Control of instability development in multibeam systems

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The development of instability in multibeam systems is investigated theoretically and experimentally. It is shown that during the linear stage the mutual influence of the electron beams is relatively weak and leads to certain changes in the increments of the unstable modes. The nonlinear stage, during which resonant particles are trapped and the amplitude growth is limited, is quite sensitive to changes of the initial velocities and of the beam density. This makes it possible to control the instability development and to vary the intensity of the excited oscillations in a wide range while leaving the average beam parameters and the total current practically unchanged.

1. INTRODUCTION

By now, as a result of extensive experimental and theoretical research, and also as a result of numerical computer calculations, many details of beam-plasma interactions have become known[1)]. During the initial stage, the beam instability develops in accordance with the linear theory, namely, an exponential growth of the unstable-wave amplitude takes place, accompanied by bunching of the beam particles. Subsequently, when the wave amplitude becomes large enough,

$$E \geqslant \frac{km}{e}(v_b - v_{ph})^2 \approx \frac{km}{e} v_b^2 \left(\frac{n_b}{n_0}\right)^{2/3} \qquad (1)$$

(v_b is the beam velocity, v_{ph} is the wave phase velocity, n_b is the beam density, n_0 is the plasma density, and k is the wave number), the nonlinear stage of the beam instability sets in: the beam is captured by the wave, is bunched, and the bunches move in the potential wells of the wave field, exchanging energy with the wave and becoming "smeared out" slowly in phase space. The exponential growth of the wave amplitude gives way to a slower growth, and then to oscillations, so that the maximum value of the amplitude does not exceed a certain value whose order of magnitude is given by (1).

The characteristic time scale of the processes connected with the capture is comparable with the characteristic time of development of the linear stage, $\tau = 1/\gamma$ (γ is the linear increment of the beam instability), since the oscillation period of the captured particles is of the order of

$$T \sim \left(\frac{e}{m}kE\right)^{-1/2} \sim \left(\frac{n_b}{n_0}\right)^{-1/3} \omega_p^{-1} \sim \tau.$$

Thus, capture of resonant particles is one of the strongest nonlinear mechanisms influencing the development of beam instability. This circumstance suggests that by varying the initial distribution function of the electron flow it is possible to vary the development of the beam instability in a wide range, i.e., to control this process.

The simplest system that makes it possible to realize control of beam instability is a beam-plasma system with several electron beams, the velocity of which can be varied in a wide range. In this paper we report the results of a study of the possibility of controlling the instability in a multibeam system.

2. THEORETICAL ANALYSIS

We consider a beam-plasma system with several electron beams with initial velocities v_i and densities n_i, placed in a strong magnetic field ($\omega_H \gg \omega_p$, with ω_H and ω_p the electron cyclotron and plasma frequencies), the direction of which coincides with the direction of the initial particle velocities. We assume that the beams are weak, $n_i \ll n_0$. In the linear approximation, this system is described by the well-known dispersion relation

$$\sum_{i=1}^{N} \frac{\omega_i^2}{(\omega - k_\parallel v_i)^2} + \frac{\omega_p^2}{\omega^2} = 1 + \frac{k_\perp^2}{k_\parallel^2}, \qquad (2)$$

where $k_\perp$ and $k_\parallel$ are the transverse and longitudinal components of the wave vector,

$$\omega_i^2 = 4\pi n_i e^2/m, \quad \omega_p^2 = 4\pi n_0 e^2/m.$$

It follows from the above dispersion relation that the number of unstable oscillation modes is equal to the number of beams whose velocities do not exceed the critical values $v_{cr} = \omega_p/k_\perp$. We assume that the condition $v_i < v_{cr}$ is satisfied by the initial velocities of all N beams, so that there are N unstable oscillation modes. It follows from (2) that if the velocities of these beams v_i and v_k are spaced sufficiently far apart,

$$|v_i - v_k| \gg \eta_i v_i + \eta_k v_k, \quad \eta_i = (n_i/n_0)^{1/3},$$

then an intense excitation of waves by each of the beams takes place, i.e., each beam corresponds to an unstable mode whose dispersion characteristics differ little from the characteristics of a single-beam system.

