.20). Therefore, in the sequel, in the analysis of the collision integral of a parametrically excited plasma, we shall consider only the diffusion coefficient (2.12). Thus, we finally obtain

$$\frac{\partial F_a}{\partial t}=\frac{\partial}{\partial p_{ai}}D_{ij}^a\frac{\partial F_a}{\partial p_{aj}}=J_a, \tag{2.23}$$

where the diffusion coefficient D_{ij}^a (2.12) is determined by the fluctuation spectrum of the longitudinal field (2.15)—(2.20).

We note that from the kinetic equation (2.23) it follows that the following conservation law holds for the total energy density:

$$\frac{d}{dt}\sum_a n_a\int d\mathbf{p}_aF_a\frac{p_a^2}{2m_a}=\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}\frac{(\mathbf{EE})_{\omega,\mathbf{k}}}{8\pi}\omega\,\mathrm{Im}\,\varepsilon. \tag{2.24}$$

It is easy to understand that the quantity which stands in the right-hand side of Eq. (2.23) represents the heat produced in the plasma per unit time and unit volume. We stress that the heat produced is to a significant degree determined by the external field, since it is the latter which determines the anomalous character of the electromagnetic fluctuations.

3. It is known[5] that in an isothermal plasma ($T_e \gg T_i$) the interaction of particles with the ion-acoustic oscillations can yield a substantial contribution to the collision integral. We therefore consider the collision integral (2.23) that takes into account the dynamical polarization, due to the interaction of the particles with the oscillations, of the nonisothermal plasma placed in a high-frequency electric field. The largest effect of such an interaction appears in the electronic diffusion coefficient. We consider a plasma state which differs little from those characterized by Maxwellian states. Then, substituting the permittivities in which Maxwellian distributions have been used into Eqs. (2.17)—(2.20) we obtain the following approximate expression for the electronic diffusion coefficient, which describes the interaction with the oscillations ($|\epsilon_{-1}|^2 \ll |\epsilon_1|^2$):

$$\begin{aligned}D_{ij}^e=D_{ij}^0+\delta D_{ij}=4(2\pi)^{1/2}v_{Te}e^4n_eI\int d\mathbf{p}'\frac{[\mathbf{vv}']_i[\mathbf{vv}']_j}{|[\mathbf{vv}']|^3}F_e(\mathbf{p}')\\+\pi e^4n_er_{De}^4\int d\mathbf{p}'F_e(\mathbf{p}')\int d\mathbf{k}\,k_ik_j\frac{\omega_s^2}{\gamma}\Big\{\delta(\omega_s-\mathbf{kv})\delta(\omega_s-\mathbf{kv}')\\+\frac{1}{16}\frac{(\mathbf{kr}_E)^2}{k^4r_{De}^4}\frac{\omega_0^2}{\tilde\gamma^2+(\Delta\omega_0-\omega_s)^2}[\delta(\omega_s-\omega_0-\mathbf{kv})\delta(\omega_s-\mathbf{kv}')\\+\delta(\omega_s-\omega_0-\mathbf{kv}')\delta(\omega_s-\mathbf{kv})]\Big\}.\end{aligned} \tag{3.1*}$$

The first term in the expression (3.1) corresponds to the interaction of particles with short-wave oscillations $kr_{De} \gg 1$ ($r_{De} = v_{Te}/\omega_{Le}$), for which the presence of HF fields is unimportant, and is determined by the usual asymptotic collision integral under conditions of strong isothermicity $T_e \gg T_i$[5]:

$$I=\frac{1}{2}\left|\frac{e_i}{e}\right|\frac{T_e}{T_i}\left[\ln\frac{e_i^2m_iT_e^3}{e^2m_eT_i^3}\right]^{-1}.$$

