Difference-frequency excitation in nonlinear optics and the conditions for Cerenkov-radiation emission

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A theory of nonlinear optical effects is developed for the case when the transverse dimension of the region occupied by the nonlinear-polarization wave is of the order of the wavelength of the emitted wave. There arise in this case specific emission conditions similar to those under which the Vavilov-Cerenkov radiation is emitted. This case is of special interest in connection with the problem of the generation of far-infrared radiation as a difference frequency. In contrast to other nonlinear-optics problems in which the quasioptical approximation is widely used, the solution of the indicated problem requires the solution of the complete wave equation. The emission of Cerenkov difference-frequency radiation (DFR) by a monochromatic-wave doublet, as well as by an ultrashort light pulse, is considered. In the latter case the Cerenkov radiation, which is inevitably generated in many media, can be treated as additional nonlinear losses. It is shown that in ordinary anisotropic media Cerenkov-radiation emission should be possible for both the forward and backward waves. This radiation possesses interesting distinctive features in optically active media, in which four Cerenkov angles can exist simultaneously. It is also shown that in DFR generation the Cerenkov condition can, besides the usual interference interpretation, also be accounted for on the basis of the vector interaction between the angular components of the beams. The advantage of the Cerenkov DFR generation over the synchronous vector interaction of the beams is noted.

1. INTRODUCTION

Definite progress has recently been made in the generation of coherent radiation in the 10–150-cm^{-1} band. In a number of investigations significant advances in this direction were achieved by mixing specially chosen optical doublets (see the proceedings of the conferences [1,2]) and through rectification of picosecond laser pulses [3-5]. The generation of coherent difference-frequency radiation (DFR) by these methods is, besides the interesting applications, of considerable interest for nonlinear optics itself. The point is that in the indicated cases the transverse dimensions a of the region occupied by the nonlinear polarization turn out to be of the order of the wavelength Λ of the generated wave (i.e., $a \sim \Lambda$). Owing to this, $\nu = 1/\Lambda$ DFR can be detected outside the induced-polarization region. On the one hand, this renders unsuitable [6-8] the standard method of nonlinear optics—the parabolic-equation method—and, on the other, it opens up new possibilities for the matching of the phase velocities of interacting waves.

Below we shall show that the effective DFR is emitted in many media at an angle $\theta_0 = \cos^{-1}(k_i/k)$ to the direction of propagation of the induced polarization (where k and k_i are the wave numbers of the natural wave and the induced polarization respectively); this phenomenon can be interpreted as a Cerenkov emission [9,10] of a nonlinear-polarization wave. The Cerenkov DFR emission occurs under conditions when there is no synchronous collinear interaction between the beams (i.e., when $k \neq k_i$); up till now, however, DFR generation has been investigated [6-8] only under conditions when the collinear-synchronism condition, $k = k_i$, is fulfilled.

The possibility of a Cerenkov emission of electromagnetic waves from a nonlinear medium polarization propagating with a "superlight" velocity ($v_i = \Omega/k_i > v = \Omega/k$) has been demonstrated by Askar'yan [11]. Before that [10], it had been pointed out that any type of wave could, in principle, emit Vavilov-Cerenkov radiation. Later, the Cerenkov generation of light harmonics in optical wave guides [12,13] and crystals [14,15] was studied. This case, however, is of little practical interest, since the magnitude of the effect is proportional to the ratio λ/a, which, for the visible spectral region, is extremely small. It is evident that the yield of the Cerenkov DFR in the submillimeter wave band can be substantial.

The object of the present article is to analyze in detail DFR emission by a narrow beam of induced nonlinear medium polarization. We consider the cases of DFR excitation by a monochromatic wave doublet and by a narrow light pulse. Besides the ordinary anisotropic medium, the phenomenon is analyzed in optically active media. It is shown that besides the well-known interference interpretation, the Cerenkov radiation in this case can be interpreted as a condition for vector synchronism for extremely narrow beams.

