Quantum-kinetic equation and magnetoresistance of metals with allowance for magnetic breakdown

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The effect of magnetic breakdown (MB) on the static electric-conductivity tensor (σ_{ik}) of metals is investigated. The analysis is carried out in the limit $kT \gg \hbar\Omega_0$ (T is the temperature and Ω_0 is the characteristic Larmor frequency) for the case when the electron scattering is mainly by static lattice defects. It is shown that under MB conditions the tensor σ_{ik} is strongly influenced by weak microscopic fields produced by extended defects such as dislocations. The basis of the entire analysis is the exact quantum kinetic equation (QKE) describing the simultaneous relaxation of the electrons on the impurities and extended defects. To derive the QKE, an iteration procedure based on the formalism of generating functionals is developed. A detailed investigation of the QKE and the calculation of σ_{ik} are carried out in the limit $t_{imp} \gg \Omega_0^{-1}$ (t_{imp} is the electron-impurity relaxation time) at an arbitrary ratio of Ω_0 and the characteristic frequency $\bar{t}^{-1}$ of the "collisions" of the electron with the extended defects. To describe the extreme quantum coherent situations, characterized by the inequality $\Omega_0\bar{t} \gg 1$, an effective kinetic equation is derived, the collision integral of which is constructed by averaging over the random MB spectrum and depends significantly on the MB probability w. With the aid of this equation it becomes possible, in particular, to obtain a closed analytic dependence of σ_{ik} on w at $t_{imp} \gg \bar{t} \gg \Omega_0^{-1}$. It is also shown that in this case some of the components of σ_{ik} depend only on the impurity parameters, while others depend only on t. We investigate in detail the situation $\Omega_0\bar{t} \ll 1$, which is characterized by a complete destruction of the quantum processes by extended defects. It is shown that it is described completely by the usual classical Boltzmann equation (with an electron-impurity collision integral), supplemented at the MB sites by boundary conditions that do not depend on $\bar{t}$ and in which the MB probability enters as a stochastic factor. The dependence of σ_{ik} on w is obtained in this case also in closed form. The conditions for experimentally observing the coherent and stochastic situations are discussed.

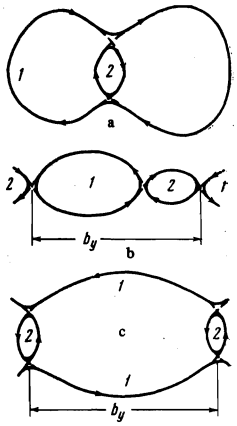
1. INTRODUCTION

1. The first calculation of magnetoresistance of metals with allowance for magnetic breakdown (MB)[1] was performed by Falicov and Sievert (FS)[2] in the relaxation-time (t_0) approximation on the basis of a hypothesis according to which the probability of interband tunneling of (MB) was ascribed a stochastic meaning, and the electron was regarded as a particle that executes random walks over a magnetic-breakdown network of classical trajectories in p-space. The FS approach is characterized by the appearance of dissipative effects, which remain finite as $t_0 \to \infty$. This stochastic result, as noted in[3], contradicts the predictions of quantum theory, which take direct account of certain general properties of the electronic magnetic-breakdown spectrum. A rigorous quantum-mechanical calculation of the kinetic coefficients carried out by the present author[4] in the t_0 approximation with allowance for MB has confirmed in the main the general premises of[3], and has shown that the influence of the MB on the kinetics of metals is determined even in the "classical" limit $kT \gg \hbar\Omega_0$ (T is the temperature and Ω_0 is the characteristic Larmor frequency) by the quantum phenomenon of multiple coherent scattering of the electron by the MB regions. The quantum coherence of this process leads to the contradiction indicated above, which cannot be eliminated within the framework of the t_0 approximation.

It was shown in[4] at the same time that weak inhomogeneous macroscopic fields (which are of no importance in the usual quasiclassical approach) can come forth as unique "latent" parameters that ensure applicability of the stochastic FS approach. This statement is proved in[4] for the particular case of an inhomogeneous macroscopic field. This field, while small from the classical point of view, nevertheless produces in the phase of the quasiclassical wave function an increment with a characteristic value $\beta_0 \gg 1$.

The method used in[4], while demonstrating the important role of the weak macroscopic fields for the magnetic-breakdown kinetics of metals, did not lead to any definite conclusions concerning the structure of the kinetic coefficients in fields with $\beta_0 \lesssim 1$. (As a rule, it is precisely these weak fields that are produced in experiments by dislocations and other extended crystal defects). In this paper we construct the magnetic breakdown tensor of the static conductivity σ_{ik} (i, k = x, y, z) at arbitrary values of β_0, and show that in fields characterized by $\beta_0 \lesssim 1$ this tensor can have an entirely different form than in the t_0 approximation. The entire investigation is carried out here in the limit $kT \gg \hbar\Omega_0$, on the basis of an exact quantum-kinetic equation that takes into account electron scattering both by impurity centers and by small macroscopic inhomogeneities.