The second term in (3.1) describes the interaction of the particles with the ion acoustic frequency $\omega_s = \omega_{Li}kr_{De} = kv_s$ (which is considerably smaller than the ionic Langmuir frequency $\omega_{Li} = (4\pi e_i^2n_i/m_i)^{1/2}$ and a wave vector which is considerably smaller than the reciprocal of the Debye radius r_{De}) and Langmuir waves of frequency $\omega_0 - \omega_s$. The damping decrement of the ionic sound has the form[1]

$$\gamma=\gamma_s[1-\cos^2\vartheta F(k)], \tag{3.2}$$

where ϑ is the angle between the direction of the external field and the propagation vector of the wave, γ_s is the damping decrement of the ion-acoustic waves on electrons

$$\gamma_s=(\pi/8)^{1/2}\omega_{Li}\omega_s/\omega_{Le}.$$

In addition

$$F(k)=\frac{1}{(2\pi)^{1/2}}\frac{r_E^2}{r_{De}^2}\frac{\omega_0\Delta\omega_0\tilde\gamma(k)kv_{Te}}{[(\Delta\omega_0)^2-\omega_s^2(k)]^2+4\omega_s^2(k)\tilde\gamma^2(k)},$$

and the high-frequency damping decrement $\tilde\gamma$ is determined by the Landau damping and collisions of ions with electrons ($\nu_{ei} \ll kv_{Te}$):

$$\tilde{\gamma}(k)=\left(\frac{\pi}{8}\right)^{1/2}\frac{\omega_0}{(kr_{De})^3}\exp\left\{-\frac{1}{2}\left(\frac{\omega_0}{kv_{Te}}\right)^2\right\}+\frac{1}{2}\nu_{ei}, \quad (3.3)$$

$\Delta\omega_0$ denotes the difference between the frequency of the external field and the frequency of the longitudinal plasma wave:

$$\Delta\omega_0=\omega_0-(\omega_{Le}^2+\omega_{Li}^2+2k^2v_{Te}^2)^{1/2}.$$

As an illustration of the obtained formula we consider a situation when, for states of the plasma which differ from Maxwellian ones ($F_e = F_e^0 + \delta F_e$), one can retain only the first term in the linear collision integral

$$\delta J_e=\frac{\partial}{\partial p_i}\left[\delta D_{ij}(F_e^0)\frac{\partial\delta F_e}{\partial p_j}+\delta D_{ij}(\delta F_e)\frac{\partial F_e^0}{\partial p_j}\right]$$

i.e., one considers the case of sufficiently rapid particles. Then, making use of the following expression for the spectral density of the energy of excited oscillations:

$$W(\mathbf{k})=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{(\mathbf{EE})_{\omega,\mathbf{k}}}{8\pi}\frac{\partial}{\partial\omega}(\omega\,\mathrm{Re}\,\tilde{\varepsilon}),$$

we obtain that for fast particles ($v > v_{Te}$) the electron diffusion coefficient $\delta D_{ij} \equiv \delta D_{ij}^S + \delta D_{ij}^L$ is determined both by the interaction with the ionic sound and with Langmuir waves

$$\delta D_{ij}=\frac{2e^4n_er_{De}^4}{\varkappa T_e}\int d\mathbf{k}\,k_ik_jW_s(\mathbf{k})\Big[\delta(\omega_s-\mathbf{kv})+\frac{1}{16}\frac{(\mathbf{k}r_E)^2}{k^4r_{De}^4}\frac{\omega_0^2}{\tilde{\gamma}^2+(\Delta\omega_0-\omega_s)^2}\delta(\omega_s-\omega_0-\mathbf{kv})\Big]. \quad (3.4)$$

Here $W_s(\mathbf{k})$ is the spectral density of the energy of the ionic sound:

$$W_s=\varkappa T_e\gamma_s/\gamma. \quad (3.5)$$

As can be seen from Eqs. (3.4), (3.5), the anomalous enhancement of the collision integral is due to the possibility that the decrement (3.2) vanishes, which is realized for a threshold field strength [1]

$$\frac{E_{\text{thr}}}{4\pi n_e\varkappa T_e}=16\frac{\gamma_s(k_0)\tilde{\gamma}(k_0)}{\omega_0\omega_s(k_0)}. \quad (3.6)$$

