Electromagnetic excitation of ultrasound and the transparency of semimetals

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A self-consistent theory of electromagnetic excitation of ultrasound in semimetals is developed. It is assumed that the skin effect is normal and that $\omega R_L/s \ll 1$ (R_L is the carrier Larmor radius and s is the velocity of sound). It is shown that the excitation of secondary electromagnetic fields by ultrasound leads to an additional "surface" absorption which under certain conditions may exceed the volume absorption. Near an acoustic resonance, neither the transparency nor the additional surface impedance is proportional to the small interaction constant coupling the ultrasonic and electromagnetic waves.

The resonance excitation of ultrasonic waves in semimetals by electromagnetic fields was discovered experimentally by Gantmakher and Dolgopolov[1] and has been discussed several times since [2–5]. In [2–4] it was shown that deformation interaction between the carriers and the ultrasonic waves predominates in semimetals, the coefficient for conversion of electromagnetic waves into ultrasonic waves was calculated, and the behavior of the resonance amplitude as a function of the magnetic field strength was examined in broad outline. It was assumed that the penetration of the electromagnetic field into the metal could be calculated by a successive-approximation method without allowing for the electromagnetic field excited by the ultrasonic waves, and the acoustic quality factor Q_a that limits the amplitude of the ultrasonic wave at a resonance had to be introduced as a phenomenological parameter. Although the method developed in [2–4] leads to some important results, it is not entirely consistent. We therefore feel that the theoretical study of the electromagnetic excitation of sound in semimetals should be continued.

In a more rigorous approach one would have to calculate both the acoustic quality factor Q_a and the observed resonance contribution to the surface impedance of the semimetal, and to do this one must take into account the excitation of coupled electromagnetic and ultrasonic oscillations in a semimetal plate by an incident electromagnetic wave. Although the coupling parameter $\epsilon = \Lambda^2 Q/\rho s^2$ is small (here ρ is the density of the semimetal, s is the velocity sound, Q is the density of states, and $\Lambda = \Lambda_e + \Lambda_h$, where $\Lambda_{e,h}$ are the deformation constants for the electrons and holes), the coupling of the electromagnetic and ultrasonic oscillations becomes important near the acoustic resonances and must be taken accurately into account.

In this paper we obtain a self-consistent solution to the problem of the electromagnetic generation of long-wavelength ultrasound in a semimetal and calculate the resonance contributions to the surface impedance of the plate for various excitation modes. Simple physical considerations show that the secondary electromagnetic waves excited by an ultrasonic wave must lead to two related effects that were not considered in the earlier work [2–5].

Let us suppose that an electromagnetic wave is incident from one side onto a semimetal plate whose thickness d greatly exceeds the penetration depths for field and carrier density disturbances, i.e., is greater than both "skin" depths [6]. Then if no ultrasound were generated, the electromagnetic field reaching the far face of the plate, and the field emerging into the vacuum, would be exponentially small. However, the ultrasonic wave excited in the plate gives rise to secondary electromagnetic-field and carrier-density oscillations. Thus, an ultrasonic wave is accompanied by forced volume oscillations, and these give rise to volume absorption of sound. Further, the reflection of the ultrasonic wave from the surface of the plate perturbs the field and the carrier density near the surface in regions whose thicknesses are of the order of the skin depths, and this means that the electromagnetic field partly penetrates through the plate. The transparency of the plate to the electromagnetic field transported by ultrasonic waves is not exponentially small.[1)]

We note that near the acoustic resonances, neither the transparency nor the change in the surface impedance of the plate due to the change in the field at the front face of the plate is proportional to the weak coupling between the electromagnetic and ultrasonic oscillations. Of course if the field E is incident on the surface of a semi-infinite metallic body, the secondary field E' excited by the ultrasonic wave will be of the order of ϵE. In the plate, however, owing to the multiple reflection of the ultrasonic wave from the faces (and to the consequent multiple infiltration of the field), the field getting through in the vicinity of an acoustic resonance will be $E'' \sim E' Q_a \sim E'/\epsilon$, i.e., it will be independent of the interaction constant ϵ.

Another effect associated with the surface perturbations of the field and carrier density is an additional ultrasonic absorption. The basic cause for the absorption of sound in a semimetal is the work done by the elastic waves against the electron pressure[2)]: $\delta A = \Lambda \int \mathbf{u} \cdot \nabla n dV$. The volume perturbations of the carrier density n, which have the spatial period $\lambda_s = 2\pi/k_1$, give rise to volume absorption (absorption coefficient independent of the thickness of the plate) of the ultrasonic wave. The presence of density perturbations and lattice deformations localized at the surfaces of the plate leads to an additional absorption of ultrasound. This "surface" part of the absorption depends on the thickness of the plate and may be comparable with or greater than the volume absorption.

