

# On the theory of precession of the polarization vector of $\mu^-$ mesons in mu-nucleonic atoms

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The precession of the spin of a  $\mu^-$  meson in a magnetic field perpendicular to the initial polarization is considered. The mu-nucleonic atom with charge  $Z - 1$  formed after the capture of a  $\mu^-$  meson in a  $K$  orbit can have an electron shell of total angular momentum  $j$ . The cases  $j = 1/2$  and  $j = 3/2$  are considered in detail. Attention is drawn to the fact that in the case  $j = 3/2$  three stopping points for the spin precession can be observed. In particular, the investigation of the stopping points of the precession appears to be of considerable interest as a possible precision method for the study of hyperfine structure. It is proposed to utilize the analysis of precession of the spin of a  $\mu^-$  meson for the investigation of the hyperfine structure of acceptor impurity levels. The nature of the expected precession picture and the possibilities of the method are analyzed. It is noted that in the "quasimuonium" formed when a  $\mu^-$  meson is captured in a semiconductor the scale of the field for the hyperfine splitting can be diminished by a factor of  $10-10^2$  compared with muonium, and effects for the observation of which in muonium superstrong magnetic fields of the order of  $10^5-10^6$  G were required (stopping of precession) can be observed in fields of the order of  $10^2-10^3$  G. It is pointed out that the proposed method provides a possibility of measuring the relaxation time for spin of a hole and of observing the tensor nature of hyperfine splitting.

1. At the present time a large number of experimental investigations are available in which mesic atoms and mesic molecules of light elements have been investigated with the aid of decay electrons. A detailed list of references can be found in<sup>[1-4]</sup>.

At the JINR in recent years systematic investigations are being developed of the precession of spins of  $\mu^-$  mesons in a magnetic field perpendicular to the initial polarization with the aim of studying chemical properties of atoms and molecules of light elements<sup>[3,5]</sup>. One should also note separately the work of the group at MIFI (Moscow Engineering Physics Institute)<sup>[6]</sup> in which the precession of the spin of  $\mu^-$  mesons captured by neon nuclei was studied, and the coupling of the spin of a  $\mu^-$  meson to the magnetic moment of the electron shell was observed directly for the first time. In the present paper we wish to discuss certain possibilities for the study of precession which, apparently, have not been mentioned in the literature.

2. As is well known, at first the  $\mu^-$  meson is captured into a high-lying orbit of a mesic atom (mesic molecule) and quite rapidly (characteristic times of  $10^{-10}-10^{-12}$  sec) passes to the ground state as a result of conversion and dipole transitions. The ion arising as a result (generally speaking a multiply-charged one), as is shown by experiment, picks up in a broad class of cases the missing electrons during times comparable with the time of the cascade, so that as a result from the point of view of a chemist an atom of atomic number  $Z - 1$  is formed. In future we shall call such atoms mu-nucleonic.

In accordance with the accepted terminology the nucleus of a mu-nucleonic atom is a mesic atom of atomic number  $Z$ . Indeed, as a rule, in discussing the properties of mesic atoms one almost always has in mind the system of a nucleus  $Z + \mu^-$  meson<sup>[1]</sup>, pp. 239-252). For example, the mesic atom of hydrogen ( $p\mu^-$ ), the mesic atom of helium ( $\alpha\mu^-$ ) etc. Attempts to utilize this term more broadly having in mind the complete resulting atom including its electron shell lead to an obvious inconvenience: the mesic atom of atomic number  $Z$  has the electron shell and the chemical

properties of the element  $Z - 1$ . The proposed term "mu-nucleonic atom" allows one, it seems to us, while retaining the accepted terminology to get rid of the confusion existing in the literature (cf., for example<sup>[3]</sup>). It can also be easily generalized to the case of atoms formed as a result of the capture by a nucleus of other elementary particles.

During the cascade process the  $\mu^-$  meson loses an appreciable portion of its initial polarization, however, as is shown both by theoretical estimates and by experiment it retains 10-20% of the initial value.

The further behavior of the spin of a  $\mu^-$  meson in an external magnetic field is dictated by the magnetic properties of the resultant atom. In other words, everything is determined by the spin and the magnetic moment of the mother nucleus (atomic number  $Z$ ), and also by the Landé factor of the electron shell containing  $Z - 1$  electrons. Indeed, in all atoms in order to break the (LS) coupling external fields are required of the order of  $10^5-10^6$  G, so that at least in fields up to  $10^4$  G the total angular momentum of the electron shell remains a very good quantum number. It is also clear that in the  $\mu^-$  meson + nucleus system the total angular momentum is an immeasurably "better" quantum number since characteristic fields which determine the coupling of the  $\mu^-$  meson to the nuclear magnetic moment have tremendous values of the order of  $10^9-10^{13}$  G.