In the opposite case

$$|v_i - v_k| \ll \eta_i v_i + \eta_k v_k$$

we get, as it were, a merging of two beams into one with velocity $\overline{v} = (\eta_i v_i + \eta_k v_k)(\eta_i + \eta_k)^{-1}$. In this case there exists an unstable mode with a large increment $\gamma \sim [(n_i + n_k)/n_0]^{1/3}\omega_p$, and a small-increment mode that vanishes in the limit as $v_i \to v_k$.

To ascertain the character of the mutual influence of the beams at

$$|v_i - v_k| \sim \eta_i v_i + \eta_k v_k,$$

we examine the dispersion equation (2), assuming that the beam velocities v_i and v_k are close to each other. In the case of the system with a single beam of velocity v_i, the maximum of the instability increment is reached at $k_\parallel \approx \omega_p'/v_i$, where $\omega_p' = \omega_p(1 + k_\perp^2/k_\parallel^2)^{-1/2}$. The corresponding frequency is $\omega_i^{(0)} = 2^{-4/3} e^{2\pi i/3} \eta_i \omega_p'$. Injection of two beams leads to deformation of this mode by the dispersion curve, and it turns out that

$$\omega(k_\parallel = \omega_p'/v_i) \approx \omega_i^{(0)} + \sum_k \frac{2^{1/3} n_k}{6 n_i} \frac{\omega_p' \eta_i}{[e^{2\pi i/3} + 2^{1/3}(v_k - v_i)/\eta_i v_i]^2}. \qquad (3)$$

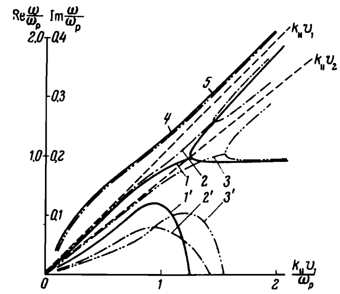

FIG. 1. Dispersion curves for space-charge waves in a plasma with two electron beams at an initial frequency difference $v_1 - v_2 = 0.2v_1$: 1–$\mathrm{Re}(\omega/\omega_p)$ at $v_1 = v_2$; 1′–$\mathrm{Im}(\omega/\omega_p)$ at $v_1 = v_2$; 2–$\mathrm{Re}(\omega_1/\omega_p)$ at $v_1 - v_2 = 0.2v_1$; 2′–$\mathrm{Im}(\omega_1/\omega_p)$ at $v_1 - v_2 = 0.2v_1$; 3–$\mathrm{Re}(\omega_2/\omega_p)$ at $v_1 - v_2 = 0.2v_1$; 3′–$\mathrm{Im}(\omega_2/\omega_p)$ at $v_1 - v_2 = 0.2v_1$; 4–constant-intensity wave at $v_1 = v_2$; 5–same, at $v_1 - v_2 = 0.2v_1$.

This formula is valid when the second term is small in comparison with the first. It follows from the foregoing relation that the correction to the increment is proportional to the beam-density ratio n_k/n_i and depends strongly on v_k/v_i. In particular, at $v_1 - v_2 = 2^{-1/3}\eta_1 v_1$ we have for a system of two beams

