Theory of the current-voltage characteristics of one-dimensional *S-N-S* and *S-N* junctions

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The distribution of the fields and currents and also the current-voltage characteristics of a one-dimensional Josephson junction of the *S-N-S* type or a Notarys bridge is calculated on the basis of the time-dependent Ginzburg–Landau equations. In the model under consideration the phase of the order parameter changes over a characteristic distance $\xi I_c/I_{cs} \ll \xi$ (I_c is the critical Josephson current, and I_{cs} is the critical current of the superconductor S), but the electric potential and $|\Psi|$ vary over the coherence length. The resistance of an S-N junction and its dependence on the current passing through it are also calculated.

INTRODUCTION

The generalization of the Ginzburg-Landau equations to the time-dependent case[1-2] makes it possible to investigate, in addition to other problems, the problem of the transmission of current through systems where the order parameter Ψ varies in space and time. For example, weakly-coupled superconductors in which the weak coupling is not due to a dielectric layer but to a local decrease in $|\Psi|$ caused by either proximity effects (S-N-S junction, Notarys bridge) or the inhomogeneity of the current density distribution (point-contacts, the Dayem bridge) belong to such systems. The theory of the Josephson effect for the latter systems is given in the article by Aslamazov and Larkin.[3] The fact that, for $a \ll \xi$ (a is the characteristic dimension of the variation of the current density, and ξ is the coherence length) the variations of Ψ and of the electric potential Φ are described by Laplace's equation ($\nabla^2\Psi = 0$ and $\nabla^2\Phi = 0$) was utilized. Therefore, it follows from the obtained solutions that Ψ and Φ vary substantially over the same distance $\sim a$. Their own specific characteristics, associated with the fact that Ψ and Φ are described by different equations, arise in one-dimensional systems of the S-N-S type and in Notarys bridges. As will be shown, this leads, for example, to the result that the potential Φ changes substantially in a region where the phase of Ψ can be regarded as essentially constant. Below in a definite model we find the solution for Ψ and Φ in the one-dimensional case of weakly-coupled superconductors, and we obtain the shape of the I-V characteristics.

In addition, the passage of sufficiently weak (in comparison with the critical current I_c) currents I through a one-dimensional S-N system will be investigated. The solutions for Ψ and Φ will be analytically determined, and also the dependence of the system's resistance R on the current I is found (for $I \ll I_c$).

ONE-DIMENSIONAL WEAKLY COUPLED SUPERCONDUCTORS

Let us consider a one-dimensional superconductor in the form of a thin film or wire, in which $|\Psi|$ is smaller at the point $x = 0$ than for $x \neq 0$. This decrease of Ψ may be related to proximity effects or to the local effect of a depairing mechanism. We shall assume that the range a of influence of the depairing mechanism (for example, the thickness of the N layer) is considerably smaller than ξ. Then the effect of this mechanism can be described by introducing a term of the form $\delta(x)\Psi/\lambda$ into the equation for Ψ, where λ is a parameter related to the critical current I_c of the system, which we shall regard as small in comparison with the critical current of the superconductor S. We note that the steady-state Josephson effect in such a system was investigated in[4-6], and the nonstationary effect was considered in[7], where the calculation (for different ratios between a and ξ) was done by a numerical method. The additional assumption (starting from intuitive considerations) of phase "slip," which is not contained in the time-dependent equations obtained for gapless superconductors, was made in this last article. We shall not introduce such an assumption, and we assume that the system under consideration is described by the equations

$$-12(\dot\Psi+i\Phi\Psi)+\Psi''+\Psi-\Psi|\Psi|^2-\frac{\delta(x)}{\lambda}\Psi=0, \quad (1)$$

$$I=\frac{1}{2i}(\Psi^*\Psi'-\text{c. c})-\Phi'. \quad (2)$$

Here all quantities are dimensionless: Length is measured in units of ξ, time is measured in units of $t_0 = (2\tau_S\Delta_0^2)^{-1}$, the potential in units of $\hbar/2et_0$, and the current density in units of $\sigma\hbar/2e\xi t_0$; the primes denote differentiation with respect to the coordinate x, and the dot denotes differentiation with respect to time (see[6]). The dimensionless parameter $\lambda \ll 1$ describes the depairing mechanism; for example, in the case of an N layer

$$\lambda\approx\frac{\xi}{a}\left(\frac{T_n-T}{T-T_s}\right)^{-1},$$

where T_n and T_S are the superconducting transition temperatures of the N and S metals, respectively (compare with[6]).