2. THE CERENKOV EMISSION OF A NONLINEAR-POLARIZATION WAVE EXCITED BY A MONOCHROMATIC-WAVE DOUBLET

We shall consider the DFR-generation problem in the approximation that the exciting radiation fields $\mathbf{E}_2(\mathbf{r}, t)$ and $\mathbf{E}_1(\mathbf{r}, t)$ are given. The system of Maxwell equations reduces in this case to the following equation for the excited field $\mathbf{E}(\mathbf{r}, t)$:

$$\Delta\mathbf{E} - \frac{n^2}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = -\frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}\hat{\chi}\mathbf{E}_2\mathbf{E}_1^*, \quad (1)$$

where Δ is the Laplacian and $\hat{\chi}$ is the nonlinear-susceptibility tensor of the medium. Let the exciting monochromatic laser beams propagate along the z axis. Then

$$\mathbf{E}_j(\mathbf{r}, t) = \mathbf{e}_j A_j(\mathbf{r}) \exp\{i(\omega_j t - k_j z)\} \quad (2)$$

(ω_j is the frequency), and we obtain for the complex DFR amplitude the equation

$$\Delta A + k^2 A = -4\pi\chi\left(\frac{\Omega}{c}\right)^2 A_2(\mathbf{r}) A_1^*(\mathbf{r}) e^{-i(k_2-k_1)z}, \quad (3)$$

where $\chi = \mathbf{e}\hat{\chi}\mathbf{e}_2\mathbf{e}_1$ is the nonlinear-coupling constant, $\mathbf{e}_j$ is the unit polarization vector, $k = \Omega n/c$, and $\Omega = \omega_2 - \omega_1$ is the generated frequency.

Let us first assume that the induced polarization propagates along the positive direction of the z axis ($k_2-k_1=k_i>0$). Let us write the solution to Eq. (3) for the angular DFR spectrum in the form:

$$A(\alpha,\beta,z)=\frac{1}{(2\pi)^2}\iint\limits_{-\infty}^{+\infty}A(x,y,z)\,e^{i(\alpha x+\beta y)}\,dx\,dy. \qquad (4)$$

its variation in the case of forward-wave generation is described by the expression

$$A(\alpha,\beta,z=l)=-i\frac{\gamma l}{k_i+g}F(\alpha,\beta)\exp\left\{-i\left(\frac{k_i+g}{2}\right)l\right\}\operatorname{sinc}\frac{k_i-g}{2}l. \qquad (5)$$

In deriving (5), we took into account the emission condition and the condition at the entrance to the crystal: $A(\alpha,\beta,z=0)=0$. In (5) we have introduced the notations

$$\gamma=4\pi\chi\Omega^2/c^2,\quad k_i=k_2-k_1,\quad g=(k^2-\alpha^2-\beta^2)^{1/2},\quad \operatorname{sinc}x=\sin x/x. \qquad (6)$$

the function $F(\alpha,\beta)$ is determined by the convolution of the angular spectra of the exciting beams:

$$F(\alpha,\beta)=\iint\limits_{-\infty}^{+\infty}A_{20}(\varkappa_2)A_{10}(\varkappa_1)\,\delta(\varkappa+\varkappa_1-\varkappa_2)\,d\varkappa_1\,d\varkappa_2, \qquad (7)$$

where $d\kappa_j=d\alpha_j d\beta_j$ and

$$A_{j0}(\varkappa)=A_j(\alpha,\beta,z=0)=A_j(\alpha,\beta,z). \qquad (8)$$

On account of the relation (8), we assume that the parameters of the laser beams in the nonlinear crystal remain constant.

The angular DFR components for which

$$k_i=k_2-k_1=g=k\cos\theta_0, \qquad (9)$$

grow in proportion to the length l on the nonlinear crystal; here θ_0 is the angle between the vector **k** and the direction of the z axis. The condition (9) is the condition for the Cerenkov emission of a forward wave with a difference frequency Ω. In the case of low-frequency ($\Omega\ll\omega_2,\omega_1$) generation by waves of the same polarization, this condition can be reduced to the standard form[1)]

$$\cos\theta_0=v/u_1, \qquad (10)$$

where v is the phase and u_1 the group velocities at the frequencies Ω and ω_1 respectively; $k_2-k_1=\Omega/u_1$.

