Effects of inhomogeneity of optical pumping in lasers and in stimulated scattering. Self-excitation due to distributed feedback

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(Submitted June 25, 1973)
Zh. Eksp. Teor. Fiz. **66**, 96–108 (January 1974)

In laser-pumped amplifying systems, the inhomogeneity of the pumping is the reason for the onset of distributed feedback (DFB). A theory of coherent DFB in a three-level laser with inhomogeneous pumping is developed. The self-excitation threshold is determined, and the role of the spatial harmonics of the inverted-population modulation is analyzed. The effects of the DFB connected with the inhomogeneity of the pumping appear also in SRS. The conditions for self-excitation of a Raman laser with coherent DFB are determined, and the stationary nonlinear regime is discussed. The conditions for the onset of SRS instability due to the DFB that can be produced by weak reflections of the pump are obtained. The thresholds of the instability due to coherent DFB in amplifying systems are compared with the thresholds of the instability due to stochastic DFB caused by random inhomogeneity of the pump or by thermodynamic fluctuations of the density. The DFB produced upon inhomogeneous burning out of the inverted population are analyzed.

1. INTRODUCTION

The feedback needed for laser self-excitation (for the onset of absolute instability) can be produced by various methods. In addition to the classical mirror resonator[1] (in which case regular "lumped" feedback is produced, wherein the opposing waves interact at fixed sections in space), feedback via scattering from a rough screen, called nonresonant feedback[2] was also used (from the point of view of the terminology employed below, it is natural to name it stochastic lumped feedback); it was also proposed (see[3]) to use reflection from a three-dimensional grating induced by the amplified wave (autoresonant feedback).

Much attention has been paid recently to systems with distributed feedback (DFB), in which the interaction of the opposing waves is continuous in all sections of the amplifying medium. As applied to optics, DFB was first considered in connection with investigations of parametric light generators and stimulated Mandel'shtam-Brillouin scattering[4]. In the field of an intense pump wave of frequency ω_p, opposing waves of frequencies ω_1 and ω_2 that satisfy the relation $\omega_1 + \omega_2 = \omega_p$ can interact. Absolute instability can set in at the frequencies ω_1 and ω_2 when a certain threshold pump power is reached.

Although many aspects of the indicated interactions are understood by now (both linear and nonlinear theories of such generators have been developed, see[4]), they have not yet been realized experimentally in optics. In[5] they proposed and realized a different system with DFB, namely a dye laser in which self-excitation is due to opposing-wave interaction caused by regular static modulation of the gain G or of the refractive index n (distributed Bragg reflections from an "active" or "reactive" grating):

$$G=G_0+\Delta G\cos Kz,\qquad n=n_0+\Delta n\cos Kz. \tag{1}$$

Distributed feedback of this type is of considerable interest for a large class of laser systems. Linear theory[5,6] shows that systems with DFB have high spectral selectivity owing to the frequency sensitivity of the conditions for Bragg reflection.

A phenomenological theory of lasers with DFB effected by three-dimensional gratings of the type (1) was developed in[6]; its results can be used directly to interpret experiments in which active media with previously produced "frozen" gratings are used.

An interesting class of systems with DFB are systems with inhomogeneous coherent optical pumping; in these systems, modulation of the gain in accordance with the law (1) can be effected by interference between pumping beams. Such schemes are already in use in dye lasers[5,7]. They are of particular interest for the technology of short-wave lasers, where the construction of resonators entails great difficulties. We develop below a theory for DFB effected by inhomogeneous coherent optical pumping, for the case of a three-level system and stimulated Raman scattering. The results can be used directly to analyze the conditions of self-excitation and the spectral characteristics of "resonatorless" three-level and Raman lasers.

Another important aspect of the problem at hand is the determination of the conditions under which amplifiers with coherent optical pumping are unstable. The instability can be due not only to the usually considered uncontrollable reflections of the amplified signal, but also weak reflections of the pump, which lead to the onset of distributed feedback. Whereas in the former case a resonator at the signal frequency is essential for the self-excitation, the realization of DFB requires only weak pump inhomogeneity caused by a single reflection of the pump.

The characteristics of coherent DFB are compared with the characteristics of stochastic (incoherent) DFB due to thermodynamic fluctuations of the density of the medium and to random inhomogeneity of the pumping.