2. Before we proceed to the exact formulation of the problem, we shall describe briefly, following[4,5], the main features of the magnetic-breakdown dynamics of the electron. Under MB conditions the entire system of electron orbits $\epsilon_m(p) = E$, $p_z = \text{const}$ ($\epsilon_m(p)$ is the dispersion law, p is the quasimomentum, m is the number of the energy band, E is the electron energy, and p_z is the projection of its momentum on the direction of the magnetic field $\mathbf{H} = \{0, 0, H\}$) breaks up into a network of classical sections (here designated by the letters $\alpha, \beta, \ldots$; the symbol $m(\alpha)$ indicates that the section α belongs to the band m), which are interconnected by small quantum regions (MB nodes), where the usual quasiclassical Hamiltonian $\epsilon_m(\hat{p})$ ($\hat{p} = \hat{P} - (e/c)\hat{A}$, $\hat{P}$ is



The arrows indicate the directions of motion along the classical trajectories, and the intersections of the dashed lines show the centers of the MB regions: 1 and 2 are the numbers of the bands, and b_y is the period in p-space.

the generalized momentum operator and $\mathbf{A} = \{-Hy, 0, 0\}$ is the vector potential) is not valid. Each MB node joins four classical sections (see the examples in the figure) and is a unique center of quantum scattering, as a result of which an electron situated in a localized state ($|\alpha\rangle$ or $|\beta\rangle$) in one of the sections α or β ($m(\alpha) \neq m(\beta)$) that enter into the MB node goes over into a superposition of states $|\alpha'\rangle$ or $|\beta'\rangle$ belonging to the outgoing sections α' and β' ($m(\alpha') = m(\alpha)$, $m(\beta') = m(\beta)$):

$$|\alpha\rangle \to s_{\alpha'\alpha}|\alpha'\rangle + s_{\beta'\alpha}|\beta'\rangle, \quad |\beta\rangle \to s_{\alpha'\beta}|\alpha'\rangle + s_{\beta'\beta}|\beta'\rangle; \tag{1}$$
$$s_{\alpha\alpha'} = s^*_{\beta\beta'} = \sqrt{1-w}\, e^{i\omega}, \quad s_{\alpha'\beta} = -s_{\beta'\alpha} = \sqrt{w}, \quad w = \exp\{-H_0(E, p_z)/H\}.$$

The form of the parameter H_0 was found in [5], the phase shift $\omega(H)$ is inessential in what follows, $w(H)$ is the MB probability, and the four scattering amplitudes (1) form a unitary s matrix, which "joins together" in the stationary case the quasiclassical wave functions on the sections α, α', β, and β' [5]. According to [5], the time of the considered scattering processes is $\tau_0 \sim \sqrt{\kappa}\Omega_0^{-1}$ where $\kappa = e\hbar H/cp_0^2 \ll 1$ is the parameter of the quasiclassical theory and p_0 is the characteristic momentum.

The eigenvector ($|\Psi\rangle$) of the magnetic-breakdown Hamiltonian $\hat{\mathcal{H}}$, determined by the quantum numbers E, p_z, P_x, can be expressed, accurate to quantities of order $\sim \Omega_0\tau_0 \ll 1$, in the form

$$|\Psi\rangle = \sum_{\alpha=1}^{N} a_\alpha(E, p_z\, P_x)|\psi_\alpha\rangle, \quad \varepsilon_{m(\alpha)}(\mathbf{p})|\psi_\alpha\rangle = E|\psi_\alpha\rangle. \tag{2}$$

Formula (2) describes both closed and open one-dimensional periodic configurations; in the former case N is the number of all the sections α, while in the latter it is the total number of sections within one unit cell. The quasiclassical functions $|\psi_\alpha\rangle$ are constructed in accordance with the standard rules [4,5] in terms of the classical motion over the section α. They are so normalized that $\langle\psi_\alpha|\psi_\alpha\rangle = T_\alpha$, where T_α is the total time that the classical particle stays on the section α. The coefficient a_α is the probability amplitude of finding the particle on the α-th section. The amplitudes a_α are connected with one another by a system of linear equations, which in the case of closed configurations take the form [4]

$$a_\gamma = \sum_{\alpha=1}^{N} U_{\gamma\alpha}(E, p_z)a_\alpha, \quad U_{\gamma\alpha} = V_{\gamma\alpha}\exp\{iS_\alpha(E, p_z)\hbar^{-1}\}, \tag{3}$$

$$\sum_{\alpha=1}^{N} |a_\alpha|^2 T_\alpha = 1.$$

Here s_α is the change of the classical action

$$-\frac{c}{eH}\int_{p_y} p_x\, dp_y'$$

during the time of motion along the entire section α. The matrix $V_{\gamma\alpha}$ in (3) is made up in accordance with the following rule: in the α-th column of $V_{\gamma\alpha}$, only two elements differ from zero, $V_{\alpha'\alpha} = s_{\alpha'\alpha}$ and $V_{\beta'\alpha} = s_{\beta'\alpha}$, where $s_{\alpha'\alpha}$ and $s_{\beta'\alpha}$ are the s-matrix elements of that magnetic-breakdown node for which α is the incoming section, and α' and β' are the indices of the outgoing sections. The last equation of (3) is a consequence of the normalization of $|\Psi\rangle$ to unity. For the case of open periodic configurations it is necessary to make in (3) the substitution

$$V_{\gamma\alpha} \to V_{\alpha\gamma}(\mathcal{H}) \equiv V_{\gamma\alpha}\exp\{ir_\alpha\mathcal{H}\}, \quad \mathcal{H} \equiv cP_x b_y/e\hbar H. \tag{3a}$$

Here r_α is equal to 1 and -1 if the incoming section α crosses the right and left boundaries of the unit cell, respectively (see Figs. b anc c), and $r_\alpha = 0$ in all the remaining cases. This dependence of $V_{\gamma\alpha}$ on the quantum number P_x ($V_{\gamma\alpha}(\mathcal{H}) = V_{\gamma\alpha}(\mathcal{H} + 2\pi)$) is the result of the requirement that $|\Psi\rangle$ be periodic in p_y (the corresponding period is equal to b_y).

