Dissipation of the energy of a dislocation moving in a random field of internal stresses

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The energy dissipated when a dislocation moves in a random field of internal stresses is determined. Expressions for the average energy dissipated under various laws governing the loading of the crystal are obtained on the basis of a statistical analysis of the relief of the internal stresses along the dislocation-motion direction.

Lattice defects, which are contained in all real crystalline bodies, produce internal-stress fields in these bodies[1]. A characteristic feature of crystal structure is the random disposition of the defects in their volume. The statistical properties of the internal-stress field $\sigma(\mathbf{x})$ produced in the crystal by the randomly disposed defects have been considered in[2-4]. The presence of random internal-stress fields has a strong influence on dislocation motion in crystals and on the evolution of crystal structure under mechanical, thermal, or other action. Allowance for the randomness of the dislocation disposition reveals qualitatively new regularities[5], which cannot in principle be derived from any predetermined scheme. An important characteristic of the defect structure is the energy dissipated when the dislocations move through the crystal. Estimation of the energy dissipation is important in the study of dislocation mobility[6], strain hardening[7], and other problems in crystal physics.

In this paper we determine the mean values of the energy released by motion of a linear dislocation, for the case when the field $\hat{\sigma}(\mathbf{x})$ does not vary along the dislocation. This case is realized, for example, in a crystal containing an ensemble of linear parallel dislocations. This model of the dislocation structure makes it possible to describe the characteristic features of the structure of thoroughly annealed crystals. The methods developed in the paper can be used to analyze the motion of charged particles, vortex lines, domain walls, etc. in random force fields.

1. DETERMINATION OF ENERGY DISSIPATION UNDER MONOTONIC LOADING OF THE CRYSTAL

We consider a linear dislocation with a Burgers vector **b**; the dislocation is oriented along the z axis and glides in the y = 0 plane along the x direction under the influence of a stress

$$S(x, t)=\sigma(x)+\tau(t), \tag{1}$$

where $\sigma(x)$ and $\tau(t)$ are the components (in the y = 0 plane in the **b** direction) of the internal and external tangential stresses, respectively, and t is a parameter of the external loading (e.g., the time). The equilibrium positions of the dislocation are determined from the conditions

$$S(x, t)=0, \quad \partial S(x, t)/\partial x>0. \tag{2}$$

We confine ourselves to the case where the dissipative forces acting on the dislocation are large in comparison with the inertial forces, i.e., the inertial forces do not influence the equilibrium position of the dislocation.

The total energy dissipated as the dislocation moves in the course of the loading, in the interval $[t_1, t_2]$, is determined by the functional

$$W[\tau(t)]=b\int\limits_{x(t_1)}^{x(t_2)} S[x(t), t]dx, \tag{3}$$

where $x(t)$ is the law of dislocation motion, $x(t_1)$ is the initial position of the dislocation, and $x(t_2)$ is the final position of the dislocation, defined as the point closest to $x(t_1)$, at which the conditions (2) are satisfied. The mechanisms whereby the energy is drawn away from the moving dislocation[6] are not specified concretely.

We distinguish between two simplest loading methods: infinitely rapid (pulsed)

$$\tau(t)=\begin{cases}\tau_1 & t\leqslant t_1\\ \tau_2 & t_1<t\leqslant t_2\end{cases} \tag{4}$$

and infinitely slow

$$\tau(t)=-\sigma[x(t)], \quad t_1\leqslant t\leqslant t_2, \quad \tau_1=\tau(t_1), \quad \tau_2=\tau(t_2). \tag{5}$$

The energy dissipated after a dislocation traverses a unit length under pulsed loading (Fig. 1a) is determined from (3) and (4):

$$W_P(\tau_1, \tau_2)=b\tau_2 l(\tau_1, \tau_2)+b\int\limits_{x_1}^{x_2}\sigma(x)dx, \tag{6}$$

where $x_1 = x(\tau_1)$ and $x_2 = x(\tau_2)$ are respectively the initial and final positions of the dislocation; $l(\tau_1, \tau) = x(\tau) - x(\tau_1)$ is the dislocation mean free path when the external stress is monotonically increased from τ_1 to τ. The energy dissipated in the case of infinitely slow loading (5) (Fig. 1b) is

$$W_S(\tau_1, \tau_2)=W_P(\tau_1, \tau_2)-b\int\limits_{\tau_1}^{\tau_2} l(\tau_1, \tau)d\tau. \tag{7}$$