In this section we find a self-consistent solution to the problem of the excitation of longitudinal ultrasonic oscillations by an electromagnetic wave incident normally onto a semimetal plate of thickness d occupying the region $0 \le y \le d$, the external magnetic field H_0 and the magnetic field H of the incident wave being assumed to be parallel to the z axis. For simplicity we consider an isotropic model of a semimetal with single electron and single hole valleys and equal carrier concentrations

$N_e = N_h$. We assume the deformation potential tensor Λ_{ik} to be isotropic and neglect the momentum dependence of its components. We also assume that the intravalley relaxation times $\tau_{e,h}$ are small compared with the intervalley recombination time $\tau_M = \omega_M^{-1}$. The equations for the problem include the elasticity equations, Maxwell's equations, and the equations of continuity (all quantities are proportional to $\exp i(\omega t - ky)$):

$$\rho(k^2s^2-\omega^2)u=-i\Lambda kn, \quad k^2E_x+4\pi i\omega c^{-2}j_x=0,$$
$$j=j^e+j^h, \quad -ie\omega n\mp ikj_y^{e,h}=e\omega_M(n-n_p), \tag{1}$$
$$j_y=\pm\sigma\beta E_x\mp iekD(n-n_p), \quad j_x=\sigma'E_x+ikeD\beta(n-n_p).$$

Here we use the following notation:

$$\sigma'=\sigma_{xx}^e+\sigma_{xx}^h, \quad \sigma=\sigma_{yy}^e\sigma_{yy}^h(\sigma_{yy}^e+\sigma_{yy}^h)^{-1},$$
$$\beta=\beta_e+\beta_h, \quad \beta_{e,h}=eH_0\tau_{e,h}/m_{e,h}c, \quad D=\sigma/e^2Q,$$
$$Q=Q_eQ_h(Q_e+Q_h)^{-1}, \quad n_p=ik\Lambda Qu;$$

$n_e = n_h = n$ are the deviations of the electron and hole concentrations from their equilibrium values, and D is the ambipolar diffusion coefficient in the magnetic field. Expressions for the currents and the form of the continuity equation were obtained in [9], where it was shown that $n_p = -Q(\delta\epsilon_e + \delta\epsilon_h)$ is the quasistatic change in the carrier density accompanying the sound wave ($\delta\epsilon = \Lambda_{ik}u_{ik}$). For a static deformation, $n = n_p$.

Expressions (1) for the current are valid when $\omega\tau_{e,h} \ll 1$. Here we have used the condition $\operatorname{div} \mathbf{j} = 0$ and have eliminated the longitudinal field E_y. The standard boundary conditions, namely that E_x and H_z be continuous at the surface of the semimetal and that $j_y^e \mp eVn = 0$ and $\partial u/\partial y = -\Lambda n/\rho s^2$ at $y = 0$ and $y = d$ (here V is the carrier recombination rate at the surface), must be imposed. Solving Eqs. (1), we obtain the dispersion equation for the coupled oscillations:

$$(k^2-\omega^2/s^2-\varepsilon k^2)\Delta(k)=-i\varepsilon\omega k^2(k^2+4\pi i\omega\sigma'c^{-2}),$$
$$\Delta(k)=(i\omega+\omega_M)(k^2+4\pi i\omega\sigma'c^{-2})+Dk^2(k^2+4\pi i\omega\sigma_0c^{-2}) \tag{2}$$
$$=D(k^2-k_2^2)(k^2-k_3^2).$$

Here $\sigma_0 = \sigma_0^e + \sigma_0^h$ is the conductivity in the absence of a magnetic field, $\epsilon = \Lambda^2Q/\rho s^2$ is the coupling constant, and we have assumed that $kR_L \ll 1$ ($R_L^{e,h}$ are the electron and hole Larmor radii). For $\epsilon = 0$, Eq. (2) breaks up into the dispersion equation for sound and the equation for electromagnetic oscillations in semimetals that was investigated in [6]. To the first order in ϵ we have

$$k_1=\frac{\omega}{s}+\delta k_1, \quad \delta k_1=\frac{\varepsilon k_1}{2}-\frac{i\varepsilon\omega k_1}{2\Delta(k_1)}(k_1^2+4\pi i\omega\sigma'c^{-2}). \tag{3}$$