Formulas (1)–(3a) play the principal role in the proposed theory. They reflect the coherent quantum character of the multiple scattering of the electron by all the magnetic-breakdown centers, and show that the probability amplitudes a_α depend essentially on the phases of the quasiclassical wave functions on the different sections α. This circumstance becomes significantly manifest in the properties of the magnetic-breakdown spectrum, which is determined by the equations

$$\mathrm{Det}\,\|\delta_{\alpha\gamma} - U_{\alpha\gamma}(E, p_z)\| = 0, \quad \mathrm{Det}\,\|\delta_{\alpha\gamma} - U_{\alpha\gamma}(E, p_z, \mathcal{H})\| = 0. \tag{4}$$

The first equation pertains to closed configurations and the second to open configurations. In the first case the spectrum is of the Landau type, namely a discrete set of terms $E_n(p_z)$ (n is the number of the terms and $|E_{n+1} - E_n| \sim \hbar\Omega_0$), which depends on one continuous parameter p_z. In the second case, the spectrum takes the form of "magnetic" bands $E_n(p_z, \mathcal{H}) = E_n(p_z, \mathcal{H} + 2\pi)$ of broadened Landau levels, with a band width $\lesssim \hbar\Omega_0$. The average transverse velocity in the stationary states $|n, p_z, P_x\rangle$ is directed along the x axis and is equal to $\partial E_n/\partial P_x$. In the general case $|\partial E_n/\partial P_x|$ is approximately equal to the characteristic Fermi velocity v_0.

Both types of spectra are characterized by the appearance of rapidly oscillating functions $\exp[iS_\alpha(E, p_z)\hbar^{-1}]$ in (4). The quantities $S_\alpha(E, P_z)$ and their derivatives with respect to p_z and E are generally speaking noncommensurate, so that the relative disposition of the terms E_η ($\eta \equiv \{n, p_z, P_x\}$), which are the zeroes of the spectral equations (4), depends in random fashion on n, p_z, and P_x, and undergoes abrupt changes when p_z or P_x is varied by a small amount $\sim \kappa p_0$. The same behavior is exhibited by $a_\alpha(\eta)$ and the matrix elements of the physical quantities, expressed in the η representation. In particular, for an arbitrary operator $\hat{b} = b_m(\hat{\mathbf{p}})$, whose classical analog $b_m(\mathbf{p})$ varies over an interval $\gg \kappa p_0$, we have [4]

$$b_{\eta\eta'} = \sum_{\alpha=1}^{N} a_\alpha(\eta)a_\alpha^*(\eta')\int_0^{T_\alpha} b_{m(\alpha)}(\mathbf{p}(\tau_\alpha, E_\eta, p_z))\exp\{i(E_{\eta'} - E_\eta)\tau_\alpha\}d\tau_\alpha\delta_{p_z p_z'}\delta_{P_x P_x'}. \tag{5}$$

The variable τ_α is the time of classical motion over the

section α ($\tau_\alpha = 0$ if the particle is located at the start of the section).

2. FORMULATION OF PROBLEM

In the treatment of low-temperature electric conductivity of metals it is customary to take into consideration only the scattering of the electrons by point defects of the lattice (impurity atoms), and the small large-scale fields ($U(\mathbf{r})$) produced by extended crystal defects are neglected. In the absence of MB, this neglect is justified, since the characteristic change of the electron momentum q_0 due to $U(\mathbf{r})$ is much smaller than p_0. The relaxation due to this scattering occurs within "diffusion" times $t_{dif} \sim \bar{t}(p_0/q_0)^2$, where $\bar{t}^{-1}$ is the characteristic frequency of the "collisions" of the electron in the field $U(\mathbf{r})$. In spite of the fact that under real conditions $\bar{t}$ can be even smaller than the electron-impurity relaxation time t_{imp}, the value of t_{dif} usually exceeds $\bar{t}$ by several orders of magnitude, owing to the smallness of q_0/p_0.

Under MB conditions, the effectiveness of electron scattering by extended defects is determined by the relation between q_0 and the small momentum κp_0, which is the characteristic scale of the rapid oscillations of E_η and $b_{\eta\eta'}$ (see Sec. 1). At the typical $q_0 \gtrsim \kappa p_0$, the scattering effectiveness becomes maximal and the characteristic time of the relaxation processes due to the perturbation $U(\mathbf{r})$ coincides with $\bar{t} \ll t_{dif}$. The ratio of the times $\bar{t}$ and t_{imp} is arbitrary. It follows therefore that in the magnetic-breakdown theory of kinetic phenomena there appears a new dimensionless quantity $\bar{\gamma} = (\Omega_0 \bar{t})^{-1}$ alongside the usual parameter $\gamma_{imp} = (\Omega_0 t_{imp})^{-1}$ that characterizes the classical analysis[6]. In the limiting case of strong fields ($\gamma_{imp} \ll 1$), the structure of the magnetic-breakdown electric-conductivity tensor depends essentially on the "competition" between $\bar{\gamma}$ and γ_{imp}. Three cases must be distinguished here:

A. $\bar{\gamma} \ll \gamma_{imp} \ll 1$. The role of extended defects is negligible.

B. $\gamma_{imp} \ll \bar{\gamma} \ll 1$. The lifetime of the states (2), in analogy with the preceding case, is $\gg \Omega_0^{-1}$, but it is now determined by the extended defects and coincides with $\bar{t}$.