Let us first briefly consider the time-independent effect. Let us introduce the amplitude and phase of Ψ, that is, $\Psi = fe^{i\chi}$. Then Eq. (1) breaks up into two equations, one of which expresses the fact that (for $\Phi = \dot\Psi = 0$) the superconducting current

$$I_s=f^2\chi', \quad (3)$$

is independent of x, and the second equation has the following form for $x > 0$:

$$f''+f-f^3-I_s^2f^{-3}=0, \quad (4)$$

here we have utilized Eq. (3). Equation (4) must be supplemented by a boundary condition, which we obtain by integrating Eq. (1) from -0 to $+0$:

$$2f'(0)=f(0)/\lambda. \quad (5)$$

We have taken the continuity of χ, which follows from Eq. (3), into consideration. Integrating Eq. (4) with the boundary condition as $x \to +\infty$ taken into account, namely, $f(\infty) = 1$ ($I_S \ll 1$), we obtain

$$f'^2+f^2-\tfrac{1}{2}f^4+I_s^2f^{-2}=\tfrac{1}{2}. \qquad (6)$$

Setting x = 0 in Eq. (6) and substituting $f'(0)$ from Eq. (5), to the required accuracy we find:[1)]

$$f^2(0)=\lambda^2[1\pm\lambda^{-1}\sqrt{\lambda^2-4I_s^2}]. \qquad (7)$$

Hence the critical current is given by

$$I_c=\lambda/2. \qquad (8)$$

Integrating Eq. (6) once, we determine f(x) for x > 0 (we note that f(x) = f(−x)):

$$f^2=2I_s^2+\text{th}^2\zeta, \qquad (9)$$

where $\zeta = (x+x_0)/2$, $x_0 \lesssim \lambda$ and is determined from Eq. (7). It is clear from Eq. (9) that the amplitude of the order parameter varies from $f(0)\approx\lambda$ at x = 0 to $f\approx 1$ for $x\to\infty$ over distances ~1 (in dimensional units—over distances ~ξ). However, the phase varies over considerably smaller distances ~λ. In fact, for $\zeta \ll 1$ (i.e., $x \ll 1$) we have

$$f^2\approx 2I_s^2+\zeta^2, \qquad (9')$$

and therefore from Eq. (3) we obtain

$$\chi(\zeta)=\text{arctg}\frac{\zeta}{\sqrt{2}I_s}-\text{arctg}\frac{x_0}{\sqrt{2}I_s} \qquad (10)$$

(for the sake of argument, we assume $\chi = 0$ for x = 0). Hence it follows that $x_0 \sim I_S \sim \lambda$ serves as the characteristic distance over which $\chi(x)$ varies. It is not difficult to see that for $I_S = I_C$ we have $\chi(\zeta) = \pi/4$ for $\zeta \gg \lambda$, that is, the phase difference $\varphi = \chi(\zeta) - \chi(-\zeta) = 2\chi(\zeta) = \pi/2$, just as should occur. For $\zeta \gg \lambda$ the phase $\chi(\zeta)$ changes form to

$$\chi(\zeta)=\sqrt{2}I\int^{\zeta}\frac{d\zeta_1}{\text{th}^2\zeta_1}=\sqrt{2}I\left(-\text{cth}\,\zeta+\zeta+\frac{\pi}{2}-\text{arctg}\frac{x_0}{2\sqrt{2}I_s}\right). \qquad (11)$$

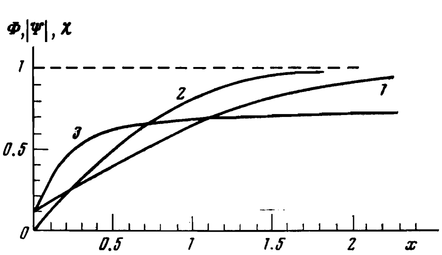
Thus, for $I_S \sim I_C$ the phase χ changes from $\chi = 0$ at x = 0 to $\chi \sim 1$ over distances ~λ, and then it slowly increases for $x \gtrsim 1$: $\chi \sim I_S x$ (see Eq. (11) and the Figure).