2. DISTRIBUTED FEEDBACK IN A LASER WITH INHOMOGENEOUS PUMPING

One-dimensional scalar generation in a three-level system is described by the quasiclassical equations[8] for the polarization P, the inverted population N, and the electric field intensity E (in the standard notation):

$$\frac{\partial^2 P}{\partial t^2}+\frac{2}{T_2}\frac{\partial P}{\partial t}+\left(\omega_0^2+\frac{1}{T_2^2}\right)P=-\frac{2|d|^2\omega_0}{\hbar}NE, \tag{2}$$

$$\frac{\partial N}{\partial t}+\left(W+\frac{1}{T_1}\right)N-\left(W-\frac{1}{T_1}\right)N_0=\frac{2}{\hbar\omega_0}E\left(\frac{\partial P}{\partial t}+\frac{1}{T_2}P\right),$$

$$\frac{\partial^2 E}{\partial t^2}-v^2\frac{\partial^2 E}{\partial z^2}+2\delta\frac{\partial E}{\partial t}=-4\pi\frac{\partial^2 P}{\partial t^2}.$$

The modulation of the gain (in space) can be effected by inhomogeneous distribution of the pump intensity. If two coherent light beams crossed at an angle 2θ are used as the pump (Fig. 1), then the interference field in the active medium is

$$E_p/E_0=\exp[-ik_p(y\cos\theta+z\sin\theta)]+\exp[-ik_p(y\cos\theta-z\sin\theta)] \tag{3}$$

and the probability of stimulated transition to the upper level, which is proportional to the pump intensity, is given by

$$W={}^1/_2W_0[1+\cos(2k_pz\sin\theta)]. \tag{4}$$

The pump frequency must satisfy two conditions. For an effective inversion it is necessary to have $\omega_p\approx\omega_{31}$, and to produce Bragg feedback at the generation frequency it is necessary to have $\omega_p\sin\theta\approx\omega_0$.

We determine first the generation threshold at the frequency $\omega=\omega_0+\epsilon$ in an active medium of length l in the steady state:

$$t\gg(\varepsilon^2+T_2^{-2})^{-1/2};\quad (W+T_1^{-1})^{-1};\quad lv^{-1}. \tag{5}$$

Neglecting saturation with respect to the signal field,

$$W+T_1^{-1}\gg\sigma|E|^2,\quad \sigma=4|d^2|/\hbar^2T_2(\varepsilon^2+T_2^{-2}),$$

the inverted population produced by the modulated pumping (4) is described by a given function of the spatial coordinate

$$N=N_0\frac{W-T_1^{-1}}{W+T_1^{-1}}. \tag{6}$$

Accordingly, the spatial modulation of the population difference does not duplicate the waveform of the pump modulation; the spectrum of N contains higher harmonics of the spatial frequency $K=2k_p\sin\theta$:

$$\frac{N}{N_0}=F_0+2\sum_{m=1}^{\infty}F_m\cos(mKz). \tag{7}$$

The spatial harmonics of the inverted population, as functions of the parameter $\eta=(1+W_0T_1)^{1/2}$, are given by

$$F_0=\frac{\eta-2}{\eta},\qquad F_m=-\frac{2}{\eta}\left(\frac{1-\eta}{1+\eta}\right)^m. \tag{8}$$

As $\eta\to\infty$ the spatial distribution of the inversion tends to become homogeneous, i.e., the feedback of all orders is vanishingly small at sufficiently large pump rates.

Introducing the slow amplitudes of the forward and backward waves

$$E=A_1\exp[i(\omega t-kz)]+A_2\exp[i(\omega t+kz)]+\text{c.c.} \tag{9}$$

with wave number $k=\omega/v$, taking into account only the first spatial harmonic in the population difference, and averaging (2) over the spatial period, we obtain a system of coupled linear equations describing the behavior of a laser with DFB in a specified pump field:

$$\pm\frac{dA_{1,2}}{dz}=\left(a-\frac{\delta}{v}\right)A_{1,2}+\alpha A_{2,1}e^{\mp 2i\Delta z}, \tag{10}$$

where $\Delta=(\Omega-\omega)/v$ characterizes the deviation of the lasing frequency from the Bragg frequency $\Omega=\omega_p\sin\theta$. The coefficients of the system are given by

$$a=GF_0/(1-i\varepsilon T_2),\quad \alpha=aF_1/F_0;\quad G=2\pi\omega_0|d|^2N_0T_2/\hbar v. \tag{11}$$

