Some parity-nonconservation effects in emission by hydrogenlike atoms

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Parity-nonconservation effects arising as a result of the parity-violating weak interaction between the neutral electron and proton currents in the emission of electromagnetic radiation by hydrogenlike atoms undergoing the $2S_{1/2}\to 1S_{1/2}$ transition in the presence of an external magnetic field are considered. The Zeeman splitting and the shift of the atomic levels in the magnetic field allow the degree of circular polarization of the photons to be increased by roughly a factor of five (for $H\sim 1$ kG) as compared to the value in zero field. Furthermore, the external magnetic field makes it possible to observe another *P*-odd correlation, namely, anisotropy in the photon emission with respect to the direction of the magnetic field. The study of the dependence of the *P*-odd correlations on the magnetic-field strength reveals, in principle, the form of the weak electron-proton interaction. The influence of external electromagnetic fields on the $2S_{1/2}\to 1S_{1/2}$ transition is also considered. Limitations on the random external fields in which the observation of the indicated parity-nonconservation effects is possible are obtained: $D<10^{-5}$ V/cm, $H<10^{-1}$ G (in the absence of a strong external magnetic field), or $D<10^{-9}$ V/cm (in an external field $H\sim 1$ kG).

1. INTRODUCTION

Zel'dovich, Michel[1], and one of the present authors[2] have shown that the existence in the hydrogen atom of two neighboring levels $2S_{1/2}$ and $2P_{1/2}$ with opposite parity affords us a unique opportunity for an experimental investigation of the weak electron-proton interaction. Such an interaction should, as is well known, be present in theoretical schemes that include neutral weak currents, whose existence is currently being discussed in the most lively fashion both from the theoretical[3] and experimental[4] points of view. The parity-nonconserving weak interaction leads to the mixing of the $2P_{1/2}$ state with the extremely metastable $2S_{1/2}$ state. As a result, there is a coherent E1-radiation admixture in the M1 radiation emitted in the $2S_{1/2}\to 1S_{1/2}$ transition. If we assume that the weak electron-proton interaction has the usual V – A structure (other forms of the interaction are also discussed below) with the standard value of the Fermi coupling constant, then there arises as a result of the mixing of the states quite a large circular polarization of the photons emitted from the $2S_{1/2}$ level: $\sim 2\times 10^{-4}$. We can realize how large a value this number is if we remember that the natural scale of the effect of weak interactions in atomic physics is the quantity $G/a_0^2\sim 10^{-16}$, where G is the weak-interaction constant ($\sqrt{G}\sim 10^{-16}$ cm is the characteristic length for the weak interaction) and $a_0 = 0.5\times 10^{-8}$ cm is the Bohr radius. The approximately 10^{12} (parametrically, α^{-6}, where $\alpha = e^2/\hbar c = 1/137$) enhancement ratio for the effect is due, first, to the small value of the Lamb shift $E(2S_{1/2}) - E(2P_{1/2})$ and, second, to the high degree of forbiddenness of the $2S_{1/2}\to 1S_{1/2}$ single-photon transition. (The decay of the $2S_{1/2}$ state is usually a two-photon process.) Thus, there arises an opportunity for formulating the problem of the existence of neutral currents in a field that is seemingly very far removed from high-energy physics.

In the second section of the present paper we shall consider again some effects connected with the mixing of the $2S_{1/2}$ and $2P_{1/2}$ levels. If the hydrogen atom is located in a magnetic field, then, because of the Zeeman effect, each of the $2S_{1/2}$ and $2P_{1/2}$ levels splits up into levels with $m_J = \pm 1/2$ (where m_J is the component of the total angular momentum of the electron). Because the S and P states have different g factors, the S and P levels with $m_J = -1/2$ intersect in a magnetic field $H\approx 1.2$ kG. We could now expect total mixing of the S and P levels when the natural line widths are neglected. Allowance for the widths leads, however, to a situation in which the degree of mixing remains much less than unity. In the expression for the polarization, there arises in place of the energy denominator $1/E_0$ (where E_0 is the Lamb shift), which occurs in the expression in the absence of a magnetic field, the typical "dispersion" factor

$$\frac{E-E_0}{(E-E_0)^2+\Gamma^2/4}.$$

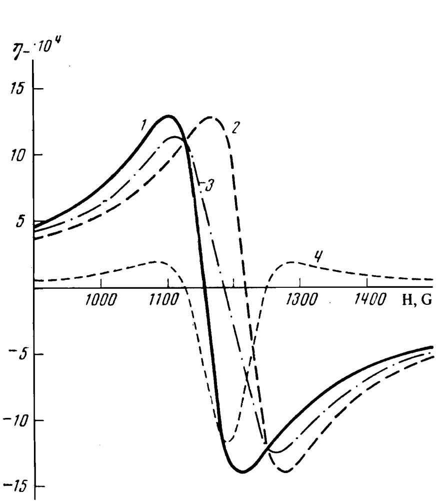
Here E is the difference between the Zeeman shifts for the S and P levels and Γ is the natural width of the $2P_{1/2}$ level (the width of the $2S_{1/2}$ level is negligibly small). If we follow the variation of the polarization as a function of the magnetic field (or of the energy E, which is linear in the magnetic field), we obtain a characteristic dependence of the type shown in the figure. The maximum value of the polarization turns out to be higher than its value in zero magnetic field by approximately a factor of $\frac{1}{2}(E_0/\Gamma)\sim 5$.

In order to avoid any misunderstanding, we must make one reservation. The photons connected with the radiation of one definite Zeeman line (e.g., the line with $m_J = -1/2$) and emitted along the direction of the magnetic field generally possess, of course, a hundred-percent circular polarization. In the present case, however, the photons emitted from the level $2S_{1/2}$ with $m_J = -1/2$ are registered together with the photons emitted from the level $2S_{1/2}$ with $m_J = +1/2$, since the Zeeman energy difference is very small. Thus, in spite of the presence of the magnetic field, we deal (without allowance for the P-state admixture) with an unpolarized initial state, as a result of which photon polarization not connected with the weak interaction does not arise. The effect of the magnetic field is to make the mixing ratios for mixing with the $m_J = -\frac{1}{2}$ and $m_J = +\frac{1}{2}$ $2P_{1/2}$ states (ratios which are proportional to the weak-interaction constant) unequal.