The value of the wave vector $\mathbf{k}_0$ corresponds to the maximum of $F(\mathbf{k})$ which occurs for

$$\omega_s(k_0)=\Delta\omega_0(k_0), \quad (3.7)$$

and equals

$$k_0=\frac{\omega_{Li}}{3v_{Te}}[(1+\Delta)^{1/2}-1],\quad \Delta=6\frac{\omega_{Le}^2}{\omega_{Li}^2}\frac{\omega_0-\omega_p}{\omega_p},\quad \omega_p=(\omega_{Le}^2+\omega_{Li}^2)^{1/2}. \quad (3.8)$$

In order to compute the tensor δD_{ij} it is convenient to introduce spherical coordinates p, ϑ, φ in momentum space (the angle ϑ being measured from the direction of the field $\mathbf{E}_0$). In these variables the collision integral has the form:

$$\delta J_e=\Big[\frac{1}{p^2}\frac{\partial}{\partial p}p^2\left(\delta D_{pp}\frac{\partial}{\partial p}+\frac{\delta D_{p\vartheta}}{p}\frac{\partial}{\partial\vartheta}\right)+\frac{1}{p\sin\vartheta}\frac{\partial}{\partial\vartheta}\sin\vartheta\left(\delta D_{\vartheta p}\frac{\partial}{\partial p}+\frac{\delta D_{\vartheta\vartheta}}{p}\frac{\partial}{\partial\vartheta}\right)\Big]\delta F_e. \quad (3.9)$$

The results of the corresponding calculations can be found in the Appendix.

We first consider the influence of the interaction of the particles with the ionic sound on the collision integral (3.9). Taking into account the fact that the velocity of the particles is large compared to the phase velocity of the ion-acoustic waves, the analysis of the expressions for the diffusion coefficient given in the Appendix yields

$$\delta D_{pp}^s/\delta D_{p\vartheta}^s\sim\delta D_{p\vartheta}^s/\delta D_{\vartheta\vartheta}^s\sim v_s/v\ll 1.$$

This implies that the diffusion occurs essentially within the angle. The contribution to the collision integral of the interaction of the particles with the ionic sound is then determined by the coefficient $\delta D_{\vartheta\vartheta}^S$:

$$\delta D_{\vartheta\vartheta}^s(v,\vartheta)=\frac{4\pi e^4n_er_{De}^4}{v\sin^2\vartheta}\int dk\frac{k^3}{F(k)}\{[1-F(k)\sin^2\vartheta]^{-1/2}-1\}. \quad (3.10)$$

For fields with an intensity close to the threshold value (3.6) the main contribution to the integration with respect to the wave numbers comes from wave numbers satisfying the decay condition (3.7). Carrying out an expansion of the quantity under the square root in (3.10) at that point k_0 one can see that for $E_0 \lesssim E_{\text{thr}}$ for angles $\vartheta = \pi/2$, the diffusion coefficient increases anomalously:

$$\delta D_{\vartheta\vartheta}^s\left(v,\vartheta=\frac{\pi}{2}\right)=\frac{4\pi e^4n_e}{v}\frac{\tilde{\gamma}(k_0)(k_0r_{De})^3}{\omega_{Li}(1+\Delta)^{1/2}}\ln\left|1-\frac{E_0^2}{E_{\text{thr}}^2}\right|^{-1}. \quad (3.11)$$

Owing to such a logarithmic dependence the anomalous enhancement of the collision integral due to the interaction with the ionic sound appears only in a narrow region near the threshold. Thus, if the fundamental contribution to the collision integral introduces a static polarization, the expression (3.9) which takes into account only the interaction with the ion-acoustic waves, is comparable with the Landau collision integral only for

$$\frac{E_{\text{thr}}^2-E_0^2}{E_{\text{thr}}^2}\lesssim\exp\left\{-\Lambda\frac{\omega_{Li}(1+\Delta)^{1/2}}{(k_0r_{De})^3\tilde{\gamma}(k_0)}\right\}.$$

For typical plasmas the Coulomb logarithm Λ ranges from 6 to 20, which confirms what was said above about the narrowness of the band of anomalous enhancement of the collision integral, determined by the interaction with the ionic sound.