These obvious considerations considerably simplify the subsequent analysis. From general considerations one can expect that observation of the manifestations of precession for isolated atoms can be most conveniently carried out in gas targets of chemically inert substances; however, they can also be observed in liquids and, possibly, in certain crystals. Indeed, the situation regarding the observation of precession in the case of isolated mu-nucleonic atoms is in principle absolutely analogous to that which exists in the case of muonium.

In an exactly similar manner the precession picture may not become manifest as a result of the mu-nucleonic atom entering into a chemical reaction with the formation of a diamagnetic compound, or as a result of the

complete depolarization of the  $\mu^-$  meson brought about by the relaxation of the spin of the electron shell. However, as the example of muonium shows, in a broad class of substances and in a broad range of physico-chemical conditions the manifestation of precession ought to be observable. The factors noted above can be taken into account in a manner completely analogous to the theory developed for muonium. However, we shall not concern ourselves with this in the present paper in order to turn our attention principally to the possibilities provided in principle by the investigation of precession of mu-nucleonic atoms.

3. We shall first of all give the trivial formula for the polarization of a  $\mu^-$  meson captured in the 1s state of a nucleus of spin  $i$ , in the case that the resultant mu-nucleonic atom is diamagnetic. The effects of hyperfine splitting have been investigated in a number of papers<sup>[7-11]</sup>. It was shown in the course of these investigations that in the process of cascade transitions the nucleus becomes partially polarized, so that in the formation of states of hyperfine structure in the  $\mu^-$  meson + nucleus system one can not assume that the states with different components of nuclear spin are formed with equal probability. However, an exact calculation of the process of cascade transitions is difficult. At the same time the simple formula for the component of the polarization vector in a magnetic field  $B_z$  perpendicular to the  $x$  axis (the direction of the initial polarization) obtained on the assumption that all components of the nuclear spin along the  $x$  axis are equally probable, is sufficiently informative. It has the form<sup>[7]</sup>

$$\langle P(t) \rangle = \frac{P(0)}{6(2i+1)^2} [2(2i+1)(2i+3) \cos \omega_+ t + 2i(2i-1) \cos \omega_- t]. \quad (1)$$

Here  $P(0)$  is the polarization of the  $\mu^-$  meson which it has when it is captured into the K shell of a mesic atom, while

$$\hbar \omega_{\pm} = B \left[ \frac{2i+1 \mp 1}{i(2i+1)} \mu_n \pm \frac{2\mu_{\mu^-}}{2i+1} \right], \quad (2)$$

where the plus and minus signs correspond to states of total angular momentum  $i = i + \frac{1}{2}$ . Averaging denotes that we have omitted terms in the polarization which depend on high (of the order of  $10^{15}$ – $10^{18}$  Hz) frequencies characterizing the hyperfine interaction in the  $\mu^-$  meson + nucleus system. It is evident that in the experimentally observed polarization the dependence on these frequencies is averaged. Naturally, as a result of what has been said  $\langle P(0) \rangle \neq P(0)$ . Both here and everywhere in subsequent discussion we have omitted the subscripts  $x, z$ .

4. If the mu-nucleonic atom that has been formed is paramagnetic, then in defining  $\langle P(t) \rangle$  one should take into account the interaction of the magnetic moment  $\mu_1$  of the  $\mu^-$  meson + nucleus system with the total magnetic moment  $\mu_j$  of the electron shell. Since the formula for the polarization of the  $\mu^-$  meson for arbitrary  $i$  and  $j$  is not susceptible to an easy overview, we consider important special cases.

Let the total angular momentum of the nucleus be zero, then  $i = \frac{1}{2} = i$ . The Hamiltonian for the interaction of magnetic moments has the form

$$\hat{H} = A(\hat{i}\hat{j}) - (\mu_j \hat{j} / j + 2\mu_{\mu^-} \hat{i}_z) B. \quad (3)$$

We introduce the operator for the total angular momentum  $\hat{F} = \hat{i} + \hat{j}$ . The energy levels in an external magnetic field  $B$  for the Hamiltonian (3) are given by the

well known Breit-Rabi formula (cf., for example<sup>[12,13]</sup>), which we reproduce in dimensionless form:

$$y_{\pm}(m_F) = \frac{E_{F=\pm 1/2, m_F}}{1/2 |A| (j+1/2)} = -\frac{\text{sign } A}{2j+1} + \frac{2m_F x}{1-\alpha} \pm \left[ 1 - \frac{4m_F x \text{sign } A}{2j+1} + x^2 \right]^{1/2} \text{sign } A; \quad (4)$$

here

$$x = \frac{B}{B_c} = \frac{B(\mu_B g(jLS) + 2\mu_{\mu^-})}{|A| (j+1/2)},$$

$$\alpha = -2j \frac{\mu_{\mu^-}}{\mu_B g(jLS)},$$

$g(jLS)$  is the Landé factor,  $B_c$  is the characteristic field for the hyperfine interaction. The square root in the formula is extracted in accordance with  $\sqrt{x^2} = x$ . The formula for the component of the polarization vector of the  $\mu^-$  meson along the  $x$  axis (direction of the initial polarization) is the following:

$$P(t) = \frac{P(0)}{2j+1} \sum_{m_j=-j}^j \left\{ R_+^2 \left( m_j + \frac{1}{2} \right) Q_+^2 \left( m_j - \frac{1}{2} \right) \times \cos \left[ y_+ \left( m_j + \frac{1}{2} \right) - y_+ \left( m_j - \frac{1}{2} \right) \right] \tilde{t} + R_+^2 \left( m_j + \frac{1}{2} \right) \times Q_-^2 \left( m_j - \frac{1}{2} \right) \cos \left[ y_+ \left( m_j + \frac{1}{2} \right) - y_- \left( m_j - \frac{1}{2} \right) \right] \tilde{t} \right. \\ \left. + R_-^2 \left( m_j + \frac{1}{2} \right) Q_+^2 \left( m_j - \frac{1}{2} \right) \cos \left[ y_+ \left( m_j - \frac{1}{2} \right) - y_- \left( m_j + \frac{1}{2} \right) \right] \tilde{t} \right. \\ \left. + R_-^2 \left( m_j + \frac{1}{2} \right) Q_-^2 \left( m_j - \frac{1}{2} \right) \cos \left[ y_- \left( m_j + \frac{1}{2} \right) - y_- \left( m_j - \frac{1}{2} \right) \right] \tilde{t} \right\}. \quad (5)$$

Here  $\tilde{t} = t(j + \frac{1}{2}) |A| / \hbar$  is a dimensionless time,  $R_{\pm}^2 = 1 - Q_{\pm}^2$ ,

The coefficients  $R_{\pm}, Q_{\pm}$  can be easily related to the elements of the density matrix at the instant when  $\mu^-$  lands in a K orbit. However, as has been pointed out earlier, it is complicated to carry out a calculation of the density matrix, which is to any degree reliable, already because we are necessarily forced to utilize one or another model concept concerning the nature of cascade transitions on the one hand, and concerning the state of the electron shell of the atom which has captured the  $\mu^-$  meson on the other hand. In particular, it is certainly not possible to exclude the possibility that the atom is ionized during the cascade.

In the case under consideration a large number of model concepts leads to the conclusion that the components of the magnetic moment of the electron shell along the  $x$  axis at the instant of capture of the  $\mu^-$  meson into a K orbit are all of equal probability. However, one can also develop schemes which are in contradiction to this. Therefore, the most sensible approach is to consider  $R_{\pm}$  and  $Q_{\pm}$  to be parameters of the theory, all the more so since their values can give us information only concerning the cascade process, while the most interesting part of the information is determined by the frequencies of precession of the polarization. For estimates it is useful to indicate the value of  $Q^2$  in the case of equally probable components of the total angular momentum of the electron shell  $j$ :

$$Q_{\pm}^2(m_F) = \frac{1}{2} \mp \left( \frac{2m_F}{2j+1} - x \text{sign } A \right) \frac{1}{2} \left( 1 - \frac{4m_F x \text{sign } A}{2j+1} + x^2 \right)^{-1/2}. \quad (6)$$

For  $x \gg 1$ , i.e., in the case of fields  $B \gg B_c$  one can significantly simplify the formula for the polarization since in this case a number of the coefficients tends to unity, while other coefficients tend to zero. We then have

$$P(t) \approx \frac{P(0)}{2j+1} \sum_{m_j=-j}^j \cos \left[ \frac{2\alpha x}{1-\alpha} - \frac{4m_j}{2j+1} - \frac{(2j+1)^2 - 4m_j^2 - 1}{(2j+1)^2 x} \right] \tilde{t}. \quad (7)$$

In the table are given data concerning the hyperfine splitting for certain mu-nucleonic atoms with  $j = 1/2$  and  $j = 3/2$ . The hyperfine splitting constant was recalculated on the basis of spectroscopic data for ordinary atoms, and, in the course of this, corrections associated with the fact that the mu-nucleonic nucleus is not a point charge, and with relativistic effects, were not taken into account. Accordingly all the values are quoted with an accuracy of 0.1%.

