Weakly damped electromagnetic waves in thin conductors under conditions of strong spatial dispersion

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The propagation of weakly damped electromagnetic waves in metals located in a magnetic field is investigated. It is shown that in a thin plate, for which $2R > d$ (R is the maximum cyclotron radius and d is the plate thickness), electromagnetic waves can propagate under conditions of strong spatial dispersion $kR \gg 1$ (k is the wave vector). The wave spectrum, the damping, and the polarization are found. The dependence of the surface impedance of a bounded sample on the magnetic field strength is obtained. The possibility of experimental observation of the effect is discussed.

It is known that in metals placed in a strong magnetic field $\Omega\tau \gg 1$, weakly damped electromagnetic oscillations can be propagated under conditions of weak spatial dispersion $kR < 1$.[1)] In uncompensated metals (number of conduction electrons not equal to the number of holes), these excitations are helicons, and in compensated metals they are magnetohydrodynamic waves.[1,2]

Under conditions in which the spatial dispersion is significant, cyclotron damping, a mechanism of collisionless absorption of the wave, comes into play and makes propagation of the wave impossible for $kR \gtrsim 1$. Electrons participate effectively in wave absorption if they are close to isolated cross sections of the Fermi surface, namely, those cross sections which are perpendicular to the magnetic field and on which $\langle v_H \rangle = 0$, where $\langle v_H \rangle$ is the projection, averaged over the cross section, of electron velocity onto the direction of the magnetic field.

It will be shown below that electromagnetic oscillations of the helicon type can exist in thin plates even in the case of strong spatial dispersion, $kR \gg 1$. The physical reason for this phenomenon lies in the possibility of a cutoff of the cyclotron orbits responsible for collisionless absorption by the boundaries of the sample.

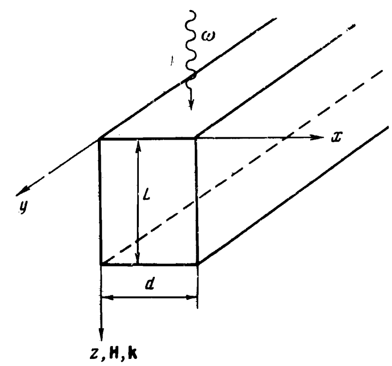
Let the magnetic field (see the figure) extend parallel to one of the surfaces of the plate. Then, under the condition

$$2r(p_z^0) > d \tag{1}$$

($2r = cD(p_z)/eH$, where $D(p_z)$ is the maximum dimension of the orbit of electrons in a magnetic field in momentum space), the electron orbits pertaining to cross sections with $\langle v_z(p_z^0) \rangle = 0$ do not fit into the thickness d of the plate. Cutoff of these electron orbits takes place because of scattering from the boundaries of the sample (for nonspecular reflection). This means that, upon satisfaction of the inequality (1), collisionless damping is suppressed and the electromagnetic wave can be propagated in the metal under conditions of strong spatial dispersion.

We now consider the case in which a monochromatic, circularly polarized electromagnetic wave of frequency ω is excited from the lateral surface of the plate and is propagated parallel to the magnetic field $k \parallel H$. It is evidently necessary that $k_z \gg k_x$ in order to satisfy the condition $k \parallel H$ (the choice of the coordinate axes is clear from the figure) and, inasmuch as $k \cong k_z \sim \lambda^{-1}$, $k_x \sim d^{-1}$, the wavelength λ should be much smaller than the thickness d of the sample:

$$kd \gg 1. \tag{2}$$



Since it is necessary for observation of the effect that $2R > d$, the inequality (2) means automatically that $kR \gg 1$ and the weakly damped electromagnetic excitations produced here are naturally called helicons in the limiting nonlocal propagation mode.