$$\mathrm{Im}\,\omega_1 = \mathrm{Im}\,\omega_1^{(0)}\,(1 - n_2/6n_1),$$

and at $v_1 - v_2 = -2^{-1/3}\eta_1 v_1$, we have

$$\mathrm{Im}\,\omega_1 = \mathrm{Im}\,\omega_1\quad(1 + n_2/18n_1).$$

These conclusions agree with the results of a numerical solution of (2), carried out at $N = 2$, $n_1/n_0 = n_2/n_0 = 0.005$, $k_\perp v_1/\omega_p = 1/2$, for the velocities (a) $v_1 - v_2 = 0$ and (b) $v_1 - v_2 = 0.2v_1$. The results of the calculation are shown in Fig. 1. In the case $v_1 = v_2$ we have in essence one beam with density $n_b = 0.01n_0$. The real and imaginary parts of ω as functions of $k_\parallel$ are shown in Fig. 1 by curves 1 and 1′. When the velocity of the second beam decreases, two unstable waves appear. The phase velocity of one of them lies between v_1 and v_2, while that of the other is smaller than the velocities of both beams. The wave increments are smaller than the increment obtained for the case $v_1 = v_2$, with the slower wave ($v_{2ph} < v_2$) having the smaller increment. The dispersion curves of both waves at a beam-velocity difference $v_1 - v_2 = 0.2v_1$ are shown in Fig. 1 (curves 2 and 2′, 3 and 3′). With further increase of the velocity difference, the dispersion curves of the unstable modes differ little from those for single-beam systems, in agreement with the analytic estimates given above.

During the nonlinear stage, the distribution function of the beam particles undergoes rapid changes, the character of which is quite sensitive to changes of the initial conditions; this is a characteristic feature of systems with many degrees of freedom. We are therefore forced to resort to numerical integration of the equations of motion or to be satisfied with a nonrigorous analytic description.

Excitation of (one or several) monochromatic waves in a single-beam system was investigated by numerically integrating the equations of motion[1,2,5]. A satisfactory qualitative description of this process was also obtained[3-6]. We present below a qualitative investigation of the nonlinear stage of the excitation of narrow wave packets in a multibeam system.

The equation of motion of a charged particle in the wave field is given by

$$\ddot{x} = \frac{e}{m}E(x,t)\sin[k(x-x_0)-\omega t], \tag{4}$$

where x_0 is the particle coordinate at $t = 0$. The wave amplitude $E(x, t)$ is assumed to be a slowly varying function of x and t. In the rest system of the wave ($x = \xi + v_{ph}t$, $v_{ph} = \omega/k$), Eq. (4) takes the form

$$\ddot{\xi} = \frac{e}{m}E(\xi,t)\sin[k(\xi-\xi_0)]. \tag{5}$$

In addition, we use the continuity equation for the average energy density

$$\frac{\partial}{\partial t}(\mathscr{E}_j + \mathscr{E}_p) - \frac{\partial}{\partial x}(v_\epsilon \mathscr{E}_j + P) = 0, \tag{6}$$

where $\mathscr{E}_j$ and $\mathscr{E}_p$ are the wave and particle energy densities averaged over the wavelength:

$$\mathscr{E}_j = \frac{E^2}{16\pi},\quad \mathscr{E}_p = \frac{1}{\lambda}\int_{x-\lambda/2}^{x+\lambda/2}\int_{-\infty}^{\infty}\frac{mv^2}{2}f(y,v)\,dy\,dv,\quad \lambda = \frac{2\pi}{k};$$

P is the particle energy flux:

$$P = \frac{1}{\lambda}\int_{x-\lambda/2}^{x+\lambda/2}\int_{-\infty}^{\infty}\frac{mv^3}{2}f(y,v)\,dy\,dv,$$

and v_ϵ is the field-energy transport velocity.

In the spatially homogeneous case, the second term in (6) is equal to zero. In a steady-state time-independent regime, the first term of (6) vanishes.

The analysis of the nonlinear-wave dispersion law (2) has shown that one can, with a certain degree of caution, set each of the beams in correspondence with one mode of unstable oscillations, so that waves belonging to the i-th mode acquire energy predominantly from the i-th beam during the nonlinear stage. The development of the wave of the i-th mode proceeds in the same manner as in the single-beam system, until the wave captures some "foreign" beam, or until the i-th beam is captured by some "foreign" wave. Of course, both can occur simultaneously, and it is also possible for the i-th wave to capture several foreign beams or for particles of the i-th beam to interact resonantly with several foreign waves. We consider here only the simplest possibilities, which become manifest primarily in systems with a small number of beams that are well resolved in velocity: $|v_i - v_k| \sim \eta_i v_i + \eta_k v_k$. To this end it suffices to consider a system of two beams that are simultaneously injected into the plasma.