5. We begin with the case  $j = 1/2$ . We note at once that in this case the mu-nucleonic atom is in the spin sense very like muonium the precession of which was considered in detail previously<sup>[14,15]</sup>. However, since the hyperfine splitting constant in our case has the opposite sign to that for muonium the triplet state turns out to be lower than the singlet state (cf., Fig. 1). For  $x = B/B_C \ll 1$  (in the region of the anomalous Zeeman effect) this difference from muonium is of small importance and the polarization can be written in the same form as in<sup>[15]</sup>:

$$\langle P(t) \rangle = 1/2 P(0) \cos x\tilde{t} \cos (x^2\tilde{t}/2). \quad (8)$$

For large  $B \approx m_\mu B_C / 2m_e$  a point of the stopping of precession must be observed just as in the case of muonium, but due to the opposite signs of the magnetic moments of  $\mu^-$  and  $\mu^+$  this effect is here associated with the crossing of the singlet term and the  $y_+(1)$  term from the triplet, and not with the crossing of the components of the triplet as in the case of muonium. The point of crossing is  $x = 1/2\alpha - \alpha/2$ .

As can be seen from the table, the characteristic fields in mu-nucleonic atoms with  $j = 1/2$ , as a rule, are considerably larger than in muonium. Correspondingly investigations in "strong" fields are more difficult in comparison with muonium. However, for "weak" fields more promising perspectives open up.

Just as in the case of muonium<sup>[16]</sup>, in high fields one should again observe two-frequency precession, but for the elements shown in the table the experimental observation of this picture is even more difficult than in the case of muonium.

6. We now consider the case  $j = 3/2$ . In the table are given data for a majority of mu-nucleonic atoms which give rise to the term  $j = 3/2$ . From the data exhibited there it can be seen that the hyperfine splitting constants for this case are very small for a number of elements. This circumstance can turn out to be convenient for experiment, since "high" fields turn out to be of the order of  $10^3$ – $10^4$  G and are easily attainable.

For weak fields  $B \ll B_C$  under the same assumptions as before the polarization can be written in the form

$$P(t) \approx \frac{P(0)}{8} \left( \cos \frac{3}{8} x^2 \tilde{t} + \frac{3}{2} \cos \frac{x^2}{8} \tilde{t} \right) \cos \frac{3+\alpha}{1-\alpha} \frac{x\tilde{t}}{2} + \frac{P(0)}{16} \cos \frac{x^2}{8} \tilde{t} \cos \frac{5-\alpha}{1-\alpha} \frac{x\tilde{t}}{2}. \quad (9)$$

Formula (9), as can be easily demonstrated, can be rewritten in the form

$$P(t) \approx \frac{P(0)}{4} \cos \frac{x^2}{2} \tilde{t} \cos \frac{x^2}{4} \tilde{t} \cos \frac{3+\alpha}{1-\alpha} \frac{x\tilde{t}}{2} + \frac{P(0)}{8} \cos \frac{x^2}{8} \tilde{t} \cos \frac{2x}{1-\alpha} \tilde{t} \cos \frac{x\tilde{t}}{2}. \quad (10)$$

Accordingly a complicated picture of three-frequency beats should be observed experimentally.

Term	Symbol	Initial element*	-A, Hz	B <sub>C</sub> , kG	Term	Symbol	Initial element*	-A, Hz	B <sub>C</sub> , kG
<sup>4</sup> S <sub>1/2</sub>	<sup>1</sup> H	He <sup>+</sup> (100)	4.516	1.6	<sup>4</sup> S <sub>3/2</sub>	<sup>7</sup> N	O (99.96)	0.4599	0.33
	<sup>11</sup> Na	Mg (89.9)	10.64	3.8		<sup>15</sup> P	S (99.3)	0.4325	0.31
	<sup>19</sup> K	Ca (99.9)	15.73	5.7		<sup>33</sup> As	Se (90.4)	1.229	0.89
	<sup>37</sup> Rb	Sr (93.0)	33.21	12.0		<sup>51</sup> Sb	Te (92.1)	2.202	1.4
<sup>3</sup> S <sub>1/2</sub>	<sup>55</sup> Cs	Ba (82.1)	55.71	20.1	<sup>2</sup> P <sub>1/2</sub>	<sup>17</sup> Cl	Ne (99.7)	6.794	7.4
	<sup>63</sup> Cu	Zn (95.9)	70.36	25.0		<sup>35</sup> Br	Ar (100)	6.655	7.2
	<sup>67</sup> Ag	Cd (74.9)	134.5	48.6		<sup>81</sup> Tl	Kr (88.5)	11.23	12.2
	<sup>79</sup> Au	Hg (70.0)	565.3	204.4		<sup>83</sup> J	Xe (52.4)	13.15	14.3
<sup>2</sup> P <sub>1/2</sub>	<sup>13</sup> Al	C <sup>+</sup> (98.9)	3.628	4.0	<sup>2</sup> P <sub>3/2</sub>	<sup>13</sup> Al	C <sup>+</sup> (98.89)	0.7269	0.79
	<sup>31</sup> Ga	Si (95.3)	6.121	6.7		<sup>31</sup> Al	Si (95.3)	1.150	1.2
	<sup>49</sup> In	Ge (92.2)	17.74	19.4		<sup>81</sup> Ga	Ge (92.2)	2.528	2.7
	<sup>81</sup> Tl	Sn (83.5)	33.11	36.2		<sup>83</sup> In	Sn (83.5)	2.842	3.1
<sup>3</sup> P <sub>1/2</sub>	<sup>19</sup> F	Pb (77.4)	117.4	128.6					
	<sup>37</sup> Cl	Ne (99.7)	34.62	37.9					
		Ar (100)	33.62	36.8					

\*Brackets contain the percentage content of isotopes of zero spin.