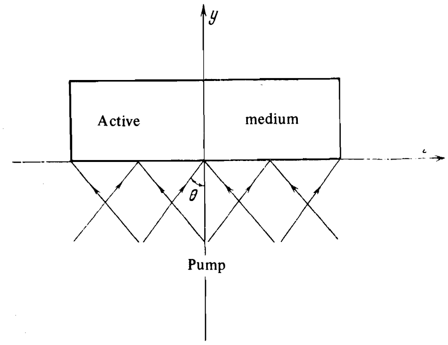

FIG. 1. Geometry of laser with distributed feedback produced by inhomogeneous pumping. Two pumping beams (the angle between which is 2θ) produce periodic modulation of the inverted population in the active medium.

The solution of the system (10) with the boundary conditions $A_1(0)=A_0$, $A_2(l)=0$ are the functions

$$A_1=A_0\frac{\operatorname{sh}[C-\Gamma(l-z)]}{\operatorname{sh}(C-\Gamma l)}e^{-i\Delta z},\quad A_2=\alpha A_0\frac{\operatorname{sh}\Gamma(l-z)}{\operatorname{sh}(C-\Gamma l)}e^{i\Delta z}, \tag{12}$$

where $\Gamma^2=(a-\delta/v+i\Delta)^2-\alpha^2$, and the constant C is determined from the relation $\Gamma\coth C=a-\delta/v+i\Delta$. The amplifier becomes self-excited ($A_{1,2}\to\infty$) if the following condition is satisfied:

$$\Gamma\operatorname{cth}\Gamma l=a-\delta/v+i\Delta. \tag{13}$$

The lasing threshold Gl_{thr} is obtained by comparing the absolute values in (13); it increases rapidly with increasing frequency deviation, thus ensuring high frequency selectivity. In the case of modulation at a frequency corresponding to the center of the luminescence line, the threshold gain (without allowance for absorption) is determined by the relation

$$2Gl_{thr}=(F_0^2-F_1^2)^{1/2}\ln\frac{F_0+(F_0^2-F_1^2)^{1/2}}{F_0-(F_0^2-F_1^2)^{1/2}}. \tag{14}$$

A plot of Gl_{thr} against the parameter η is shown in Fig. 2.

The minimal pumping rate at which lasing is possible is $W_0\approx1.4/T_1$. With increasing pump, the threshold gain first drops to $Gl\approx2.7$ at $W_0\approx5.5/T_1$, and then increases slowly to infinity, since the DFB vanishes. At a fixed gain therefore, lasing is possible in a pump interval that is bounded from above and below.

Comparison of the phases in (13) yields the spectrum of the generated frequencies. At large pumping rates, when $F_1\ll F_0$ (weak coupling), the generation frequencies $\omega=\omega_0-\epsilon$ are determined by the relation

$$\Delta\operatorname{ctg}\left(\frac{\varepsilon T_2G}{1+(\varepsilon T_2)^2}+\Delta\right)l=G+\varepsilon T_2\Delta. \tag{15}$$

If the m-th spatial harmonic of the population difference is used to produce the DFB, then the angle θ is determined from the condition $m\omega_p\sin\theta\approx\omega_0$. This, of course, increases the lasing threshold, which is given by formula (14) with F_1 replaced by F_m (Fig. 2). The minimum pump level that ensures self-excitation increases with increasing m to $W_0=2/T_1$. At higher pump levels, the relation between the thresholds of orders m and 1 is approximated by the equality

$$\frac{\exp(Gl)^{(m)}}{\exp(Gl)^{(1)}}\approx\left(1+\frac{2}{\eta}\right)^{(m-1)(1+2/\eta)}, \tag{16}$$

which shows that the difference between them vanishes as $\eta\to\infty$.

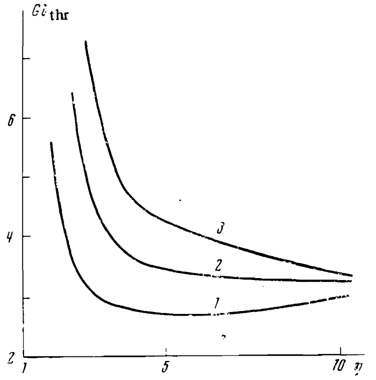

FIG. 2. Threshold Gl_{thr} of a laser without a cavity vs. the rate of the modulated pumping $\eta = (1 + W_0T_1)^{1/2}$. The feedback is produced by the m-th spatial harmonic of the inversion: 1–m = 1, 2–m = 2, 3–m = 3.