The external magnetic field gives rise to still another effect connected with parity nonconservation; an asymmetry in the photon emission with respect to the direction of the magnetic field, i.e., to a correlation of the form $\sim(1 + \eta \cos \theta)$. The coefficient η almost coincides with the value of the circular polarization of the photons.

Besides increasing the magnitudes of the effects by approximately a factor of five as compared to their magnitudes in zero field, the external magnetic field affords us still another unexpected opportunity. When the S and P levels with $m_J = -\frac{1}{2}$ come sufficiently close to each other, it becomes necessary to take the hyperfine splitting into account. It then turns out that if the weak interaction is of the V − A type, then intermixing occurs only for the S and P states with total-angular-momentum components $m_F = 0$, while if the structure of the interaction is of the V + A type, then the $m_F = -1$ states intermix. If the weak interaction is a mixture of the V − A and V + A types of interaction, or it has a different, more complicated matrix form, then both states (i.e., the $m_J = -\frac{1}{2}$, $m_F = -1.0$ $2S_{1/2}$ and $2P_{1/2}$ states) intermix. Thus, by studying the polarization or the asymmetry as a function of the magnetic field, we can judge the form of the weak interaction (for details, see the text and the figure).

In the third section of the paper we consider the one-photon emission from the $2S_{1/2}$ level of the hydrogen atom without allowance for the weak interaction, but in the presence of external electric and magnetic fields. The extremely small value of the Lamb shift and the metastability of the $2S_{1/2}$ level impose severe limitations on the admissible strengths of the external random electric field that may freely be present in the experiment. This happens because of the fact that an external electric field, like the weak interaction, mixes the $2P_{1/2}$ state into the $2S_{1/2}$ state. Of course, this mixing is somewhat different in nature from the mixing induced by a weak potential, so that it does not in the end lead to the circular polarization of the photons (otherwise there would be nonconservation of parity in a purely electromagnetic interaction). This occurs in such a way that the value of the admixture in the state with $m_J = +\frac{1}{2}$ is opposite in sign to the value of the admixture in the state with $m_J = -\frac{1}{2}$, which leads to the vanishing of the circular polarization when both states are taken into account. The mixing-in of the $2P_{1/2}$ states can, however, easily lead to the de-excitation of the $2S_{1/2}$ level, since the lifetime of the $2P_{1/2}$ state is shorter by a factor of 10^{14} than the one-photon $2S_{1/2} \rightarrow 1S_{1/2}$ transition time. As is shown in the paper, this leads to the limitation $D < 10^{-5}$ V/cm on the external electric field.



Dependence of the coefficient η_- of asymmetry in the photon emission on the external magnetic field H for different variants of the weak interaction. Curves: 1) the (V−A) variant, 2) the (V + A) variant, 3) the ½[(V−A) + ½(V + A)] variant, and 4) the [½(V−A) − ½(V + A)] variant.

In the presence of external electric D and magnetic H fields, the imitation of the effects connected with the nonconservation of parity is possible. Thus, for example, the appearance of circular photon polarization proportional to the pseudoscalar $\sim\mathbf{D}\cdot\mathbf{H}$ is possible. If the electric field strength is already bounded by the value $D < 10^{-5}$ V/cm, then the limitation on the magnetic field is that H must be less than a few gauss. Some other effects imitating circular polarization can lead to a limitation on H almost an order of magnitude more severe. If the question is the setting up of an experiment with a strong magnetic field that allows, as was indicated above, the observation of the characteristic dependence of the circular polarization or the asymmetry on the magnetic field, then the limitation on the external random electric field becomes, unfortunately, very severe ($D < 10^{-9}$ V/cm).

The main reason that impelled us to investigate the emission from the $2S_{1/2}$ level in external electric and magnetic fields was the apprehension that the random fields might have a significant influence on the possibility of an experimental investigation of the weak interaction. We can, however, point out a beautiful, purely electromagnetic effect, the measurement of which would, in our opinion, be of definite interest. In the presence of an external electric field, a correlation of the type $(1 + \alpha \cos \vartheta)$ between the direction n of emission of a photon and the direction of the electric field D turns out to be possible. A correlation $\sim\mathbf{D}\cdot\mathbf{n}$ is formally T-odd in nature, and its appearance is due to the instability of the system under consideration: the coefficient attached to this correlation is proportional to the natural line width of the $2P_{1/2}$ state. We are dealing here with the imitation of T-odd effects in unstable systems that was predicted by Zel'dovich[5]. The magnitude of the correlation attains a maximum, equal to $\alpha = \frac{1}{20}$, at an electric-field strength $D = 0.5 \times 10^{-4}$ V/cm. The investigation of the dependence $\alpha = \alpha(D)$ could, in principle, yield information about the line width and the magnitude of the Lamb shift.

In conclusion, we should like to mention the possibility of experiments with mesic atoms. These experiments have a number of advantages over those that investigate the radiation from the hydrogen atom: larger magnitudes of the effects connected with parity nonconservation, the virtual absence of limitations on the external random electric and magnetic fields because of the larger level spacing, the relative ease with which the experimental investigation of the polarization of the

γ quanta corresponding to the $2S_{1/2} \to 1S_{1/2}$ transition in mesic atoms can be carried out in comparison with the corresponding investigation of the ultraviolet radiation from the hydrogen atom. To be sure, in the case of mesic atoms the question of emission intensity is crucial, and the above-described experiments in which the levels approach each other in the magnetic field are practically impossible.

A detailed discussion on the mesic atoms is a seprate interesting problem that falls outside the framework of the present work.