C. $\gamma_{imp} \ll 1$, $\bar{\gamma} \gg 1$. The second inequality means that the magnetic-breakdown spectrum and the stationary states (2) are destroyed by the scattering of the electrons from the extended lattice defects.

We shall consider all three situations (A, B, C) from a unified point of view, simulating the field $U(\mathbf{r})$ by a system of randomly disposed force centers (called "d centers" henceforth) with a macroscopic action radius $d_0 \gg a_0$ (a_0 is the interatomic distance) on the order of the distance between them and the characteristic potential $u_0 \ll \epsilon_F$ (ϵ_F is the Fermi energy). In addition, we assume that d_0 is much smaller than the characteristic Larmor radius r_H. This condition means that $q_0 \gg \kappa p_0$, i.e., the scattering of the electron by each d-center is effective (in the scale of κp_0). The number of short-range impurities per lattice atom (c_{imp}) is assumed, as usual, to be small.

In the assumed model, the tensor σ_{ik} is expressed in terms of the single-electron density matrix $\hat{\rho}$ averaged over the positions of the impurities and the d-centers. The quantum kinetic equation (QKE) for the matrix $\hat{\rho}$ will be derived in Sec. 3 by an iteration method[1)] constituting a "static" modification of the formalism of generating functionals[11]. It automatically yields compact operator expressions for the terms of the iteration series that determines the "collision integral." These expressions are quite convenient for estimates and make it easy to determine the dimensionless parameter of the iteration expansion connected with the macroscopic d-centers. It turns out that under MB conditions, when the density matrix ρ has no classical analog, this parameter is equal to β_0^2, where $\beta_0 \equiv u_0 d_0 / \hbar v_0$ coincides with the quantity introduced in subsection 1 of Sec. 1. (In the absence of MB, the "collision integral" is expanded in powers of $(u_0/\epsilon_F)^2 \ll \beta_0^2$). Thus, a closed expression can be written for the quantum "collision integral" in the presence of d-centers ($d_0 \lesssim r_H$) and MB only in the case when $\beta_0^2 \ll 1$.

The analysis of the "coherent" situations A and B, which are characterized by a small broadening of the terms E_η, reduces to investigation of the diagonal part of the QKE, expressed in the η representation. Owing to the peculiarly irregular nature of the magnetic-breakdown spectrum, all the quantities that enter in the QKE are in this representation in fact random functions of the quantum numbers η. On the other hand, it is obvious that σ_{ik} depends analytically on the MB probability $w(H)$. This leads to the need for constructing an effective analytic procedure that makes it possible to express σ_{ik} in terms of the integral characteristics of the random spectrum E_η. The general method of solving this problem is formulated in subsection 1 of Sec. 4. The main idea of the method is to replace the random phases $S_\alpha(E_\eta, p_z)\hbar^{-1}$ by independent variables φ_α and introduce an effective distribution function $\chi(E, p_z, P_x, \varphi_\alpha)$ in terms of which σ_{ik} can be expressed. We construct for χ an effective kinetic equation that is the magnetic-breakdown analog of the classical Boltzmann equation. For the effective transition probabilities per unit time we obtain closed expressions that depend significantly on w. The general formulas for σ_{ik} are investigated in Sec. 4 for both coherent situations A and B. In case B, the inequalities $q_0/p_0 \ll 1$ and $\bar{t}/t_{imp} \ll 1$ enable us to find in closed form an analytic dependence of σ_{ik} on w for an arbitrary magnetic-breakdown configuration.

In Sec. 5 we investigate the case C under the additional condition $\bar{t} \gg \tau_0$, which means that the d_0 centers have no effect on the tunneling of the electron through the MB regions. We shall show that, accurate to quantities $\sim e^{-\bar{\gamma}} \ll 1$, it is described by the classical kinetic equation (with an arbitrary electron-impurity collision integral), supplemented at the MB nodes by special boundary conditions that contain the MB probability as a stochastic factor and do not depend on the d-center parameters. The result, which verifies and generalizes the FS approach, yields a complete description of the stochastic situation (see Sec. 1). We obtain this result in general terms, using only those properties of the QKE which are connected with the smallness of H, $\bar{\gamma}^{-1}$, and τ_0/t and do not depend on the value of the parameter β_0.