The real and imaginary parts of δk_1 respectively determine the change in the dispersion and the volume absorption coefficient $\Gamma_{vol} = -s \operatorname{Im} \delta k_1$ for the ultrasound, which we investigated previously [9]. Here we also give the well known expressions (see [6]) for the other roots of Eq. (2) at $\beta_{e,h} \gg 1$ and $\epsilon = 0$, which we shall need later on:

$$k_2^2\approx-\frac{i\omega+\omega_M}{D}\frac{4\pi i\omega\sigma'}{c^2}\left[\frac{i\omega+\omega_M}{D}+\frac{4\pi i\omega\sigma_0}{c^2}\right]^{-1},$$
$$k_3^2\approx-\left[\frac{i\omega+\omega_M}{D}+\frac{4\pi i\omega\sigma_0}{c^2}\right]. \tag{4}$$

The solution to Eqs. (1) represents a superposition of three oscillations, of which one is basically acoustic, and the other two are basically electromagnetic:

$$u=A_1e^{-ik_1y}+B_1e^{ik_1y}-\frac{\Lambda}{e\rho s^2(\omega_M+i\omega)}\sum_{r=2,3}\frac{k_r^2S_r}{k_r^2-k_1^2}(A_re^{-ik_ry}+B_re^{ik_ry}),$$

$$E=\frac{\Lambda QM}{\Delta(k_1)}(A_1e^{-ik_1y}+B_1e^{ik_1y})+\sum_r(A_re^{-ik_ry}+B_re^{ik_ry}),$$

$$n=\frac{i\Lambda Qk_1P}{\Delta(k_1)}(A_1e^{-ik_1y}-B_1e^{ik_1y})-\frac{i}{e(\omega_M+i\omega)}\sum_r k_rS_r(A_re^{-ik_ry}-B_re^{ik_ry}),$$

$$j_y^e=-\frac{i\Lambda QW}{\Delta(k_1)}(A_1e^{-ik_1y}+B_1e^{ik_1y})+\sum_r S_r(A_re^{-ik_ry}+B_re^{ik_ry}).$$

Here we have used the notation:

$$M=4\pi\omega^2e\beta Dk_1^2c^{-2}, \quad W=e\omega k_1^2D(k_1^2+4\pi i\omega\sigma_0c^{-2}),$$
$$S_r=\frac{c^2(k_r^2+4\pi i\omega\sigma_0c^{-2})}{4\pi i\omega\beta}, \quad P=\Delta(k_1)-i\omega(k_1^2+4\pi i\omega\sigma'c^{-2}).$$

Using the boundary conditions and standard procedures, we find the unknown amplitudes, the strength of the field that penetrates the plate, and the surface impedance. However, if $|k_{2,3}|d \gg 1$ and exponentially small effects can be neglected, the quantities of interest can be easily expressed in terms of the solutions of two simple auxiliary problems.

The first auxiliary problem, that of the excitation of ultrasound by an electromagnetic wave incident on a semi-infinite specimen, was solved in [3]. The amplitude u_∞ of the ultrasonic wave and the field $E_0(0)$ at the surface are given in our notation by the following expressions:

$$u_\infty=\varkappa_1H(0),$$
$$\varkappa_1=\frac{\Lambda k_0k_1S_2S_3D[k_3(1+p_2)(k_2^2-k_1^2)-k_2(1+p_3)(k_3^2-k_1^2)]}{e\rho s^2\Delta(k_1)(\omega_M+i\omega)\{k_2S_3(1+p_3)-k_3S_2(1+p_2)\}}$$
$$E_0(0)=-Z_0^0H(0), \quad Z_0^0=\frac{k_0[S_3(1+p_3)-S_2(1+p_2)]}{k_2S_3(1+p_3)-k_3S_2(1+p_2)} \tag{5}$$
$$k_0=\omega/c, \quad p_{2,3}=ik_{2,3}V(\omega_M+i\omega)^{-1};$$

here κ_1 is the amplitude conversion factor for the transformation of the electromagnetic field into an acoustic field, and Z_0^0 is the surface impedance.

The second auxiliary problem is that of the reflection of ultrasound of amplitude u_∞ from the semimetal-vacuum interface in the $y = d$ plane: its solution is of the form

$$\bar{B}=u_\infty(1-R)e^{-2ik_1d}, \quad \bar{E}=\varkappa_2u_\infty e^{-ik_1d},$$
$$\varkappa_2=8\pi\rho s\omega^2\varkappa_1/c, \tag{6}$$
$$R=\frac{2i\varepsilon k_1k_2k_3DW[S_2(k_3^2-k_1^2)-S_3(k_2^2-k_1^2)]}{e\Delta(k_1)(\omega_M+i\omega)[k_2S_3(1+p_3)-k_3S_2(1+p_2)]}$$

Here $\bar{B}$ is the amplitude of the reflected ultrasound, $\bar{E}$ is the field emerging into the vacuum, and it is natural to call κ_2 the conversion factor for transformation of ultrasonic waves into electromagnetic waves. The quantity R in (6) characterizes the change in the ultrasonic reflection coefficient associated with the excitation of electromagnetic fields at the interface.