2. THE EFFECTS OF PARITY NONCONSERVATION IN AN EXTERNAL MAGNETIC FIELD

Let us in the first place ascertain the form of the parity-nonconserving potential that can arise as a result of the weak electron-proton interaction. First, let this interaction have the standard V − A form:

$$\mathscr{H}=\frac{G}{\sqrt{2}}(\bar{\psi}_p\gamma_\mu(1+\gamma_5)\psi_p)(\bar{\psi}_e\gamma_\mu(1+\gamma_5)\psi_e). \qquad (1)$$

Limiting ourselves in this expression to only the nonrelativistic (upper) components of the proton wave function ψ_p and the lower ψ_e components (the product of the upper ψ_p and ψ_e components leads, of course, to a parity-conserving potential), we easily obtain

$$\mathscr{H}=-\frac{G}{2\sqrt{2}\,m}\{(\varphi_p{}^*\varphi_p)[\varphi_e{}^*\boldsymbol{\sigma}(\mathbf{p}+\mathbf{p}')\varphi_e] -(\varphi_p{}^*\boldsymbol{\sigma}\varphi_p)[\varphi_e{}^*(\mathbf{p}+\mathbf{p}'-i[\mathbf{q}\boldsymbol{\sigma}])\varphi_e]\}, \quad \mathbf{q}=\mathbf{p}'-\mathbf{p}, \qquad (2)$$

where p and p′ are the electron momenta before and after scattering. Let us now set $p = -i\nabla_R$ and $p' = +i\nabla_L$, where the gradients ∇_R and ∇_L act respectively on the functions standing to the right and left of them, i.e., on the wave functions of the initial and final states. It is then easy to see that to the interaction (2) corresponds the following potential:

$$V(\mathbf{r})=-\frac{G}{2\sqrt{2}\,m}\delta^3(\mathbf{r})[i(\boldsymbol{\sigma}_e-\boldsymbol{\sigma}_p)(\nabla_L-\nabla_R)-[\boldsymbol{\sigma}_e\boldsymbol{\sigma}_p](\nabla_L-\nabla_R)]. \qquad (3)$$

Here $\frac{1}{2}\sigma_e$ and $\frac{1}{2}\sigma_p$ are the electron- and proton-spin operators; it is assumed that the gradients do not act on $\delta^3(\mathbf{r})$.

What form of the potential is possible in the more general case? For the V + A interaction, for example, when $\mathscr{H}$ is of the form

$$\mathscr{H}=\frac{G}{\sqrt{2}}(\bar{\psi}_p\gamma_\mu(1-\gamma_5)\psi_p)(\bar{\psi}_e\gamma_\mu(1+\gamma_5)\psi_e), \qquad (1')$$

the potential is given by

$$V(\mathbf{r})=-\frac{G}{2\sqrt{2}\,m}\delta^3(\mathbf{r})[i(\boldsymbol{\sigma}_e+\boldsymbol{\sigma}_p)(\nabla_L-\nabla_R)+[\boldsymbol{\sigma}_e\boldsymbol{\sigma}_p](\nabla_L+\nabla_R)]. \qquad (3')$$

As for the general form in the approximation in which only the terms $\sim(\sigma\cdot p/m)$ are retained and in which the conservation of combined parity is assumed, it is given by

$$V(\mathbf{r})=-\frac{1}{2\sqrt{2}\,m}\delta^3(\mathbf{r})[G_1 i(\boldsymbol{\sigma}_e-\boldsymbol{\sigma}_p)(\nabla_L-\nabla_R) +G_2 i(\boldsymbol{\sigma}_e+\boldsymbol{\sigma}_p)(\nabla_L-\nabla_R)-G_3[\boldsymbol{\sigma}_e\boldsymbol{\sigma}_p](\nabla_L+\nabla_R)]. \qquad (4)$$

Terms of the form $(\sigma_e - \sigma_p)\cdot(\nabla_L + \nabla_R)$, $(\sigma_e + \sigma_p)\cdot(\nabla_L + \nabla_R)$, and $i[\sigma_e \times \sigma_p]\cdot(\nabla_L - \nabla_R)$ correspond to CP nonconservation, and we do not consider them in the present paper.

To compute the mixing of the $2S_{1/2}$ and $2P_{1/2}$ states, we shall use the electronic-fine-structure states and the nuclear spin functions with a definite spin component. Such a selection of wave functions implies that we assume $\mu_0 H < \Delta_F$, but $\mu_0 H > \Delta_{HF}$, where Δ_F and Δ_{HF} are the fine and hyperfine splittings ($\mu_0 = e\hbar/2mc$ and H is the magnetic field strength). We shall consider this case in the first place because, as has already been noted in the Introduction, we are interested in magnetic field strengths close to the values obtained from the condition for the crossing of the levels $2S_{1/2}$ and $2P_{1/2}$ with total angular momentum component $m_J = -\frac{1}{2}$, i.e., in the case when $\mu_0 H \sim \Delta_L$, where $\Delta_L = E(2S_{1/2} - E(2P_{1/2})$ is the Lamb shift. On the other hand, $\Delta_L \sim \frac{1}{10}\Delta_F^{(S)} \sim 10\Delta_{HF}^{(S)}$, where the fine and hyperfine splittings pertain to the $2S_{1/2}$ state. The corrections due to the influence of the magnetic field on the fine structure $\sim\mu_0 H/\Delta_F \sim \frac{1}{10}$ and the corrections due to the hyperfine interaction $\sim\Delta_{HF}/\mu_0 H \sim \frac{1}{10}$ are insignificant in the wave functions, but are taken into account below in the energy denominators, where the Lamb energy difference can be cancelled by the quantity $\mu_0 H$ (as has already been noted, this is actually only possible in the one term with $m_J = -\frac{1}{2}$).

Using the standard wave functions and the explicit form of the potential (4), we can easily obtain the exact functions $[\Psi_S^{\pm 1/2}\alpha_{\pm 1/2}]'$ with the admixture of the corresponding P states:

$$[\Psi_S^{+1/2}\alpha_{\pm 1/2}]'=\left[\Psi_S^{+1/2}+\frac{\lambda^\pm}{\varepsilon^\pm(H)}\Psi_P^{+1/2}\right]\alpha_{\pm 1/2}, \qquad (5)$$
$$[\Psi_S^{-1/2}\alpha_{\pm 1/2}]'=\left[\Psi_S^{-1/2}+\frac{\lambda^\mp}{\varepsilon^\mp(-H)}\Psi_P^{-1/2}\right]\alpha_{\pm 1/2}.$$

