The modified decay of Langmuir waves

V. D. Shapiro

Physico-technical Institute, Ukrainian Academy of Sciences
(Submitted September 26, 1973)
Zh. Eksp. Teor. Fiz. **66**, 945–953 (March 1974)

The nonlinear phase of the decay of Langmuir waves is investigated under conditions when the decay-instability increment exceeds the sound frequency. The possibility of appreciable transformation of energy into the low-frequency mode is demonstrated for decay in a system of three waves, as well as in the case of the modulation instability of a plasmon gas. A self-similar solution is obtained which describes the nonlinear dynamics of the modulation instability. The various plasmon-dissipation mechanisms connected with such an instability are discussed.

1. The elucidation of the mechanisms underlying the absorption of a plasma condensate is one of the basic questions in the problem of the initiation of a pulsed thermonuclear reaction in a D—T target[1]. Such an absorption may be connected with the decay instability of Langmuir waves, which was discovered by Oraevskiĭ and Sagdeev[2], and which leads to the excitation of ion sound. So far, however, the investigation of the nonlinear dynamics of this instability has been limited essentially to the limiting case of low Langmuir-wave amplitudes $w < \omega_S/\omega_L$ ($w = W/n_0T_e$, where $W = E_0^2/4\pi$ is the Langmuir-oscillation energy, ω_S is the sound frequency, and ω_L is the plasma frequency), when the decay only decreases the wave vectors of the plasmons, practically not changing their total energy. As a result, the plasmons are concentrated in the long-wave section of the spectrum, where the Landau-damping—induced absorption of the plasmon energy by the particles is negligibly weak. Under these conditions the primary mechanism responsible for the dissipation of the plasmons is the phenomenon of collapse—a distinctive phenomenon connected with the self-focusing of the Langmuir waves and leading to the formation in the plasma of regions of field localization, regions which collapse in a finite time to sizes at which the electron trajectories intersect[3].

For Langmuir waves of large amplitude $w > \omega_S/\omega_L$, there obtains the so-called modified decay[3,4]—the increment of the instability at such amplitudes exceeds the sound frequency, and the dispersion properties of the low-frequency perturbations excited during the decay are determined by the spectral characteristics of the Langmuir waves. The first thoroughly investigated example of such a decay is the modulation instability of the plasmon gas[5]. Another important example is the aperiodic instability of the Langmuir waves[6]; below (in Sec. 2) we shall show that this instability develops into the modulation instability at long wavelengths of the low-frequency perturbation.

The primary aim of the present paper is to investigate the nonlinear phase of the instabilities arising during the modified decay. The pertinent investigation is carried out in Secs. 3 and 4. In Sec. 3 we solve a model problem—the decay of a monochromatic Langmuir wave into a Langmuir satellite and a low-frequency perturbation of the acoustic type. Such a problem in the case of ordinary decay has been considered by Bloembergen[7]. The solution obtained in[7] describes the transfer of the energy of the wave motions between the high-frequency modes, the energy in the low-frequency mode being smaller by a factor of ω_S/ω_L. For the modified decay, of greatest interest is the case of short-wave satellite excitation, when the instability is aperiodic. In this case, after the phase of exponential growth of the amplitude, the instability goes over into an asymptotic regime in which the amplitudes of the Langmuir waves remain approximately constant, while the amplitude S of the low-frequency perturbation grows with the time as t^2. The instability stabilizes as a result of the fact that the effective increment $\gamma \sim S^{-1}dS/dt$ becomes less than ω_S at large t. At the maximum, the energy of the low-frequency mode constitutes an appreciable ($\sim$w) fraction of the Langmuir-wave energy.