The amplitudes of the ultrasonic waves in the plate and the field that has traversed the plate can now be easily found by considering the multiple reflection of the ultrasound at the faces:

$$A_1=\frac{u_\infty}{2i}\frac{e^{ik_1d}}{\sin k_1d-iR\cos k_1d}, \quad B_1\approx A_1e^{-2ik_1d},$$
$$E(d)=\frac{\bar{E}e^{ik_1d}}{2i[\sin k_1d-iR\cos k_1d]}, \quad Z=Z_0^0+\Delta Z, \tag{7}$$
$$\Delta Z=\frac{4\pi i\rho s\omega^2}{c}\frac{\varkappa_1^2\cos k_1d}{\sin k_1d-iR\cos k_1d};$$

here ΔZ is the resonance increment of the surface impedance for $|k_{2,3}|d \gg 1$.

We note the following simple relation between the amplitude E(d) of the field that has traversed the plate and the surface-impedance increment ΔZ:

$$E(d)\cos k_1 d = -H(0)\Delta Z.$$

Thus, by measuring the resonance increment of the surface impedance one automatically measures the amplitude of the field that has traversed the plate.

As is evident from (7), the acoustic resonances correspond to the minima of $\sin k_1 d - iR\cos k_1 d$. If there is no external field ($H(0) = 0$), the equation

$$\sin k_1 d = iR\cos k_1 d \qquad (8)$$

determines the resonant frequencies $\omega_N = sN\pi d^{-1} + \Delta\omega_N$ and the damping constant Γ for free acoustic oscillations in the plate. We have

$$\Delta\omega_N = \omega_N \operatorname{Re}\left(\frac{iR}{k_1 d} - \frac{\delta k_1}{k_1}\right),$$
$$\Gamma = \frac{\omega_N}{Q_a} = \Gamma_v + \Gamma_{sur}, \quad \Gamma_{sur} = \frac{s}{d}\operatorname{Re} R. \qquad (9)$$

This result means that, as we mentioned above, the excitation of secondary electromagnetic fields at the surface of the semimetal by the ultrasound leads to an additional ultrasonic absorption Γ_{sur} and to an additional shift of the resonance frequencies by the amount $-(s/d)\operatorname{Im} R$.

We note that in the considered case $|k_{2,3}|d \gg 1$ the ratio Γ_{sur}/Γ_v does not exceed $4/|k_2|d$ for any value of the frequencies and the other parameters. Thus, the surface absorption can be neglected when $|k_{2,3}|d \gg 1$. However, the surface absorption may be substantial for thin plates in weak magnetic fields under the condition that $|k_2^0|d \ll 1$; $|k_2^0|^2 \approx (i\omega + \omega_M)\sigma'/\sigma_0 D$. This situation apparently obtained in the low-temperature experiments on antimony reported in [3].

Finally, for the transparency η (defined as the ratio of the transmitted flux to the incident flux) near a resonance (detuning $\delta\omega \sim \Gamma$), we have

$$\eta \approx 4|\Delta Z|^2 = \frac{32\pi^2\rho^2 s^4\omega^4}{c^2 d^2}\frac{|\varkappa_1|^4}{\Gamma_v^2}, \qquad (10)$$

where, according to [9],

$$\Gamma_v \approx \frac{\varepsilon}{2}\frac{\omega^2}{|\Delta(k_1)|^2}\left\{\omega_M\left[k_1^4 + \left(\frac{4\pi\omega\sigma'}{c^2}\right)^2\right] + Dk_1^2\left[k_1^4 + \frac{4\pi\omega\sigma'}{c^2}\frac{4\pi\omega\sigma_0}{c^2}\right]\right\}.$$

Let us consider the dependence of η on H_0 for the limiting cases in which the recombination is respectively strong ($|p_{2,3}| \gg 1$) and weak ($|p_{2,3}| \ll 1$). Since $\eta(\infty) = \eta(0)|(k_3^0)^2/(k_1^2 + k_2^0 k_3^0)|^4$, we give only the results for $\eta(0)$. At high frequencies,

$$\omega \sim \omega_{c0} = 4\pi s^2\sigma_0/c^2 \ (k_1^2 \sim |k_3^0|^2 \sim 4\pi\omega\sigma_0/c^2)$$

we have

$$\eta \sim \left|\frac{k_0}{k_3^0}\right|^2 \frac{1}{|k_3^0 d|^2} \text{ for } \frac{4\pi\sigma_0 D}{c^2}\omega\tau_M = \alpha \gg 1,$$
$$\eta \sim \left|\frac{k_0}{k_3}\right|^2 \frac{1}{|k_3 d|^2} \text{ for } \frac{4\pi\sigma_0 D}{c^2}\omega\tau_M = \alpha \ll 1. \qquad (11)$$