We consider the influence of the capture of the particles of the foreign beam on the development of the instability. Let the wave belong to an unstable mode that vanishes as $n_1 \to 0$; then $v_{ph} < v_1$. The velocity of the second beam can be arbitrary. If

$$(v_2 - v_{ph})^2 < \frac{2e}{km}E_{max}^{(1)},$$

then particles of the second beam can be captured. When $v_2 > v_{ph}$, the development of the nonlinear stage is practically the same as in the case of a single-beam system. The oscillations of the wave amplitude are then more complicated than in the single-beam system, but an estimate of the maximum field amplitude, given by the expression

$$E_{max}^2 = 8\pi m[n_1(v_1^2 - v_{ph}^2) + n_2(v_2^2 - v_{ph}^2)], \tag{7}$$

is apparently not far from the true value.

If $v_2 < v_{ph}$, the picture is significantly different. After the capture, the particles of the second beam begin to be accelerated by the wave and limit its growth rate.

Let us examine with the aid of (5) and (6) the later stage of development of the instability, when the wave captures particles from both beams and the field amplitude varies within a certain range about the mean value $\overline{E}$. As already noted, the captured particles are gathered into bunches. The bunch can be regarded as a macroparticle with

$$N_i=\int_0^{\lambda} n_i(x)\,dx=n_i\lambda$$

electrons in it (λ is the wavelength) and with an initial average velocity $\tilde{v}_i$. If $v_1 > v_{ph} > v_2$, then the wave draws energy only from the particles of the first beam, and to determine the initial velocities $\tilde{v}_k$ of the bunches we can use the expression

$$\tilde{v}_k^2=v_k^2-\frac{\overline{E^2}}{8\pi n_1 m}\delta_{ik},$$

where δ_k is the Kronecker symbol.

The solutions of Eqs. (5), which describe the motion of the bunches, are

$$\dot{\xi}_i=\tilde{\dot{\xi}}_i\cos(\Omega_i t+\theta_i),$$

where

$$\tilde{\dot{\xi}}_i=\tilde{v}_i-v_{ph},\quad \Omega_i\sim(ek\overline{E}/m)^{1/2}, \tag{8}$$

and θ_i is the average phase of the bunch particles. At the instant of capture, the distribution of the bunch particles in space is almost homogeneous, and the average phase of the particles is equal to zero, but changes subsequently during the course of bunch formation.

The solutions (8) enable us to trace the variation of the field energy with the aid of Eq. (6), from which it follows that

$$\begin{gathered}\mathscr{E}_f=\overline{\mathscr{E}}_f-\frac{mv_{ph}}{2}\sum_{i=1,2} n_i\tilde{\dot{\xi}}_i\cos(\Omega_i t+\theta_i),\\ \overline{\mathscr{E}}_f=\frac{m}{2}\sum_{i=1,2} n_i(v_i^2-v_{ph}^2).\end{gathered} \tag{9}$$

If we neglect the frequency difference $\Omega_1-\Omega_2$, then (9) can be represented in the form

$$\begin{gathered}\mathscr{E}_f=\overline{\mathscr{E}}_f-{}^1/{}_2 mv_{ph}[n_1^2\tilde{\dot{\xi}}_1^2+n_2^2\tilde{\dot{\xi}}_2^2+2n_1n_2\cos(\theta_1-\theta_2)\tilde{\dot{\xi}}_1\tilde{\dot{\xi}}_2]^{1/2}\cos(\Omega t+\theta),\\ \theta=\operatorname{arctg}\left(\frac{n_1\tilde{\dot{\xi}}_1\sin\theta_1+n_2\tilde{\dot{\xi}}_2\sin\theta_2}{n_1\tilde{\dot{\xi}}_1\cos\theta_1+n_2\tilde{\dot{\xi}}_2\cos\theta_2}\right).\end{gathered} \tag{10}$$

We see therefore that if $\tilde{\dot{\xi}}_2 < 0$, then the changes of $\mathscr{E}_f$ can be negligible. Thus, the capture of a slow foreign beam stabilizes the instability.

An essential question is whether this stabilization can lead to establishment of stability of a linear wave of the Bernstein-Green-Kruskal type [9]. The possible establishment of such waves in two-stream systems was investigated by numerical methods in [10, 11] and was considered for multibeam systems in [12].