3. THE QUANTUM KINETIC EQUATION

We assume first that the electron gas moves through a system of n_0 identical randomly-disposed scattering centers. The current density $\mathbf{j}$ is determined, accurate

to quantities $\lesssim \hbar\Omega_0 kT^{2)}$, by the following equations:

$$\mathbf{j}=\frac{e}{V}\operatorname{Sp}\hat{\rho}\hat{\mathbf{v}},\quad \hat{\rho}=\langle\!\langle\hat{f}\rangle\!\rangle,\quad \frac{i}{\hbar}\Big[\hat{\mathscr{H}}+\sum_{l=1}^{n_0}u(\hat{\mathbf{r}}-\mathbf{R}_l),\hat{f}\Big]=e(\mathscr{E}_k\hat{v}_k)f_0'(\hat{\mathscr{H}}). \quad (6)$$

Here V is the volume of the system, $\hat{\mathbf{v}}$ is the electron-velocity operator, $f_0(E)$ is the Fermi distribution function, $f_0'(E)\equiv df_0/dE$, $\mathscr{E}_k$ (k = x, y, z) is the electric field intensity, $u(\hat{\mathbf{r}}-\mathbf{R})$ is the potential of an individual center located at the lattice site R; the brackets $\langle\!\langle\ldots\rangle\!\rangle$ denote averaging over all the possible configurations $\{\mathbf{R}_1,\ldots,\mathbf{R}_{n_0}\}$. The averaged density matrix $\hat{\rho}$ satisfies the QKE

$$(\tilde{\mathscr{H}}+\tilde{J})\hat{\rho}=-e(\mathscr{E}_k\hat{v}_k)f_0'(\hat{\mathscr{H}}),\quad \tilde{\mathscr{H}}\equiv i\hbar^{-1}[\hat{\mathscr{H}},\ldots]. \quad (7)$$

The tilde denotes throughout linear transformation in the space of "ordinary" operators, and $\tilde{J}\hat{\rho}$ is the quantum analog of the collision integral and is to be determined.

The operator $\hat{\rho}$ is expressed in terms of the right-hand side of (7) with the aid of the Green's function of the QKE ($\tilde{G}$). From the definition of $\tilde{G}$ we have

$$(\tilde{\mathscr{H}}+\tilde{J})\tilde{G}=\tilde{I},\ \tilde{I}\hat{a}\equiv\hat{a},\ \tilde{G}=\langle\!\langle\tilde{\mathscr{G}}\rangle\!\rangle, \quad (8)$$

$$\Big(\tilde{\mathscr{H}}+\sum_{l=1}\tilde{u}_{\mathbf{R}_l}\Big)\tilde{\mathscr{G}}=\tilde{I},\quad \tilde{u}_{\mathbf{R}}\hat{a}\equiv i\hbar^{-1}[\hat{u}_{\mathbf{R}},\hat{a}],\quad \hat{u}_{\mathbf{R}}\equiv u(\hat{\mathbf{r}}-\mathbf{R}), \quad (9)$$

where $\hat{a}$ is an arbitrary operator and $\tilde{\mathscr{G}}$ is the Green's function of the "microscopic" equation (6). The product $\tilde{A}\tilde{B}$ is used throughout in its matrix meaning. Applying to (9) the operation $\langle\!\langle\ldots\rangle\!\rangle$, we can easily express $\tilde{G}$ in terms of $\tilde{G}_{\mathbf{R}_1}$, which is a linear transformation obtained by averaging $\tilde{\mathscr{G}}$ over $\mathbf{R}_2,\ldots,\mathbf{R}_{n_0}$ ($\mathbf{R}_1$ is fixed). In turn, $\tilde{G}_{\mathbf{R}_1}$ is connected with $\tilde{G}_{\mathbf{R}_1\mathbf{R}_2}$, and so on. The infinite chain of linear transformations $\tilde{G}$, $\tilde{G}_{\mathbf{R}_1}$, $\tilde{G}_{\mathbf{R}_1,\mathbf{R}_2}$, ... can be replaced by one generating functional

$$\tilde{G}_\Phi=\sum_{(\mathbf{R}_1,\ldots,\mathbf{R}_{n_0})}\tilde{\mathscr{G}}(\mathbf{R}_1,\ldots,\mathbf{R}_{n_0})\exp\Big\{\sum_{l=1}^{n_0}\Phi(\mathbf{R}_l)\Big\}\Big[\sum_{(\mathbf{R}_1,\ldots,\mathbf{R}_{n_0})}\exp\Big\{\sum_{l=1}^{n_0}\Phi(\mathbf{R}_l)\Big\}\Big]^{-1} \quad (10)$$

The summation in (10) is over all the configurations $\{\mathbf{R}_1,\ldots,\mathbf{R}_{n_0}\}$, and $\Phi(\mathbf{R})$ is an arbitrary real function with the meaning of the external field, in which the scattering centers have a Boltzmann distribution. The functional $\tilde{G}_\Phi$ is connected with the linear transformation

$$J_\Phi\equiv\tilde{G}_\Phi^{-1}-\tilde{\mathscr{H}},\quad \tilde{G}_\Phi\tilde{G}_\Phi^{-1}=\tilde{I}, \quad (11)$$

which coincides with $\tilde{J}$ of formula (7) at $\Phi(\mathbf{R})\equiv 0$ and which satisfies the following functional-derivative equation

$$J_\Phi=c_0A_\Phi\sum_{\mathbf{R}}\tilde{u}_{\mathbf{R}}e^{\Phi(\mathbf{R})}-\sum_{\mathbf{R}}\tilde{u}_{\mathbf{R}}\tilde{G}_\Phi(\delta J_\Phi/\delta\Phi_{\mathbf{R}}),$$

$$A_\Phi=\sum_{\mathbf{R}}e^{\Phi(\mathbf{R})}N_0^{-1}. \quad (12)$$

The summation here is over all N_0 sites of the crystal lattice, and $c_0 = n_0/N_0$. In the derivation of (12), we took (9)—(11) into account and used the identity

$$\delta\tilde{G}_\Phi/\delta\Phi_{\mathbf{R}}=-\tilde{G}_\Phi(\delta\tilde{G}_\Phi^{-1}/\delta\Phi_{\mathbf{R}})\tilde{G}_\Phi.$$