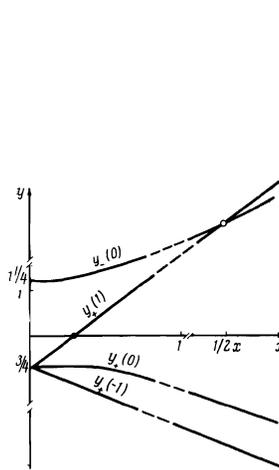


FIG. 1

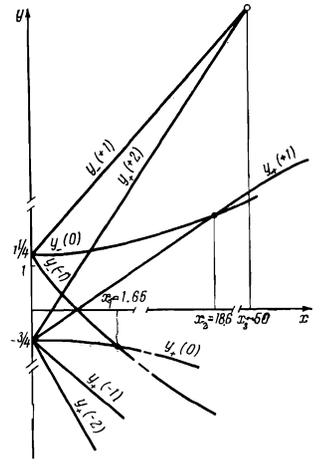


FIG. 2

FIG. 1. Dependence of the energy levels on the magnetic fields (Breit-Rabi diagram) for  $j = 1/2$ . Cf. the text for notation (formula (4)).

FIG. 2. Schematic dependence of the energy levels on the magnetic field for  $j = 3/2$ . Actually the pairs of curves  $y_+(-1)$ ,  $y_+(-2)$  and  $y_-(0)$ ,  $y_+(-1)$  intersect at very small angles.

In the domain of "superweak" fields ( $B \sim 1$ – $10$  G) formula (9), naturally, turns out to be absolutely analogous to formula (1), and two frequencies of precession of the spin of the  $\mu^-$  meson associated with the upper (-) and the lower (+) states (cf., Fig. 2) will be observed. A characteristic feature of the  $j = 3/2$  term turns out to be the possibility of observing the stoppage of precession in relatively low fields  $B \approx 1.65B_C$ , determined by the crossing of the  $y_-(-1)$  and  $y_+(0)$  terms. Near this point the slowly oscillating part of the polarization, as can be easily verified with the aid of formulas (4) and (5), has the form

$$P(t) \approx 0.22P(0) \cos 0.3(x-1.65)\tilde{t}. \quad (11)$$

In addition to this point of the stoppage of precession, in high fields in accordance with formula (7) we can observe two more such points: in high fields near  $B \approx m_\mu B_C / 12m_e$  and  $B \approx m_\mu B_C / 4m_e$ . The first is determined by the crossing of the  $y_-(0)$  and  $y_+(1)$  terms at the point

$$x_2 = 18.6, \quad (12)$$

and the second by the crossing of the  $y_-(1)$  and  $y_+(2)$  terms at the point

$$x_3 = 3/4\alpha^{-1} - 3/2 - 1/4\alpha. \quad (13)$$

As can be seen from the table, in the case of N, As, B, Al all the points of stoppage of precession turn out to be situated in fields easily attainable experimentally.

Two points of stoppage can be practically observed for all the elements shown in the table.

In very high fields  $B \gg B_C$  for such elements as N and B, apparently, one can successfully observe beats of two integrally related frequencies  $A/2$  and  $A \approx 5 \times 10^8$  Hz:

$$P(t) \approx P(0) \cos \frac{2\alpha x}{1-\alpha} \bar{t} \cos \frac{\bar{t}}{2} \cos \bar{t} \quad (14)$$

In this case the first frequency can be made larger than the other two, and one can observe the phenomenon of two-frequency beats. However, one should remember that formula (14) is obtained only on the assumption of equally probable components of the angular momentum of  $\mu^-$  along the x axis.

7. An investigation of the points of stoppage of precession appears to be of considerable interest, in particular, as a possible precision method for the study of hyperfine splitting. Indeed, although in the case of ordinary spectrometric measurements in atoms the constant itself can be measured with a high accuracy, nevertheless, since the magnetic moments of many nuclei are measured with an accuracy not greater than  $10^{-3}$ , the density of the wave function at the origin which determines the hyperfine splitting constant for the S term can be determined only with the same accuracy.

