Determination of the electron velocity distribution by the Thomson scattering method

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It is shown that the electron velocity distribution can be determined by inverting the incoherent Thomson scattering function and spectrum. Formulas are given for low-temperature and relativistic plasmas.

1. The inversion of the spectrum of laser radiation scattered incoherently by low-temperature plasma can be used to determine the electron velocity distribution.[1-3] However, the dependence of the cross section for photon-electron scattering on the velocity of the electron must be taken into account in the case of high-temperature plasma.[4,5] One must distinguish between two cases: in the first, the space-time region W, in which the radiation is formed, is greater than the scattering volume and the duration of the light pulses and, in the second, it is smaller. The quantity W is of the order of the period and wavelength of the wave whose frequency is the smaller of the two frequencies ω_2 and $|\omega_2-\omega_1|$, where ω_1 and ω_2 are the frequencies of the incident and scattered radiations, respectively. The relativistic case, when the space-time region W is less than or of the order of the scattering volume is considered in[4,6]. This analysis presupposes the use of very small scattering volumes or very short laser pulses. This condition is not usually satisfied in practice (see[7,8]).

The present paper is devoted to the determination of the relativistic distribution function for pulses of arbitrary length and large scattering volumes, which is based on the inversion of the spectrum or the Thomson scattering function.[5,9] The results are general and can be used for low-temperature plasmas.

2. We shall suppose that the electron velocity distribution is isotropic, the scattering is incoherent, the incident wave is plane, and its plane of polarization is perpendicular to the plane of scattering. If we then take into account the light-scattering cross section of an electron as a function of its velocity v, using our earlier results,[5] we find that

$$\sigma(\omega,\theta)=\frac{r_0^2\omega^2}{4\sin(\theta/2)}\int\limits_{p(\omega,\theta)}^{1}\frac{f(v)(1-v^2)dv}{v[1-v^2\cos^2(\theta/2)]^{1/2}}\tag{1}$$

$$p(\omega,\theta)=\frac{|\omega-1|}{(1+2\omega\cos\theta+\omega^2)^{1/2}},\quad \omega=\frac{\omega_2}{\omega_1}.\tag{2}$$

In these expressions, $\sigma(\omega,\theta)$ is the light scattering cross section in plasma with an electron distribution function f(v), v is the dimensionless velocity of the electrons in units of the velocity of light c, θ is the scattering angle, and r_0 the classical electron radius.

Multiplying both sides of the integral equation (1) by $4\sin(\frac{1}{2}\theta)/r_0^2\omega^2$ and differentiating with respect to ω, we obtain the velocity distribution function in terms of known scattered spectrum $\sigma(\omega)$ at given θ:

$$f(v)=\frac{(\omega-1)(1-2\omega\cos\theta+\omega^2)}{2r_0^{-2}\omega\sin^2(\theta/2)}\frac{d}{d\omega}\left[\frac{\sigma(\omega)}{\omega^2}\right],\tag{3}$$

where

$$v\equiv\frac{\omega-1}{(1-2\omega\cos\theta+\omega^2)^{1/2}}=p(\omega,\theta),\quad \omega\geqslant1.$$

The result given by Eq. (3) is valid for arbitrary V. If the electron velocities are low, or the scattering angle θ is small (but collective processes can still be neglected), then $\Delta\omega=\omega-1\ll1$. Substituting $\omega=1+\Delta\omega$ in (3), and neglecting terms of the order of $\Delta\omega^2$, we obtain

$$f(v)=f\left(\frac{\Delta\omega}{2(1+\Delta\omega)\sin(\theta/2)}\right)=-4r_0^{-2}\sin\frac{\theta}{2}\Delta\omega(1+\Delta\omega)\frac{d}{d\Delta\omega}\left[\frac{\sigma(\Delta\omega)}{(1+\Delta\omega)^2}\right].\tag{4}$$

When the temperature is not too high, corrections of the order of v become unimportant and (4) can be rewritten in the form

$$f(v)=f\left(\frac{\Delta\omega}{2\sin(\theta/2)}\right)=-4r_0^{-2}\sin\frac{\theta}{2}\Delta\omega\frac{d\sigma(\Delta\omega)}{d\Delta\omega}.\tag{5}$$

We note that the form of the resultant distribution function is very dependent on the relativistic corrections.

3. The distribution function f(v) can also be found from a known scattering function $\sigma(\theta)$ determined at a fixed frequency $\omega\neq1$. Analytic inversion of (1) is impossible, but the second iteration suffices for the determination of f(v) to within v^2.