In anisotropic nonlinear media, the condition (9) also remains valid when the angle θ varies in the interval $\pi/2\le\theta\le\pi$. It then corresponds to the Cerenkov emission of a backward wave, since in this case the direction of propagation of the DFR is opposite to that of the exciting waves. Thus, for

$$|k_2-k_1|<k \qquad (11)$$

the condition (9) is always fulfilled either for the forward or for the backward wave with the difference frequency.

In the case of the backward-wave generation, the boundary conditions are prescribed for $z=l$ and, furthermore, the angular DFR spectrum for an arbitrary distance z in the nonlinear crystal is determined by the expression

$$A(\alpha,\beta,z)=-i\gamma\frac{l-z}{k_i+g}F(\alpha,\beta)\exp\left\{i(g-k_i)\frac{l}{2}-i(g+k_i)z\right\}\operatorname{sinc}\left[\frac{k_i-g}{2}(l-z)\right]. \qquad (12)$$

The exit of the crystal now corresponds to $z=0$. If the difference-frequency wave is a normally polarized wave, then the radiation has the form of a circular cone; for the extraordinary wave this cone is deformed, since the wave number k depends on the azimuthal angle φ between the directions of **k** and the z axis.

If the condition (11) is fulfilled, then there obtains a vector synchronism between the interacting wave components. Indeed, the expression (9), multiplied by the Planck constant $\hbar$, implies the conservation of the longitudinal components of the momenta; the condition for the conservation of the transverse components follows from (7):

$$\varkappa=\varkappa_2-\varkappa_1. \qquad (13)$$

The latter condition constitutes together with (9) precisely the vector-interaction condition for the corresponding wave components in the paraxial approximation for the exciting beams. Thus, the conditions (9) and (10) under which the Vavilov-Cerenkov radiation is emitted can be interpreted as the result of the synchronous interaction between the waves, or as the usual result of the interference of the difference-frequency waves.

The possibility of Cerenkov emission in an optically nonlinear medium is determined by the dispersion properties of the medium. The emitted DFR power at the Cerenkov angle depends on the angular distribution of the intensity of the laser beams. From this point of view, the optimum conditions for the excitation of DFR are those conditions under which the convolution (7) of $F(\alpha,\beta)$ differs from zero only for given angular components α and β. In the ideal case, this requires plane exciting waves propagating at the indicated angles. Under real conditions, we can use for this purpose the beams of a laser operating in the higher transverse mode regime.

Let us now proceed to calculate the DFR power for certain types of laser beams, considering, for definiteness, the generation of the forward wave (5); the angular distribution of the DFR intensity is given here by the expression

$$S(\alpha,\beta,l)=\left(\frac{\gamma l}{k_i+g}\right)^2|F(\alpha,\beta)|^2\operatorname{sinc}^2\left[\frac{l}{2}(k_i-g)\right]. \qquad (14)$$

2.1. Gaussian Laser Beams

In the case under consideration the complex amplitudes of the laser beams have the form

$$A_{j0}(\mathbf{r})=A_{j0}\exp\{-(a_j^{-2}+ih_j)(x^2+y^2)\}, \qquad (15)$$

where a_j is the beam radius and the parameter $h_j=k_{j0}/2R_j$ characterizes the divergence of the beam (k_{j0} is the wave number in the linear medium and R_j is the equivalent focal length of the lens). For the function $F(\alpha,\beta)$, (7), we obtain

$$F(\alpha,\beta)=\frac{a^2}{4\pi(1+iha^2)}A_2(0)A_1(0)\exp\left\{-\frac{a^2(\alpha^2+\beta^2)}{4(1+iha^2)}\right\};\quad a^{-2}=a_2^{-2}+a_1^{-2},\quad h=h_2-h_1. \qquad (16)$$

It follows from (16) that the divergence of the exciting beams has no effect on the generation process when $h=0$.