Here the index $\pm\frac{1}{2}$ on the electronic wave functions $\Psi_S^{\pm 1/2}$ and $\Psi_P^{\pm 1/2}$ indicates the component of the total electronic angular momentum, $\alpha_{\pm 1/2}$ are the nuclear spin functions, while

$$\varepsilon^\pm(H)=\Delta_L+i\frac{\Gamma}{2}+\frac{2}{3}\mu_0 H+\frac{2}{9}\frac{(\mu_0 H)^2}{\Delta_F}\pm\frac{1}{4}(\Delta_{HF}{}^S-\Delta_{HF}{}^P),$$
$$\Delta_L=E(2S_{1/2})-E(2P_{1/2}),\quad \Gamma=\Gamma(2P_{1/2})-\Gamma(2S_{1/2})\approx\Gamma(2P_{1/2}), \qquad (6)$$
$$\Delta_F=E(2P_{3/2})-E(2P_{1/2}),$$
$$\Delta_{HF}{}^S=E(2S_{1/2}, F=1)-E(2S_{1/2}, F=0),\quad \Delta_{HF}{}^P=E(2P_{1/2}, F=1)-E(2P_{1/2}, F=0),$$
$$\lambda^\pm=\frac{iG^\pm\sqrt{6}}{32\pi m a_0{}^4},\quad G^+=\frac{1}{3}[G_1+2G_2-G_3],\quad G^-=\frac{1}{3}[2G_1+G_2+G_3].$$

The Lamb splitting Δ_L has, as usual, been defined without allowance for the hyperfine splitting, so that in the absence of a magnetic field the energies of the $2S_{1/2}$ and $2P_{1/2}$ levels with F = 0.1, which will henceforth be denoted by E_{S0}, E_{S1}, and E_{P0}, E_{P1}, are equal to $E_{S0} = E(2S_{1/2}) - \frac{3}{4}\Delta_{HF}^S$, $E_{S1} = E(2S_{1/2}) + \frac{1}{4}\Delta_{HF}^S$, and similarly for E_{P0} and E_{P1}.

In the formulas (5) we have omitted the terms in which the states $\Psi_S^{\pm 1/2}\alpha_{\pm 1/2}$ contain admixtures of the states $\Psi_P^{\pm 1/2}\alpha_{\pm 1/2}$. This is due to the fact that such an admixture does not make any contribution to the final expression for the emission probability after summation over the nuclear spin components. The energy denominators $\epsilon^\pm(H)$ evidently contain the following terms: 1) the Lamb shift Δ_L, 2) the natural $2P_{1/2}$-line width Γ, 3) the difference $(\frac{2}{3}\mu_0 H)$ between the Zeeman shifts for the S and P states, 4) the hyperfine splittings $\Delta_{HF}^{S,P}$, and 5) the correction $\frac{2}{9}(\mu_0 H)^2/\Delta_F$ of the second approximation to the energy in the magnetic field. In this case, as has already been noted, we are largely interested in the region where $\mu_0 H \sim \Delta_L$, while the remaining terms are approximately an order of magnitude smaller. The combinations G^+ and G^- of the weak-potential constants entering into (5) and (6) possess an interesting property: in the purely V − A variant of the interaction, in which $G_1 = G_3 = G$ and $G_2 = 0$, only the constant $G^- = G$ survives, while $G^+ = 0$; in the V + A variant ($G_1 = 0$, $G_2 = -G_3 = G$), on the other hand, $G^+ = G$ and $G^- = 0$. Thus, G^- and G^+ can conditionally be understood to be the constants of the V − A and V + A interactions (although in fact other variants of the four-fermion interaction can make a contribution to them). For $G^- \neq 0$ and $G^+ = 0$, λ^- is different from zero, while $\lambda^+ = 0$. In this case only the energy ϵ^-, which contains the hyperfine splitting in the form $-\frac{1}{4}(\Delta^S_{HF} - \Delta^P_{HF})$, enters into the formulas (5). This corresponds to a situation in which in the pure V − A variant of the interaction only the S and P states with total angular momentum component $m_F = 0$ intermix. In the V + A theory, on the other hand, $\lambda^+ \neq 0$ and $\lambda^- = 0$, and into the answer enters the energy ϵ^+, where the shift due to the hyperfine interaction is equal to $+\frac{1}{4}(\Delta^S_{HF} - \Delta^P_{HF})$. Here the S and P states with $m_F = -1$ for $m_J = -\frac{1}{2}$ and $m_F = +1$ for $m_J = +\frac{1}{2}$ intermix.

The amplitude of the E1 radiation corresponding to the $2P_{1/2} \to 1S_{1/2}$ transition can be written in the form $b\sigma_e \cdot e^*$, where, in contrast to the foregoing, $\frac{1}{2}\sigma_e$ is the total angular momentum operator for the electron, while e is the polarization vector of the emitted photon. The total emission probability W_P (which coincides here with the total width Γ_P of the level) is the quantity $8\pi|b|^2$, which is equal to

$$8\pi|b|^2 = W_P = (2/3)^8\alpha^5 m = 0.63\cdot 10^9 \text{ sec}^{-1}. \quad (7)$$

The amplitude of the M1 radiation from the one-photon $2S_{1/2} \to 1S_{1/2}$ transition has the structure $a\sigma_e[e^*n]$, where n is the direction of emission of the photon. The probability W_S of this one-photon transition is

$$8\pi|a|^2 = W_S = \frac{1}{2^4 3^3}\alpha^{11}m = 0.57\cdot 10^{-5} \text{ sec}^{-1}. \quad (8)$$

The total amplitude of the $2S_{1/2} \to 1S_{1/2}$ decay with allowance for the admixture of the $2P_{1/2}$ states can now be written in the form

$$M = a\sigma_e[e^*n] + b\sigma_e e^*\left[\frac{\lambda^+}{\varepsilon^+(H)}\Pi_e^+\Pi_p^+ + \frac{\lambda^-}{\varepsilon^-(H)}\Pi_e^+\Pi_p^- + \frac{\lambda^-}{\varepsilon^-(-H)}\Pi_e^-\Pi_p^+ + \frac{\lambda^+}{\varepsilon^+(-H)}\Pi_e^-\Pi_p^-\right]. \quad (9)$$

Here $\Pi_e^\pm$ and $\Pi_p^\pm$ are the electron and proton operators of projection onto the states with angular momentum components $\pm\frac{1}{2}$ along the direction of the magnetic field **h**:

$$\Pi_e^\pm = \frac{1}{2}(1 \pm \sigma_e h), \quad \Pi_p^\pm = \frac{1}{2}(1 \pm \sigma_p h), \quad h = \frac{H}{H}. \quad (10)$$