In weak fields η is independent of H_0, while in strong fields it decreases as H_0^{-4}. At low frequencies ($\omega\tau_M \ll 1$), η has a maximum at the magnetic field strength determined by the condition

$$\alpha = \left|\frac{k_1}{k_2^0}\right|^2\left(1 + \left|\frac{k_1}{k_2^0}\right|^2\right)^{-1/2} = \alpha_{ext}.$$

As the magnetic field increases, η increases as H_0^4 when $\alpha \ll \alpha_{ext}$ and decreases as H_0^{-4} when $\alpha \ll \alpha_{ext}$. The value $\eta(0)$ of the transparency at the extremum is given by

$$\eta = \left|\frac{k_0}{k_3^0}\right|^2 \frac{1}{|k_3^0 d|^2}\frac{|k_1^2 + k_2^0 k_3^0|^4}{\{k_1^4 + 2|k_2^0||k_3^0|^2(k_1^2 + |k_2^0|^2)^{1/2}\}^2}$$

We note again that the transparency reaches its greatest value when the frequency is low and the surface recombination is strong.

Now let us discuss the excitation of ultrasonic oscillations in a semimetal plate by electromagnetic waves incident simultaneously onto both faces ($H(0) = H(d)$, $E(0) = -E(d)$), since such excitation has been investigated experimentally[1,3]. The solution for the case in which the excitation is symmetric (with respect to the magnetic field) is similar to the solution given above, but here the field E, the currents $j_y^{e,h}$, and the displacement u contain the factor $\sin k_i(y - d/2)$ in place of the exponential, and the density n contains the factor $\cos k_i(y - d/2)$, with $i = 1, 2, 3$. The impedance of the plate is given by $Z = Z_0 + \Delta Z$, with

$$Z_0 = \frac{ik_0 t_3 t_2[S_3(1+\bar{p}_3) - S_2(1+\bar{p}_2)]}{k_2 S_3(1+\bar{p}_3)t_3 - k_3 S_2(1+\bar{p}_2)t_2},$$
$$\Delta Z = \frac{4\pi i\varepsilon k_0^3 k_1^3\sigma_0 D s_1}{\Delta^2(k_1)[c_1 + {}^1/_2 iR_c s_1]}$$
$$\times\left[\frac{k_2(k_3^2 - k_1^2)t_3 - k_3(k_2^2 - k_1^2)t_2 + k_2 k_3 V(k_3^2 - k_2^2)(\omega_M + i\omega)^{-1}}{k_2(k_3^2 + 4\pi i\omega\sigma_0 c^{-2})t_3 - k_3(k_2^2 + 4\pi i\omega\sigma_0 c^{-2})t_2 + k_2 k_3 V(k_3^2 - k_2^2)(\omega_M + i\omega)^{-1}}\right]^2$$
$$R_c = \frac{2\varepsilon\omega D^2 k_1^3 k_2 k_3(k_3^2 - k_2^2)(k_1^2 + 4\pi i\omega\sigma_0 c^{-2})^2(\omega_M + i\omega)^{-1}}{\Delta^2(k_1)\{k_2(k_3^2 + 4\pi i\omega\sigma_0 c^{-2})(1+\bar{p}_3)t_3 - k_3(k_2^2 + 4\pi i\omega\sigma_0 c^{-2})(1+\bar{p}_2)t_2\}}$$
$$\bar{p}_{2,3} = k_{2,3}V/(\omega_M + i\omega)t_{2,3};$$
$$t_i \equiv \operatorname{tg}\frac{k_i d}{2}, \quad s_i \equiv \sin\frac{k_i d}{2}, \quad c_i \equiv \cos\frac{k_i d}{2}. \qquad (12)$$

The behavior of $\operatorname{Re}\Delta Z$ near the acoustic resonances at $\omega_N = (2N + 1)s\pi/d$ with $N = 0, 1, 2$ was studied experimentally in [1,3].