The solution of (12) can be represented in the form of the series

$$A_\Phi^{-1}J_\Phi=c_0J_\Phi^{(1)}+c_0^2J_\Phi^{(2)}+\ldots, \quad (13)$$

$$J_\Phi^{(r)}=\sum_{(\mathbf{R}_1,\ldots,\mathbf{R}_r)}J^{(r)}_{\mathbf{R}_1,\ldots,\mathbf{R}_r}\exp\Big\{\sum_{l=1}\Phi(\mathbf{R}_l)\Big\},$$

in which $\tilde{J}^{(r)}_{\{\mathbf{R}_1,\ldots,\mathbf{R}_r\}}$ does not depend on $\Phi(\mathbf{R})$ or c_0. Substituting (13) in (12) and simultaneously expanding $\tilde{G}_\Phi=(\tilde{\mathscr{H}}+\tilde{J}_\Phi)^{-1}$ in powers of $\tilde{J}_\Phi$, we obtain after simple calculations

$$J_{\mathbf{R}}^{(1)}=\tilde{u}_{\mathbf{R}}-\tilde{u}_{\mathbf{R}}\tilde{G}_0J_{\mathbf{R}}^{(1)},\quad \tilde{G}_0\equiv(\tilde{\mathscr{H}}+i0\cdot\tilde{I})^{-1}, \quad (14)$$

$$J_{12}^{(2)}=\tfrac{1}{2}(J+(\tilde{u}_1+\tilde{u}_2)\tilde{G}_0)^{-1}(\tilde{u}_1\tilde{G}_0J_2^{(1)}\tilde{G}_0J_1^{(1)}+\tilde{u}_2\tilde{G}_0J_1^{(1)}\tilde{G}_0J_2^{(1)}), \quad (15)$$

where the subscripts 1 and 2 stand for $\mathbf{R}_1$ and $\mathbf{R}_2$. The smallness of c_0 or $\tilde{u}_{\mathbf{R}}$ (see below) enables us to confine ourselves to the first term of the iteration series (13). Expressing the solution (14) in the form

$$J_{\mathbf{R}}^{(1)}=\tilde{u}_{\mathbf{R}}(\tilde{I}+(\tilde{\mathscr{H}}+\tilde{u}_{\mathbf{R}})^{-1}) \quad (16)$$

and taking into account the identities

$$(\hat{\mathscr{H}}+\tilde{u}_{\mathbf{R}})^{-1}\hat{a}\equiv\frac{\hbar}{2\pi}\int_{-\infty}dz\,\hat{g}_{\mathbf{R}}^{(+)}(z)\hat{a}\hat{g}_{\mathbf{R}}^{(-)}(z),\quad \hat{g}_{\mathbf{R}}^{(\pm)}=(z-\hat{\mathscr{H}}-\hat{u}_{\mathbf{R}}\pm i0)^{-1}, \quad (17)$$

$$\hat{g}_{\mathbf{R}}^{(\pm)}(z)=\hat{g}^{(\pm)}(z)+\hat{g}^{(\pm)}(z)\hat{t}_{\mathbf{R}}^{(\pm)}(z)\hat{g}^{(\pm)}(z),\quad \hat{g}^{(\pm)}(z)=(z-\hat{\mathscr{H}}\pm i0)^{-1}, \quad (18)$$

where $\hat{t}_{\mathbf{R}}^{(\pm)}(z)$ is the scattering operator defined by the equation

$$\hat{t}_{\mathbf{R}}^{(\pm)}=\hat{u}_{\mathbf{R}}+\hat{u}_{\mathbf{R}}(\hat{\mathscr{H}}-z\pm i0)^{-1}\hat{t}_{\mathbf{R}}^{(\pm)}, \quad (19)$$

we obtain, after substituting (17)—(19) in (16) and changing over to the diagonal η representation, the sought "collision integral":

$$(J\hat{\rho})_{\eta\eta'}=\frac{i}{\hbar}c_0\sum_{\mathbf{R},\xi,\xi'}\Big\{\frac{(\hat{t}_{\mathbf{R}}^{(+)}(E_\xi))_{\eta\xi}\rho_{\xi\xi'}(\hat{t}_{\mathbf{R}}^{(-)}(E_\xi))_{\xi'\eta'}}{E_\xi-E_{\eta'}-i0}$$

$$-\frac{(\hat{t}_{\mathbf{R}}^{(+)}(E_\xi))_{\eta\xi}\rho_{\xi'\xi}(\hat{t}_{\mathbf{R}}^{(-)}(E_\xi))_{\xi'\eta'}}{E_\xi-E_\eta+i0}+(\hat{t}_{\mathbf{R}}^{(+)}(E_{\eta'}))_{\eta\xi}\rho_{\xi\eta'}-\rho_{\eta\xi}(\hat{t}_{\mathbf{R}}^{(-)}(E_\eta))_{\xi\eta'}\Big\}. \quad (20)$$

In the derivation of (20) we have discarded integrals of the type

$$\int dz\,\hat{t}^{(+)}(z)[\hat{g}^{(+)}(z),\hat{\rho}]\hat{t}^{(-)}(z)\hat{g}^{(-)}(z).$$

This is valid with quasiclassical accuracy, since the poles of $\hat{t}^{(\pm)}(z)$ have imaginary parts $\gg\hbar\Omega_0$, and $\rho_{\eta\eta'}\approx 0$ at $|n-n'|\gg 1$.