For the angular distribution of the DFR in spherical coordinates we have (when $h_1=h_2$) the expression

$$S(\theta,\varphi,z)=\frac{1}{2}\left[\frac{1}{8\pi}\gamma A_2(0)A_1(0)a^2l\right]^2\frac{k^2\sin2\theta}{(k_i+k\cos\theta)^2}\times\exp\left\{-\frac{(ak\sin\theta)^2}{2}\right\}\operatorname{sinc}^2\left[\frac{l}{2}(k_i-k\cos\theta)\right]. \qquad (17)$$

The dependence of the function (17) on the angle θ is shown in Fig. 1. The angular width $\Delta\theta_0$ of the Cerenkov radiation (for $\theta_0\neq0$) is primarily determined by the last term in (17):

$$\Delta\theta_0 \approx \frac{2\pi}{k_i l}\operatorname{ctg}\theta_0. \qquad (18)$$

The expression (18) is valid for a crystal length l greater than the diffraction length $L_d = ka^2/2$ for the frequency Ω ($l > L_d$).

The power of the Cerenkov DFR is equal to

$$P_0 \approx 2^5\pi^3\frac{n}{n_1 n_2}\left(\frac{a_1 a_2}{a_1^2+a_2^2}\right)^2\frac{\chi^2 P_1 P_2 \Omega^4}{c^5 k_i}\, l \exp\left\{-\frac{(ak\sin\theta_0)^2}{2}\right\}, \qquad (19)$$

where P_j is the exciting power ($P_j = cn_j a_j^2 A_j^2(0)/16$). The plot of P_0, (19), as a function of Ω is shown in Fig. 2. The DFR power P' in the case of a one-dimensional synchronous interaction ($k = k_2 - k_1$) is determined in, for example, [8]. For crystal lengths $l > L_d$ the ratio of the powers of the Cerenkov and synchronous DFR is equal (for $\theta_0 \neq 0$) to

$$\eta = P_0/P' \approx {}^1/_4\pi^{-1/2}\exp\{-{}^1/_2(ak\sin\theta_0)^2\}. \qquad (20)$$

The formula (20) clearly shows how the parameters of the interacting beams affect the power ratio in the case under consideration.

2.2. Multimode Laser Beams

Let us consider the case of DFR generation when one laser beam is the lower transverse mode defined by (15), while the other beam, although its transverse intensity distribution is Gaussian, contains a large number of transverse modes. The spatial statistics of the second beam is, as has been shown in [17], similar to the statistics of thermal radiation. Taking the foregoing into account, we obtain for the angular distribution of the DFR intensity in the same approximation in which the expression (5) was obtained the expression (14), where $|F(\alpha, \beta)|^2$ should, however, be replaced by

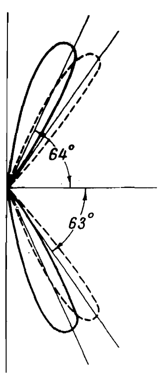

FIG. 1. The indicatrix of DFR for a nonsynchronous collinear interaction between the bounded beams. For $\ell > L_d$ the peak of the radiation corresponds to a Cerenkov angle of $\theta_0' = 64°$, while for $\ell \approx L_d$ the angle $\theta_0'' = 63°$. The angle scale in the vicinity of the angle θ_0' has been increased by a factor of ten.

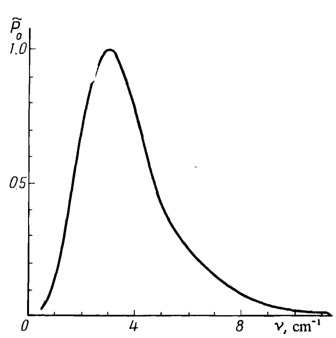

FIG. 2. The dependence of the DFR power P_0 on the radiation frequency $\nu = 1/\Lambda$ for a fixed transverse dimension a of the induced-polarization region. For a Gaussian transverse distribution of the polarization the maximum value is attained at the wave number $k = \sqrt{3}/a\sin\theta_0$.

$$\overline{|F(\alpha,\beta)|^2} = \frac{a^4}{16\pi^2(1+2a^2/r_c^2)}\exp\left\{-\frac{(\alpha^2+\beta^2)a^2}{2(1+2a^2/r_c^2)}\right\}. \qquad (21)$$

The bar denotes statistical averaging and r_c is the correlation length of the multimode beam at the level $1/e$.

