Space-time focusing of an ion beam that excites transverse Langmuir ion plasma oscillations

L. P. Katsubo, V. P. Kovalenko, and I. A. Soloshenko

Physics Institute, Ukrainian Academy of Sciences
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We investigate the mechanism of the nonlinearity produced upon excitation of transverse ion Langmuir plasma oscillations modulated by an ion beam whose velocity exceeds considerably that of the ion sound. It is shown that the limitation on the oscillation amplitude is due to intersection of the beam-particle trajectories, resulting from space-time focusing of the oscillations in a direction transverse to that of the unperturbed beam velocity.

It was recently observed[1,2] that a fast ion beam excites short-wave ion oscillations in a plasma that are transverse to the beam velocity. The frequency and the growth rate of the oscillations conform to the linear theory[3]. At the same time, the experiments indicate that excitation of an initially monochromatic wave gives rise to essentially nonlinear effects whose nature has not been established.

The present study was made to ascertain the mechanism of the nonlinearity produced when transverse plasma ion oscillations are excited by a fast ion beam.

We present first an approximate theoretical analysis of the motion of the ion beam under the conditions of the investigated instability. For simplicity we confine ourselves to the two-dimensional case, which can be easily realized in experiment. Assume that an ion beam with velocity v_0, average density ρ_0, and no random velocity scatter propagates along the z axis in a plasma with cold ions. We postulate furthermore at the entry to the plasma ($z = 0$) a certain perturbation of the transverse beam velocity v_x, in the form

$$v_x|_{z=0} = v_1 \sin kx \sin \omega t. \tag{1}$$

If the role of the electrons is restricted only to cancellation of the averaged positive charge, which is permissible at $k^2 d_e^2 \gg 1$ (d_e is the Debye radius), then the potential electric field in the plasma is described by the equation

$$\frac{\partial^2 \nabla E}{\partial t^2} + \omega_{pi}^2 \nabla E = 4\pi \frac{\partial^2 \rho_b}{\partial t^2}, \tag{2}$$

where ω_{pi} is the ion Langmuir frequency and ρ_b is the alternating beam-charge density.

Equation (2) was obtained in an approximation in which the plasma density $\rho_p \gg \rho_0$, and the plasma ion oscillations are accordingly linear. At a large beam velocity, when $\omega/v_0 \ll k$, the transverse field component E_x greatly exceeds E_z and we can neglect the latter henceforth. Equation (2) takes the form

$$\frac{\partial^3 E_x}{\partial t^2 \partial x} + \omega_{pi}^2 \frac{\partial E_x}{\partial x} = 4\pi \frac{\partial^2 \rho_b}{\partial t^2}. \tag{3}$$

We assume that in the case of periodic beam modulation (1) the steady-state process is also periodic in t and in x, so that the following Fourier expansions are permissible:

$$\rho_b = \sum_{n=1}^{\infty} A_n(z,t) \sin nkx + B_n(z,t) \cos nkx, \tag{4}$$

$$\int \rho_b\, dx = \sum_{s=1}^{\infty} C_s(z,x) \sin s\omega \left(t - \frac{z}{v_0}\right) + D_s \cos s\omega \left(t - \frac{z}{v_0}\right) \tag{5}$$

with coefficients

$$A_n = \frac{k}{\pi} \int_0^{2\pi/k} \rho_b \sin nkx\, dx, \quad B_n = \frac{k}{\pi} \int_0^{2\pi/k} \rho_b \cos nkx\, dx. \tag{6}$$

$$C_s = \frac{\omega}{\pi} \sum_{n=1}^{\infty} \int_0^{2\pi/\omega} \left(-\frac{A_n}{nk} \cos nkx + \frac{B_n}{nk} \sin nkx \right) \sin s\omega \left(t - \frac{z}{v_0}\right) dt,$$
$$D_s = \frac{\omega}{\pi} \sum_{n=1}^{\infty} \int_0^{2\pi/\omega} \left(-\frac{A_n}{nk} \cos nkx + \frac{B_n}{nk} \sin nkx \right) \cos s\omega \left(t - \frac{z}{v_0}\right) dt. \tag{7}$$

The solution of (3) then takes the form

$$E = -\sum_{s=1}^{\infty} \frac{4\pi s^2 \omega^2}{\omega_{pi}^2 - s^2\omega^2} \left[C_s \sin s\omega \left(t - \frac{z}{v_0}\right) + D_s \cos s\omega \left(t - \frac{z}{v_0}\right) \right], \quad \omega_{pi} \neq s\omega. \tag{8}$$