An analysis of the hyperfine splitting in mu-nucleonic atoms opens up new perspectives for this problem. Of course, we must keep in mind that theoretical formulas given in the present paper do not take into account the fine effects and in the first instance the correction to the hyperfine splitting constant due to the fact that the mu-nucleonic nucleus is not a point. The nonrelativistic correction to the hyperfine splitting constant can be obtained comparatively easily with the aid of ordinary perturbation theory. A calculation of the hyperfine splitting constants for mu-nucleonic atoms was given in the paper by Otten<sup>[17]</sup>. The author correctly notes that the standard perturbation theory can not pretend to be very accurate, but the method of calculation proposed by him is not sufficiently consistent and well founded.

We should note that the ratio of fields at which different points of the stoppage of precession should be observed is a function of a single quantity - the ratio of the magnetic moment of the electron to the magnetic moment of the  $\mu^-$  meson in the bound state. Thus, precision measurements of the points of stoppage of precession can provide a new method for the measurement of the magnetic moment of the  $\mu^-$  meson in a bound state. This method is all the more convenient for terms with  $j = \frac{3}{2}$  because utilizing all the points of stoppage of precession one can have control relations for determining the magnetic moment of the  $\mu^-$  meson in a bound state.

We further note that by investigating the precession one can obtain information concerning the final reaction products of the capture of a  $\mu^-$  meson. It should be stated that even in the case when the  $\mu^-$  meson is captured in a monatomic gas, different variants are, generally speaking, possible. As is well known,<sup>[2-4, 18, 19]</sup> in the initial stage of the cascade process the probability of Auger transitions is much greater than of radiation transitions, and therefore by the time the cascade ends a singly charged or a multiply charged ion can be formed. During times of the order of collision times ( $10^{-11}$  sec) this ion takes electrons away from atoms of

the surrounding medium. However, cases are possible when the ionization potential of the mu-nucleonic ion is lower than the ionization potential of the atoms of the medium<sup>[5, 20]</sup>. Then during its whole lifetime ( $10^{-7}$  sec) the mu-nucleonic ion remains ionized and a picture of the precession will be observed which corresponds to the term of the ion. In order to remove this possibility an impurity is introduced into the medium consisting of a monatomic inert gas with a low ionization potential<sup>[5]</sup>.

In the case that the  $\mu^-$  meson is slowed down in molecular gases the number of possible variants increases sharply. Firstly, as was first noted in<sup>[18]</sup>, a molecule multiply ionized as a result of the Auger-effect can break up ("fly apart") into ions and subsequently to this the situation does not differ from the behavior of a  $\mu^-$ -meson when captured in a monatomic gas. This hypothesis is developed in the work of the group at the JINR<sup>[3]</sup>. Generally speaking, one can also discuss the possibility of the molecule breaking up because the nucleus acquires a recoil momentum as a result of emitting a  $\gamma$  quantum in transitions at the lower steps of the cascade.

As an elementary estimate shows, in the radiation transition  $2p - 1s$  the nucleus acquires the kinetic energy  $E \approx 10^{-3} Z^3$  eV. Subsequently to this vibrational degrees of freedom can become excited and the molecule can be dissociated. Corresponding general estimates of similar processes can be found, for example, in the book by Migdal and Kraĭnov<sup>[21]</sup>. In the case of light elements  $Z \sim 20-30$  the probability of dissociation is very small. However, more rigorous estimates of the probability of dissociation are difficult, and one can only conclude that in the case of heavy elements  $Z \sim 50-60$  this possibility has to be taken into account.

By analyzing the precession one can establish in what chemical compound does the  $\mu^-$  meson finally find itself, and this may enable one to determine the fate of the mother molecule.

8. In solids the precession picture, naturally, depends on the electronic structure of the crystal. We here consider only the beautiful possibility which the  $\mu^-$  meson provides for the study of the properties of semiconductors.

In recent years the attention of the experimenters is being more and more attracted to the investigation of the structure of the wave functions of impurity centers and, in particular, to the determination of  $|\psi(0)|^2$  - the density of the wave function for an electron (or a hole) at the nucleus<sup>[22-25]</sup>. To achieve this aim the so-called method of double electron paramagnetic resonance (DEPR) is utilized.

It should be noted that the corresponding theoretical calculations which have their origin in the papers of Kohn and Luttinger<sup>[26]</sup> are of necessity based on one or another model picture. On the other hand, experimental investigation of acceptor centers in semiconductors by the DEPR method is in a number of cases associated with definite difficulties<sup>[27]</sup> and, therefore, the number of experimental investigations in which values of  $|\psi(0)|^2$  would be obtained for acceptors is, as far as we know, very small.