The expression which allows us to find the spectrum and the damping of the helicon in an unbounded metal has the form[2]

$$\omega = k^2c^2/4\pi\sigma_\pm(k, \omega), \tag{3}$$

where the Fourier component of the conductivity $\sigma_\pm$ is determined by the relation

$$\sigma_\pm(k, \omega) = \pm\sigma_{xy}(k, \omega) + i\sigma_{xx}(k, \omega).$$

The possibility of application of Eq. (3) to consideration of boundary problems is based on the "indifference" of electrons, under conditions of strong spatial dispersion ($kR \gg 1$), to the boundary conditions when the magnetic field H is perpendicular to the skin layer.[3,4] The physical nature of the insensitivity of the effect to the boundary conditions is connected with the fact that the basic contribution to the current is made by electrons which slide parallel to the skin layer and consequently do not collide with the surface of the conductor.

Inasmuch as the dependence of the impedance of the sample on the magnetic field in the case of propagation of a helicon in the metal depends weakly on the form of the dispersion law of the charge carriers, there is no necessity of taking the anisotropy of the Fermi surface into account.[2)] In the case of a quadratic isotropic dispersion law, the expression for the Fourier component of the conductivity can be written in the following form:

$$\sigma_\pm(k, \omega, d) = i\frac{2e^2v^3}{(2\pi\hbar)^3}\frac{m^{*2}}{\Omega}\int_{-1}^{1} d\mu\, \Theta(\mu^2-\mu_0^2)(1-\mu^2) \times \int_0^{2\pi} d\varphi \cos\varphi \int_{-\infty}^{\varphi} d\varphi' \exp\left[\mp i\varphi' + \frac{1}{\Omega}(\tau^{-1}-i\omega+ikv\mu)(\varphi'-\varphi)\right]. \tag{5}$$

Here $\Theta(x)$ is a unit step function, v the Fermi velocity of the electron, and

$$\mu_0=(1-(d/2R)^2)^{1/2}. \tag{6}$$

The expression (5) differs from the corresponding formula (111) from [2] for $\sigma_\pm$ in a bulky sample by the presence of the step function $\Theta(\mu^2-\mu_0^2)$ under the integral over $d\mu$. Here, the fact that the effective contribution to the high-frequency current in the case of nonspecular reflection is made by electrons which do not collide with the boundaries of the sample is automatically taken into account.

Carrying out the elementary integration, we get, for the condition $kR \gg 1$, the following formula for $\sigma_\pm$:

$$\sigma_\pm(k,\omega,d)\cong\frac{3Ne^2}{m^*kv}\left[\mp\ln\left|1-\frac{\xi^{(\pm)}}{\mu_0}\right|+i\arg\left(1-\frac{\xi^{(\pm)}}{\mu_0}\right)\right], \tag{7}$$

where

$$\xi^{(\pm)}=\frac{1}{kR}\left(1\pm\frac{\omega+i\tau^{-1}}{\Omega}\right).$$

Since the expression in the logarithmic term is less than unity, we have $\operatorname{Re}\sigma_- < 0$ and $\operatorname{Re}\sigma_+ > 0$, and in the limiting nonlocal regime the helicon with $\mathbf{k}\parallel\mathbf{H}$ also represents a circularly polarized wave, but the direction of rotation of the vector of the high-frequency electric field is opposite to that which is the case for helicons in the local regime. As is seen from Eq. (7), for $\xi^{(+)} < \mu_0$, the imaginary part is small because of the smallness of the quantity $(\Omega\tau)^{-1}$. As a consequence of this, $\operatorname{Re}\sigma_+ \gg \operatorname{Im}\sigma_+$, i.e., the Hall conductivity σ_{xy} is much greater than the dissipative σ_{xx}, and consequently electromagnetic waves can propagate in the sample.

As has already been noted, everything that has been said above can easily be transferred to the case of a metal with a complicated dispersion law for the charge carriers. We note here that, inasmuch as the high-frequency Hall conductivity, in contrast with the static case, and for the condition $kR \gg 1$, is not determined by the difference in the concentrations of electrons and holes, the given result applies in equal measure both to noncompensated and compensated metals.