In Sec. 4 we show that the distinctive features of the nonlinear dynamics of the modified decay that were previously investigated for a system of three waves remain unchanged for a plasmon gas. In this section we obtain a self-similar solution in which the plasma-density modulation due to the low-frequency oscillations grows in proportion to t^2 and in which a spectral transfer of the plasmons to the short-wave section of the spectrum ($k \sim t$) occurs. The latter is connected with the fact that the depth of the potential wells in which the plasmons are trapped and the maximum plasmon kinetic energy ($k^2\lambda_D^2 \sim \delta n/n$) increase with increasing modulation of the plasma density. As in Sec. 3, the maximum value of the energy of the low-frequency oscillations is of the order of W^2/n_0T_e, the wave vectors of the plasmons increasing then to the value $k_{max} \sim w^{1/2}/\lambda_D$, where

$$\lambda_D = \omega_L^{-1}\sqrt{T_e/m_e}.$$

According to Rosenbluth and Sagdeev[8], under these conditions two nonlinear mechanisms limiting the growth of the plasmon energy become important in the problem of laser heating. One of these mechanisms is connected with the decrease (due to the violation of the conditions for resonance interaction of the waves when the plasma-density modulation is sufficiently deep) of the increments of the parametric instability that leads to plasmon production. The other mechanism is connected with the transfer of plasmons to the region of low phase velocities (large k), for which their resonance absorption by the particles is essential. Such an absorption leads to the "elongation" of the tail of the electron-distribution function and, eventually, to the establishment of a quasi-stationary state in which the growth of the energy of the plasmons owing to the parametric instability of the electromagnetic radiation is balanced by the absorption of their energy by the resonant electrons.

2. In the modified decay the instability increment is comparable to the frequency of the slow motions, and we cannot use the random-phase approximation and the kinetic-equation method for the waves. To describe such strongly nonlinear processes as the modified-decay process, the method used by Zakharov in[3] and based on the averaging of the dynamical plasma equations over a "fast" time of the order of $1/\omega_L$ [1)] proves to be extremely convenient.

In the present paper we limit ourselves to the consideration of one-dimensional oscillations. Representing the plasma-electron and plasma-ion densities in the form

$$n_e=n_0+{}^1/_2(\delta n_e(t,z)e^{-i\omega_L t}+\text{c.c.})+\delta n(t,z),\quad n_i=n_0+\delta n(t,z) \tag{1}$$

($\delta n_e = -(4\pi e)^{-1}\partial E/\partial z$ is the high-frequency perturbation of the electron density, $\delta n = -n_0\partial\xi/\partial z$ is the quasineutral perturbation of the particle density in the low-frequency oscillations, and ξ is the particle displacement in these oscillations), we have after averaging over the "fast" time the following system of equations for E and ξ (see[3]):

$$\frac{\partial E}{\partial t}-\frac{3}{2}i\frac{T_e}{m_e\omega_L}\frac{\partial^2 E}{\partial z^2}=\frac{i\omega_L}{2}E\frac{\partial\xi}{\partial z}, \tag{2}$$

$$\frac{\partial^2\xi}{\partial t^2}-\frac{T_e}{m_i}\frac{\partial^2\xi}{\partial z^2}=-\frac{1}{16\pi n_0 m_i}\frac{\partial|E|^2}{\partial z} \tag{3}$$

(T_e is the electron temperature; the plasma is assumed to be strongly nonisothermal, i.e., $T_e \gg T_i$.

It is not difficult to investigate with the aid of Eqs. (2) and (3) the stability of a Langmuir wave of large amplitude:

$$E_0(t,z)={}^1/_2\{E_0\exp[ik_0z-i\omega_Lt(1+{}^3/_2k_0^2\lambda_D^2)]+\text{c.c.}\}.$$

Setting

$$E=\exp[ik_0z-{}^3/_2ik_0^2\lambda_D^2\omega_Lt]\{E_0+E_+\exp[ikz-i\omega t]+E_-\exp[i\omega t-ikz]\},$$

$$\xi={}^1/_2\{\xi_k\exp[ikz-i\omega t]+\text{c.c.}\}$$

and linearizing with respect to the amplitude of the test waves, we obtain from (2) and (3) the following dispersion equation:

$$\omega^2-k^2\frac{T_e}{m_i}=-\frac{e^2E_0^2k^2}{4m_em_i}\left[\frac{1}{\omega_L^2\delta_+-2\omega_L\omega}+\frac{1}{\omega_L^2\delta_-+2\omega_L\omega}\right],\quad \delta_\pm=3[(k_0\pm k)^2-k_0^2]\lambda_D^2. \tag{4}$$