We note that the solution (9′) and (10) can be obtained by solving the equation

$$\Psi''=0. \qquad (12)$$

Thus, for small λ the solution of the problem can be approximately obtained by investigating Eq. (12) for $x \ll 1$. As one can easily verify, the term $\chi'^2 f$ in the equation for f can be neglected for $x \gg \lambda$, that is, one can neglect the influence of the current $I \sim \lambda$ on the amplitude of Ψ. The integration constants are found by matching the obtained solutions. Being guided by these ideas, let us now solve the time-dependent problem.

We shall seek the solution of Eqs. (1) and (2) for x > 0 in the form of an expansion in powers of the



The potential Φ (curve 2) and also the amplitude |Ψ| (curve 1) and the phase χ (curve 3) of the order parameter Ψ as functions of the coordinate x for λ = 0.1, I = 3/4, λ = (3/2)I_C, and Φ = π/2. For Φ the scale along the y axis should be multiplied by $\lambda\sqrt{2}F(1/2)/F_Z'(1/2)$.

parameter λ ($\Psi = \Psi_1 + \Psi_2 + \dots$, $\Phi = \Phi_1 + \dots$), considering the intervals $0 < x \ll 1$ and $\lambda \ll x$. In the first order approximation in λ and for $x \ll 1$, Eq. (1) reduces to Eq. (12) just as in the time-independent case (below we see that $\Psi\Phi \sim \dot\Psi \sim \lambda\Psi$). Therefore, we have

$$\Psi_1=(A_1+iB_1)x+f(0),\quad 0<x\ll 1, \qquad (13)$$

where we again assume $\arg\Psi(0) = 0$. The coefficients A, B, and f(0) are functions of the time, and from Eq. (5) it follows that

$$2A_1=f(0)/\lambda. \qquad (14)$$

Substituting (13) into expression (2) for the current, we find

$$\Phi_1=-(I-B_1f(0))x,\quad 0<x\ll 1. \qquad (15)$$

Just like the phase χ, we assume the potential Φ_1 to be an odd function of x. The found solutions (13) and (15) can be substituted into Eq. (1) to obtain the correction Ψ_2, then the correction Φ_2 to the potential can be determined from Eq. (2), and so forth.

Now let us consider $x \gg \lambda$. In accordance with what was said earlier, we seek a solution in the form

$$\Psi=e^{i(\varphi/2+\delta\chi)}f, \qquad (16)$$

where φ is a quantity that depends only on the time ($\varphi \sim 1$), and $\delta\chi = \delta\chi(x, t) \sim \lambda$. Substituting (16) into (1), we obtain the following equation for the amplitude:

$$f''+f-f^3=(\delta\chi')^2f+12\dot f, \qquad (17)$$

and for the phase:

$$12f^2(\dot\varphi/2+\delta\dot\chi+\Phi)-(f^2(\delta\chi)')'=0. \qquad (18)$$

Expressing the superconducting current $f^2(\delta\chi)'$ from (2) and substituting it into Eq. (18), we arrive at the following equation for Φ:[2)]

$$12f^2(\dot\varphi/2+\delta\dot\chi+\Phi)-\Phi''=0. \qquad (19)$$

To the first-order approximation in λ, we obtain the following expression for f_1 from Eq. (17):

$$f_1(\zeta)=\text{th}\,\zeta, \qquad (20)$$

where

$$\zeta=(x+x_0)/\sqrt{2},\quad x_0=x_0(t).$$

Noting that $\delta\dot\chi \sim \lambda\dot\varphi/2$, and introducing $\Phi_1 + \dot\varphi/2 = \mu_1$, we obtain from (19) the following result to first-order and with the form of f_1 (given by Eq. (20)) taken into account:

$$\mu_1''-12\mu_1\,\text{th}^2\zeta=0. \qquad (21)$$

The solution of this equation, which decreases at infinity, can be expressed in terms of a hypergeometric function.[9] Thus, Φ_1 is of the form

$$\Phi_1=-\tfrac{1}{2}\dot\varphi+C\eta_1(\zeta), \qquad (22)$$

where

$$\eta_1(\zeta)=\text{ch}^{-2\sqrt{6}}\zeta\,F\left(\alpha,\beta,\gamma,\frac{1-\text{th}\,\zeta}{2}\right),$$

$$\alpha=\frac{-\sqrt{97+4\sqrt{6}}+1}{2}\qquad \beta=\frac{\sqrt{97+4\sqrt{6}}+1}{2},\qquad \gamma=1+2\sqrt{6}.$$

C is an arbitrary constant (depending on the time).

Let us find the correction to the phase from Eq. (2), by substituting the solutions (20) and (22):

$$\text{th}^2\zeta\cdot\delta\chi_1'=I+\Phi_1'.$$

Integrating this equation, we determine the correction $\delta\chi_1$, with the aid of which one can find f_2 from Eq. (12) and thus extend the iteration procedure. We shall confine our attention to the first approximation. Matching the solutions for Ψ given by Eqs. (13), (16), and (20) gives

$$A_1=\frac{1}{\sqrt{2}}\cos\frac{\varphi}{2},\quad B_1=\frac{1}{\sqrt{2}}\sin\frac{\varphi}{2}. \tag{23}$$

Matching expressions (15) and (22) for the potential leads to the following relationships:

$$\dot\varphi=2CF(1/2),\quad I-B_1f(0)=CF_z'(1/2)/2\sqrt{2}, \tag{24}$$

where $F_z'(1/2)=[\partial F(\alpha,\beta,\gamma,z)/\partial z]_{z=1/2}$. Combining relations (14), (23), and (24) we find the following expression for the current:

$$I=I_c\sin\varphi+\dot\varphi[F_z'(1/2)/F(1/2)4\sqrt{2}], \tag{25}$$

where I_c is determined by expression (8). Consequently, the expression for the current has the same form as in the case of a point contact, but the quantity (in dimensional form) $R=4\sqrt{2}\,F_z(1/2)\xi/F_z'(1/2)\sigma S$ (S denotes the cross-sectional area) plays the role of the effective resistance. This is associated with the fact that, as is clear from expressions (15) and (22), the potential Φ varies over distances $x\sim 1$. At the same time, the characteristic distance over which the phase varies is $x\sim\lambda$ (see the figure), just as in the time-independent case. The I-V characteristics of the system have the usual form: $I=(I_c^2+V^2/R^2)^{1/2}$. Thus, in the model under consideration the asymptotic resistance intrinsically depends on the temperature ($R\sim\xi\sim(T_c-T)^{-1/2}$).

S-N JUNCTION

Let us again consider a one-dimensional system consisting of a superconductor ($x>0$) and a normal metal ($x<0$). We shall determine the steady-state solutions, describing a small, constant current in such a system. Such a problem was solved numerically in[10]. The system under consideration is again described by Eq. (1) in which, however, it is necessary to replace $\delta(x)$ by $\theta(-x)$ and to assume $\dot\Psi=0$ ($\theta(x)=1$ for $x>0$ and $\theta(x)=0$ for $x<0$). For small values of λ the amplitude of Ψ will be small in the N-region (it is not difficult to verify that, in the absence of current, $f=\sqrt{\lambda/2}\exp(x/\sqrt{\lambda})$ for $x<0$); therefore, in general we may neglect f in the N-region and assume that

$$f=0,\quad x\leqslant 0. \tag{26}$$

Then the problem reduces to the solution of the following equations for $x>0$:

$$f''+f-f^3=\chi'^2f, \tag{27}$$

$$12f^2\Phi-\Phi''=0, \tag{28}$$

$$I=-\Phi'+\chi'f^2 \tag{29}$$

with the boundary condition (26).