For the value of the magnetic field $H = H_0$, when the equality

$$\xi^{(+)}=\mu_0, \tag{8}$$

is satisfied, the real part of the conductivity σ_+ has a logarithmic singularity. Taking into account that μ_0 is determined by the formula $2r = 2R(1-\mu^2)^{1/2} = d$, we find that Eq. (8) is equivalent to the following two equations:

$$kv_z(p_1)-\omega=\Omega(H_0),\quad v_z(p_1)=v\mu_0(H_0), \tag{9}$$

from which it is seen that the singularity in the conductivity for $H = H_0$ is connected with the Doppler-shifted cyclotron resonance on the cross section $p_z = p_1$. Thus, the threshold of the helicon spectrum in the limiting nonlocal regime has a resonance character, which is analogous to the situation which takes place when $kR < 1$. We shall limit ourselves below to the range of frequencies ω which are less than or of the order of the cyclotron frequency Ω. Then $\xi^{(+)} \cong 1/kR$ and, solving Eq. (8) for the magnetic field, we obtain the following expression for H_0:

$$H_0\cong\frac{2cp_F}{ed}\left(1-\frac{2}{(kd)^2}\right). \tag{10}$$

Inasmuch as $kd \gg 1$, the value of the magnetic field H_0 is very close to that for which $2R = d$, and the cross section $p_z = p_1$ is close to the central cross section of the Fermi surface.

The expression for the spectrum of the spiral wave that is determined by Eqs. (3) and (7) is simplified considerably on a decrease in the magnetic field intensity, when $\xi^{(+)}$ becomes much less than μ_0:

$$\omega=\frac{k^4c^2l^2\mu_0}{12\pi\sigma_0\Omega\tau}[1-i\mu_0(\Omega\tau)^{-1}], \tag{11}$$

here $\sigma_0 = Ne^2\tau/m^*$ is the static conductivity of the unbounded sample in the absence of a magnetic field and l is the free path.

Upon excitation of a circularly polarized electromagnetic wave in a plate, the frequency ω of the external excitation is fixed and Eq. (10) should be solved for the wave number k:

$$k^4=k_0^4(1+i\Gamma), \tag{12}$$

where $k_0 = (4\pi\omega\sigma_0/\mu_0c^2lR)^{1/4}$ and Γ is the relative damping decrement

$$\Gamma=\mu_0(\Omega\tau)^{-1}\ll 1. \tag{13}$$

The quantity k_0^{-1} determines the characteristic length of the weakly damped electromagnetic wave in the metal, having the order of $\lambda\cong(\delta^2lR)^{1/4}$. Taking into account that $2R \gtrsim d$, the inequality (2), which is necessary for observation of the effect, takes the form

$$d\gg(\delta^2l)^{1/3}=\delta_a, \tag{14}$$

where δ_a is the penetration depth of the electromagnetic wave into the metal for the case of the anomalous skin effect, and $\delta\cong(c^2/\omega\sigma)^{1/2}$ is the thickness of the normal skin layer.

Using the expression for the conductivity (7) for $\xi^{(+)} \ll \mu_0$, it is easy to compute the distribution of the electromagnetic field of the wave in the sample, which is determined by the following considerations:[2]

$$E_+(z)=-\frac{2}{\pi}T_+(z)\frac{\partial E_+(0)}{\partial z}, \tag{15}$$

$$T_+(z)=\int_0^\infty dk\frac{\cos(kz)}{k^2-4\pi\omega c^{-2}\sigma_+(k,\omega)} \tag{16}$$

Carrying out the integration, we obtain an expression for $T_+(z)$:

$$T_+(z)=-\frac{\pi i}{4k}\left(\exp\left\{-ik_0z-\frac{\Gamma}{4}k_0z\right\}-i\exp\left\{-k_0z+ik_0\frac{\Gamma}{4}z\right\}\right) \tag{17}$$

It is seen from Eq. (17) that $T_+(z)$ contains a weakly damped component with the characteristic damping length $l_0\cong k_0^{-1}(\Omega\tau)$, which is $\Omega\tau\cong l/d$ times greater than the wavelength λ.