From this equation follows for $k \ll k_0$ the dispersion equation for the modulation instability[5]:

$$\omega^2-k^2\frac{T_e}{m_i}=\frac{3e^2E_0^2}{8m_em_i}\frac{k^4\lambda_D^2}{(\omega-3kk_0\lambda_D^2\omega_L)^2}. \tag{5}$$

The solution to this equation for $\omega \gg k\sqrt{T_e/m_i}$ has the form

$$\left(\omega-\frac{3}{2}kk_0\lambda_D^2\omega_L\right)^2=\frac{9}{4}k^2k_0^2\lambda_D^4\omega_L^2\left[1\pm\frac{1}{k_0^2\lambda_D^2}\left(\frac{2}{27}\frac{m_e}{m_i}w\right)^{1/2}\right]. \tag{6}$$

The condition for the appearance of the instability is $k_0 \le (2/27)^{1/4}g$, where

$$g=\frac{1}{\lambda_D}\left(w\frac{m_e}{m_i}\right)^{1/4}\quad\left(w=\frac{E_0^2}{4\pi}\frac{1}{n_0T_e}\right). \tag{7}$$

The instability increment defined by (6) grows linearly with k. The last circumstance is connected with the assumption made in going over from (4) to (5) that the terms in $\delta_\pm$ proportional to k^2 are small, and is valid only for $k \ll g$. At higher k the increment becomes saturated, and the instability becomes aperiodic. Simple analytical formulas for this instability can be obtained in the limiting case when $k_0 \ll k$. Then $\delta_+ = \delta_-$, and from (4) follows the equation previously obtained by Silin[6] for ω^2:

$$\omega^2=\frac{\delta^2\omega_L^2}{8}\pm\left[\frac{\delta^4\omega_L^4}{64}+\frac{e^2E_0^2k^2\delta}{8m_em_i}\right]^{1/2},\quad \delta=3k^2\lambda_D^2 \tag{8}$$

For $k \ll g$, the instability increment is, in accordance with (6), proportional to k:

$$\omega=i\omega_Lk\lambda_D\left(\frac{3}{8}w\frac{m_e}{m_i}\right)^{1/4};$$

for $k \gtrsim g$, the increment levels out:

$$\omega=\frac{i}{\sqrt{6}}\omega_L\left(w\frac{m_e}{m_i}\right)^{1/2}. \tag{9}$$

The relations (7) and (9) determine the characteristic spatial and time scales of the instability during a modified decay. By going over to the dimensionless variables

$$\zeta=\frac{z}{\lambda_D}\left(\frac{m_e}{m_i}w\right)^{1/4},\quad \tau=\frac{\omega_Lt}{2}\left(\frac{m_e}{m_i}w\right)^{1/2},\quad \mathcal{E}=\frac{E}{E_0},\quad \eta=\frac{\xi}{\lambda_D}\left(\frac{m_e}{m_i}w\right)^{-1/4} \tag{10}$$

(here E_0 is the amplitude of the hf field at t = 0), we can write the basic system of equations (2), (3) in a universal form containing, as a parameter, only the quantity $\Gamma = 4(m_e/m_iw)^{1/2}$:

$$\frac{\partial\mathcal{E}}{\partial\tau}-3i\frac{\partial^2\mathcal{E}}{\partial\zeta^2}=i\mathcal{E}\frac{\partial\eta}{\partial\zeta}, \tag{11}$$

$$\frac{\partial^2\eta}{\partial\tau^2}-\Gamma\frac{\partial^2\eta}{\partial\zeta^2}=-\frac{\partial|\mathcal{E}|^2}{\partial\zeta}. \tag{12}$$

It follows from (7) and (9) that the parameter $\Gamma \sim \omega_s^2/\gamma^2$ (γ is the increment), the condition for a modified decay being $\Gamma \ll 1$, i.e., $w \gg m_e/m_i$.