The loading can occur also at a rate intermediate between (4) and (5), and then the plot of the dislocation loading $\tau[t(x)]$ Fig. 1 would be a line intermediate between lines 1 and 3. Since the field $\sigma(x)$ is random, the quantities $x(\tau)$ and $l(\tau_1, \tau)$ are also random. If the dislocation moves in the course of infinitely slow loading in a region where $\sigma(x)$ decreases ($-\sigma(x)$ increases) monotonically, then the dissipation is equal to zero. In the case of a nonmonotonic relief of $\sigma(x)$, the region of values of X in which (5) is satisfied becomes multiply-connected, and then $x(\tau)$ and $l(\tau_1, \tau)$ are multiple-valued functions of τ and the dissipation is not equal to zero.

We confine ourselves below to the case of stationary random fields $\boldsymbol{\sigma}(\mathbf{x})$ with mean value $\langle\sigma(\mathbf{x})\rangle = 0$. We denote by $N^+(\tau, L)$ the number of times that the relief $-\sigma(x)$ reaches the level τ on the length L. The average number

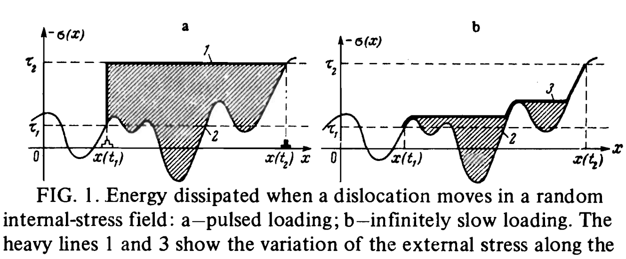

FIG. 1. Energy dissipated when a dislocation moves in a random internal-stress field: a—pulsed loading; b—infinitely slow loading. The heavy lines 1 and 3 show the variation of the external stress along the path of dislocation motion; 2—relief of $-\sigma(x)$. The shaded area is equal to the released energy W divided by the Burgers vector b.

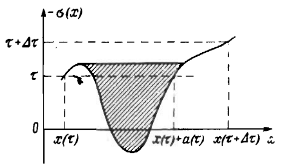

FIG. 2. Change of dislocation mean free path in the case of infinitely slow increases of the external stress from τ to $\tau + \Delta\tau$. The shaded area is equal to the energy released under such loading, divided by the Burgers vector b.

of such events per unit length[8] is

$$\mu^+(\tau) = \lim_{L\to+\infty} \frac{N^+(\tau, L)}{L}. \tag{8}$$

On the set of all the events when the level τ is reached, we define the random quantity $I(\tau, \Delta\tau)$ (the indicator of the maxima of the $-\sigma(x)$ relief), which is equal to unity if $-\sigma(x)$ has a maximum in the strip τ, $\tau + \Delta\tau$ following attainment of the level τ, and to zero if there is no such maximum. For the loading (5), the change in the mean free path $l(\tau_1, \tau)$ following an increase of the external stress from τ to $\tau + \Delta\tau$ is

$$l(\tau_1, \tau+\Delta\tau) - l(\tau_1, \tau) = -\left\{\frac{\partial\sigma[x(\tau)]}{\partial x}\right\}^{-1} \Delta\tau + a(\tau) I(\tau, \Delta\tau), \tag{9}$$

where $a(\tau)$ is the distance from the point where $-\sigma(x)$ reaches the level τ (and which is followed by a maximum in the strip τ, $\tau + \Delta\tau$) to the next point where τ is reached (Fig. 2).

The first term in (9) determines the mean free path of the dislocation motion on the sections of X where $\sigma(x)$ decreases monotonically[9]. For such a motion, the potential energy of the crystal increases. The second term in (9) is determined mainly by the jumplike motion of the dislocations, during the course of which the energy released is

$$W_s(\tau_1, \tau+\Delta\tau) - W_s(\tau_1, \tau) = bQ(\tau) I(\tau, \Delta\tau), \tag{10}$$

where $Q(\tau)$ is the area of the dip of the $-\sigma(x)$ relief under the level τ (Fig. 2), following the maximum of $-\sigma(x)$.