To determine the coefficients A_n and B_n, we write down the equation for the beam-charge conservation in the volume element dzdx, recognizing that this volume is deformed only along x:

$$\rho_0 dx_0 = \rho(z, x, t)\, dx, \tag{9}$$

where x_0 is the coordinate of a certain beam ion at $z = 0$. Coordinates x and x_0 are related by the equation of motion

$$x = x_0 + \frac{e}{v_0^2 M} \int_0^z dz \int_0^z E_x(x_0, t_0, z)\, dz + \frac{kzv_1}{v_0} \sin kx_0 \sin \omega t_0, \tag{10}$$

where e and M are the charge and mass of the ion, and $t_0 = t - z/v_0$. Putting

$$g(x_0, t_0, z) = \frac{ke}{v_0^2 M} \int_0^z dz \int_0^z E_x(x_0, t_0, z)\, dz + \frac{kzv_1 \sin kx_0 \sin \omega t_0}{v_0}, \tag{11}$$

substituting (9) and (10) in (6) and (7), and integrating (8) twice, we obtain an equation for the function $g(x_0, t_0, z)$:

$$g = -\frac{\omega_b^2 k \omega^3}{v_0^2 \pi^2} \sum_{s=1}^{\infty} \sum_{n=1}^{\infty} \frac{s^2}{n(\omega_{pi}^2 - s^2\omega^2)} \Big\{ \sin \omega t_0 \int_0^{2\pi/\omega} \Big[\sin n(kx+g) \times \sin s\omega t \int_0^{2\pi/k} \cos n(kx+g)\, dx \Big] dt + \cos s\omega t_0 \int_0^{2\pi/\omega} \Big[\sin n(kx+g) \cos s\omega t \times \int_0^{2\pi/k} \cos n(kx+g)\, dx \Big] dt \Big\} + \frac{kv_1 \sin \omega t_0 \sin kx_0}{v_0} z, \tag{12}$$

where ω_b is the natural frequency of the beam.

Equation (12) already takes into account the fact that it is satisfied by a function g that is odd in x_0, and consequently, the coefficients A_n vanish.

In the zeroth approximation, under the condition $|g| \ll 1$, the solution of (12) can easily be obtained in the form

$$g = X(z) \sin kx_0 \sin \omega t_0;$$
$$X(z) = \frac{kv_1}{\gamma_n v_0} \operatorname{sh} \gamma_n z, \quad \gamma_n = \frac{\omega_b}{v_0[1 - (\omega/\omega_{pi})^2]^{1/2}}, \tag{13}$$

where γ_n is the spatial growth increment of the oscillations. An estimate of the additional nonlinear terms that appear in the right-hand side of the nonlinear equation (12) following the substitution of the solution (13) has shown that this solution satisfies (12) with sufficient accuracy up to values $X \sim 1$.

Thus, the ion beam should break up into clusters during the nonlinear stage of the considered instability, and should acquire a distinctive space-time structure:

$$\rho=\rho_0 \Big/ \frac{\partial x}{\partial x_0} \approx \frac{\rho_0}{1+X\cos kx_0 \sin \omega t_0}. \tag{14}$$

At the point with the coordinate $z = S$ determined from the condition

$$kv_1 \,\mathrm{sh}\, \gamma_n S/\gamma_n v_0 \approx 1, \tag{15}$$

the charge density in the clusters tends to infinity. It is here that the intersection of the ion trajectories sets in.

We note that in contrast to the case of excitation of longitudinal electron oscillations by a beam, when phase focusing takes place because certain particles overtake others[4,5], in our problem the clusters are produced because of transverse focusing of the beam.

The possibility of particle focusing in transverse fields was pointed out earlier by Krasovitskiĭ[6] in an analysis of the problem of the interaction of a bounded electron beam with a plasma.