If the system contains two types of scatterers (impurities (I) and d-centers (II)), then the QKE, apart from inessential crossing terms, can be expressed in the form:

$$(\tilde{\mathscr{H}}+J_*)\rho=-e(\mathscr{E}_k\hat{v}_k)f_0'(\mathscr{H}),\quad J_*=J_{\mathrm{I}}+J_{\mathrm{II}}. \quad (21)$$

Here $\tilde{J}_{\mathrm{I}}$ and $\tilde{J}_{\mathrm{II}}$ are obtained from $\tilde{J}$ of formulas (8)—(13) by the substitutions $c_{\mathrm{I,II}}\to c_0$ and $u_{\mathrm{I,II}}\to\hat{u}$ (the subscripts I and II denote throughout quantities pertaining to impurities and d-centers, respectively). The "collision integrals" $\tilde{J}_{\mathrm{I,II}}\hat{\rho}$ are determined in closed form by formula (20) under the condition that the series (13) converge rapidly enough. We shall make the corresponding estimates later on.

For short-range impurities we have $\tilde{J}_{\mathrm{I}}^{(1)}\sim\tilde{J}_{\mathrm{I}}^{(2)}$, and consequently the iteration expansion of $\tilde{J}_{\mathrm{I}}\hat{\rho}$ is in powers of $c_{\mathrm{I}}\ll 1$. To obtain an estimate for the case of d-centers, we expand $\tilde{J}_{\mathrm{II}}^{(1)}\rho$ and $\tilde{J}_{\mathrm{II}}^{(2)}\rho$ (see (13)—(15)) in powers of $\hat{u}_{\mathrm{II}}$ and retain the first two nonvanishing terms in u_0. Since the magnetic-breakdown operator $\hat{\rho}$ cannot be represented, generally speaking, in the "quasiclassical" form (5), while the matrix elements $\rho_{\eta\eta'}$ as functions of $p_Z P_X$ vary significantly over the interval $\sim\kappa p_0\ll\hbar d_0^{-1}$ (see Secs. 1 and 2), it follows that

$$|(\tilde{u}_{\mathrm{II}}\hat{\rho})_{\eta\eta'}|\sim|\hbar^{-1}(\hat{u}_{\mathrm{II}}\hat{\rho})_{\eta\eta'}|. \quad (22)$$

Substituting (22) in $\tilde{J}_{II}^{(1,2)}\hat{\rho}$ and noting that $c_{II} \sim a_0^3 d_0^3$, we get[3)]

$$c_{II}^2 J_{II}^{(2)}\hat{\rho} \sim \beta_0^2 c_{II} J_{II}^{(1)}\hat{\rho}, \quad \beta_0 = u_0 d_0/\hbar v_0.$$

It is interesting to note that β_0 is a parameter of the iteration expansion not only for $\tilde{J}_{II}\hat{\rho}$, but also for $\hat{t}_{II}^{(\pm)}$: if $\beta_0 \ll 1$, then $\hat{t}_{II}^{(\pm)}$ can be calculated, as is well known (see[12]), by the Born approximation, and at $\beta_0 \gg 1$ the quasiclassical approach is valid. It follows from the foregoing that in the calculation of $\tilde{J}_{II}\hat{\rho}$ we can put $\hat{t}_{II}^{(\pm)} = \hat{u}_{II}$ in the first two terms of (20). In the remaining terms it is necessary to take into account the second Born approximation since $\sum_R \tilde{u}_R \hat{\rho} = 0$.

We emphasize that the estimates obtained for the d-centers are valid under the condition $d_0 \lesssim r_H$ ($\hbar d_0^{-1} \gtrsim \kappa p_0$). If $d_0 \gg r_H$, then the criterion for the applicability of (20) begins to depend on H.

The transformations $\tilde{J}_{I,II}$ satisfy the quasiclassical correspondence principal, which is formulated in the following manner: if $\hat{b} = b_m(\hat{p})$ is an arbitrary operator given by (5), then $\tilde{J}_{I,II}\hat{b} = B_m^{(I,II)}(\hat{p})$ where

$$B_m^{(I,II)}(p) = \hat{J}_{I,II}^{(cl)} b_m(p) \equiv \sum_{m'} \int J_{mm'}^{(I,II)}(p, p')\delta(\varepsilon_{m'}(p) - \varepsilon_m(p))(b_m(p) - b_{m'}(p'))dp', \quad (23)$$

$$J_{mm'}^{(I,II)}(p, p') = (2\pi)^{-2}\hbar^{-4} c_V^{(I,II)} |\tau_{mm'}^{(I,II)}(p, p')|^2. \quad (23a)$$