It is seen from (15) that if $\theta_1\approx\theta_2$ and $n_1\tilde{\dot{\xi}}_1\approx n_2\tilde{\dot{\xi}}_2$, then $\mathscr{E}_f\approx\overline{\mathscr{E}}_f$, thus indicating that a finite-amplitude wave can be established[2)]. Relation (10) enables us to estimate the amplitude of the steady-state wave. Thus, it follows from (10) that

$$\overline{E}^2=16\pi mv_1[n_1(v_1-v_{ph})+n_2(v_2-v_{ph})]. \tag{11}$$

We see that when v_2 decreases $\overline{E}$ also decreases. It should be noted that our analysis is valid only so long as $\overline{E}$ exceeds the amplitude for the capture of particles from the first and second beams, and also so long as the conditions $\Omega_1\approx\Omega_2$ and $\theta_1\approx\theta_2$ are satisfied, which is possible only if

$$|v_1-v_{ph}|\sim|v_2-v_{ph}|.$$

We now examine how the capture of the second beam by the first wave affects the instability of waves belonging to the second mode. As a result of this capture, the particles of the second beam initiate an intensive exchange of energy with the wave that has captured them, as a result of which the coherence of their interaction with the waves of the second mode is disturbed. This should lead to a limitation on the growth of the second-mode waves.

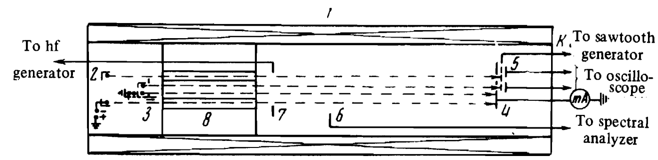

FIG. 2. Diagram of experimental setup: 1—solenoid; 2 and 3—electron guns; 4—collector; 5—electrostatic analyzers; 6—high-frequency probe; 7—supply electrode; 8—channel producing pressure drop.

3. EXPERIMENT

The experiments were performed using a setup with two and three electron beams. A diagram of the setup with two beams is shown in Fig. 2. Hollow electron beams with radii 1 and 0.25 cm pass through a metallic tube of 9 cm diameter placed in a homogeneous magnetic field H = 400 Oe. The electrons are injected with the aid of guns 2 and 3. The electron emitters are tungsten filaments 0.05 cm thick. The beam energies range from 80 to 300 eV, corresponding to the velocity interval $(5.3-10.4)\times 10^8$ cm/sec. The total current $i = i_1 + i_2$ was maintained at 8 mA in all the experiments. The pressure in the working region was 8×10^{-5} Torr, and was lower by one order of magnitude in the cathode region. The pressure drop is made possible by channel 8 and by admission of xenon into the working part of the chamber. The plasma was a result of ionization of the gas by the electron beam.

At equal velocities of the two beams ($v_1 = v_2 = 10.4\times 10^8$ cm/sec), oscillations are observed in the system, in a band 30 MHz wide with a maximum intensity at 280 MHz. This is lower than the plasma frequency and much lower than the electron cyclotron frequency. The oscillations are amplified as they propagate along the plasma column. The intensity distribution of the oscillations at 280 MHz along the length of the system was investigated with a moving probe 6, located 1.5 cm away from the surface of the external beam.

In the subsequent experiments, the velocity of the external beam remained at 10.4×10^8 cm/sec, while that of the internal beam was varied between 5.3×10^8 and 10.4×10^8 cm/sec. The current of the internal beam ranged from 6 to 25% of the total current. This was accompanied by a change in the oscillation intensity, whereas the bandwidth of the excited frequencies remained constant.