3. In this section we investigate with the aid of Eqs. (11) and (12) the modified decay in a three-wave system. Let us accordingly set

$$\mathcal{E}=\mathcal{E}_0(\tau)\exp[i\varkappa_0\zeta-3i\varkappa_0^2\tau+i\alpha_0(\tau)]+\mathcal{E}_1(\tau)\exp[i(\varkappa+\varkappa_0)\zeta-3i(\varkappa+\varkappa_0)^2\tau+i\alpha_1(\tau)],$$

$$\eta=\frac{i}{\varkappa}\{S(\tau)\exp[i\varkappa\zeta+i\Phi(\tau)]-\text{c.c.}\}.$$

We then have from (11) and (12) the following system of equations for the amplitudes and phases of the waves participating in the decay:

$$\frac{d}{d\tau}[\mathcal{E}_0\exp(i\alpha_0)]=-i\mathcal{E}_1S\exp(i\alpha_1-i\Delta_+\tau-i\Phi), \tag{13}$$

$$\frac{d}{d\tau}[\mathcal{E}_1\exp(i\alpha_1)]=-i\mathcal{E}_0S\exp(i\alpha_0+i\Delta_+\tau+i\Phi), \tag{14}$$

$$\frac{d^2}{d\tau^2}[Se^{i\Phi}]+\Gamma\varkappa^2Se^{i\Phi}=-\varkappa^2\mathcal{E}_0\mathcal{E}_1\exp(i\alpha_1-i\alpha_0-i\Delta_+\tau), \tag{15}$$

where $\Delta_+ = 3[(\kappa+\kappa_0)^2-\kappa_0^2]$. In these equations we have neglected the excitation of the Langmuir satellite with the wave number $\kappa_- = \kappa_0 - \kappa$, assuming that the "detuning" Δ_- corresponding to it is sufficiently large.

Assuming that for the given pump wave $\mathcal{E}_0 = 1$, and setting $Se^{i\Phi} \sim e^{-i\nu\tau}$, we obtain from (14) and (15) the dispersion equation of the linear theory:

$$[\nu^2-\Gamma\varkappa^2][\nu-\Delta_+]=\varkappa^2. \tag{16}$$

This equation describes a continuous transition, as the parameter Γ decreases, from ordinary decay, which is investigated in[2], to a "modified" decay. For $\Gamma \gg 1$, we have in accordance with[2]:

$$\nu=\pm\varkappa\sqrt{\Gamma}+\varkappa/\sqrt{2\Delta_+},\quad \Delta_+=\pm\varkappa\sqrt{\Gamma}.$$

The instability in this case arises when $\Delta_+ < 0$, i.e., leads to the excitation of a long-wave satellite. For $\Gamma \ll 1$ (i.e., for "modified" decay) all the "detuning" values $\Delta_+ \ge -3(\kappa/2)^{2/3}$ in (16) are unstable, the values $\Delta_+ > 0$ corresponding to the excitation of a short-wave satellite. In the case when $\Delta_+ \gg 1$, there develops an aperiodic instability with an increment $\text{Im}\,\nu \approx \kappa/\sqrt{\Delta_+}$.

Of great importance for the investigation of the nonlinear dynamics of the "modified" decay are the integrals of Eqs. (13)—(15), solutions which are similar to the Manley-Rowe relations for the ordinary decay:

$$\mathscr{E}_0^2+\mathscr{E}_1^2\approx 1,\quad \mathscr{E}_1^2-2\frac{S^2}{\varkappa^2}\frac{d\Phi}{d\tau}\approx 0. \tag{17}$$