EXPERIMENTAL RESULTS

The experiments were performed with the setup illustrated in Fig. 1. The H_2^+ ions were extracted from a "duoplasmatron" source 1 by the field of electrode 2 and were then focused by magnetic lens 3 into a beam 4. The maximum beam current was $I_i = 40$ mA and the maximum energy $-eU_0 = 40$ keV. The average ion density in the beam usually amounted to $\sim 10^7$ cm^{-3}, and could be varied with the aid of a magnetic lens. The plasma was produced by ionization of the gas by the beam itself. A change in the argon or air pressure from 3×10^{-5} to 3×10^{-4} mm Hg changed the plasma density in the range from 10^7 to 10^8 cm^{-3}. The interaction chamber was 500 cm long and ~35 cm in diameter. Prior to entering the chamber, a perturbation periodic in time and in the transverse coordinate x was imparted to the transverse beam velocity with the aid of modulator 5. The modulator was a grid of tungsten wires of 0.2 mm diameter, placed in the plane of the beam cross section. Alternate wires in the grid were electrically interconnected in such a way that the beam ions could be imparted an alternating velocity in the form (1) with the aid of an external generator (the coordinate x was perpendicular to the wires). To ensure satisfaction of the condition $k^2 d_e^2 \gg 1$, which is necessary for the excitation of ion Langmuir oscillations, the distance between wires was chosen to be sufficiently small, ~0.4 cm. In the principal experiments, this resulted in $k^2 d_e^2 \approx 4$. We investigated the oscillations of the beam current and of the plasma potential. The current was measured with a probe comprising a collector, screened from the plasma, with an entrance diaphragm of 1 mm diameter, which determined the spatial resolution along the x axis. The potential was measured with insulated probes[2].

In the absence of external modulation, the ion beam excited in the plasma oscillations whose amplitude was maximal at a frequency equal to the ion Langmuir fre-

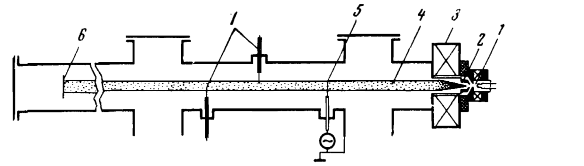

FIG. 1. Experimental stetup: 1—"duoplasmatron" ion source, 2—drawing electrode, 3—magnetic lens, 4—ion beam, 5—modulator, 6—collector, 7—measuring probes.

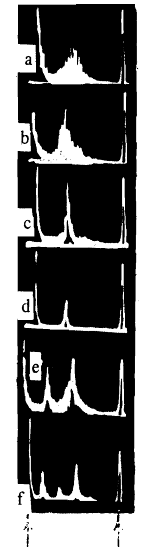

FIG. 2. Spectra of plasma-potential oscillations, photographed from the screen of the C4-8 analyzer. The H_2^+ ion current was $I_i = 15$ mA, the energy $eU_0 = 25$ keV, the air pressure in the chamber $P = 5 \times 10^{-5}$ mm Hg. a) Generator voltage $U_1 = 0$; b) $U_1 = 5$ V, modulation frequency f = 370 kHz; c) $U_1 = 12$ V, f = 370 kHz; d) $U_1 = 150$ V, f = 370 kHz; e) $U_1 = 40$ V, f = 200 kHz; f) $U_1 = 50$ V, f = 140 kHz. The left-hand marker represents 0 MHz, and the right-hand markers represent 1 MHz for Figs. 2a-2d and 850 kHz for Figs. 2e and 2f.

quency of the plasma (Fig. 2a). When an alternating voltage of frequency $\omega \lesssim \omega_{pi}$ was applied to the modulator, a signal corresponding to the generator frequency appeared in the spectrum (Fig. 2b). At sufficiently large modulation amplitude, the oscillations at frequencies different from the generator frequency were suppressed, and only the signal corresponding to the modulation frequency remained in the spectrum (Figs. 2c, d). An explanation of the field-amplitude decrease in Fig. 2d in comparison with Fig. 2c, and also a discussion of Figs. 2e and 2f, will be presented below. Thus, at sufficiently large modulation amplitudes, it can be assumed that the beam interacts with monochromatic oscillations, i.e., the assumptions made in the theoretical part of the paper are realized. Modulation of the beam at a frequency $\omega > \omega_{pi}$ produced no change whatever in the oscillation spectrum, even at the maximum generator voltage, and the amplitude of the signal corresponding to the generator frequency was very small. This agrees qualitatively with the theory, which predicts amplification of the oscillations only at $\omega \leq \omega_{pi}$. Since the behavior of the system at $\omega > \omega_{pi}$ was determined by the spontaneously excited oscillations, all the subsequent measurements were carried out under conditions $\omega \lesssim \omega_{pi}$.