In experiments it is convenient to produce DFB by crossing two beams of unequal intensity. If one of them is weaker than the other by a factor $M^2 \ll 1$, the gain is determined, as before, by the total pump intensity

$$F_0 = (\xi - 1)/(\xi + 1), \quad \xi = W_0T_1 \tag{17}$$

whereas the coupling coefficient is proportional to M:

$$F_1 = M\xi/(\xi+1)^2. \tag{18}$$

The M needed for lasing in this case is given as a function of the parameters of the medium and of the pumping rate by

$$M_{thr} = \frac{2(\xi^2-1)}{\xi} \times \exp\left[-\frac{G(\xi-1)l}{\xi+1}\right]. \tag{19}$$

At $W_0T_1 = 4$ and $Gl = 10$, for example, we have $M_{thr} = 2 \times 10^{-2}$.

Owing to the inhomogeneity of the pump beams in the cross section, the interference pattern produced when they are superimposed is more complicated than in the plane-wave case, and the efficiency of the DFB is decreased. If we assume, in particular, that the pump intensity has a Gaussian cross-section distribution with half-width r_0, then calculation yields a lowering of the effective pump rate by a factor

$$W_{0\,eff}/W_0 \approx \exp[-(l\cos\theta/r_0)^2]. \tag{20}$$

Focusing of the pump influences the angular spectrum of the laser by altering the effective period of the modulation as a function of the lasing angle φ (lasing along the direction $z\cos\varphi$):

$$K_{eff}(\varphi, f) \approx K(\varphi, f\to\infty)(1 - \varphi Kl\cos\theta/f), \tag{21}$$

where f is the focal distance of the cylindrical lens.

3. DISTRIBUTED FEEDBACK IN STIMULATED RAMAN SCATTERING

1. Spatial Modulation of the Gain in an Inhomogenous-Pump Field. Raman Laser Without Cavity

Just as in a three-level laser, spatial modulation of the pump in stimulated scattering leads to interaction of the opposing waves of the Stokes component; under certain conditions, this interaction causes absolute instability. Let us examine these effects as applied to stimulated Raman scattering (SRS). Assuming that the pump field is produced in the active medium in the manner shown in Fig. 1, we express the field in the form

$$E_p/E_0 = \exp[i(\omega_p t - k_{pz}z)] + M\exp[i(\omega_p t + k_{pz}z)] + \text{c.c.} \tag{22}$$

Here M is a complex coefficient, $|M| \leq 1$ (the intensities of the interfering pump beams are in general not equal), and $k_{pz} = k_p \sin\theta$. We represent the field of the first Stokes component in the form

$$E_S = A_{S1}\exp[i(\omega_S t - k_S z)] + A_{S2}\exp[i(\omega_S t + k_S z)] + \text{c.c.}, \tag{23}$$

$\omega_p - \omega_S = \Omega$, where Ω is the natural frequency of the optical phonon.

Substituting (20) and (21) in the wave equation

$$\frac{\partial^2 E}{\partial t^2} - v^2\frac{\partial^2 E}{\partial z^2} = -4\pi\frac{\partial^2 P}{\partial t^2}, \tag{24}$$

where the polarization is represented in the form $P = \kappa E + \chi E^3$ (the Raman gain is connected with the imaginary component of the nonlinear susceptibility χ), and using the standard procedure for deriving the abbreviated equations, we arrive at first-order equations for the complex amplitude of the forward and backward Stokes waves (we write down for simplicity the stationary equations in a given pump field):

$$\begin{aligned} \frac{1}{\gamma}\frac{dA_{S1}}{dz} &= (1+MM^*)A_{S1} + M^*A_{S2}\exp(-2i\Delta_S z), \\ -\frac{1}{\gamma}\frac{dA_{S2}}{dz} &= (1+MM^*)A_{S2} + MA_{S1}\exp(2i\Delta_S z). \end{aligned} \tag{25}$$

Here $\Delta_S = k_{pz} = k_S$, and γ is expressed in terms of the standard Raman gain g and pump intensity I_p, namely, $\gamma = gI_p/2$.