Here we give the expression for the oscillating addition to the impedance, calculated under the following simplifying assumptions, which are valid for the experiments of [1,3]: $|k_3|d \gg 1$ and $k_1^2 \ll |k_2 k_3|$. The second inequality sets an upper bound ω_{max} of the order of 10^8 sec^{-1} to the frequency of the ultrasonic wave. No limitations are imposed on $|k_2|d$. Under these conditions we have

$$\Delta Z = \frac{s}{d}|\Delta Z_0|\frac{e^{i\xi}}{\delta\omega - i\Gamma}, \quad \operatorname{tg}\xi = \frac{\operatorname{Im}\Delta Z_0}{\operatorname{Re}\Delta Z_0}$$
$$\delta\omega = \omega - \omega_{res}, \ \Gamma = \Gamma_v + (s/d)\operatorname{Re} R_c, \qquad (13)$$
$$\Delta Z_0 \approx \frac{8\pi i\varepsilon k_0^3\sigma_0 D k_2^2 k_1^3[(\omega_M + i\omega) + ik_3 V]^2}{(k_1^2 - k_2^2)^2\{k_2(\omega_M + i\omega)^2 - k_3(\omega_M + i\omega)4\pi\sigma_0 D\omega c^{-2}t_2 - ik_2 k_3^3 VD\}^2}.$$

Except for a numerical factor, $\operatorname{Re}\Delta Z_0$ is proportional to the energy lost by the electromagnetic wave in exciting ultrasound; these losses were investigated in [4].

To investigate the quantity $\partial^2 X/\partial\omega^2 = (\operatorname{Re}\Delta Z)''$, which was measured in [1,3], it is convenient to express X'' in the form

$$X'' = \frac{2s}{d}\frac{|\Delta Z_0|}{\Gamma^3}f(\xi, x), \quad x = \frac{\delta\omega}{\Gamma},$$
$$f(\xi, x) = [\cos\xi(x^3 - 3x) - \sin\xi(3x^2 - 1)](1 + x^2)^{-2}. \qquad (14)$$

Following [1,3], we define the amplitude of the resonance as $X''_{max} - X''_{min}$. Direct calculations show that $f(x_{max}) - f(x_{min})$ depends only very weakly on ξ, so that the frequency, magnetic-field, and temperature dependences of the resonance amplitude are fully determined by the factor $|\Delta Z_0|/\Gamma^3$.

Let us first discuss the resonance line shape, which is determined only by the parameter ξ. When $\xi = \pi/2$, the line has one maximum at the center and two symmetric minima. When $\xi=0$ or $\xi=\pi$, the line has one maximum and one minimum, with $x_{min}=-x_{max}$ and $f(x_{max}) = -f(x_{min})$; in intermediate cases there is one maximum and two unsymmetric minima. Let us consider the dependence of the line shape in weak magnetic fields ($k_2 \approx k_2^0$, $k_3 \approx k_3^0$) on various parameters in certain limiting cases. For definiteness we assume that $|k_2^0|d \gtrsim 1$. If $|p_{2,3}| \gg 1$ (strong surface recombination), $\xi=\pi/2$ and the line is always symmetric. For $|p_{2,3}| \ll 1$, we have $\xi = 2\tan^{-1}(\omega\tau_M)$, and the asymmetry of the line depends on the magnitude of the volume recombination, the asymmetry increasing as $\omega\tau_M$ decreases. Finally, if $|p_2| \ll 1$ but $|p_3| \gg 1$, we have $\xi = \pi/2 - \tan^{-1}(\omega\tau_M)$. In this case the resonance line becomes less asymmetric as $\omega\tau_M$ decreases. Further, when $\omega\tau_M \ll 1$, we have $\xi = 2\tan^{-1}(p_3(1+|p_3|\cos(\pi/4))^{-1}\sin(\pi/4)$ and the line shape is entirely determined by the surface recombination. Thus, a detailed study of the resonance line shape could yield interesting information about the parameters of the semimetal.

Let us consider the dependence of the resonance amplitude $\partial^2X/\partial\omega^2$ on the magnetic field H_0 and the temperature T. The H_0 dependence of the amplitude is determined mainly by the H_0 dependence of $|\Delta Z_0|$ and therefore differs little from the dependence predicted in [3,4]. For $\omega\tau_M \ll 1$ in weak fields ($\alpha = 4\pi\sigma_0 Dc^{-2}\omega\tau_M \ll 1$) we have $\Delta Z_0 \sim H_0^2$ and Γ_{sur} is independent of H_0, whereas Γ_V is independent of H_0 for $k_1 \lesssim |k_2^0| \approx (\omega_M\sigma'/\sigma_0 D)^{1/2}$. In strong fields ($\alpha \ll 1$), $\Gamma_V \sim \epsilon\omega^2\tau_M/2$, Γ_V is again independent of H_0, $\Gamma_{sur} \ll \Gamma_V$, and the resonance amplitude decreases as H_0^{-2}. The amplitude reaches its maximum at $\alpha \sim \alpha_{ext}$. For the higher harmonics, $k_1 \gtrsim |k_2^0|$ and the sound absorption increases with H_0 at $\alpha \lesssim k_1/|k_2^0|$, so the amplitude maximum may shift toward smaller H_0 with increasing frequency. Such a shift was noted in [1].