The complete analytical solution to the problem of the nonlinear phase of the decay can be obtained only for $\Delta_+ \gg 1$, when we can use the fact that in this case the instability increment $\mathrm{Im}\,\nu \ll \Delta_+$ and, accordingly, that the amplitude of the Langmuir perturbations varies significantly more slowly with time than the phase (see[10]). In this case we obtain from Eqs. (13) and (14), besides the first solution in (17), the following approximate relations:

$$\mathscr{E}_1=\frac{\mathscr{E}_0 S}{d\alpha_1/d\tau},\quad \frac{d\alpha_1}{d\tau}\frac{d\alpha_0}{d\tau}=S^2,$$

$$\alpha_0-\alpha_1+\Phi+\Delta_+\tau=\pi-\frac{1}{\mathscr{E}_0 S}\frac{d}{d\tau}\left[\frac{\mathscr{E}_0 S}{d\alpha_1/d\tau}\right]\approx\pi.$$

It is not difficult to determine from these relations the amplitudes and phases of the Langmuir waves:

$$\frac{d\alpha_1}{d\tau}=\left(\frac{\Delta_+^2}{4}+S^2\right)^{1/2}+\frac{\Delta_+}{2},\quad \frac{d\alpha_0}{d\tau}=\left(\frac{\Delta_+^2}{4}+S^2\right)^{1/2}-\frac{\Delta_+}{2}$$
$$\mathscr{E}_0=\left[1+S^2\Big/\left(\frac{d\alpha_1}{d\tau}\right)^2\right]^{-1/2}. \tag{18}$$

With the aid of these formulas, Eq. (15) can be written in the form

$$\frac{d^2}{d\tau^2}[Se^{i\Phi}]+\Gamma\varkappa^2 Se^{i\Phi}=\frac{\varkappa^2 Se^{i\Phi}}{2}\left(\frac{\Delta_+^2}{4}+S^2\right)^{-1/2}.$$

The first integral of this equation is

$$\left(\frac{dS}{d\tau}\right)^2+S^2\left(\frac{d\Phi}{d\tau}\right)^2=\left(\frac{dS_0}{d\tau}\right)^2+\varkappa^2\left[\left(\frac{\Delta_+^2}{4}+S^2\right)^{1/2}-\left(\frac{\Delta_+^2}{4}+S_0^2\right)^{1/2}\right]-\Gamma\varkappa^2S^2 \tag{19}$$

where

$$S_0=S(\tau=0),\quad \frac{dS_0}{d\tau}=\frac{dS}{d\tau}\Big|_{\tau=0}$$

Using the second equation in (17), we can easily show that in the case $\Delta_+ \gg 1$ under consideration the term $S^2(d\Phi/d\tau)^2$ is negligibly small. As $\Gamma \to 0$, the function $S(\tau)$, which is the solution of Eq. (19), increases without restriction with the time: the exponential growth $S \sim \exp(\kappa\tau/\sqrt{\Delta_+})$, which is valid for $S \ll \Delta_+$, is subsequently replaced by the power law $S \approx \kappa^2\tau^2/4$ when $S \gg \Delta_+$; for such large S the quantity $\mathscr{E}_0 \approx \mathscr{E}_1 \approx 1/\sqrt{2}$.

Allowance in Eq. (19) for the term proportional to ΓS^2 leads to the limitation of the growth of S and to the appearance of a periodic solution with the period

$$\tau_0=\frac{2\pi}{\varkappa\sqrt{\Gamma}}+\frac{2\sqrt{\Delta_+}}{\varkappa}K\left[1-\frac{1}{2\Delta_+}\left|\frac{S_0^2}{\Delta_+}-\frac{1}{\varkappa^2}\left(\frac{dS_0}{d\tau}\right)^2\right|\right] \tag{20}$$

(K is the elliptic integral of the first kind). In this solution S varies from the minimum value

$$S_{min}=\left[S_0^2-\frac{\Delta_+}{\varkappa^2}\left(\frac{dS_0}{d\tau}\right)^2\right]^{1/2}$$

(the quantity $S_{min} = 0$ for $(\Delta_+/\kappa^2)(dS_0/d\tau)^2 > S_0^2$) to the maximum value

$$S_{max}\approx 1/\Gamma,$$

corresponding to a density perturbation $\delta n_{max} = n_0 w/2$ in the low-frequency mode. The maximum value of the energy that can be acquired in the oscillations of the ion component also turns out to be quite considerable:

$$\overline{W}_i=\frac{n_0 m_i\overline{v_i^2}}{2}=\frac{W}{4\varkappa^2}\left(w\frac{m_e}{m_i}\right)^{1/2}\left(\frac{dS}{d\tau}\right)^2_{max}=\frac{W^2}{64n_0T_e} \tag{21}$$

(the bar denotes averaging over ζ).