At $M \neq 0$, the opposing Stokes waves are coupled. The lasing condition for the system (25), which determines the threshold pump power and the spectrum of the natural modes of a Raman laser with DFB, is obtained by the same method as used for (10). It takes the form

$$\begin{gathered} \Gamma\,\mathrm{cth}\,\Gamma l = \gamma(1+R) + i\Delta_S; \\ \Gamma^2 = [\gamma(1+R) + i\Delta_S]^2 - \gamma^2 R, \quad R = MM^*. \end{gathered} \tag{26}$$

An essential circumstance is that the lasing condition does not depend on the pump reflection phase shift, and contains only the intensity reflection coefficient.

Relation (26) yields a prescription for the construction of a "cavityless" Raman laser. The optimal conditions are obviously $R = 1$ and $\Delta S = 0$, from which we obtain the optimal angle $\theta_{opt} = \sin^{-1}(k_S/k_p)$ and the threshold value of the total gain $gI_p l \approx 0.76$. By way of example, let us consider a Raman laser with DFB using liquid nitrogen ($\Omega = 2326\ \text{cm}^{-1}$, $g = 2\cdot 10^{-2}$ cm/MW), pumped at a wavelength $\lambda = 0.5\ \mu$. The optimal angle is in this case 61°, and the interaction length is $l = 1$ cm if the pump beams have diameters 5 mm. The threshold pump power under these conditions is $P_{thr} = 7.6$ MW. The transfer of energy from each of the pump beams into the accompanying Stokes wave remains negligibly small (gain $gI_p l/\sin\theta \approx 0.9$). The possibility of exceeding the threshold (in high-gain media) in small volumes of the active medium makes it possible to produce DFB Raman lasers pumped by pulses with durations down to 10^{-10} sec.

2. Coherent Distributed Feedback as the Cause of Instabilities in SRS

Another aspect of our problem is connected with the instabilities that arise in stimulated Raman scattering. Attention has been called many times in reports of experiments with solids, liquids, and gases[9-12] to the fact that the intensity of the Stokes components, when measured as a function of the pump intensity, exhibits clearly pronounced jumps. Whereas for liquids, which have a large Kerr constant, the jumps are usually attributed to self-focusing of the principal beam (we note at the same time that to our knowledge, no concrete calculations based on this model have been performed), for gases and for media with small Kerr constants there is no correct quantitative explanation. In our opinion, the cause of the instabilities may be uncontrollable reflections of the pump, which lead to the onset of coherent DFB.

To estimate the pump power necessary for self-excitation as a result of DFB due to weak pump reflections, we write down the condition (26) for $R \ll 1$. We have

$$R=4\left[1+\left(\frac{\Delta_S}{gI_p}\right)^2\right]\exp(-2gI_pl),\quad \mathrm{tg}(\Delta_S l)=\frac{\Delta_S}{gI_p}. \tag{27}$$

In substances with large gain $gI_pl = 15$, $l = 25$ cm, assuming that the angle between the incident and reflected pump beams is close[1)] to 180° and that the detuning is $\Delta_S \approx 10^4$ cm^{-1}, the reflection coefficient needed for self-excitation is $R \sim 10^{-3}$. Under optimal conditions, ($\Delta_S = 0$), the threshold value of R is much smaller and can drop to $R \sim 10^{-5}$–10^{-6}.

In the scheme considered here, the reflection of the second (and higher) Stokes components from the gratings induced by the pump and by the first Stokes component are nonresonant, owing to the dispersion of the refractive index (the threshold of the generation of the second Stokes component is determined from (27) by substituting the corresponding wave detuning). We can therefore expect an appreciable transfer of the radiation into the first Stokes component.