Figure 3 shows the dependence of the fundamental harmonic of the beam current density on the coordinate x, obtained during a stage when noticeable focusing took place. We see that this dependence has a periodic structure in which the distance between maxima is equal to the distances between the modulator filaments. (The distance between the outermost pairs of filaments was chosen for control purposes to be approximately double the remaining distances, at 1 cm.) We note that the beam-current density varies monotonically along the

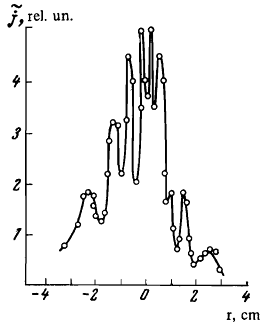

FIG. 3

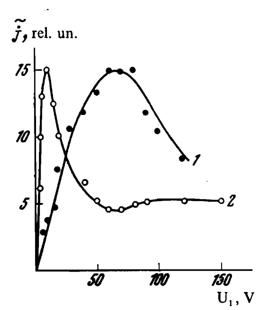

FIG. 4

FIG. 3. Radial distribution of the amplitude of the fundamental current harmonic. H_2^+ ion current I = 20 mA, energy eU_0 = 25 keV, air pressure P = 6 × 10^{-5} mm Hg, distance between probe and modulator z = 270 cm, modulation frequency f = 380 kHz, U_1 = 100 V.

FIG. 4. Dependence of the amplitude of the fundamental current harmonic on the generator voltage. H_2^+ ion current I_i = 15 mA, energy eU_0 = 25 keV, air pressure P = 6 × 10^{-5} mm Hg, modulation frequency f = 380 kHz, distance from modulation: 1) z = 50 cm, 2) z = 270 cm.

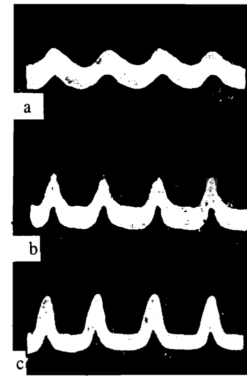

FIG. 5. Beam-current oscillograms. H_2^+ ion current I_1 = 50 mA, energy eU_0 = 25 keV, distance from modulator z = 270 cm, modulation frequency f = 300 kHz. a) Air pressure P = 5 × 10^{-5} mm Hg, generator-voltage amplitude U_1 = 5 V; b) P = 5 × 10^{-5} mm Hg, U_1 = 15 V; c) P = 8 × 10^{-6} mm Hg, U_1 = 50 V.

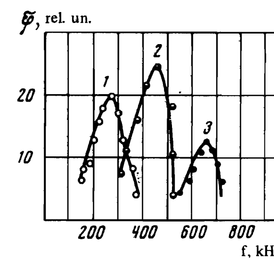

FIG. 6. Plots of the amplitude of the fundamental potential harmonic in the focal region against the modulation frequency. H_2^+ ion beam current I_i = 20 mA, energy eU_0 = 30 keV, argon pressure: 1) P = 4 × 10^{-5} mm Hg; 2) 1 × 10^{-4} mm Hg, 3) 2 × 10^{-4} mm Hg.

radius in the absence of modulation. On going from the region of one maximum to the region of another, the phase of the oscillations is reversed. Thus, the space-time structure described by expression (14) is observed in the experiment.

In order to determine the coordinate at which the beam-ion trajectories intersect, limiting the oscillation amplitude, we measured the dependence of the amplitude of the fundamental current harmonic on the generator voltage amplitude. Figure 4 shows these dependences, obtained at different distances from the modulator. We see that, in agreement with the theory, at a definite generator voltage, i.e., at a definite value of the bunching parameter X, a maximum is observed in the current-density oscillation amplitude. The existence of this maximum explains, in particular, the decrease in the amplitude of the oscillations of the potential in Fig. 2d in comparison with Fig. 2c. Substituting in (15) the values $k = 7.8\ \mathrm{cm}^{-1}$, $U_0 = 25$ kV, the experimentally measured value $\gamma_n \approx 1 \times 10^{-2}\ \mathrm{cm}^{-1}$. and also $v_1 \sim U_1 v_0/2U_0$, where $U_1 = 10$ V is the generator voltage, we find that at the given U_1 the distance from the modulator to the focus is z = 290 cm, in agreement with the experimentally obtained z = 270 cm (see curve 2 in Fig. 4). The calculations are also in agreement with the fact that the optimal value of the modulation amplitude increases with decreasing distance.