Now let us consider the temperature dependences of the resonance line width and amplitude for the case of weak magnetic fields. As regards the surface recombination, we shall assume that $|p_2| \ll 1$ and $|p_3| \gg 1$; the last condition was apparently satisfied in the experiments reported in [3]. Under these assumptions the resonance amplitude $\partial^2X/\partial\omega^2$ and absorption coefficient Γ are given respectively by

$$\frac{\partial^2 X}{\partial\omega^2} \approx \frac{2s}{d}\frac{|\Delta Z_0|}{\Gamma^3}, \quad |\Delta Z_0| \approx \frac{2\varepsilon k_1{}^3 c^2 |k_2{}^0|^2 V^2 \tau_M{}^2 k_0}{(k_1{}^2+|k_2{}^0|^2)^2 4\pi\sigma_0 D T_2{}^2}, \tag{15}$$

$$\Gamma \approx \frac{\varepsilon}{2}\omega\tau_M\left\{\frac{\omega\sigma'}{\sigma'+\sigma_0 D\tau_M k_1{}^2} + \frac{4s}{d}\frac{k_1{}^3|k_2{}^0|}{(k_1{}^2+|k_2{}^0|^2)^2 T_2}\right\}, \tag{16}$$

$$T_2 \equiv \text{th}\,(|k_2{}^0|d/2).$$

The first term in (16) describes the volume absorption, and the second term, the surface absorption. We must call attention to the following circumstance. In the experiments reported in [1,3], the condition $kl_{e,h} \gg 1$ was sometimes satisfied ($l_{e,h}$ are the electron and hole mean free paths). When $kl \gtrsim 1$, the nonlocal corrections

$$\sigma_{xx}^{e,h} \approx \frac{\sigma_0}{\beta_{e,h}^2} + \frac{2}{5}\sigma_0^{e,h}(kR_{e,h})^2 = \frac{\sigma_0}{\beta_{e,h}^2}\left(1+\frac{2}{5}k^2 l_{e,h}^2\right)$$

contribute significantly to the conductivity $\sigma' = \sigma_{xx}^e + \sigma_{xx}^h$. It is necessary to take spatial dispersion into account when calculating the volume damping of the ultrasound, and it seems legitimate to do so. At the same time, a small correction of the order of $(kR_{e,h})^2$ to σ' changes the roots $k_{2,3}^2$ of the dispersion equation (2) by a small quantity of the order of $|k_3^0|^2 R_{e,h}^2$, and in our approximation these roots should be neglected.

Let us first consider the temperature dependence of the resonance line width, which is determined by the ultrasonic absorption coefficient Γ. The behavior of Γ as a function of T depends essentially on the frequency (the number of the resonant harmonic). Suppose the frequencies are such that the condition $k_1 d < (\tau_M/\tau)^{1/2}$ is satisfied. In this case the main contribution to Γ at low temperatures where

$$|k_2{}^0|d \sim \frac{d}{l}\left(\frac{\tau}{\tau_M}\right)^{1/2} \ll 1,$$

comes from $\Gamma_{sur} \sim \epsilon s^2 d^{-2}\tau_M$ ($k_1 \gg |k_2^0|$). As the temperature rises, $|k_2^0|$ increases as $(\tau_M\tau)^{-1/2}$ and Γ_V increases. We have $\Gamma_{sur} \sim \Gamma_V$ when $|k_2^0|d \sim 1$, and as T increases further, Γ_{sur} falls while Γ_V rises as long as $|k_2^0| < k_1$, and the total absorption may remain almost constant. When $|k_2^0| > k_1$, we have $\Gamma \sim \Gamma_V \sim \epsilon\omega^2\tau_M$.