Earlier Otten<sup>[17, 28]</sup> has pointed out that a study of the precession of the spin of a  $\mu^-$  meson slowed down in superconductors can turn out to be convenient for the

investigation of acceptor impurity centers. However, the principal possibilities of the method and the characteristic features of the precession of a  $\mu^-$  meson in superconductors have not been analyzed. As of today investigation of  $\mu^-$  mesons in semiconductors is only beginning. In JINR<sup>[29]</sup> a study has been made of the residual polarization of a  $\mu^-$ -meson in Si and CuO. During times shorter than  $10^{-10}$  sec the resulting atom is thermalized. One can quite simply choose physicochemical conditions in such a manner that the given acceptor should not be ionized (for example, low temperatures or an appropriate alloying of the semiconductor). Then the hydrogenlike atom: mu-nucleonic acceptor + hole will in the spin sense be similar to an atom of muonium and the precession picture will be similar to the case of muonium<sup>[14,15]</sup>.

We note the significant differences. Firstly  $|\psi(0)|^2$  can turn out to be smaller by a factor of  $10-10^2$  than in the case of muonium<sup>[27,14]</sup>. Secondly, the details of the picture of hyperfine splitting become more complicated in view of the tensor nature of the spin-spin interaction in a crystal.

The smallness of  $|\psi(0)|^2$  opens up broad possibilities for the study of precession in "strong" fields comparable with the magnetic field produced by the spin of the hole at the position of the  $\mu^-$  meson. In the case of muonium this field is of the order of  $10^4 - 3 \times 10^5$  G, but here it is lower by a factor of  $10-10^2$ . Correspondingly the two-frequency precession in strong fields<sup>[16]</sup>, the stoppage of precession<sup>[15]</sup>, etc., can be observed in fields of  $10^2-10^3$  G. Of particular interest is the possibility of investigating asymmetry in the hyperfine structure constant. For example, for As in ZnSe in the case of different crystallographic orientations the values of the constant vary from 108 to 28 MHz<sup>[30]</sup>. In this connection we note that, just as in the case of muonium, additional important information can be obtained by observing the reestablishment of residual polarization in longitudinal fields. It is also of interest that for a theoretical analysis the situation is in one respect more advantageous than in the case of muonium. The lifetime of muonium prior to entering a chemical reaction is unknown, and is a parameter of the theory<sup>[31,14]</sup>. The lifetime of "quasimuonium" is known - it is the lifetime of a  $\mu^-$  meson captured by a given nucleus.

One should have in mind that if the relaxation time  $\tau$  for the spin of a hole is very small and  $1/\tau \gg \omega_0$  ( $\omega_0$  is the frequency of the hyperfine splitting), then the coupling between the hole and the muon is broken and damped precession should be observed at the frequency of the free muon<sup>[14]</sup>. As DEPR data show<sup>[27]</sup> under normal conditions  $\tau$  is very small. In order to reduce the relaxation time the sample is cooled to  $2-10^\circ\text{K}$  and sometimes is subjected to strong deformations<sup>[27]</sup>. In such a case the relaxation time can be successfully reduced to  $10^{-3}-10^{-6}$  sec. For acceptors  $\omega_0 \sim 10^7-10^9$  sec<sup>-1</sup>. Accordingly "quasimuonium" can be observed when  $\tau \sim 10^{-8}-10^{-10}$  sec. We note that in this sense the proposed method, apparently, has definite advantages compared to the DEPR method. It is also very convenient for an exact measurement of the relaxation time. For As in ZnSe<sup>[30]</sup> and B in SiC<sup>[27]</sup> it turned out to be possible to carry out observations by the DEPR method. Therefore these systems can be conveniently taken as starting points for investigations by the proposed method.

It appears that a systematic study of acceptor cen-

ters with the aid of negative muons can significantly extend the available information.