The distribution of the oscillation intensity along the plasma column at different velocities of the internal beam and at a current $i_2 = 0.06i$ is shown in Fig. 3. At an initial velocity of the internal-beam particles $v_2 = v_1 = 10.4\times 10^8$ cm/sec, the wave was amplified over the

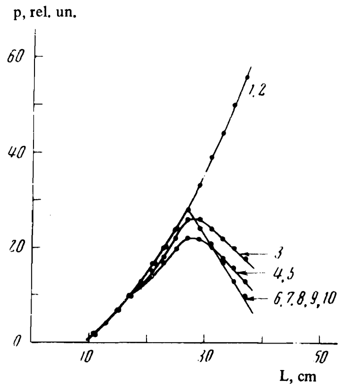

FIG. 3. Dependence of the oscillation intensity on the length along the plasma column for different internal-beam velocities v_2 (in cm/sec) at an external beam velocity $v_1 = 10.4 \times 10^8$ cm/sec and at $i_2 = 0.06i$: 1–10.4×10^8, 2–9.6×10^8, 3–8.8×10^8, 4–8.4×10^8, 5–8×10^8, 6–7.5×10^8, 7–7×10^8, 8–6.5×10^8, 9–6×10^8, 10–5.4×10^8.

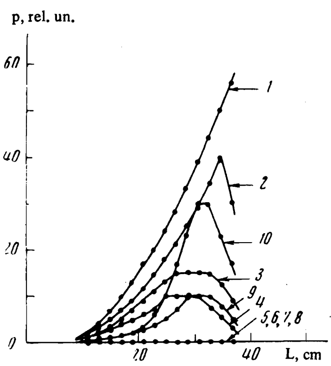

FIG. 4. The same as Fig. 3, but at $k_2 = 0.25i$.

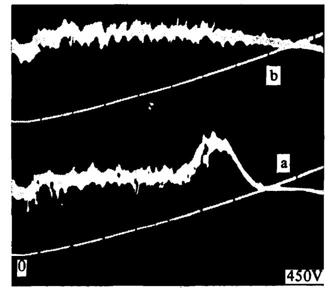

FIG. 5

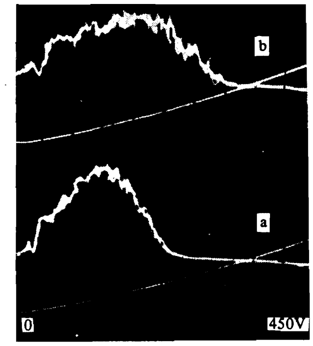

FIG. 6

FIG. 5. Energy distribution of the external-beam electrons: a–in the case of maximum suppression of the oscillations ($v_1 = 10.4 \times 10^8$ cm/sec, $v_2 = 8 \times 10^8$ cm/sec; $i_2 = 0.25i$), b–in the absence of suppression ($v_2 = v_1 = 10.4 \times 10^8$ cm/sec).

FIG. 6. Energy distribution of internal-beam electrons at $v_1 = 10.4 \times 10^8$ cm/sec and $v_2 = 8 \times 10^8$ cm/sec: a–in the absence of an external beam ($i_1 = 0$, $i_2 = 2$ mA), b–in the presence of an external beam ($i_1 = 6$ mA, $i_2 = 2$ mA).

entire length of the plasma column. The amplitude along the setup does not reach a maximum value, although at the end of the plasma column the growth of the amplitude differs noticeably from exponential, and therefore the largest attainable amplitude is apparently close to E_{max} (7).

The phase velocity of the amplified wave was measured by comparing the phase of the wave with the phase of a reference signal. To this end, a reference signal at 280 MHz was applied to the diaphragm 7 (Fig. 2). According to the measurements, the phase velocity is 8.7×10^8 cm/sec.

At $v_2 = 8.8 \times 10^8$ cm/sec, the increment decreases, and at a certain distance from the start of the system the gain gives way to damping. The transition from gain to damping was also observed at smaller v_2. With decreasing v_2, the maximum amplitude attainable in the system first decreases, and then begins to increase, after going through a minimum in the internal-beam velocity region $(8.0–8.4) \times 10^8$ cm/sec.

At a current $i_2 = 0.25i$, the external probe 6 registers almost complete suppression of the instability over the entire length of the system in the velocity range $v_2 = (6.4–7.9) \times 10^8$ cm/sec (Fig. 4).