In analyzing the contribution of the interaction with Langmuir waves to the electronic diffusion coefficient near the threshold we consider two cases: $k_0v \gg \omega_0$ and $|1 - \omega_0^2/k_0^2v^2| \equiv \delta^2 \ll 1$.

For sufficiently rapid particles $k_0v \gg \omega_0$, and by analogy with the above we obtain

$$\delta D_{pp}^L/\delta D_{p\vartheta}^L\sim\delta D_{p\vartheta}^L/\delta D_{\vartheta\vartheta}^L\sim\omega_0/k_0v\ll 1$$

and, accordingly

$$\delta D_{\vartheta\vartheta}^L(v,\vartheta)=\frac{\pi e^4n_er_E^2}{4v\sin^2\vartheta}\int dk\frac{\omega_0^2}{\tilde{\gamma}^2}\frac{k}{F^2(k)}\left\{[1-F(k)\sin^2\vartheta]^{-1/2}-1-\frac{\sin^2\vartheta}{2}F(k)\right\}. \quad (3.12)$$

For field strengths close to the threshold value and $\vartheta = \pi/2$ Eq. (3.12) takes the form

$$\delta D_{\vartheta\vartheta}^L\left(v,\vartheta=\frac{\pi}{2}\right)=\frac{\pi\sqrt{2\pi}\,e^4n_e}{v}\frac{k_0r_{De}}{(1+\Delta)^{1/2}}\ln\left|1-\frac{E_0^2}{E_{\text{thr}}^2}\right|^{-1} \quad (3.13)$$

and the anomalous enhancement of the diffusion coefficient is observed as before in a narrow region near the threshold:

$$\frac{E_{\text{thr}}^2-E_0^2}{E_{\text{thr}}^2}\lesssim\exp\left\{-\Lambda\frac{(1+\Delta)^{1/2}}{k_0r_{De}}\right\}.$$

However, in spite of the identical functional dependence on the strength of the external field in (3.11) and (3.13), the interaction with the high-frequency oscillations exceeds by far the interaction with ion-acoustic waves:

$$\delta D_{\vartheta\vartheta}^L\left(v,\vartheta=\frac{\pi}{2}\right)\Big/\delta D_{\vartheta\vartheta}^s\left(v,\vartheta=\frac{\pi}{2}\right)=\sqrt{\frac{\pi}{8}}\frac{1}{(k_0r_{De})^2}\frac{\omega_{Li}}{\tilde{\gamma}(k_0)}\gg 1.$$

In the resonance case $k_0v \sim \omega_0$ the determining effect is diffusion with respect to momentum

$$\delta D_{\vartheta\vartheta}^L/\delta D_{p\vartheta}^L\sim\delta D_{p\vartheta}^L/\delta D_{pp}^L\sim\delta\ll 1.$$

Corresponding to this we have

$$\delta D_{pp}{}^{L}(v,\vartheta)=\frac{\pi e^4 n_e r_E{}^2}{4v}\int dk\,\frac{\omega_0{}^2}{\tilde{\gamma}^2}\frac{k}{F(k)}\{[1-F(k)\cos^2\vartheta]^{-1}-1\}. \qquad (3.14)$$