The decay probability per unit time is $\frac{1}{4}\mathrm{Tr}\, MM^+$, where the trace is computed with respect to both the electronic and nuclear (proton) variables. A simple computation then leads to the following expression for the probabilities (the terms proportional to the square of the weak-interaction constant are, of course, omitted):

$$W = |a|^2[1 + \eta_+ sn + \eta_- hn],$$
$$s = i[ee^*],$$
$$\eta_\pm = \eta_0 G^+\left\{\frac{\mathrm{Re}\,\varepsilon^+(H)}{[\mathrm{Re}\,\varepsilon^+(H)]^2 + \Gamma^2/4} \pm \frac{\mathrm{Re}\,\varepsilon^+(-H)}{[\mathrm{Re}\,\varepsilon^+(-H)]^2 + \Gamma^2/4}\right\}$$
$$+ \eta_0 G^-\left\{\frac{\mathrm{Re}\,\varepsilon^-(H)}{[\mathrm{Re}\,\varepsilon^-(H)]^2 + \Gamma^2/4} \pm \frac{\mathrm{Re}\,\varepsilon^-(-H)}{[\mathrm{Re}\,\varepsilon^-(-H)]^2 + \Gamma^2/4}\right\}, \quad (11)$$
$$\eta_0 = -\frac{1}{32\pi}\sqrt{\frac{3}{2}\frac{r}{ma_0^4}}, \quad r = \left|\frac{b}{a}\right| = \frac{2^8}{9\sqrt{3}}\alpha^{-3} = 1{,}06\cdot 10^7.$$

The energies $\epsilon^\pm(\pm H)$ and the constants $G^\pm$ are given by the formulas (6). If we measure the energies $\epsilon^\pm$ and Γ in units of the Lamb shift ($\Delta_L = 7.8\alpha^5 m/6\pi$)—which is the natural unit here—and the weak-interaction constants $G^\pm$ in units of the standard Fermi constant $G_F = 10^{-5}m_p^{-2}$, then all the quantities in the expressions (11) for $\eta_\pm$ can be considered to be dimensionless, and η_0 assumes the form

$$\eta_0 = -\frac{2\sqrt{2}}{3\cdot 7.8}G_F m^2\alpha^{-4} = -1.27\cdot 10^{-4}. \quad (12)$$

This number defines the scale of the effects. As can be seen from the formulas (11), two correlations connected with parity nonconservation are possible; asymmetry in the photon emission with respect to the direction of the magnetic field and the circular polarization of the photons. The photon spin $s = i[e \times e^*]$ is directed along the direction n of the momentum of the right circularly polarized photons: $s \cdot n = +1$, while the oppositely directed spin is along the direction of the momentum of the left circularly polarized photons: $s \cdot n = -1$. Therefore, the degree of circular polarization is

$$\delta = (N_+ - N_-)/(N_+ + N_-) = \eta_+. \quad (13)$$

In the absence of a magnetic field, $\eta_+ \approx 2\eta_0(G^+ + G^-)$. If the weak interaction has the V − A form with the standard Fermi constant ($G^- = 1$ in units of G_F and $G^+ = 0$), then $\delta = -2.5 \times 10^{-4}$ [2]. In the Weinberg model[6] for the e-p interaction without allowance for the renormalization of the axial constant, we have $G^- = -\frac{1}{2}(1 - 4\sin^2\theta_W)$ and $G^+ = 0$. The CERN neutrino experiment yields $\sin^2\theta_W \approx 0.35$[4]. In this case $G^- \approx 0.2$.

The formulas (11) are valid only for $\mu_0 H > \Delta^{S,P}_{HF}$. Therefore, the transition in them to zero magnetic field can be effected only approximately: the hyperfine splitting $\Delta^{S,P}_{HF}$ should be neglected at first.

The formula for η_+ for H = 0 but with allowance for the hyperfine interaction can easily be derived if the states of the hyperfine structure are used. It has the form

$$\eta_+ = \eta_0\left[\frac{3G^+(E_{S1} - E_{P1})}{(E_{S1} - E_{P1})^2 + \Gamma^2/4} + \frac{(2G^- - G^+)(E_{S0} - E_{P0})}{(E_{S0} - E_{P0})^2 + \Gamma^2/4}\right]. \quad (14)$$

Here E_{SF} and E_{PF} are the energies of the $2S_{1/2}$ and $2P_{1/2}$ levels with a definite total angular momentum F. As before, upon the neglect of the hyperfine splitting and Γ, $\eta_+ \approx 2\eta_0(G^+ + G^-)$. If $G^- \neq 0$ and $G^+ = 0$ (V − A theory), then only the spacing between the singlet levels enters into the last formula.

The asymmetry in the magnetic field (i.e., the coefficient η_-) is equal to zero when H = 0, then it increases and attains a maximum at magnetic-field strengths close to those at which $\mathrm{Re}\,\epsilon^\pm(-H) = 0$. In this region η_- as a function of the magnetic field has a characteristic "dispersion" dependence (see the figure). It can be seen from the explicit expression (6) for the energies $\epsilon^\pm(\pm H)$ that since $\Delta_L > 0$, the quantities that can vanish are precisely the quantities $\mathrm{Re}\,\epsilon^\pm(-H)$, but not the quantities $\mathrm{Re}\,\epsilon^\pm(+H)$ (i.e., the levels that can intersect are those with $m_J = -\frac{1}{2}$, but not those with $m_J = +\frac{1}{2}$). As has already been noted, $G^+ = 0$ for the pure V − A variant, so that in this case there remain in the expression (11) for $\eta_\pm$ only the last two terms, in the last one of which $\mathrm{Re}\,\epsilon^-(-H)$ can, as noted above, vanish. For the V + A variant, in which $G^- = 0$, there remain in (11) only the first two terms, the second one of which has a resonance character. The resulting curve for $\eta_-(H)$ turns out to be shifted in comparison with the curve for the V − A variant. The shift occurs because Re $\epsilon^+(-H)$ differs from Re $\epsilon^-(-H)$ by the constant value $\frac{1}{2}(\Delta_{HF}^{S} - \Delta_{HF}^{P}) \approx 60$ MHz, which corresponds to a magnetic field strength difference ~50 G. When the G^+ and G^- terms are both present, both resonances contribute to the shift. The corresponding curves are shown in the figure. (In constructing the graphs, it was assumed that $G^- = 1$ in the first case and $G^+ = 1$ in the second.) Thus, the form of the dependence of the asymmetry parameter $\eta_-(H)$ (as, indeed, of $\eta_+(H)$) allows us, in principle, to judge the nature of the weak interaction between the electron and the proton.