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- <sup>1</sup>A. O. Vaĭsenberg, Myu-mezon (Mu Meson), "Nauka", 1964.
- <sup>2</sup>S. S. Gershtein, V. I. Petrukhin, L. N. Ponomarev, and Yu. D. Prokoshkin, Usp. Fiz. Nauk 97, 3 (1969) [Sov. Phys.-Usp. 12, 1 (1969)].
- <sup>3</sup>A. A. Dzhuraev and V. S. Evseev, Zh. Eksp. Teor. Fiz. 62, 1166 (1972) [Sov. Phys.-JETP 35, 615 (1972)].
- <sup>4</sup>A. I. Babaev, V. S. Evseev, G. G. Myasishcheva, Yu. V. Obukhov, V. S. Roganov, and V. A. Chernogorova, Yad. Fiz. 10, 964 (1969) [Sov. J. Nucl. Phys. 10, 554 (1970)].
- <sup>5</sup>V. S. Evseev, Preprint OIYaI (JINR) P14-4052, 1968.
- <sup>6</sup>V. Varlamov, Yu. Dobretsov, B. Dolgoshein, V. Kirillov-Ugrymov and A. Rogozhin, IV Internat. Conf. on High Energy Phys. and Nuclear Structure, Abstracts of papers, Dubna, 1971; ZhETF Pis. Red. 17, 4 (1973) [JETP Lett. 17, 2 (1973)].
- <sup>7</sup>H. Uberall, Phys. Rev. 114, 1640 (1959).
- <sup>8</sup>A. P. Bukhvostov and I. M. Shmushkevich, Zh. Eksp. Teor. Fiz. 41, 1895 (1961) [Sov. Phys.-JETP 14, 1347 (1962)].
- <sup>9</sup>A. P. Bukhvostov and N. P. Popov, Materialy chetvertoĭ zimnei shkoly po teorii yadra i fizike vysokikh energiĭ (Materials of the Fourth Winter School on Nuclear Theory and High Energy Physics), Leningrad, 1969.
- <sup>10</sup>A. P. Bukhvostov, Yad. Fiz. 9, 107 (1969) [Sov. J. Nucl. Phys. 9, 65 (1969)].
- <sup>11</sup>É. V. Tammet, Yad. Fiz. 11, 840 (1970) [Sov. J. Nucl. Phys. 11, 470 (1970)].
- <sup>12</sup>Yu. A. Plis and L. M. Soroko, Usp. Fiz. Nauk 107, 281 (1972) [Sov. Phys.-Usp. 15, 318 (1972)].
- <sup>13</sup>N. Ramsey, Molecular Beams (Russ. Transl., IIL, 1960, Ch. 2-3).
- <sup>14</sup>I. G. Ivanter and V. P. Smilga, Zh. Eksp. Teor. Fiz. 54, 559 (1968) [Sov. Phys.-JETP 27, 301 (1968)].
- <sup>15</sup>I. I. Gurevich, I. G. Ivanter, E. A. Meleshko, B. A. Nikol'skiĭ, V. S. Roganov, V. I. Selivanov, V. P. Smilga, B. V. Sokolov, and V. D. Shestakov, Zh. Eksp. Teor. Fiz. 60, 471 (1971) [Sov. Phys.-JETP 33, 253 (1971)].
- <sup>16</sup>I. I. Gurevich, B. A. Nikol'skiĭ, and V. S. Roganov, ZhETF Pis. Red. 15, 640 (1972) [JETP Lett. 15, 453 (1972)].
- <sup>17</sup>E. W. Otten, Z. Phys. 228, 393 (1969).
- <sup>18</sup>E. Fermi and E. Teller, Phys. Rev. 72, 999 (1947) (Russ. Transl. in E. Fermi, Nauchnye trudy (Scientific Works) 2, Nauka, 1972, p. 400).
- <sup>19</sup>G. Birbide and de Berde, Phys. Rev. 89, 189 (1953).
- <sup>20</sup>P. P. Hutchinson, Y. Menes, and G. Shapiro, Phys. Rev. 131, 1362 (1963).
- <sup>21</sup>A. B. Migdal and V. P. Kraĭnov, Priblizhennye metody kvantovoi mekhaniki (Approximate Methods in Quantum Mechanics), Nauka, 1966.
- <sup>22</sup>G. Feher, Phys. Rev. 114, 1219 (1959) (Russ. Transl. in collection of articles "Elektronnyi spinovyi rezonans v poluprovodnikakh" (Electron Spin Resonance in Semiconductors), IIL, 1962).
- <sup>23</sup>V. B. Gindoman, P. S. Gladkov, B. G. Zhurkin, and

- B. V. Kornikov, *FiTPP* 5, 2214 (1971) [*Sov. Phys.-Semicond.* 5, 1930 (1972)].
- <sup>24</sup> G. L. Bir and G. E. Pikus, in: *Simmetriya i deformatsionnye efekty v poluprovodnikakh* (Symmetry and Deformation Effects in Semiconductors), Mir, 1964.
- <sup>25</sup> S. E. Barnes, J. Dupraz, and R. Orbach, *J. Appl. Phys.* 42, 1659 (1971).
- <sup>26</sup> W. Kohn and J. Luttinger, *Phys. Rev.* 98, 915 (1955).
- <sup>27</sup> D. Ludwig and G. Woodbury, *Electron Spin Resonance in Semiconductors* (Russ. Transl., Mir, 1964).
- <sup>28</sup> E. W. Otten, *The meeting on muons in solid state physics*, Bürgenstock, 1971.
- <sup>29</sup> A. A. Dzhuraev, V. S. Evseev, G. G. Myasishcheva, Yu. V. Obukhov, and V. S. Roganov, *Zh. Eksp. Teor. Fiz.* 62, 1424 (1972) [*Sov. Phys.-JETP* 35, 748 (1972)]. Preprint OIYaI (JINR) P1-6020, 1971.
- <sup>30</sup> R. K. Watts, W. C. Holton, and M. Wit, *Phys. Rev. B* 3, 404 (1971).
- <sup>31</sup> V. G. Nosov and I. V. Yakovleva, *Zh. Eksp. Teor. Fiz.* 43, 1751 (1962) [*Sov. Phys.-JETP* 16, 1236 (1962)].

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