An experimentally measured quantity is the surface impedance of the plate, the expression for which in the case $L \gg l_0$ is determined by the value of the function $T_+(z)$ at $z = 0$:

$$Z_+=-\frac{8i\omega}{c^2}T_+(0)=\frac{2\pi\omega}{c^2k_0}(1-i). \tag{18}$$

It is of interest to note that the relation between the real part of the impedance R_+ and its imaginary part X_+ has the form $R_+ = -X_+$, while for excitation of helicons in the local regime, $R_- \gg X_-$.

For experimental detection of the effect considered above, it is necessary that the length of the plate L be less than the damping length l_0. Here the impedance of the plate is a nonmonotonic function of the magnetic field and has a singularity when an integral number of wavelengths fits into the length of the sample. As is seen from the expression for the length of the helicon wave (12), the Z(H) dependence is not periodic in the magnetic field H or in its powers H^{α}, in contrast to the case in which helicons propagate in the sample in the local regime and Z(H) depends periodically on $H^{-1/2}$.

Following [5], it is easy to show, using the conductivity formula (7) in the case $\xi^{(+)} \ll \mu_0$, that, close to resonance, $k_0(\omega, H_n, L)L = \pi n$, the behavior of the impedance as a function of the magnetic field is described by the formula

$$Z_+^{(n)} \cong \frac{2\pi\omega}{c^2 k_0}\left\{\frac{{}^1/_4\pi n\Gamma + {}^1/_2 i\alpha_n L\Delta}{({}^1/_4\pi n\Gamma)^2 + ({}^1/_2\alpha_n L\Delta)^2} - if\left(\frac{k_0 L}{2}\right)\right\}, \qquad (19)$$

where

$$\alpha_n = \left(H\frac{\partial k_0}{\partial H}\right)_{H=H_n}, \qquad \Delta = \frac{H-H_n}{H_n},$$

$$f\left(\frac{k_0 L}{2}\right) = \begin{cases} \operatorname{cth}(k_0 L/2), & n = 2m \\ -\operatorname{th}(k_0 L/2), & n = 2m+1 \end{cases}.$$

It is seen from (19) that at resonance, the maxima of the real and imaginary parts of the impedance have the order

$$Z_+^{(n)} \cong \omega L k_0 l_0 / c^2 n^2$$

and decrease with the number in proportion with n^2, while satisfaction of the inequality $L \ll l_0$ is necessary to obtain a clear resonance peak.

For experimental observation of the given effect, it is necessary to choose the dimensions of the sample by starting from the inequalities $\lambda \ll L \ll l_0$, $\lambda \ll d \ll l$. The magnetic field here is determined by the condition $2R \cong d$, and the choice of frequency for the external electromagnetic excitation is dictated by the inequality $d \gg (\delta^2 l)^{1/3}$. The surfaces of the sample $x = 0$ and $x = d$ should reflect the electrons in a nonspecular way.

In pure samples, at liquid-helium temperatures, when the free path of the charge carriers can reach 1 cm, with plate dimensions $d \cong 10^{-2}$ cm, $L \cong 10^{-2}$ cm and a frequency range $\omega \cong 10^{7-8}$ sec^{-1}, the effect should be observed in magnetic fields of the order of 10^3 Oe.

In conclusion, the authors consider it their pleasant duty to thank G. E. Zil'berman, É. A. Kaner and V. G. Peschanskiĭ for attention to the work and useful discussions.

[1)]Here we introduce the following notation: $\Omega = |e| H/m^*c$ is the cyclotron frequency, m* the effective mass of the charge carriers, c the velocity of light, |e| the absolute value of the charge on the electron, $R = cp_F/eH$ the cyclotron radius, p_F the Fermi momentum of the charge carriers, τ their free path time, and k the wave number.

[2)]For electrons with an anisotropic dispersion law, the considered effect will take place in the case of cutoff of all cross sections with $\langle v_z \rangle = 0$.

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