An electrostatic analyzer was used to plot the energy distribution of the electrons in the beams. To this end, a sawtooth retarding voltage with amplitude 450 V was applied to the analyzer grid. The signal from the analyzer collector was differentiated and fed to an oscilloscope. Figure 5 shows oscillograms of the energy distribution of the external-beam electrons in the regime where the oscillations are suppressed (a) and when there is no suppression (b). The figure also shows the oscillogram of the sawtooth voltage applied to the analyzer grid. It is seen from the oscillogram that the energy scatter of the external-beam electrons is large in the instability regime and becomes much smaller in the regime when the oscillations are suppressed.

Figure 6 shows oscillograms characterizing the energy distribution of the electrons of a small-diameter beam in the absence (a) and in the presence (b) of the external beam, the beam current and velocities being such that the oscillation-suppression regime is realized. We see that the internal beam remains unstable in the suppression regime. The oscillations generated by this beam are not registered by the external probe 6 (see Fig. 2), but an antenna placed in the space between the beams registered oscillations of 200 MHz frequency.

As shown by oscillogram b of Fig. 6, an appreciable fraction of the internal-beam electrons is accelerated. The acceleration effect becomes more clearly manifest at a larger velocity difference, when oscillations again appear in the circuit of the external measuring probe.

The absence of oscillations in the region of the external probe is evidence that the oscillations belonging to the unstable branch connected with the external beam are suppressed. However, oscillations excited by the internal beam are still present and can be registered by a probe placed between the beams. The fact that the internal beam generates oscillations is also evidenced by the smearing of the energy distribution function of the particles of this beam in the regime when the external-beam oscillations are suppressed (Fig. 6). The power level of the oscillations excited by the second beam is relatively low. It can be made much lower by injecting a weak third beam into the system (the beam currents are then $i_1 = 6$ mA, $i_2 = 2$ mA, and $i_3 = 1$ mA), with a velocity $v_3 < v_2$ chosen such as to suppress the oscillations generated by the second beam. No oscillations were observed in the system in this case—their level was below the sensitivity of the recording apparatus, and the beam-particle energy distribution functions were slightly smeared out.

This experiment shows that in a multiple-beam system it is possible, by varying the velocities and densities of the beams, to control the level of the excited oscillations and to limit the beam instability at very small oscillation amplitudes.

4. DISCUSSION

Let us analyze the experimental results and compare them with the developed theoretical representations.

We note first that in the experiment the beam and plasma densities, and probably also the wave amplitude, depend strongly on the radius. The beams have annular cross sections and therefore $n_i = \tilde{n}_i \delta(r - r_i)$. The radius of the plasma column in the setup is ≈ 1.5 cm. As to the plasma density and the electric-field intensity, their radial dependences are unknown. It can be assumed that the quantities $\mathscr{E}_p$ and $\mathscr{E}_f$, like the plasma density $n_0(r)$, are smooth monotonic functions at $r \leq R$, and decrease quite rapidly at $r > R$ ($R = 1.5$ cm). By averaging over the cross section

$$\left(f(r) \to f = \frac{1}{\pi R^2} \int_0^R f(r) r\, dr \right)$$

we can obtain corresponding quantities that do not depend on r. By applying the formulas given in Sec. 2 to these quantities we can obtain certain estimates. Thus, to determine the average beam densities we can use the formulas

$$n_{1,2} = i_{1,2} (e v_{1,2} \pi R^2)^{-1},$$

where i_k is the current of the k-th beam. We assume that $k_\perp \sim \pi/R \sim 1$, and then $\omega_p' = 0.8\omega_p$ at $k_\parallel = 2$ cm^{-1}. From the excited-wave phase velocity $v_{ph} = 8.7 \times 10^8$ cm/sec, measured at $v_1 = v_2 = 1.04 \times 10^9$ cm/sec, it is easy to determine the linear increment, namely $\gamma = \omega_p'(v_1 - v_{ph}) v_1^{-1} \approx 0.17\omega_p'$. At[3] $v_{gr} = 2v_{ph}/3$, the spatial increment during the linear stage is $\gamma_d = \gamma v_{gr}^{-1} \approx 0.25 k_\parallel = 0.5$ cm^{-1}. A similar estimate is obtained from the spatial dependence of the oscillation intensity at $v_1 = v_2$ (see curve 1 in Fig. 3), based on the formula $\gamma_d = (2E^2)^{-1} \Delta E^2 / \Delta x$.