3. Allowance for the Reaction of the Stokes Radiation on the Pump

At large Raman conversion coefficients it is also necessary to take the change in the pump intensity into account. Whereas in the analysis of the co-moving interaction this problem can easily be solved (in the one-dimensional case), the introduction of opposing waves complicates the complete solution to a considerable degree. Substitution of the pump fields and of the Stokes frequency in the form (23) into the wave equation leads to a system of five coupled nonlinear equations:

$$\begin{gathered}\mp\frac{2}{g}\frac{dA_{p1,2}}{dz}=A_{p1,2}(A_{S1}^2+A_{S2}^2)+A_{p2,1}A_{S1}A_{S2}\cos\Phi,\\ \pm\frac{2}{g}\frac{dA_{S1,2}}{dz}=A_{S1,2}(A_{p1}^2+A_{p2}^2)+A_{S2,1}A_{p1}A_{p2}\cos\Phi,\\ \frac{d\Phi}{dz}=2\Delta_S-\mathrm{tg}\,\Phi\frac{d}{dz}\ln(A_{p1}A_{p2}A_{S1}A_{S2}).\end{gathered} \tag{28}$$

Here $A_{p1,2}$ and $A_{S1,2}$ are the real amplitudes of the forward and backward waves of the fundamental and Stokes frequencies, $\Phi = 2\Delta_S z - \varphi_1 + \varphi_2 + \psi_1 - \psi_2$, where $\varphi_{1,2}$ and $\varphi_{1,2}$ are the phase shifts of the pump and Stokes-component waves.

The system (28) satisfies the Manley-Rowe relation

$$A_{p1}^2-A_{p2}^2+A_{S1}^2-A_{S2}^2=C_1 \tag{29}$$

and admits of a solution in the form

$$A_{p1}^2=A_{S2}^2=X_1,\quad A_{p2}^2=A_{S1}^2=X_2,$$

corresponding, for specified $A_{p1}(0)$, $A_{p2}(l)$, $A_{S1}(0)$, and $A_{S2}(l)$ on the boundaries to total energy exchange between the pump and the Stokes component. The system (28) now simplifies to

$$\mp\frac{1}{g}\frac{dX_{1,2}}{dz}=X_{1,2}+X_1X_2(1+\cos\Phi). \tag{30}$$

In the optimal scheme ($\Delta_S = 0$), it has two additional integrals

$$X_1X_2\sin\Phi=C_2,\quad X_1X_2(X_1+X_2)(1+\cos\Phi)=C_3, \tag{31}$$

and its solution can be obtained in terms of hyperelliptic functions. The second integral in (31) shows that the total energy-transfer regime is realized, in particular, under the condition

$$\left[\frac{A_{p1}(0)}{A_{p2}(l)}\right]^2=\frac{A_{S2}(l)}{A_{S1}(0)}$$

(the intensity of the Stokes "primers" is assumed to be small). The characteristic spatial scale is inversely proportional here to the quantity $g(I_{in.p}^2 I_{in.S})^{1/3}$, and the transformation has a monotonic character, as shown by a phase-plane analysis of (30).

4. COHERENT AND INCOHERENT (STOCHASTIC) DFB. ABSOLUTE INSTABILITIES IN SYSTEMS WITH LARGE GAIN

Stimulated Raman scattering in media with a large scattering cross sections is not the only example of a situation wherein a rather small pump inhomogeneity suffices to produce self-excitation via DFB. In dye lasers, self-excitation has been realized in experiments at $\Delta G/G \sim 10^{-6}$ [7]. The gains Gl obtained by Bjorkholm and Shank[7] are still far from the limit; therefore, in accordance with (19), self excitation is also possible at much lower values of ΔG.

It should be noted, however, that at high gains an important role is assumed by feedback mechanisms not accounted for in (2). If we neglect effects on the boundary of the amplifying medium[2)], the most significant among these mechanisms are density fluctuations that lead to stochastic spatial modulation of the gain and of the refractive index. This results in positive DFB, a distinguishing feature of which is its incoherent character. The phase shifts of waves reflected even from relatively closely located sections of the random grating are not correlated. Consequently, there is actually no difference here between the spatial modulation of the refractive index and the gain (their effects are additive).

We represent the dielectric constant of the medium in the form

$$\varepsilon=\bar{\varepsilon}[1+\tilde{\beta}(z)],\quad \tilde{\beta}(z)=\tilde{\beta}_1(z)+i\tilde{\beta}_2(z). \tag{32}$$

We write for the real random functions $\tilde{\beta}_{1,2}$

$$\tilde{\beta}_{1,2}(z)=\beta_{1,2}(z)e^{2ikz}+\text{c.c.} \tag{33}$$

We now obtain for the amplitudes (9) of the opposing waves in the amplifying medium

$$\begin{gathered}\frac{dA_1}{dz}=\frac{G}{2}A_1+\frac{k}{2}(\beta_2^*-i\beta_1^*)A_2,\\ -\frac{dA_2}{dz}=\frac{G}{2}A_2+\frac{k}{2}(\beta_2-i\beta_1)A_1.\end{gathered} \tag{34}$$

The equations for the complex amplitudes of the coupled waves now turn out to be equations with random coefficients. If the gain (or the damping) over a length on the order of the correlation radius z_c is small, $Gz_c \ll 1$, we can assume that total mixing of the phases takes place.