The integration of (9) determines the mean free path of the dislocation when the load changes from τ_1 to τ:

$$l(\tau_1, \tau) = \int_{\tau_1}^{\tau} \left\{-\frac{\partial\sigma[x(u)]}{\partial x}\right\}^{-1} du + \int_{\tau_1}^{\tau} a(u) I(u, du). \tag{11}$$

The first integral in (11) is determined on segments with random ends in the region $[x(\tau_1), x(\tau)]$, where $x(u)$ is single-valued; the second integral in (11) is stochastic[10]. Integration of (10) in the interval $[\tau_1, \tau]$ determines the energy dissipation in the course of infinitely slow loading from τ_1 to τ, of a crystal with a dislocation:

$$W_s(\tau_1, \tau) = b \int_{\tau_1}^{\tau} Q(u) I(u, du). \tag{12}$$

From (12), (7), and (11) we can also determine the energy dissipated in the case of pulsed loading.

2. DETERMINATION OF MEAN VALUES OF DISSIPATED ENERGY

To determine the energy dissipated when a dislocation moves in a crystal it is necesary to find the average values of (12), (7), and (11):

$$\langle W_s(\tau_1, \tau)\rangle = b \int_{\tau_1}^{\tau} \langle Q(u)\rangle \langle I(u, du)\rangle, \tag{13}$$

$$\langle W_p(\tau_1, \tau_2)\rangle = \langle W_s(\tau_1, \tau_2)\rangle + b \int_{\tau_1}^{\tau_2} \langle l(\tau_1, \tau)\rangle\, d\tau, \tag{14}$$

$$\langle l(\tau_1, \tau)\rangle = \int_{\tau_1}^{\tau} \left\langle \left\{-\frac{\partial\sigma[x(u)]}{\partial x}\right\}^{-1} \right\rangle du + \int_{\tau_1}^{\tau} \langle a(u)\rangle \langle I(u, du)\rangle, \tag{15}$$

where $\langle I(\tau, \Delta\tau)\rangle$ is the probability of detachment of the dislocation from the restraining peak of the internal stresses when the external stress is increased from τ to $\tau + \Delta\tau$ (the fraction of the number of times that the relief $-\sigma(x)$ reaches the level τ followed by a maximum of $-\sigma(x)$ in the strip τ, $\tau + \Delta\tau$).

To calculate $\langle I(\tau, \Delta\tau)\rangle$ we introduce the quantities $N_1(\tau, \Delta\tau, L)$ and $N_2(\tau, \Delta\tau, L)$—the numbers of maximum and minimum points, respectively, of the relief $-\sigma(x)$ on the length L. The quantity $\langle I(\tau, \Delta\tau)\rangle$ is defined as follows:

$$\langle I(\tau, \Delta\tau)\rangle = \lim_{L\to+\infty} \frac{N_1(\tau, \Delta\tau, L)}{N^+(\tau, L)}. \tag{16}$$

The following relation holds:

$$N^+(\tau+\Delta\tau, L) - N^+(\tau, L) = N_2(\tau, \Delta\tau, L) - N_1(\tau, \Delta\tau, L). \tag{17}$$

We divide (17) by L and, taking the limit as $L \to +\infty$, we obtain with allowance for (8)

$$\Delta\mu^+(\tau) = \mu_2(\tau, \Delta\tau) - \mu_1(\tau, \Delta\tau), \tag{18}$$

where

$$\mu_1(\tau, \Delta\tau) = \lim_{L\to+\infty} \frac{N_1(\tau, \Delta\tau, L)}{L}, \quad \mu_2(\tau, \Delta\tau) = \lim_{L\to+\infty} \frac{N_2(\tau, \Delta\tau, L)}{L}$$

are respectively the average numbers of the maximum and minimum points of the relief $-\sigma(x)$ in the interval $[\tau, \tau + \Delta\tau]$ per unit length. The probability distribution density of the values of the relief at the extremal points is