This system of equations can be solved by iterations for small currents I. In the first approximation the right-hand side of Eq. (27) can be neglected, and we find

$$f_1(\zeta)=\operatorname{th}\zeta,$$

here $\zeta=x/\sqrt{2}$. Substituting this solution into Eq. (28), we obtain the following equation for Φ_1:

$$\Phi_1''-12\Phi_1\operatorname{th}^2\zeta=0. \tag{30}$$

In analogy to the preceding discussion (see Eq. (22)) we have

$$\Phi_1(\zeta)=-I\eta_I(\zeta)/\eta_I'(0). \tag{31}$$

Here $\zeta=x/\sqrt{2}$ and the integration constant is chosen so as to satisfy the boundary condition $\Phi_1'(0)=-1$, which follows from Eq. (29). As follows from Eq. (30) itself, for small values of x ($x\ll 1$) the potential Φ_1 has the form

$$\Phi_1(x)=\Phi_1(0)-Ix+{}^1/_2x^4\Phi_1(0).$$

For large values of x, the potential falls to zero exponentially ($\Phi_1\sim e^{-x\sqrt{12}}$). From Eq. (29) we find the superconducting velocity

$$\chi_1'(\zeta)=I\left[1-\frac{\eta_I'(\zeta)}{\eta_I'(0)}\right]\operatorname{th}^{-2}\zeta. \tag{32}$$

Let us find the corrections to f and to Φ in the next approximation. This enables us to determine the nonlinear dependence of the junction's resistance on I, which apparently is easily measured in an experiment. From Eq. (27) we obtain the following equation for f_2:

$$f_2''+f_2(1-3f_1^2)=\chi_1'^2f_1. \tag{33}$$

Its solution, satisfying the conditions $f_2(0)=0$ and $f_2(\infty)=-I^2/2$ (the latter follows from (33) and the asymptotic form of χ_1' which, according to Eq. (32), has the form $\chi_1'=I$ as $x\to\infty$) is given by the function

$$f_2(\zeta)=I^2\sqrt{2}\int_0^\zeta d\zeta_1[1-\eta_I'(\zeta_1)/\eta_I'(0)][y_I(\zeta_1)y_{II}(\zeta)-y_I(\zeta)y_{II}(\zeta_1)]\operatorname{th}^{-3}\zeta_1+Cy_{II}(\zeta)=I^2f_2^0(\zeta), \tag{34}$$

where

$$C=-I^2\sqrt{2}\int_0^\infty d\zeta_1[1-\eta_I'(\zeta_1)/\eta_I'(0)]y_I(\zeta_1)\operatorname{th}^{-3}\zeta_1,$$

and y_I and y_{II} are solutions of the homogeneous equation. They have the form

$$y_I(\zeta)=\sqrt{2}f_1'=\operatorname{ch}^{-2}\zeta,$$

$$y_{II}(\zeta)=y_I\sqrt{2}\int^\zeta\frac{d\zeta_1}{y_I^2(\zeta_1)}=\frac{y_I(\zeta)}{2\sqrt{2}}\left[\frac{3}{2}\zeta+\operatorname{sh}2\zeta+\frac{1}{8}\operatorname{sh}4\zeta\right].$$

For small x the function f_2 has the form

$$f_2=-|C|x,$$

and for large x we have $f_2\approx-(1/2)I^2$; thus, the function f_2^0, which was introduced in Eq. (34), is negative and assumes a value of the order of unity.

We obtain the equation for the desired correction Φ_2 from Eq. (28):

$$\Phi_2''-12\Phi_2\operatorname{th}^2\zeta=-24I^3f_1f_2^0\eta_I(\zeta)/\eta_I'(0). \tag{35}$$