The plots of the functions $S(\tau)$, $\mathscr{E}_0(\tau)$, and $\mathscr{E}_1(\tau)$ are shown in Fig. 1. The invertible character of the solution, which is demonstrated in this figure, is maintained only when $w < 1$. Upon increase of the pump-wave amplitude to values at which the parameter $w > 1$, it, generally speaking, becomes necessary to take into account in the basic equations (2) and (3) the electronic nonlinearity of the Langmuir oscillations, since at such amplitudes the vibrational velocity v_e of the electrons in the Langmuir-wave field satisfies the condition $k^2v_e^2/\omega_L^2 \sim \gamma/\omega_L$ (the characteristic values of the wave number k and the increment γ are determined by the relations (7) and (9)). This does not, however, change the qualitative nature of the solution, and, as a result of the growth of the amplitude of the low-frequency perturbation, values of $S \sim (m_i/m_e w)^{1/2} < \Gamma^{-1}$ are attained at which breaking of the front of this perturbation ($\delta n/n \sim 1$) and the dissipation of the pump-wave energy owing to the intersection of the ion trajectories occur[2)].

In conclusion of this section, let us note that a numerical solution on an electronic computer of Eqs. (13)—(15) with $\Gamma = 0$ was carried out in[10]. It follows from this solution that the development of the instability into an asymptotic regime in which S grows as τ^2, while $\mathscr{E}$ oscillates about the mean value $\overline{\mathscr{E}} = 1/\sqrt{2}$, occurs at all $\Delta_+ > 0$, i.e., in all those cases in which the instability leads to the excitation of a short-wave satellite.

4. A similar asymptotic regime occurs also in the case of the modulation instability of a plasmon gas. In fact, we can show that for $\tau \gg 1$ and $\Gamma = 0$ the system of equations (11) and (12) has the following self-similar solution:

$$\mathscr{E}(\tau,\zeta)=f(\zeta)\exp\left[i\tau\int\varkappa\,d\zeta-i\lambda\tau^3\right],\quad \eta=-\frac{\tau^2}{2}\frac{df^2}{d\zeta}. \tag{22}$$

Here λ = const and, as in (18), the phase of the Langmuir waves grows with the time as τ^3. The solution given above describes a modulation, which grows with the time in proportion to τ^2, of the plasma density in the low-frequency oscillations:

$$\delta n=n_0\frac{\tau^2}{2}\left(\frac{m_e}{m_i}w\right)^{1/2}\frac{d^2f^2}{d\zeta^2} \tag{23}$$

and the spectral transfer, due to such a modulation, of plasmons to the short-wave section of the spectrum:

$$\bar{k}\approx\frac{\tau}{\lambda_D}\left(\frac{m_e}{m_i}w\right)^{1/4}. \tag{24}$$

For $f(\zeta)$ and $\kappa(\zeta)$, separating the imaginary (proportional to τ^2) and the real (proportional to τ) terms in Eq. (11), we obtain

$$\frac{1}{2}\frac{d^2f^2}{d\zeta^2}+3(\varkappa^2-\lambda)=0,\quad f\frac{d\varkappa}{d\zeta}+2\varkappa\frac{df}{d\zeta}=0. \tag{25}$$