$$\varphi(\tau) = f(\tau, 0)/f_1(0), \tag{19}$$

where $f(y, z)$ is the joint distribution density of the quantities $\sigma(x)$ and $\partial\sigma(x)/\partial x$ at an arbitrary point x; $f_1(z)$ is the distribution density of the random quantity $\partial\sigma(x)/\partial x$. The average number, per unit length, of the extremal points that fall in the band $[\tau, \tau + \Delta\tau]$ is

$$\mu_1(\tau, \Delta\tau) + \mu_2(\tau, \Delta\tau) = \varphi(\tau)\Delta\tau/\lambda(0), \tag{20}$$

where $\lambda(0)$ is the average distance between the neighboring extremal points of the relief $-\sigma(x)$.

From (18) and (20) we obtain

$$\mu_1(\tau, \Delta\tau) = \frac{\varphi(\tau)\Delta\tau}{2\lambda(0)} - \frac{\Delta\mu^+(\tau)}{2}, \tag{21}$$

$$\mu_2(\tau, \Delta\tau) = \frac{\varphi(\tau)\Delta\tau}{2\lambda(0)} + \frac{\Delta\mu^+(\tau)}{2}. \tag{22}$$

Relation (16), with (8) taken into account, assumes the form

$$\langle I(\tau, \Delta\tau)\rangle = \mu_1(\tau, \Delta\tau)/\mu^+(\tau). \tag{23}$$

From (23) and (21) it follows that

$$\langle I(\tau, \Delta\tau)\rangle = \varphi(\tau)\Delta\tau/2\lambda(0)\mu^+(\tau) - {}^1/_2\Delta \ln \mu^+(\tau). \tag{24}$$

To find the mean values (13)—(15) it is necessary to determine the mean values $\langle a(\tau)\rangle$ and $\langle Q(\tau)\rangle$. For sufficiently large dislocation free paths ($a(\tau) > 1/2\mu^+(0)$), the event $-\sigma[x + a(\tau)] > \tau$ (stopping of the dislocation) is practically independent of the value of the derivative at the point x, so that the following relations hold:

$$\langle a(\tau)\rangle = \langle \xi(-\tau)\rangle, \quad \langle Q(\tau)\rangle = \langle R(-\tau)\rangle, \tag{25}$$

where $\xi(\tau)$ and $R(\tau)$ are respectively the width and area of the spike of the relief $-\sigma(x)$ over the level τ, following an arbitrary attainment of this level by the relief $-\sigma(x)$ (Fig. 3). The quantity $\langle \xi(\tau)\rangle$ was defined earlier[4].

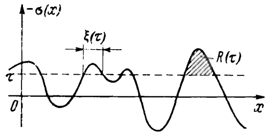

FIG. 3. Spikes of the relief $-\sigma(x)$ over the level τ. $\xi(\tau)$ is the width of the spike of the relief over this level. The area $R(\tau)$ of the spike is shown shaded.

To calculate $\langle R(\tau)\rangle$ we introduce the random quantities

$$\sum_L \xi(\tau), \quad \sum_L R(\tau)$$

representing respectively the total width and the total area of the spikes of the relief $-\sigma(x)$ over the level τ on the length L. According to [4] we have

$$\langle \xi(\tau)\rangle = \lim_{L\to\infty} \frac{1}{N^+(\tau, L)} \sum_L \xi(\tau) = \frac{1}{\mu^+(\tau)}\left[1-\int_{-\infty}^{\tau} f(y)\,dy\right], \tag{26}$$

where $f(y)$ is the distribution density of the random quantity $-\sigma(x)$ at the arbitrary point x. Since the relief $-\sigma(x)$ is ergodic on sections of x where $-\sigma(x) > \tau$, the average height $h(\tau)$ of the spike of the relief over the level τ satisfies the relation

$$\lim_{L\to+\infty}\left(\sum_L R(\tau)\Big/\sum_L \xi(\tau)\right) = \left[1-\int_{-\infty}^{\tau} f(y)\,dy\right]^{-1}\int_{\tau}^{+\infty}(y-\tau)f(y)\,dy = h(\tau). \tag{27}$$