Assuming the functions $\beta_{1,2}$ to be δ-correlated:

$$\overline{\beta_{1,2}(z)\beta_{1,2}(z')}=\frac{B_{1,2}}{k}\delta(z-z') \qquad (35)$$

and using the method described earlier[14,15], we arrive at equations for the average intensities[3)]:

$$\pm\frac{d}{dz}\bar{I}_{1,2}=\bar{G}\bar{I}_{1,2}+\bar{B}\bar{I}_{2,1}; \qquad (36)$$
$$\bar{G}=G+(B_2-B_1)/4, \quad \bar{B}=(B_2+B_1)/4.$$

In an amplifying medium ($G > 0$ - laser amplifier, stimulated scattering) the stochastic coupling can lead to self-excitation if the condition $\Gamma \coth \Gamma l = \bar{G}$, with a growth increment $\Gamma^2 = \bar{G}^2 - \bar{B}^2$, is satisfied. If $G \gg B_{1,2}$ (weak coupling), the latter condition takes the approximate form

$$B/G=e^{-Gl}, \qquad (37)$$

where $B = \bar{B}/2$ has the meaning of the coefficient of one-dimensional back scattering (see[16]). It follows from (37) that in the presence of stochastic DFB, self-excitations are produced by fluctuations of both the imaginary and the real part of the refractive index, whereas in systems with regularly varying parameters the active and reactive types of DFB differ in character: the pure reactive regular DFB cannot lead to self-excitation. Figure 3 shows the dependence of the threshold value of Bl on the total gain of the system. At $Gl = 10$, for example (such gains are realistically attainable in dye lasers and in Raman-active media over lengths on the order of 10 cm), the threshold value of the coefficient B, which is proportional to the dispersion of the refractive index, is of the order of $10^{-4} - 10^{-5}$, whence $(\overline{\Delta n^2})^{1/2}/\bar{n} \sim 10^{-9} - 10^{-10}$. Much larger modulation coefficients have already been obtained by using coherent pumping[7]. It therefore appears that the use of noise pumping is also feasible in DFB systems.

The system (36) also describes the instability of SRS with account taken of the reflections from the random grating produced by the thermodynamic density fluctuations (backward Rayleigh scattering). The threshold condition in the form (37) for backward Rayleigh scattering as the feedback mechanism was derived by Elyutin[17] by another method[4)]. In this case $B = R\Omega_0$, where R is the Rayleigh-scattering coefficient and Ω_0 is the lasing solid angle, determined by the geometry of the system; in most liquids and gases we have $B \sim 10^{-5} - 10^{-7}$.

5. DFB SELF-INDUCED BY THE SATURATION EFFECT IN A THREE-LEVEL LASER

Spatial inhomogeneity of inverted population in an active medium can also result from the saturation ef-

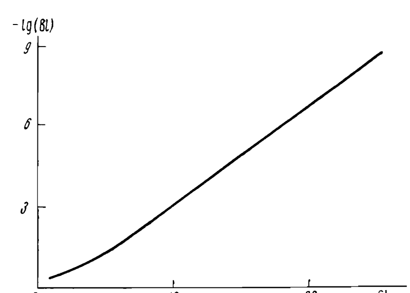

FIG. 3. Threshold characteristics of stochastic DFB in the form of a plot of Gl against the logarithm of Bl, where B is the back-scattering coefficient and l is the length of the active medium.