For the mean DFR power when $a > r_c$ we obtain

$$\bar{P}_0 \approx 2^5\pi^3\frac{n}{n_1 n_2}\frac{r_c^2}{2(a_1^2+a_2^2)}\frac{\chi^2 P_1 P_2\Omega^4}{c^5 k_B}\, l\exp\left\{-\frac{1}{4}(kr_c\sin\theta_0)^2\right\}. \qquad (22)$$

The ratio of the Cerenkov-DFR powers (22) in the case under consideration here and (19) in the case of generation by Gaussian beams is

$$\frac{\bar{P}_c}{P_0} \approx \frac{r_c^2}{2a^2}\exp\left\{-\frac{1}{2}k^2\left(\frac{r_c^2}{2}-a^2\right)\sin^2\theta_0\right\}. \qquad (23)$$

Thus, for the same exciting-beam powers and radii the multimode "filling" of one of the beams leads to a more efficient DFR generation at the Cerenkov angle [2)].

3. THE CERENKOV EMISSION OF ULTRASHORT LIGHT PULSES; THE OPTICAL RECTIFICATION OF PULSES

In the present section we analyze DFR excitation by ultrashort laser pulses; the difference frequencies are, in essence, the spectral components of the light pulse detected in the optically nonlinear crystal. Other possible cases of Cerenkov DFR generation by temporally and spatially modulated radiation can easily be analyzed, using the procedure developed in the present article.

The generation of DFR by an intense light beam of finite duration is described by Eq. (1), in which the following substitution should be made:

$$E_2 E_1^* \to 2I(r)f(t - z/u_1), \qquad (24)$$

where $I(r)$ is the transverse distribution of the beam intensity, $f(t)$ describes the pulse shape, and u_1 is the group velocity of the pulse in the crystal. The frequency-angular spectrum of the DFR for a Gaussian beam in spherical coordinates has the form

$$E(\Omega,\theta,\varphi,l) = -i\frac{\chi I(0)a^2 l\sin 2\theta}{2v\pi c^2(v/u_1+\cos\theta)}\exp\left\{-\frac{(a\Omega\sin\theta)^2}{8v^2} - i\Omega\left(\frac{1}{u_1}-\frac{\cos\theta}{v}\right)\frac{l}{2}\right\}\Omega^3 f(\Omega)\operatorname{sinc}\left(\left[\frac{1}{u_1}-\frac{\cos\theta}{v}\right]\frac{\Omega l}{2}\right]. \qquad (25)$$

Here $I(0)$ is the maximum value of the intensity, a is the radius of the exciting-radiation beam, and $f(\Omega)$ is the Fourier spectrum of the pulse $f(t)$.

The frequency spectrum (25) differs significantly from the initial spectrum, which is due to the dependence of the nonlinear wave-wave coupling and the additional (for $\theta \neq \theta_0$ given by (10)) dependence of the radiation yield on the frequency Ω. Naturally, the shape of the excited pulse is also very different from that of $f(t)$. The radiation pulse generated at the Cerenkov angle θ_0 defined by (10) is determined by the expression

$$F(t,\theta_0,l) = C\frac{d^3}{dt^3}\int_{-\infty}^{\infty} f(\tau)\exp\left\{-2\frac{(t-l/u_1-\tau)v^2}{a^2\sin^2\theta_0}\right\}d\tau, \qquad (26)$$

where the coefficient $C > 0$ (we have neglected the dispersion of the crystal in the difference-frequency region).

Along the z axis, the pulse $F(t, \theta_0, l)$ propagates with the velocity u_1 of the exciting wave; the increase of its width in comparison with the initial pulse is due to the finite spatial dimensions of the latter. The shape of the pulse $F(t)$ is illustrated by the curves in Fig. 3.