In distinction from the preceding discussion the main contribution to the diffusion coefficient comes from particles with velocities directed along or against the direction of the electric field ($\vartheta = 0, \pi$). Then for $E_0 \lesssim E_{thr}$

$$\delta D_{pp}{}^{L}(v,\vartheta=0,\pi)=\frac{\pi^2\sqrt{2\pi}\,e^4 n_e}{v}\frac{k_0 r_{De}}{(1+\Delta)^{1/2}}\left[1-\frac{E_0{}^2}{E_{thr}^2}\right]^{-1/2}. \qquad (3.15)$$

i.e., for resonant particles the E_{thr} strong dependence of the diffusion coefficient on the field strength, (3.15) leads to a widening of the region of anomalous enhancement of the collision integral.

$$\frac{E_{thr}^2-E_0{}^2}{E_{thr}^2}\lessdot\frac{(k_0 r_{De})^2}{\Lambda^2(1+\Delta)},$$

4. We now turn to the case of a plasma with developed fluctuations of the fields. We first note that the results obtained above can be used for the more general case of the stationary fluctuation spectrum. In particular, the general expression (3.4) allows one to consider the problem of behavior of the electronic diffusion coefficient of a weakly turbulent plasma with a stationary level of ion-acoustic noise, which is attained as a result of the induced scattering of the ionic sound on the ions[1]. We neglect the influence of the induced scattering of waves on particles in the collision integral.

Keeping in mind that the fundamental contribution to the diffusion coefficient for fast particles ($k_0 v \gg \omega_0$) is determined by velocity vectors which are perpendicular to the direction of the external field one can write down the following approximate expression, describing the interaction of the electron with the ion-acoustic and Langmuir waves in the turbulent state:

$$\delta D_{\vartheta\vartheta}(v,\vartheta=\pi/2)\equiv\delta D_{\vartheta\vartheta}{}^{s}(v,\vartheta=\pi/2)+\delta D_{\vartheta\vartheta}{}^{L}(v,\vartheta=\pi/2)$$
$$=\frac{16\pi^2 e^4 n_e}{v}k_0 r_{De}{}^4\left(\frac{E_0{}^2}{E_{thr}^2}-1\right)^{-1/2}\frac{W}{\varkappa T_e}\left[1+\left(\frac{\pi}{8}\right)^{1/2}\frac{E_0{}^2}{E_{thr}^2}(k_0 r_{De})^{-2}\frac{\omega_{Li}}{\tilde{\gamma}(k_0)}\right]. \qquad (4.1)$$

Then the total (with respect to the spectrum) stationary energy density of the ionic sound[1]

$$W=\int\frac{d\mathbf{k}}{(2\pi)^3}W_s(\mathbf{k})=\frac{1}{(2\pi)^{1/2}}\frac{n_e\varkappa T_e}{1+\Delta}\left(\frac{E_0{}^2}{E_{thr}^2}-1\right)\frac{r_{De}{}^2\omega_{Li}\tilde{\gamma}^2(k_0)}{r_{Di}{}^2\omega_{Le}\omega_s{}^2(k_0)} \qquad (4.2)$$

must satisfy the condition that it be small compared to the energy density of the thermal motion of the plasma particles

$$W/n_e\varkappa T_e \ll 1.$$

In Eqs. (4.1), (4.2) the threshold value of the electric field, E_{thr} the decay wave number k_0 and the frequency shift Δ are determined by the expressions (3.6), (3.8), and strictly speaking, the formulas are valid only if the field strength E_0 exceeds the threshold value by a small amount ($E_0 \gtrsim E_{thr}$).