It follows from this that $f = f_0/\sqrt{\kappa}$ (f_0 = const), while the

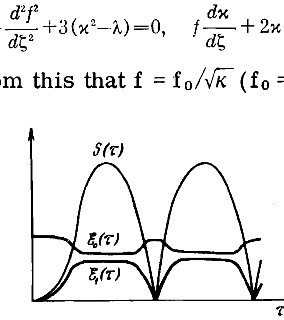

FIG. 1

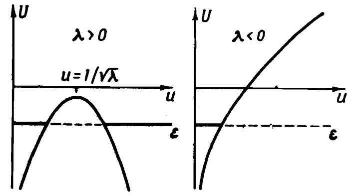

FIG. 2

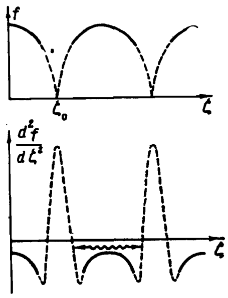

FIG. 3. The dependence of f and $d^2f^2/d\zeta^2$ on ζ.

function $u = 1/\kappa(\zeta)$ satisfies the equation of the nonlinear oscillator:

$$\frac{d^2u}{d\zeta^2}+\frac{6}{f_0^2}\left(\frac{1}{u^2}-\lambda\right)=0. \qquad (26)$$

The law of "conservation of energy" for such an oscillator can be written in the form

$$\frac{1}{2}\left(\frac{du}{d\zeta}\right)^2=\varepsilon+\frac{6}{f_0^2}\left(\frac{1}{u}+\lambda u\right), \quad \varepsilon=\text{const}.$$

The plot of the potential energy

$$U(u)=-\frac{6}{f_0^2}\left(\frac{1}{u}+\lambda u\right)$$

is shown in Fig. 2. The condition that E should be bounded as $\zeta \to \infty$ is satisfied by the solution in which u varies periodically in the interval $0 < u < u_{max}$; such a solution, as is easy to see, exists for either sign of λ.

The solution (22) corresponds to the geometrical-optics approximation, when the plasmons have a wavelength significantly shorter than that of the low-frequency perturbation, i.e., $|\eta^{-1}d\eta/d\zeta| \ll \kappa\tau$. In the vicinity of the points u = 0, where $\eta \sim (\zeta-\zeta_0)^{-1/3}$, such a solution is inapplicable, and it becomes necessary to take into account the highest space derivatives in Eqs. (11) and (12), i.e., $\partial^2 f/\partial\zeta^2$ and $\Gamma\partial^2\eta/\partial\zeta^2$. For $\Gamma = 0$, the solution in the neighborhood of the point ζ_0 can be obtained with the aid of the self-similar substitution

$$\mathscr{E}=\tau^{-1}\varphi(y), \quad \eta=\tau^3\psi(y), \quad y=(\zeta-\zeta_0)\tau^3, \qquad (27)$$

where the functions $\varphi(y)$ and $\psi(y)$ are determined from the equations

$$\frac{d^2\varphi}{dy^2}=-\frac{\varphi}{3}\frac{d\psi}{dy}, \quad 3y^2\frac{d^2\psi}{dy^2}+8y\frac{d\psi}{dy}+2\psi=-\frac{1}{3}\frac{d\varphi^2}{dy}. \qquad (28)$$

The contribution of this solution to the general Langmuir-oscillation energy balance is negligibly small, being of the order of $1/\tau^5$. In Fig. 3 we show a typical Langmuir-field amplitude distribution $\sim f(\zeta)$ and a typical plasma-density modulation distribution $\sim d^2f^2/d\zeta^2$ in the low-frequency oscillations; the dashed sections of the curves correspond to the regions of applicability of the solution (27). Thus, the modulation of the plasma leads to the appearance of potential wells in which the plasmons are trapped, the short-wave pumping of the plasmons occurring as a result of the increase of the depth of these wells.

The region in which allowance for the spatial transfer of the low-frequency oscillations (i.e., for the term $\Gamma\partial^2\eta/\partial\zeta^2$ in Eq. (12)) is essential expands in time according to the law $|\zeta-\zeta_0| \sim \tau(m_e/m_i w)^{1/4}$. As in the model problem considered in the preceding section, the transport of the low-frequency oscillations leads here to the limitation of the growth of δn during times $\tau_0 \sim (m_i w/m_e)^{1/4}$. During such times τ, the plasma-density perturbation, the energy transferable to the ion component, and the maximum value of the wave number of the plasmons increase to the values

$$\delta n\sim\frac{W}{T_e}, \quad W_i=\frac{n_0 m_i v_i^2}{2}\sim\frac{W^2}{10 n_0 T_e}, \quad k_{max}\sim\frac{1}{\lambda_D}\left(\frac{W}{n_0 T_e}\right)^{1/4}. \qquad (29)$$