The value of the parameters $\eta_\pm$ at the maximum with respect to the magnetic field is approximately equal to $\eta_0 G(\Delta_L/\Gamma)$, i.e., is five times greater than the value of the parameter $\eta_+ \approx 2\eta_0 G$ in the absence of a magnetic field. Notice, finally, that the effects under consideration depend on the sign of the weak-interaction constant[1)].

3. THE ONE-PHOTON $2S_{1/2} \to 1S_{1/2}$ TRANSITION IN EXTERNAL ELECTRIC AND MAGNETIC FIELDS

As has already been explained in the Introduction, the presence in the experiment of an external random electric should lead to strong intermixing of the S and P states and, as a result of this, to the rapid de-excitation of the metastable $2S_{1/2}$ level. The de-excitation can also occur owing to collisions, the van der Waals interaction between the atoms, the weak electric field that arises during the motion of the atoms in the magnetic field, etc. We shall not consider the entire set of these problems, which pertain rather to the competence of the experimenter; instead, we shall compute the radiation emitted in the one-photon $2S_{1/2} \to 1S_{1/2}$ transition in the presence of external electric and magnetic fields. As we shall see, the presence of both fields makes possible not only the de-excitation of the atom, but the imitation of effects connected with parity nonconservation as well.

If the quantization axis (the z axis) is chosen, as before, in the direction of the magnetic field, while the electric field **D** is directed obliquely to the z axis, then in the state $\Psi_S^{\pm 1/2}$ will be mixed not only $\Psi_P^{\pm 1/2}$, but $\Psi_P^{\mp 1/2}$ as well (the component of the angular momentum along the z axis will not be conserved). The formulas similar to (5), but connected with the electric-field induced admixture of the P states, have the form

$$[\Psi_S{}^{1/2}\alpha_{\pm 1/2}]' = \left[\Psi_S{}^{1/2} + \frac{\sqrt{3}\,ea_0D_z}{\varepsilon^\pm(H)}\Psi_P{}^{1/2} + \frac{\sqrt{3}\,ea_0(D_x+iD_y)}{\varepsilon_1{}^\pm(H)}\Psi_P{}^{-1/2}\right]\alpha_{\pm 1/2},$$

$$[\Psi_S{}^{-1/2}\alpha_{\pm 1/2}]' = \left[\Psi_S{}^{-1/2} - \frac{\sqrt{3}\,ea_0D_z}{\varepsilon^\mp(-H)}\Psi_P{}^{-1/2} + \frac{\sqrt{3}\,ea_0(D_x-iD_y)}{\varepsilon_1{}^\mp(-H)}\Psi_P{}^{1/2}\right]\alpha_{\pm 1/2}. \tag{15}$$

The energy differences $\epsilon^\pm(H)$ are given by the formula (6), while the differences $\epsilon_1^\pm(H)$ are equal to

$$\varepsilon_1{}^\pm(H) = \Delta_L + \frac{i\Gamma}{2} + \frac{4}{3}\mu_0 H + \frac{2}{9}\frac{(\mu_0 H)^2}{\Delta_F} \pm \frac{1}{4}(\Delta_{HF}{}^S + \Delta_{HF}{}^P). \tag{16}$$

If we now replace $\Psi_P^{\mp 1/2}$ in the last terms of the Eqs. (15) by $\frac{1}{2}(\sigma_x \pm i\sigma_y)\Psi_P^{\pm 1/2}$, then it becomes easy to write out the radiation amplitude in a matrix form similar to (9):

$$M' = a\boldsymbol{\sigma}_e[\mathbf{e}^*\mathbf{n}] + b\sqrt{3}\,ea_0\boldsymbol{\sigma}_e\mathbf{e}^*\Big\{\frac{1}{2}(1+\boldsymbol{\sigma}_e\mathbf{h})(\mathbf{Dh})\Big(\frac{\Pi_p{}^+}{\varepsilon^+(H)} + \frac{\Pi_p{}^-}{\varepsilon^-(H)}\Big)$$
$$-\frac{1}{2}(1-\boldsymbol{\sigma}_e\mathbf{h})(\mathbf{Dh})\Big(\frac{\Pi_p{}^+}{\varepsilon^-(-H)} + \frac{\Pi_p{}^-}{\varepsilon^+(-H)}\Big) + \frac{1}{2}((\boldsymbol{\sigma}_e\mathbf{D}) - (\mathbf{Dh})(\boldsymbol{\sigma}_e\mathbf{h})$$
$$+i[\mathbf{Dh}]\boldsymbol{\sigma}_e)\Big(\frac{\Pi_p{}^+}{\varepsilon_1{}^+(H)} + \frac{\Pi_p{}^-}{\varepsilon_1{}^-(H)}\Big) + \frac{1}{2}((\boldsymbol{\sigma}_e\mathbf{D}) - (\mathbf{Dh})(\boldsymbol{\sigma}_e\mathbf{h})$$
$$-i[\mathbf{Dh}]\boldsymbol{\sigma}_e)\Big(\frac{\Pi_p{}^+}{\varepsilon_1{}^-(-H)} + \frac{\Pi_p{}^-}{\varepsilon_1{}^+(-H)}\Big)\Big\}. \tag{17}$$