When the intensity of the plasma oscillations is sufficiently large, so that the interaction with the oscillations can lead to an effective collision frequency

$$\nu_{eff}=2\tilde{\gamma}(k_0)\left(\frac{E_0{}^2}{E_{thr}^2}-1\right)\left[1+\frac{2\sqrt{2\pi}}{9C}\frac{r_{Di}{}^2}{r_{De}{}^2}\frac{\omega_{Li}{}^3}{\omega_{Le}{}^3}(1+\Delta)\left((1+\Delta)^{1/2}-1\right)^2\right]^{-1} \qquad (4.3)$$

(C is a dimensionless constant of the order of one) which is larger than the usual electron-ionic one ν_{ei}, and the turbulent dissipation becomes essential for the high-frequency oscillations, one must replace $\tilde{\gamma}(k_0)$ by $\tilde{\gamma}(k_0) + \frac{1}{2}\nu_{eff}$ in the expressions for W and $\delta D_{\vartheta\vartheta}$. As a result of this replacement we obtain for the turbulent coefficient $\delta D_{\vartheta\vartheta}$

$$\delta D_{\vartheta\vartheta}\left(v,\vartheta=\frac{\pi}{2}\right)=\frac{12(2\pi)^{3/2}e^4 n_e}{v}\left(\frac{E_0{}^2}{E_{thr}^2}-1\right)^{1/2}\frac{n_e r_{De}{}^3}{(1+\Delta)[(1+\Delta)^{1/2}-1]}\frac{r_{De}{}^2}{r_{Di}{}^2}$$
$$\times\frac{[\tilde{\gamma}(k_0)+{}^1/{}_2\nu_{eff}]^2}{\omega_{Li}{}^2}\left\{1+\frac{9}{4}(2\pi)^{1/2}\frac{\omega_{Le}{}^2}{\omega_{Li}{}^2}[(1+\Delta)^{1/2}-1]^{-2}\left[\tilde{\gamma}(k_0)+\frac{1}{2}\nu_{eff}\right]^{-1}\right\}. \qquad (4.4)$$

However, the effective frequency of collisions (4.3) exceeds the usual electron-ionic collision frequency only in the interval

$$\Delta_1<\Delta<\Delta_2, \qquad (4.5)$$

in which the collisions contribute little compared to the Cerenkov effect. The upper limit Δ_2 in (4.5) is determined by the decay wave number $k_0(\Delta)$ for which the acoustic frequency becomes equal to the contribution to $\tilde{\gamma}$ produced by the Cerenkov effect

$$\Delta_2\approx\frac{9}{2}\frac{\omega_{Le}{}^2}{\omega_{Li}{}^2}\left(\ln\frac{\omega_{Le}{}^2}{\omega_{Li}{}^2}\right)^{-1},$$

and the lower limit Δ_1 is determined by equating the two terms in the expression (3.3) for $\tilde{\gamma}$, i.e., from the condition of equality of the effects of Cerenkov damping on electrons and the electron-ion collisions

$$\Delta_1\approx\frac{9}{2}\frac{\omega_{Le}{}^2}{\omega_{Li}{}^2}\left(\ln\frac{\omega_{Le}{}^2}{\nu_{ei}{}^2}\right)^{-1}.$$

It can be seen from Eq. (4.4) that in the interval of frequencies (4.5) the interaction of the particles with high-frequency oscillations is more important than the interaction with the ion sound. In this region of frequency shifts the quantity $\delta D_{\vartheta\vartheta}$ increases with the increase of the relative shift $(\omega_0 - \omega_p)/\omega_p$ and the maxima of the diffusion coefficient, the increase of which is accompanied by a corresponding enhancement of the threshold field (3.6) as a function of Δ, is attained at the upper limit $\Delta = \Delta_2$ of the frequency shifts

$$\max_{\Delta}\delta D_{\vartheta\vartheta}\left(v,\vartheta=\frac{\pi}{2}\right)=\frac{4}{9}(2\pi)^2\frac{e^4 n_e}{v}n_e r_{De}{}^3\left(\frac{E_0{}^2}{E_{thr}^4}-1\right)^{1/2}\cdot$$
$$\times\frac{r_{De}{}^2\omega_{Li}{}^3}{r_{Di}{}^2\omega_{Le}{}^3}\left(\ln\frac{\omega_{Le}{}^2}{\omega_{Li}{}^2}\right)^2\left[1+\frac{2C}{9(2\pi)^{1/2}}\frac{r_{De}{}^2\omega_{Li}}{r_{Di}{}^2\omega_{Le}}\left(\ln\frac{\omega_{Le}{}^2}{\omega_{Li}{}^2}\right)^2\left(\frac{E_0{}^2}{E_{thr}^2}-1\right)\right]^2, \qquad (4.6)$$