The mean value of $R(\tau)$ can be defined in analogy with (26):

$$\langle R(\tau)\rangle = \lim_{L\to+\infty}\frac{1}{N^+(\tau, L)}\sum_L R(\tau). \tag{28}$$

Transforming (28) and taking (26) and (27) into account, we obtain

$$\langle R(\tau)\rangle = h(\tau)\langle\xi(\tau)\rangle = \frac{1}{\mu^+(\tau)}\int_{\tau}^{+\infty}(y-\tau)f(y)\,dy. \tag{29}$$

The quantity $b\langle R(\tau)\rangle$ determines the average value of the energy barrier that the dislocation must overcome as it glides under the influence of an external stress τ. Substituting (24) and (25) in (13) and carrying out transformations with allowance for (26) and (29), we obtain

$$\langle W_s(\tau_1, \tau)\rangle = b\int_{\tau_1}^{\tau}\frac{\varphi(u)\,du}{2\lambda(0)[\mu^+(u)]^2}\int_{-u}^{+\infty}(y+u)f(y)\,dy + \frac{b}{2}\int_{\tau_1}^{\tau} d\left[\frac{1}{\mu^+(u)}\right]\int_{-u}^{+\infty}(y+u)f(y)\,dy. \tag{30}$$

To estimate the energy dissipation in pulsed loading, we determine the dislocation mean free path (15). The first term in (15) was calculated in [9], the average slope of the relief $-\sigma(x)$ being

$$\langle\{-\partial\sigma[x(\tau)]/\partial x\}^{-1}\rangle = f(\tau)/2\mu^+(\tau). \tag{31}$$

Substituting (24) in (15) and performing transformations with allowance for (26) and (31) we obtain

$$\langle l(\tau_1, \tau)\rangle = \int_{\tau_1}^{\tau}\frac{\varphi(u)\,du}{2\lambda(0)[\mu^+(u)]^2}\int_{-u}^{+\infty} f(y)\,dy + \frac{1}{2}\left[\frac{1}{\mu^+(\tau)}\int_{-\infty}^{\tau} f(y)\,dy - \frac{1}{\mu^+(\tau_1)}\int_{-\infty}^{\tau_1} f(y)\,dy\right]. \tag{32}$$

The average energy dissipation in pulsed loading is determined by substituting (30) and (32) in (14). In the particular case $\tau_1 = 0$ we get for

$$\tau_2 \gg \left[\int_{-\tau_2}^{+\infty} f(y)\,dy\right]^{-1}\int_{-\tau_2}^{+\infty} yf(y)\,dy \tag{33}$$

from (14), (30), and (32) the obvious relation

$$\langle W_s(0, \tau_2)\rangle \cong b\langle l(0, \tau_2)\rangle\tau_2. \tag{34}$$

If $f(y)$ is a Gaussian distribution with variance $D^2\sigma$, the inequality (33) is satisfied already at $\tau_2 \geq D\sigma$. We note that the presence of the random relief of the internal stresses limits the dislocation mean free path at a given external stress.

For small external stresses ($\tau_2 \ll D\sigma$) we obtain from (30), (14) and (32) in first-order approximation

$$\langle W_s(0, \tau_2)\rangle \cong \langle W_p(0, \tau_2)\rangle \cong \frac{b\varphi(0)\tau_2}{2\lambda(0)[\mu^+(0)]^2}\int_0^{+\infty} yf(y)\,dy. \tag{35}$$

The mean free path is in this case

$$\langle l(0, \tau_2)\rangle \cong \varphi(0)\tau_2/4\lambda(0)[\mu^+(0)]^2. \tag{36}$$

Comparing (35) and (36), we can write

$$\langle W_s(0, \tau_2)\rangle \cong \langle W_p(0, \tau_2)\rangle \cong b\sigma_f\langle l(0, \tau_2)\rangle, \tag{37}$$

where

$$\sigma_f = 2\int_0^{+\infty} yf(y)\,dy \tag{38}$$

has the meaning of the effective friction stress generated by the random relief of the internal stresses. For a Gaussian relief $\sigma(x)$ we have $\sigma_f = (2/\pi)^{1/2}D\sigma$.

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