Now let us consider the case of high frequencies with $k_1 d \gg (\tau_M/\tau)^{1/2}$. In this case the T dependence of Γ is more complicated. When $|k_2^0|d \ll 1$, the principal contribution to the absorption comes from the volume term $\Gamma_V \sim \epsilon\omega^2\tau$, which decreases with increasing T. The conditions $|k_2^0|d < 1$ and $k_1 d > (\tau_M/\tau)^{1/2}$ break down at higher temperatures. If the first of these conditions is the first to break down, Γ will be determined as before by Γ_V, which has a minimum at $kl \sim 1$. Actually, for $kl \sim 1$ we have $\Gamma_V \sim \epsilon\omega^2\tau$, and for $kl < 1$ (with $k_1 > |k_2^0|$) we have $\Gamma_V \sim \epsilon s^2\sigma'/\sigma_0 D \sim 1/\tau$. If the condition $k_1 d > (\tau_M/\tau)^{1/2}$ is the first to break down, however, Γ_V will be significant only as long as this condition continues to hold. When $|k_2^0|d < 1$ and $k_1 d < (\tau_M/\tau)^{1/2}$, the absorption is due mainly to the surface term ($\Gamma \sim \Gamma_{sur} \sim \epsilon s^2 d^{-2}\tau_M$), but as T rises further the absorption again becomes of volume type ($\Gamma \sim \Gamma_V \sim \epsilon s^2\sigma'/\sigma_0 D \sim 1/\tau$) when $|k_2^0|d \gtrsim 1$ and continues to rise as long as $|k_2^0| < k_1$. When $|k_2^0| > k_1$, we have $\Gamma \sim \Gamma_V \sim \epsilon\omega^2\tau_M$.

Finally, let us consider the T dependence of the resonance line amplitude $\partial^2X/\partial\omega^2$. It was found in [3] that the amplitude at the first resonance decreases with increasing T, but that the amplitude at the third harmonic has a maximum; it was also found that the line width remains constant in the first case but decreases appreciably in the second case. It is not difficult to see that Eq. (15) leads to such a temperature dependence of the amplitude provided Γ is temperature independent (i.e., does not have an electronic origin [11]) and provided further that the condition $|k_2^0| < k_1$ gives way to the condition $|k_2^0| > k_1$ as T increases, it being assumed that the condition $|k_2^0| > k_1$ holds at the first resonance for all T. Such an explanation, however, is not entirely satisfactory: First, it contradicts the temperature dependence of the line width; second, the condition $|k_2^0| > k_1$ apparently does not hold at low temperatures. Actually, $k_1 \gtrsim \pi/d$; then $|k_2^0|/k_1 \lesssim (d/\pi l)(\tau/\tau_M)^{1/2} \ll 1$, since in the experiments[1,3] $l \sim d$ but $\tau/\tau_M \ll 1$ at low frequencies.

Now let us examine the temperature dependence of Γ. At the first resonance we have $k_1 d < (\tau_M/\tau)^{1/2}$, and $\Gamma(T) = \text{const}$ provided τ_M is temperature independent. The line amplitude remains unchanged as long as $|k_2^0| < k_1$ and $|k_2^0|d \lesssim 1$, and then it falls, being proportional to $|k_2^0|^2 V^2 \tau_M^2 (k_1^2 + |k_2^0|^2)^{-2}$. At the higher harmonics we have $k_1 d > (\tau_M/\tau)^{1/2}$, and the line-amplitude maximum may be associated with an absorption coefficient minimum. For example, if the condition $|k_2^0|d > 1$ remains in effect as T rises, the amplitude maximum will be determined by the condition $kl \sim 1$. When $kl > 1$ we shall have $\partial^2X/\partial\omega^2 \sim \tau^{-2}\tau_M^3$, and when $kl < 1$ (with $k_1 > |k_2^0|$), $\partial^2X/\partial\omega^2 \sim \tau^4\tau_M^{-3}$.

It should be pointed out, however, that the condition for a possible amplitude maximum and narrowing of the line width to which our calculations lead, namely the condition $k_1 d > (\tau_M/\tau)^{1/2}$, is very rigid and is not satisfied at the third harmonic. Perhaps allowance for the anisotropy and the presence of more than one valley in the electron spectrum would lead to the appearance of numerical factors that would make it easier for this condition to be satisfied.

Thus, our results are only in qualitative agreement with the experimental data on the temperature dependences of the resonance-line amplitude and width. Quantitative agreement between the theory and experiment will be possible only when a more realistic model of a semimetal is developed and more experimental data are available.

1) Kaganov and Fiks[7] were the first to call attention to the possibility of a nonexponential transparency of metals associated with generation of ultrasound. Effects associated with the transport of an electromagnetic field as a result of its interaction with weakly damped oscillations of another kind were discussed even earlier[8].

2) According to[9], the induction mechanism for sound absorption is significant only at extremely low frequencies or in very strong magnetic fields. Estimates made in[4] show that in most cases one can also neglect the contribution from the induction forces to the electromagnetic generation of sound.

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