Differentiating (1) with respect to θ, we obtain

$$f(v)=-u(v)\frac{d\sigma_0(\theta)}{d\theta}-\frac{\sin\theta}{4}u(v)\int\limits_{p(\omega,\theta)}^{1}\frac{f(v)(1-v^2)vdv}{(1-v^2\cos^2(\theta/2))^{3/2}},\tag{6}$$

where $v=p(\omega,\theta)$ and

$$4r_0^{-2}\omega^{-2}\sin\frac{\theta}{2}\sigma(\theta)\equiv\sigma_0(\theta),\qquad \frac{v(1-v^2\cos^2(\theta/2))^{1/2}}{1-v^2}\frac{d\theta}{dv}\equiv u(v).$$

For the first approximation to f(v) we take

$$f_1(v)=-u(v)\frac{d\sigma_0(\theta)}{d\theta}=\frac{v(1-v^2\cos^2(\theta/2))^{1/2}}{1-v^2}\frac{d\theta}{dv}\frac{d\sigma_0(\theta)}{d\theta}.\tag{7}$$

The second iteration after substitution for $f_1(v)$ into the right-hand side of (6) yields

$$f_2(v)=f_1(v)+\frac{\sin\theta}{4}u(v)\left\{\frac{v^2\sigma_0(\theta)}{1-v^2\cos^2(\theta/2)}-\int\limits_{\theta}^{0}\sigma_0(\theta')\frac{d}{d\theta'}\left[v'^2\frac{(1-v^2\cos^2(\theta'/2))^{1/2}}{(1-v^2\cos^2(\theta/2))^{3/2}}\right]d\theta'\right\}.\tag{8}$$

In deriving (8), we have integrated (6) by parts, using the condition $\sigma_0(0)=0$. The integral in (8) differs from the integral

$$2\int\sigma_0(\theta')v'^2\frac{dv'}{d\theta'}d\theta'\tag{9}$$

by an amount less than the corrections to f(v) or order v^2, since it is readily seen that for the Maxwellian distribution function the difference between the integrals in (8) and (9) is of the order of v^3.

Thus, the second approximation leads to the following expression for f(v):

$$f_2(v)=f_1(v)+\frac{\sin\theta}{4}u(v)\left\{\frac{v^2\sigma_0(\theta)}{1-v^2\cos^2(\theta/2)}-2\int\limits_{\theta}^{0}\sigma_0(\theta')v'^2\frac{dv'}{d\theta'}d\theta'\right\}.\tag{10}$$

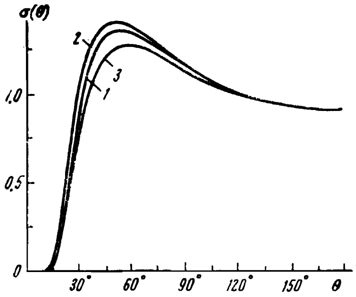

FIG. 1

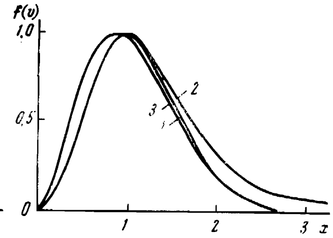

FIG. 2

FIG. 1. Scattering function $\sigma(\theta)$ for nonrelativistic distribution functions: (1) f(T); (2) f(T) + 0.2 f(0.5T); (3) f(T) + 0.2 f(2T). The scattering functions are normalized at $\theta = \pi$.

FIG. 2. Electron-velocity distributions of the form $f(v, T) = Av^2 \exp(-mv^2/2T)$ as functions of $x = v(2T/m)^{1/2}$, normalized at the maximum: 1 – f(T), 2 – f(T) + 0.2 f(0.5T), 3 – f(T) + 0.2 f(2T).

We note that the first approximation to (7) is subject to an error of the order of v^2, and the third iteration to an error of the order of v^5. Hence, to determine the distribution function to within v, we have from (7)

$$f_1(v) = -v\frac{d\theta}{dv}\frac{d\sigma_0(\theta)}{d\theta} = \frac{1-2\omega\cos\theta+\omega^2}{\omega\sin\theta}\frac{d\sigma_0(\theta)}{d\theta}, \quad v=p(\omega,\theta). \quad (11)$$

If we take $\Delta\omega = \omega - 1$, then at the minimum scattering angle θ_{min}

$$\frac{\Delta\omega}{2\sin(\theta_{min}/2)} \ll 1,$$

and we have the simplified result

$$f_1(v) = f_1\left(\frac{\Delta\omega}{2\sin(\theta/2)}\right) = 8r_0^{-2}\,\mathrm{tg}\frac{\theta}{2}\frac{d}{d\theta}\left[\sin\frac{\theta}{2}\sigma(\Delta\omega,\theta)\right]. \quad (12)$$

Figure 1 shows the scattering function calculated from (1) and (2) for low temperatures T and different distribution functions (Fig. 2). Inversion of the curves given in Fig. 1 in accordance with (11), naturally leads to the given functions in Fig. 2. The accuracy with which the distribution function can be determined from the scattering function is no worse than the accuracy achieved by spectrum inversion.

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