The strong bunching of the ions is demonstrated in Fig. 5, which shows oscillograms of the beam-current density at different modulation amplitudes. At small U_1 the current is sinusoidal (Fig. 5a), and at U_1 corresponding to the focusing of the ions at a given point z, the current density acquires, in accord with the theory, the form of individual peaks (Fig. 5b). The most probable cause of the limitation of the peak amplitude (its experimental value was approximately equal to the dc component of the current) is the scatter of the transverse velocities of the beam. A similar current waveform is obtained in the case when the pressure in the chamber is low and the "ion-beam" plasma consists mainly of electrons and beam ions (Fig. 5c). Theoretically this corresponds to the case $\gamma_n \to 0$. As follows from (15), the place of intersection of the trajectories as $\gamma_n \to 0$ is determined from the condition

$$kv_1 z/v_0 = 1. \qquad (16)$$

The generator voltage required to obtain trajectory intersection at a distance z = 270 cm in the kinematic case ($\gamma_n = 0$) is approximately three times larger than the corresponding voltage at $\gamma_n = 1 \times 10^{-2}\ \mathrm{cm}^{-1}$. This is in agreement with experiment. The distance to the trajectory-intersection point decreases in the interaction between the ion beam and the transverse plasma-ion oscillations because of the spatial growth of the ac component of the beam velocity. This phenomenon is analogous to the decrease of the distance to the phase focus in the interaction of electron[4,5] and ion[7] beams with longitudinal plasma electron oscillations.

Since the focusing of the beam causes the first harmonic of the current density to reach its limiting value, which does not depend on the modulation frequency, it follows from (8) that the field amplitude, and hence the potential amplitude, should be resonantly dependent on this frequency during the nonlinear stage. Indeed, the corresponding experimental plot (Fig. 6) has a resonant character, and the maximum is observed at $\omega = \omega_{pi}$ (the value of ω_{pi} was determined from the frequency of the spontaneously excited oscillations).

An interesting effect is observed when the ion beam is modulated at frequencies much smaller than the ion plasma frequency. It follows from (18) that if ω_{pi} is s times larger than ω, then the main contribution to the field is made to the s-th harmonic, owing to the appearance of multiple harmonics of the beam density as a result of beam focusing. This agrees with experiment. Figures 2e and 2f show the potential oscillation spectra when the modulation frequencies are one-half and one-third the ion Langmuir frequency, respectively. We see that the second harmonic is maximal in the former case and the third in the latter.

Thus, the experimental facts obtained in the present study and their comparison with the calculation, allow us to conclude that space-time focusing of the beam ions plays the decisive role during the nonlinear stage of interaction between a modulated ion beam and the transverse plasma ion oscillations excited by the beam. This effect can also play a significant role in the interaction of an unmodulated ion beam with a plasma if the wave excited in the system is regular enough.

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[1] M. D. Gabovich, A. A. Goncharov, V. Ya. Poritskiĭ, and I. M. Protsenko, Zh. Eksp. Teor. Fiz. **64**, 1291 (1973) [Sov. Phys.-JETP **37**, 655 (1973)].

[2] M. D. Gabovich, I. A. Soloshenko, and A. A. Goncharov, Zh. Tekh. Fiz. **43**, 2292 (1973) [Sov. Phys.-Tech. Phys. **18**, 1450 (1974)].

[3] A. B. Mikhailovskiĭ, Teoriya plazmennykh neustoĭchivosteĭ (Theory of Plasma Instabilities), Vol. 1, Atomizdat, 1970, p. 73.

[4] M. D. Gabovich and V. P. Kovalenko, Zh. Eksp. Teor. Fiz. **57**, 716 (1969) [Sov. Phys.-JETP **30**, 392 (1970)].

[5] V. P. Kovalenko, ibid. **60**, 2122 (1971) [**33**, 1142 (1971)].

[6] V. B. Krasovitskiĭ, ZhETF Pis. Red. **9**, 679 (1969) [JETP Lett. **9**, 422 (1969)].

[7] M. D. Gabovich, I. A. Soloshenko, and L. S. Simonenko, Zh. Eksp. Teor. Fiz. **62**, 1369 (1972) [Sov. Phys.-JETP **35**, 721 (1972)].

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