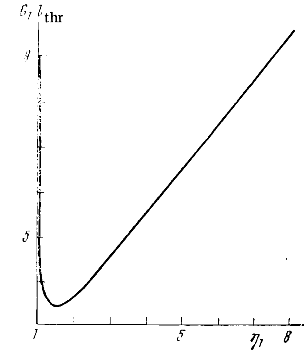

FIG. 4. Dependence of the self-excitation threshold $G_1 l_{thr}$ in self-modulation of the inverted population on the intensity of the "priming" standing wave; $\eta_1 = [1 + \sigma E_0^2/(W + T_1^{-1})]^{1/2}$.

fect[19]. The appearance of this inhomogeneity lowers the generated power[8], but can also play a favorable role by producing DFB. The inversion that determines the signal gain, with allowance for the "burnout" under the influence of the saturating standing wave E_p (in the pulse regime at not too high an off-duty factor, the saturation can be provided by the preceding pulse) is distributed in space in accordance with the law

$$\frac{N}{N_0}=\frac{W-T_1^{-1}}{W+T_1^{-1}+\sigma|E_p|^2}. \qquad (38)$$

If the group-delay effect and other transient times (5) are neglected, the changes occurring in the slow amplitudes of the generated signal are described by equations that are fully analogous to (10), and the self-excitation conditions can be satisfied here. This case, however, unlike the one described in Sec. 2, is characterized by a hard excitation regime: the absolute instability arises in the presence of a "priming" standing wave at a frequency close to the frequency of the working transition (see[3]).

The dependence of the threshold value of the gain

$$G_1 l=Gl\frac{W-T_1^{-1}}{W+T_1^{-1}} \qquad (39)$$

on the saturation parameter

$$\eta_1=[1+\sigma E_0^2/(W+T_1^{-1})]^{1/2}$$

at zero deviation from the transition frequency is characterized by upper and lower bounds on the saturation at a given gain (Fig. 4). There exists an optimal "burnout," $(\sigma E_0^2)_{opt} \approx 1.3(W + T_1^{-1})$, at which the lowest threshold $G_1 l_{thr} = 3.5$ is reached. The rapid growth of $G_1 l_{thr}$ with increasing degree of saturation is due, firstly, to the decrease of the effective gain as the result of the burnout of the inversion and, secondly, to the fact that, in analogy with (6), the inversion distribution tends to become homogeneous at large η_1 and weakens the feedback. When the population is modulated in accordance with the "preceding"-pulse scheme, the resulting DFB is of the self-resonant type, as in[3], since the Bragg frequency is in this case the generation frequency.

The requirements imposed on the intensity of the saturating wave become less stringent with increasing crystal length (Fig. 4). The approximate threshold condition, which determines the power that the transition-saturating standing wave must have for the instability to set in, takes the following form in the case of small values of the saturation parameter $\sigma E_0^2/W \ll 1$:

$$E_0^2=3(W+T_1^{-1})\sigma^{-1}e^{-Gl}. \qquad (40)$$

In media used in practice (ruby, dyes), the saturating standing-wave power necessary to ensure satisfaction of (40) is a fraction of a microwatt.

Particular interest attaches to the question of the role played in laser self-excitation by incoherent DFB due to saturation in the field of superradiance of a standing wave. Estimates performed with the aid of formulas of Sec. 4 show that this effect can be appreciable.

6. CONCLUSION

Distributed feedback due to inhomogeneity of the pump is a promising method of narrowing down the spectral line and increasing the output power of laser systems. In systems with large gain (such as dye lasers, stimulated-scattering lasers) the pump inhomogeneities needed for self-excitation turn out to be quite small. In dye lasers and SRS amplifiers, the relative gain modulation needed for self-excitation is $\Delta G/G \sim 10^{-5} - 10^{-6}$; DFB due to pumping inhomogeneity can therefore be one of the principal causes of instabilities (the latter also include, for example, instabilities of the spectrum) in the lasers. An important question is the competition between the distributed and lumped feedbacks; further research is necessary here.

An interesting question is that of the conditions under which DFB is produced in lasers with inhomogeneous two-photon pumping. It is the result of harmonics of the populations (see (8)); its use for two-photon pumping of noble gases is of interest.

The authors are sincerely grateful to Yu. E. D'yakov for a useful discussion and remarks, particularly in connection with the material in Sec. 4.

1) The threshold is altered very little by small changes of the angle near 180°. At the same time, a characteristic structure, which cannot be attributed to other mechanisms, should be observed in the angular spectrum of the first Stokes component.

2) The elimination of coherent lumped feedback is a technical problem which is not considered here. However, stochastic coupling due to scattering by the boundaries can also be appreciable [13]

3) These equations for an amplifying medium were introduced by D'yakov.

4) The effect of Rayleigh scattering on SRS was discussed also by Sorokin [18].

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Translated by J. G. Adashko
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