The emission probability is equal to $\frac{1}{4}\mathrm{Tr}\, M'M'^+$, where the trace is computed with respect to the electronic and nuclear variables. A direct calculation then yields the following expression, which may be compared with the formula (11):

$$W' = |a|^2\{1 + 3e^2a_0{}^2r^2[A(\mathbf{Dh})^2 + B[\mathbf{D}^2 - (\mathbf{Dh})^2] + C(\mathbf{Dh})(\mathbf{Ds})$$
$$+E(\mathbf{Dh})^2(\mathbf{sh}) + F[\mathbf{D}^2 - (\mathbf{Dh})^2](\mathbf{sh}) + G(\mathbf{Dh})([\mathbf{Dh}]\mathbf{s})]$$
$$+\sqrt{3}\,ea_0r[K(\mathbf{Dh})(\mathbf{sn}) + L(\mathbf{Dn}) + M([\mathbf{Dh}]\mathbf{n}) + N(\mathbf{Dh})(\mathbf{hn})]\}, \tag{18}$$

where

$$A = \frac{1}{4}\Big[\frac{1}{|\varepsilon^+(H)|^2} + \frac{1}{|\varepsilon^-(H)|^2} + \frac{1}{|\varepsilon^+(-H)|^2} + \frac{1}{|\varepsilon^-(-H)|^2}\Big],$$

$$B = \frac{1}{4}\Big[\frac{1}{|\varepsilon_1{}^+(H)|^2} + \frac{1}{|\varepsilon_1{}^-(H)|^2} + \frac{1}{|\varepsilon^+(-H)|^2} + \frac{1}{|\varepsilon_1{}^-(-H)|^2}\Big],$$

$$C = \frac{1}{2}\mathrm{Re}\Big[\frac{1}{\varepsilon^+(H)[\varepsilon_1{}^+(H)]^*} + \frac{1}{\varepsilon^-(H)[\varepsilon_1{}^-(H)]^*} - \frac{1}{\varepsilon^+(-H)[\varepsilon_1{}^+(-H)]^*}$$
$$- \frac{1}{\varepsilon^-(-H)[\varepsilon_1{}^-(-H)]^*}\Big],$$

$$E = \frac{1}{4}\Big[\frac{1}{|\varepsilon^+(H)|^2} + \frac{1}{|\varepsilon^-(H)|^2} - \frac{1}{|\varepsilon^+(-H)|^2} - \frac{1}{|\varepsilon^-(-H)|^2}$$
$$-2\,\mathrm{Re}\frac{1}{\varepsilon^+(H)[\varepsilon_1{}^+(H)]^*} - 2\,\mathrm{Re}\frac{1}{\varepsilon^-(H)[\varepsilon_1{}^-(H)]^*}$$
$$+2\,\mathrm{Re}\frac{1}{\varepsilon^+(-H)[\varepsilon_1{}^+(-H)]^*} + 2\,\mathrm{Re}\frac{1}{\varepsilon^-(-H)[\varepsilon_1{}^-(-H)]^*}\Big],$$

$$F = -\frac{1}{4}\Big[\frac{1}{|\varepsilon_1{}^+(H)|^2} + \frac{1}{|\varepsilon_1{}^-(H)|^2} - \frac{1}{|\varepsilon_1{}^+(-H)|^2} - \frac{1}{|\varepsilon_1{}^-(-H)|^2}\Big]$$

$$G = \frac{1}{2}\mathrm{Im}\Big[\frac{1}{\varepsilon^+(H)[\varepsilon_1{}^+(H)]^*} + \frac{1}{\varepsilon^-(H)[\varepsilon_1{}^-(H)]}$$
$$+ \frac{1}{\varepsilon^+(-H)[\varepsilon_1{}^+(-H)]^*} + \frac{1}{\varepsilon^-(-H)[\varepsilon_1{}^-(-H)]^*}\Big],$$

$$K = -\frac{1}{2}\mathrm{Im}\Big[\frac{1}{\varepsilon^+(H)} + \frac{1}{\varepsilon^-(H)} - \frac{1}{\varepsilon^+(-H)} - \frac{1}{\varepsilon^-(-H)}\Big],$$

$$L = -\frac{1}{2}\mathrm{Im}\Big[\frac{1}{\varepsilon_1{}^+(H)} + \frac{1}{\varepsilon_1{}^-(H)} + \frac{1}{\varepsilon_1{}^+(-H)} + \frac{1}{\varepsilon_1{}^-(-H)}\Big],$$

$$M = -\frac{1}{2}\mathrm{Re}\Big[\frac{1}{\varepsilon_1{}^+(H)} + \frac{1}{\varepsilon_1{}^-(H)} - \frac{1}{\varepsilon_1{}^+(-H)} - \frac{1}{\varepsilon_1{}^-(-H)}\Big],$$

$$N = -\frac{1}{2}\mathrm{Im}\Big[\frac{1}{\varepsilon^+(H)} + \frac{1}{\varepsilon^-(H)} + \frac{1}{\varepsilon_1{}^+(H)} + \frac{1}{\varepsilon_1{}^-(H)}$$
$$- \frac{1}{\varepsilon^+(-H)} - \frac{1}{\varepsilon^-(-H)} - \frac{1}{\varepsilon_1{}^+(-H)} - \frac{1}{\varepsilon_1{}^-(-H)}\Big]. \tag{19}$$

Like (11), the formulas (19) are literally inapplicable when $\mu_0 H < \Delta_{HF}^{S,P}$ (it is first necessary to let $\Delta_{HF}^{S,P}$ go to zero). Since we want, in the first place, to estimate the influence of the external random electric and magnetic fields in the experiment on the measurement of the circular polarization in the absence of a specially introduced external magnetic field, we can a priori expect that $H < \Delta_{HF}^{S}/\mu_0 \sim 100$ G. If the magnetic field is, on the whole, exactly equal to zero, then the emission probability has the form

$$W' = |a|^2[1 + 3e^2a_0{}^2r^2A\mathbf{D}^2 + \sqrt{3}\,ea_0rL\mathbf{Dn}], \tag{20}$$

where

$$A = \frac{1}{\Delta^2{}_L + \Gamma^2/4}, \quad L = \frac{\Gamma}{\Delta^2{}_L + \Gamma^2/4}. \tag{21}$$

The simplest limitation on the strength of the external electric field is that the term $\sim\mathbf{D}^2$ in (20) should be less than unity, which implies that the electric-field-induced admixture of the P state will not lead to the de-excitation of the atom in a time shorter than the one-photon $2S_{1/2} \rightarrow 1S_{1/2}$ transition time. Of course, in reality, the lifetime of the atoms is shorter than $W_S^{-1} \sim 2 \times 10^5$ sec. But the causes that lead to the disappearance of atoms in the metastable $2S_{1/2}$ state (e.g., the two-photon emission) and that are not connected with the admixture of the P state will, evidently, influence only the general statistics, but not the magnitude of the circular polarization.