It follows from these formulas that as the quantity $w = W/n_0T_e$ increases, two plasmon-dissipation mechanisms become possible: the transfer of the plasmon energy to the low-frequency oscillations and its absorption by the resonant electrons. At the not too large values of the parameter $w \sim 1/10$ the dominant mechanism is the absorption of the plasmons by the epithermal particles, which leads to the formation of a fast-electron "tail" in the distribution function. When $w \sim 1$, we also have coming into effect the dissipation mechanism connected with the intersection of the electron trajectories. Substituting into the condition $\partial a_e/\partial z \sim 1$ for intersection the electron displacement $a_e = eE_0\mathscr{E}/m_e\omega_L^2$ in the Langmuir wave and $\mathscr{E}(\tau, \zeta)$ from (22), we find that the time $\tau \sim (m\ /m\ )^{1/4}w^{-3/4}$ for such an intersection is, when $w \sim 1$, comparable to τ_0.

In conclusion, the author expresses his gratitude to Academician R. Z. Sagdeev for formulating the problem and for valuable advice and to A. A. Galeev, V. E. Zakharov, and V. I. Shevchenko for a discussion of the paper.

1) Actually, such an averaging was used in [5, 9], where some limiting cases of the system of equations (2), (3) were derived.

2) In a plasmon gas, the spectral pumping, during a modified decay, of the plasmon energy into the short-wavelength region leads to a situation in which the dissipation mechanisms connected with the resonance absorption of the plasmons by the particles and (for $w > 1$) with the intersection of the electron trajectories (for details, see Sec. 4) turn out to be more important.

1 R. Z. Sagdeev, Trudy 6-ĭ Evropeĭskoĭ konferentsii po UTS i fizike plazmy (Proceedings of the 6th European Conference on Controlled Thermonuclear Fusion and Plasma Physics), Atomizdat, 1974.
2 V. N. Oraevskiĭ and R. Z. Sagdeev, Zh. Tekh. Fiz. **32**, 1291 (1962) [Sov. Phys.-Tech. Phys. **7**, 955 (1963)].
3 V. E. Zakharov, Zh. Eksp. Teor. Fiz. **62**, 1745 (1972) [Sov. Phys.-JETP **35**, 908 (1972)].
4 A. A. Galeev and R. Z. Sagdeev, Nuclear Fusion **13**, 603 (1973).
5 A. A. Vedenov and L. I. Rudakov, Dokl. Akad. Nauk SSSR **159**, 739 (1964) [Sov. Phys.-Doklady **9**, 1073 (1965)].
6 V. P. Silin, Zh. Eksp. Teor. Fiz. **48**, 1679 (1965) [Sov. Phys.-JETP **21**, 1127 (1965)]; N. E. Andreev, A. Yu. Kiriĭ, and V. P. Silin, Zh. Eksp. Teor. Fiz. **57**, 1024 (1969) [Sov. Phys.-JETP **30**, 559 (1970)].
7 N. Bloembergen, Nonlinear Optics, Benjamin, New York, 1965 (Russ. Transl., Mir, 1966).
8 M. N. Rosenbluth and R. Z. Sagdeev, Comments on Plasma Physics and Controlled Fusion **1**, 27 (1973).
9 V. I. Bespalov, A. G. Litvak, and V. I. Talanov, Dokl. na II Vsesoyuznom simpoziume po nelineĭnoĭ optike (Paper presented at the 2nd All-Union Symposium on Nonlinear Optics), Novosibirsk, 1966.
10 B. A. Al'terkop, A. S. Volokitin, V. D. Shapiro, and V. I. Shevchenko, ZhETF Pis. Red. **18**, 46 (1973) [JETP Lett. **18**, 24 (1973)].

Translated by A. K. Agyei
99