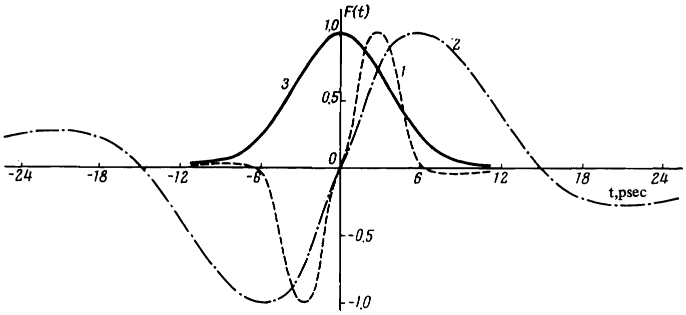

FIG. 3. The shape of the DFR pulses emitted at the Cerenkov angle for different widths a: 1) $\tau_0 = av^{-1}\sin\theta_0 = 1.6$ psec and 2) $\tau_0 = 16$ psec. The curve 3 corresponds to an exciting beam with the Gaussian shape of a pulse of width $\tau_1 = 5$ psec.

The energy radiated by the pulse

$$W=8\pi^3\frac{cn}{8\pi}\iiint_{-\infty}^{+\infty}|A^2(\Omega,\alpha,\beta,l)|\,d\alpha\,d\beta\,d\Omega \qquad (27)$$

is, within a narrow angle range near the direction θ_0 ($\Delta\theta_0 \ll 1$), given by the expression [3]

$$W=2^9\pi^{5/2}\frac{n}{n_1^2c^5\tau_1}(\chi W_1 l)^2\left(\tau_1^2+\frac{a^2\sin^2\theta_0}{v^2}\right)^{-5/2}\Delta\theta_0\,\mathrm{tg}\,\theta_0, \qquad (28)$$

where W_1 is the energy of the pulse, τ_1 is its width at, like a—the dimension of the beam—the level e^{-2} (the formula (28) was derived for a Gaussian pulse and a Gaussian beam profile). It is worth noting that here, in contrast to the generation of DFR by a doublet of spectral components (see Sec. 2.1, formula (19)), the DFR energy is proportional to l^2, and not to the crystal length l.

4. THE CERENKOV CONDITIONS IN OPTICALLY ACTIVE MEDIA

The optical activity of the medium can be taken into account by additing to the linear polarization the term $\hat{\alpha}$ curl E, where $\hat{\alpha}$ is the gyration pseudotensor. We shall not write out here the expressions for the angular distributions and the power of the DFR: they turn out to be similar to the expressions given in the preceding sections. Let us point out that in the general case of a crystal of arbitrary symmetry and elliptically polarized waves (in particular, linearly polarized waves) there exist four angles at which the Cerenkov DFR can be generated [4]:

$$\cos\theta_0=[k_2-k_1\pm(\rho_2\pm\rho_1)]/k, \qquad (29)$$

where $\rho_j = \omega_j^2\alpha(\omega_j)/2c^2$ is the specific optical rotation at the frequency ω_j.

In many highly nonlinear crystals (e.g., $LiNbO_3$ and $LiIO_3$), the quantity ρ_j (~10 cm^{-1}) in the optical band is comparable to the wave number in the far-infrared region. Therefore, some of the DFR cones can be oriented in the direction opposite to the direction of propagation of the induced polarization (cf. the formula (9)). Of greater importance, however, is the following circumstance. In the case under consideration, what is realizable on account of the natural optical activity of the crystal is the collinear synchronous interaction ($\theta_0 = 0$). The compensation by this means of the phase detuning during light-harmonics generation—something that has earlier been considered a possibility in the literature [20,21]—turns out to be difficult to realize because of the fact that the value of ρ_j is not large enough.