where the threshold field becomes

$$\frac{E_{thr}^2}{n_e\varkappa T_e}=\frac{8}{\sqrt{2}}(2\pi)^{3/2}\frac{\omega_{Li}{}^2}{\omega_{Le}{}^2}\left(\ln\frac{\omega_{Le}{}^2}{\omega_{Li}{}^2}\right)^{1/2}.$$

Thus, according to Eq. (4.6) in a plasma with sufficiently high electron temperature (the temperature is in eV, n is the number of electrons in 1 cm^3; the plasma is homogeneous):

$$10^5\frac{T_e}{T_i}T_e{}^{3/2}n_e{}^{-1/2}\left(\frac{E_0{}^2}{E_{thr}^2}-1\right)^{1/2}\gg\Lambda,$$

one can observe an enhancement of the diffusion coefficient in momentum space, for which the dependence on density and temperature of the plasma differs substantially from the usual one, and the dependence on the field strength has the threshold character.

APPENDIX

Making use of the spherical coordinates p, ϑ, φ in momentum space and k, ϑ', φ' in the space of wave numbers, we obtain the following expression for the diffusion tensor ($\delta D_{\alpha\beta} = \delta D_{\beta\alpha}$)

$$\delta D_{\alpha\beta}(v,\vartheta)=\frac{4e^4 n_e r_{De}{}^4}{\varkappa T_e v}\iint\sin\vartheta' d\vartheta' k^3 dk W_s(k,\vartheta')\Big[d_{\alpha\beta}(\omega_s)f^{-1/2}(\omega_s)\theta(f(\omega_s))$$
$$+\frac{1}{16}\frac{r_E{}^2}{r_{De}{}^2}\frac{\omega_0{}^2}{\tilde{\gamma}^2}\frac{\cos^2\vartheta'}{k^2 r_{De}{}^2}d_{\alpha\beta}(\omega_s-\omega_0)f^{-1/2}(\omega_s-\omega_0)\theta(f(\omega_s-\omega_0))\Big],$$

where

$$f(\omega)=\sin^2\vartheta\sin^2\vartheta'-[\omega/kv-\cos\vartheta\cos\vartheta']^2,$$

$$d_{pp}(\omega)=\frac{\omega^2}{k^2v^2},\quad d_{p\vartheta}=\frac{\omega[\omega\cos\vartheta/kv-\cos\vartheta']}{kv\sin\vartheta},$$

$$d_{\vartheta\vartheta}(\omega)=\frac{[\omega\cos\vartheta/kv-\cos\vartheta']^2}{\sin^2\vartheta},$$

and the function $\theta(x)$ is defined by

$$\theta(x)=\begin{cases}0, & x<0\\ 1, & x>0\end{cases}.$$

[1] V. P. Silin, Parametricheskoe vozdeĭstvie izlucheniya hol'shoi moshchnosti na plazmu (The Parametric Action of High-Power Radiation on a Plasma), Nauka, Moscow, 1973.
[2] D. F. DuBois and M. V. Gol'dman, Phys. Rev. **164**, 207 (1967).
[3] V. Yu. Bychenkov, V. P. Silin and V. T. Tikhonchuk, Kratkie soobshcheniya po fizike (FIAN) **8**, 27 (1972).
[4] Yu. L. Klimontovich and V. P. Silin, Dokl. Akad. Nauk SSSR **145**, 764 (1962) [Sov. Phys. Doklady **7**, 693 (1963)].
[5] V. P. Silin, Vvedenie v kinetichskuyu teoriyu gazov (Introduction to the Kinetic Theory of Gases), Nauka, Moscow, 1971.

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