Here we have utilized the solution (31) for Φ_1 and the notation (34). The solution of Eq. (35) is represented by the function

$$\Phi_2(\zeta)=-\frac{24\sqrt{2}}{D}I^3\int_0^\zeta d\zeta_1f_1(\zeta_1)f_2^0(\zeta_1)\frac{\eta_I(\zeta_1)}{\eta_I'(0)}\times[\eta_I(\zeta_1)\eta_{II}(\zeta)-\eta_I(\zeta)\eta_{II}(\zeta_1)]+C_1\eta_I(\zeta)+C_2\eta_{II}(\zeta), \tag{36}$$

where $D=\eta_I\eta_{II}'-\eta_{II}\eta_I'$ is the Wronskian, η_I and η_{II} are the solutions of the homogeneous equation, one of which (η_I) has already been used in Eq. (31), and the second has the form[9]

$$\eta_{II}(\zeta)=\exp(2\sqrt{6}\zeta)F\left(\beta-\gamma+1,\alpha-\gamma+1,2-\gamma,\frac{1-\operatorname{th}\zeta}{2}\right)\equiv\exp(2\sqrt{6}\zeta)F(z).$$

We note that $D=2^{2(\sqrt{6}+1)}$. We determine the constants C_1 and C_2 from the boundary conditions for Φ_2: $\Phi_2'(0)=0$ and $\Phi_2(\infty)=0$. We obtain

$$C_1=-C_2\eta_{II}'(0)/\eta_I'(0),$$

$$C_2=\frac{24\sqrt{2}}{D}I^3\int\limits_0^\infty d\zeta_1 f_1(\zeta_1)f_2^0(\zeta_1)\frac{\eta_1^2(\zeta_1)}{\eta_1'(0)}. \qquad (37)$$

Thereby the form of the potential is established correct to terms $\sim I^3$.

We note that we have $\Phi(x) = \Phi(0) - Ix$ for $x \le 0$. We shall be interested in precisely the quantity $\Phi(0) \equiv V$, i.e., the difference of potentials between two points, one of which is located on the S-N interface, and the other is located deep inside the superconductor. Setting $\zeta = 0$ in Eq. (36) and using expressions (37), we find

$$\Phi_2(0)=-\frac{24\sqrt{2}}{(\eta_1'(0))^2}I^3\int\limits_0^\infty d\zeta_1 f_1(\zeta_1)f_2^0(\zeta_1)\eta_1^2(\zeta_1)\equiv\alpha_2 I^3, \qquad (38)$$

where $\alpha_2 > 0$ since, as indicated above, $f_2^0 < 0$. Now let us set $\zeta = 0$ in Eq. (31); then

$$\Phi_1(0)=-\frac{\eta_1(0)}{\eta_1'(0)}I=\frac{2\sqrt{2}F(^1/_2)}{F_z'(^1/_2)}I.$$

Combining Eqs. (38) and (39), let us write down the relation between the voltage V and the current I in dimensionless units:

$$V=RI(\alpha_1+{}^4/_{27}\alpha_2 I^2/I_c^2),$$

where $\alpha_1 R = \xi\alpha_1/S\sigma$ is the temperature-dependent linear part of the junction's resistance, $\alpha_1 = 2\sqrt{2}\,F(1/2)/F_Z'(1/2) > 0$, and I_c is the critical current of the superconductor S. The coefficients α_1 and α_2 are numbers of the order of unity.

Yu and Mercereau[11] measured the temperature dependence of R. One can verify that it agrees qualitatively with the results obtained here. However, in this work it was asserted that the relation between V and I is linear right up to $I \sim I_c$. The calculations which have been made indicate that, in the adopted model (the validity of Eqs. (1) and (2), and a time-independent solution) the linearity between V and I is absent, which is quite easily understood physically since the form of the potential Φ in the S-region intrinsically depends on the amplitude f which, inturn, depends on the current I in a nonlinear manner. Therefore, the question of the agreement between the experimental results[11] and the theoretical results, discussed in[10] and here, remains open.

1)We note that the analytic solution in the time-independent case can be found without assuming λ to be small. Then a transcendental equation is obtained for the current I_C.

2)It follows from Eq. (19) that a time-independent state with a potential bounded over all space (for example, periodically depending on the coordinate) cannot arise in a superconductor (the opposite assertion is made by Fick [8]). In fact, let, for example, $\Phi(x_0) > 0$ at some point x_0. Then at this point $\Phi'' > 0$ and the function Φ is concave upward. It follows from the absence of an inflection point that Φ will increase (decrease) without limit upon going away from the point x_0.

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