The condition

$$3e^2a_0^2r^2D^2A \approx (\sqrt{3}ea_0rD/\Delta_L)^2 < 1 \tag{22}$$

implies that $D < D_0 = 0.5 \times 10^{-4}$ V/cm.

Although the second term in the formula (20) does not lead to additional limitations on the electric field, it is of interest in itself. A correlation of the form $\mathbf{D}\cdot\mathbf{n}$ is formally T odd. Its appearance is due to the instability of the atom in the excited state, and, as can be seen from the expression (21), the coefficient attached to this correlation is proportional to the width of the $2P_{1/2}$ level. We are dealing here with the imitation of T-parity nonconservation in unstable systems that was predicted by Zel'dovich[5]. If we rewrite the emission probability in the form

$$W' = W_0[1 + \alpha \mathbf{Dn}/D], \quad \alpha = \frac{\sqrt{3}ea_0rDL}{1 + 3e^2a_0^2r^2D^2A}, \tag{23}$$

then it is evident that the coefficient α attains a maximum at the above-obtained field strength $D = D_0 = [\sqrt{3}ea_0rA^{1/2}]^{-1} \approx 0.5 \times 10^{-4}$ V/cm, $\alpha(D_0)$ being equal to $L/2\sqrt{A} \approx \Gamma/2\Delta_L \approx 1/20$. The experimental observation of the effect, including the measurement of α as a function of D, could provide an independent method of determining the Lamb shift and the width of the $2P_{1/2}$ level.

Let us now suppose that there exists in the experimental setup, besides an external random electric field, a weak magnetic field. As can be seen from (18), there then appears in the expression for the emission probability a correlation of the form $(\mathbf{D}\cdot\mathbf{h})(\mathbf{s}\cdot\mathbf{n})$, which imitates the effect of the circular polarization connected with the weak interaction.

The magnitude of the effect is determined here by the coefficient K, which is approximately equal to

$$K = -{}^4/_3\mu_0H\Gamma/\Delta_L^3. \tag{24}$$

The necessary condition for the correlation $(\mathbf{D}\cdot\mathbf{h})(\mathbf{s}\cdot\mathbf{n})$ not to efface the effect of the weak-interaction-induced circular polarization is now given by

$$\sqrt{3}\,rea_0D\cdot{}^4/_3\mu_0H\Gamma/\Delta_L^3 < 10^{-4}. \tag{25}$$

We obtain from this that for $D < 10^{-5}$ V/cm (which is dictated by the condition (22)) the magnetic field should satisfy the condition

$$H < 3\ \mathrm{G}. \tag{26}$$

Besides the above-considered correlation, correlations of the form $(\mathbf{D}\cdot\mathbf{h})(\mathbf{D}\cdot\mathbf{s})$ and $(\mathbf{D}\cdot\mathbf{h})^2(\mathbf{s}\cdot\mathbf{h})$ are also dangerous. These terms can imitate the circular polarization $\mathbf{s}\cdot\mathbf{n}$ if the random fields D and H are inclined at certain angles to the direction n and the experiment is performed in such a way that special measures are not taken to average the quantities over these angles. For the case when $\mathbf{h} \parallel \mathbf{D}$ the correlation $\mathbf{s}\cdot\mathbf{n}$ is imitated by the terms $\sim(C + E)$ in (18). We have,

$$C + E = -{}^4/_3\mu_0H/\Delta_L^3. \tag{27}$$

The limitation on the magnetic field strength arises now from the condition

$$3e^2a_0^2D^2r^2\cdot{}^4/_3\mu_0H/\Delta_L^3 < 10^{-4}, \tag{28}$$

so that for $D < 10^{-5}$ V/cm we must have $H < 1.5$ G.

The limitations found thus far pertained to an experimental setup without a specially introduced magnetic field. As was explained in Sec. 1 of the present paper, the introduction of an external magnetic field of intensity ~1 kG allows us, in principle, to observe the beautiful effects of the dependence of the asymmetry parameter and the polarization on the magnetic field. The payment is much more rigid background conditions on the electric-field strength D than those that have thus far been formulated.

As can be seen from (18), the correlation $(\mathbf{D}\cdot\mathbf{h})(\mathbf{n}\cdot\mathbf{h})$ is the principal effect that imitates an asymmetry of the form $\mathbf{n}\cdot\mathbf{h}$. The condition

$$r\sqrt{3}ea_0DN < 10^{-4} \tag{29}$$

turns out to be very rigid, so that the coefficient N is increased at the peak of the enhancement of the effect, i.e., when $|{}^3/_2\mu_0H - \Delta_L| \approx {}^1/_2\Gamma$. At this point

$$N \approx \mathrm{Im}\frac{1}{\varepsilon^-(-H)} \approx \frac{2}{\Gamma}. \tag{30}$$

This, together with (29), leads to the condition

$$D < 2{,}5\cdot 10^{-10}\ \mathrm{V/cm}. \tag{31}$$

To be sure, the magnitude of the effect at the peak here $\sim 10^{-3}$, so that fields of intensity $D < 10^{-9}$ V/cm may still be admissible.

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Note Added in Proof (May 27, 1974). The formula (8), which corresponds to the result obtained by Breit and Teller (G. Breit and E. Teller, Astrophys, J. 91, 215 (1940), is incorrect. The correct formula contains the additional factor 4/9 (C. Drake, Phys. Rev. A3, 908 (1971)). Allowance for this factor leads to the multiplication of the quantities r, η_0, and $\eta_\pm$ (the formulas (11), etc.) by 3/2.

*$[\mathbf{q}\sigma] \equiv \mathbf{q} \times \sigma$.

[1)]The sign of our constants is opposite to that of the constants of the paper [2], i.e., G is introduced in [2] with the plus sign in the Lagrangian, and not in the Hamiltonian, as is done in the present paper. Furthermore, the definition of left and right circularly polarized photons in [2] is also opposite to ours, which leads to a formal coincidence of the expressions for the degree of polarization for H = 0.

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