5. DISCUSSION

The analysis carried out in the present paper shows that in DFR generation under conditions when the dispersion of the nonlinear medium does not allow a synchronous collinear interaction between the waves (i.e., $k_2 - k_1 - k = \Delta \neq 0$) the radiation indicatrix peaks at the Cerenkov angle defined by (9) and (10). Since in nonlinear media the exciting waves can propagate in one direction, while the medium polarization induced by them propagate in the opposite direction, both forward (the angle $\theta_0 < \pi/2$) and backward ($\theta_0 > \pi/2$) waves can be emitted. Both types of scattering obtain in that region of the nonlinear crystal where the condition for vector synchronism is fulfilled. In this connection, the conditions for the emission of Vavilov-Cerenkov radiation during DFR generation admit of two interpretations: as an interference effect, or as a synchronous vector interaction.

Compared with the ordinary anisotropic media, optically active media allow the existence in them of a large number of ordinary waves; therefore, the DFR indicatrix in the latter media is more complex: here four Cerenkov angles are simultaneously possible. The existence of a DFR peak in a direction not coinciding with the direction of propagation of the induced polarization was discovered in an analysis [22] of this process in the quasioptical approximation. However, the use of the incomplete wave equation gives only diffraction corrections to the synchronism of the plane waves, whereas in the far-infrared region the Cerenkov angles attain values of 40–60°.

Although, as follows from the formula (20), the Cerenkov-radiation yield in the submillimeter-wave region can be one-two orders of magnitude less than the DFR yield due to the synchronous collinear interaction, the presence, as a rule, of strong absorption in this spectral region can wipe out this difference. The point is that under the Cerenkov generation conditions the losses are determined not by the length of the nonlinear crystal, but by its transverse dimensions. The Cerenkov DFR generation mechanism also has an advantage over the synchronous vector generation regime: the effectiveness of the latter is limited by the finite overlap region of the exciting beams.

Estimates with the aid of the formula (19) show that for exciting-beam powers of ~1 MW the maximum power of the Cerenkov DFR of wavelength $\Lambda = 0.5$ mm in a $LiNbO_3$ crystal of length 1 cm is ~1 kW.

The results of the present paper are also applicable to the case of DFR excitation during a four-photon interaction between the waves ($2\omega_1 = \omega_2 + \Omega$); for this purpose, the quantity $\chi A_2 A_1^*$ in the above-obtained formulas should be replaced by $\hat{\Theta} A_2^2 A_2^*$, where $\hat{\Theta}$ characterizes the cubic susceptibility of the medium.

It has thus far not been possible to detect the Cerenkov radiation in experiments on the nonsynchronous generation of DFR in the far-infrared region, since in the absence of special measures it undergoes total internal reflection from the exit and lateral faces of the crystal; in order to observe this radiation, the exit end of the crystal should be ground off at an angle of $\pi/2 - \theta_0$.

For the overwhelming majority of nonlinear media (solids, liquids), the refractive indices at radio frequencies and in the far-infrared region are larger than the refractive indices in the optical region; therefore, according to the results of Sec. 3, the propagation of short and ultrashort light pulses in such media will be accompanied by the emission of waves in the indicated spectral regions. Therefore, the propagating pulse will lose energy. Thus, for typical nonlinear crystals and pulses of power 0.1 J, width 5×10^{-12} sec, and beam radius 0.01 cm the losses due to Cerenkov radiation are $\sim(0.1-1)\times10^{-3}$ J. Quite appreciable Cerenkov radiation should accompany the self-focusing of pulsed laser beams in non-centrosymmetric nonlinear media.

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1) To the collinear synchronism $k = k_2 - k_1$ ($\theta_0 = 0$) corresponds the relation $v = u_1$, which, for DFR excitation in the far-infrared region, can be fulfilled for isotropic media [16].

2) Qualitatively, this result is, apparently, also contained in [18]; however, the model of unbounded beams used there is too crude.

3) The influence of the time statistics of the exciting radiation on the DFR generation efficiency is studied in [19]; the results obtained there are applicable to the case of the Cerenkov DFR.

4) The value of ρ_j decreases substantially with decreasing frequency; therefore, we have neglected the quantity $\rho(\Omega)$.

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132