The character of the particle velocity distribution function (Figs. 5 and 6) and the development of the instability in the two-beam system (Figs. 3 and 4) show that the particles of the second beam are captured, and that this imposes a limit on the instability.

Recognizing that curves 1—3 in Fig. 4 correspond to $v_2 > v_{ph}$ and curves 4—10 correspond to $v_2 < v_{ph}$, we see that the greatest effect on the development of the beam instability is exerted by the second beam at $v_2 < v_{ph}$. However, this influence is appreciable even at $v_2 > v_{ph}$. The reason is that at $v_{ph} < v_2 < v_1$ the capture of the second beam occurs earlier than that of the first beam, and consequently, by the instant the bunches of the first beam are captured, the particles of the second beam have had time to shift in phase in such a way that $\theta_1 - \theta_2 \sim 1$, and consequently the maximum value of the amplitude, as seen from (10), should decrease. With increasing number of particles in the second beam (Fig. 4) the decrease of the maximum value of the amplitude becomes more significant and the wave field amplitude is no more than a few volts per centimeter already at $n_2/n_1 = 0.4$—0.6 (curves 5—8 on Fig. 4).

The average amplitude of the steady-state oscillations can be estimated with the aid of expression (11). It appears that at $v_1 - v_{ph} \approx v_{ph} - v_2$ the bunches of charged particles differ little in average phase, so that $\theta_1 - \theta_2 \ll 1$. Taking this circumstance into account, we obtain from (11) $\overline{E} \sim 6$—8 V/cm, which is close enough to the second-beam capture amplitude.

From the oscillograms of Fig. 6, which show the particle energy distribution functions in the suppression regime, we see that the particles of the second beam are accelerated. The particles captured by the wave can be accelerated to

$$v_{2max} = v_{ph} + (2e\overline{E}/km)^{1/2}.$$

Substituting here the obtained value of $\overline{E}$, we get $v_{2max} \approx 9.8 \times 10^8$ cm/sec. It is easy to find from the oscillograms that the maximum energy of the second beam reaches 270 eV. This energy corresponds to $v_{2max} = 9.9 \times 10^8$ cm/sec, which agrees with the calculated value and enables us to verify the validity of the estimates.

5. CONCLUSION

The theoretical and experimental investigations of the development of instability in a multibeam system show that the mutual interaction of weak beams leads during the linear stage only to a negligible change in the increments of the unstable modes, whereas the development of the nonlinear stage of instability, in which the beam particles are captured and the growth of the amplitudes of the unstable modes is limited, turns out to be quite sensitive to the change of the resonant-particle distribution function brought about by varying the initial velocities and densities of the beams. Capture of resonant particles leads to establishment of nonlinear oscillations, with amplitudes that depend strongly on the initial resonant-particle distribution function. This makes it possible, by varying the beam parameters, to control development of the instability of a multibeam system, and to vary the amplitudes of the excited waves in a wide range. The use of one or two controlled electron beams with currents amounting to small fractions of the current of the main beam makes it possible, if the velocities are appropriately chosen, to lower the intensity of the oscillations generated in this system by a factor of several times ten, i.e., practically to suppress the beam instability.

We note that by modulating the beam parameters in the multibeam system it is possible to excite amplitude-modulated electron oscillations with large depth of modulation. The latter, in turn, can serve as a means of exciting intense ionic oscillations at the parameter-modulation frequencies. Thus, it is possible to control both the high-frequency and low-frequency oscillations in multibeam systems.

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1) From among the many publications devoted to this question, we mention [1-8], which are devoted to the nonlinear stage of excitation of narrow wave packets by monoenergetic beams.

2) We exclude from consideration here the possibility of the development of slow instabilities with a characteristic time much larger than $1/\gamma_{max}$.

3) This relation holds true in a single-beam system for waves with maximum increment.

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