Here $c_V^{(I,II)}$ is the concentration per unit volume, $\tau_{mm}^{(I,II)}(p, p')$ are the scattering operators in the (p, m) representation at H = 0, and $\hat{J}_{I,II}^{(cl)} b$ are the classical collision integrals. Relation (23) is obtained from (20) in the zeroth approximation in κ after essentially straightforward but cumbersome transformations, in which account is taken of the equality

$$\hat{t}^{(\pm)}(E, R) = \hat{L}_R^+ \hat{K}_R^+ \hat{t}^{(\pm)}(E, 0)\hat{K}_R\hat{L}_R,$$
$$\hat{K}_R = \exp\left\{-i\frac{eHY}{c}\hat{x}\right\}, \quad \hat{L}_R = \exp\left(R\frac{\partial}{\partial r}\right), \quad (24)$$

($R = \{X, Y, Z\}$), which follows from (19) and from the commutation relations $[\hat{u}_R, \hat{K}_R] = [\hat{\mathcal{H}}, \hat{K}_R\hat{L}_R] = 0$, and also from the fact that $\hat{t}_{I,II}^{(\pm)}(E, 0) = \hat{\tau}_{I,II}^{(\pm)}$ accurate to quantities $\sim a_0/r_H, d_0/r_H$. The independence of $\tilde{J}_{I,II}\hat{b}$ of the MB parameters plays an important role in what follows.

The reciprocal relaxation times t_{imp}^{-1} and $\bar{t}^{-1}$ are of the same order as the factors $\nu_m^{(I)}(p)$ and $\nu_m^{(II)}(p)$ preceding $b_m(p)$ in the "departure" term of the collision integral (23). In the first order in β_0, we have $\nu_m^{(II)}(p) = \nu_{II}$, where ν_{II} is a constant that does not depend on p or m. Noting that $c_V^{(II)} \sim d_0^{-3}$, we find from (23) and (23a) that $\bar{t}^{-1} \sim \beta_0 u_0 \hbar^{-1}$, and at the same time $q_0 \sim \hbar d_0^{-1}$.

4. CALCULATION OF THE ELECTRIC CONDUCTIVITY (THE "COHERENT" SITUATIONS A AND B)

1. In the coherent situations A and B, the operator $\hat{\rho}_k$ (the subscript k = x, y, z indicates that the electric field $\mathcal{E}$ is directed along the k axis) can be sought in the form of a series in powers of $\gamma = \gamma_{imp} + \bar{\gamma}$. We consider first open periodic configurations. The average velocity is then $(\hat{v}_z)_{\eta\eta} \sim (\hat{v}_x)_{\eta\eta} \sim v_0$ (see Sec. 1) and $\rho_{z,x}$ are determined, accurate to values $\sim \gamma$, by the diagonal part of the QKE (21):

$$(J_s\hat{\rho}_{x,s}^{(0)})_{\eta\eta} = -e\mathcal{E}_{x,s}(\hat{v}_{x,s})_{\eta\eta} f_0'(E_\eta), \quad (\hat{\rho}_{x,s}^{(0)})_{\eta\eta'} = \rho_\eta^{(x,s)}\delta_{\eta\eta'}, \quad \hat{\rho}_{x,s} = \hat{\rho}_{x,s}^{(0)} + O(\gamma). \quad (25)$$

At the same time, $(\hat{v}_y)_{\eta\eta} = 0$. The series for $\hat{\rho}_y$ therefore begins with terms that do not depend on γ. Taking the relation $\hat{\mathcal{H}}p_x = eHc^{-1}\hat{v}_y$ into account, we obtain

$$\hat{\rho}_y = \hat{\rho}_y^{(0)} + \hat{\rho}_y^{(1)} + O(\gamma^2), \quad \hat{\rho}_y^{(0)} = c\mathcal{E}_y H^{-1}(\hat{p}_x f_0'(\hat{\mathcal{H}}) - \hat{C}_y),$$
$$(\hat{\rho}_y^{(1)})_{\eta\eta'} = -i\hbar c\mathcal{E}_y H^{-1}(J_s(\hat{p}_x f_0'(\hat{\mathcal{H}}) - \hat{C}_y)_{m'})_{\eta\eta'}(E_\eta - E_{\eta'})^{-1}; \quad (26)$$

$$(J_s\hat{C}_y)_{\eta\eta} = (J_s\hat{p}_x f_0'(\hat{\mathcal{H}}))_{\eta\eta}, \quad (\hat{C}_y)_{\eta\eta'} = C_\eta^{(y)}\delta_{\eta\eta'}. \quad (26a)$$

The first term in $\hat{\rho}_y^{(0)}$ has the usual "drift" form, and the diagonal operator $\hat{C}_y$ is determined by Eq. (26a), which is obtained from the condition that there be no singularities in the terms $\sim \gamma$. Applying the correspondence principle (23) to the right-hand side of (26a) and neglecting $\hat{J}_{II}^{(cl)}(p_x f_0')$ in the zeroth approximation in t_{imp}/t_{dif}, we obtain

$$J_s(\hat{p}_x\hat{f}_0') = Q_m(\hat{p})\hat{f}_0', \quad Q_m(p) = \sum_{m'}\int J_{mm'}^{(I)}(p, p')\delta(\varepsilon_m(p) - \varepsilon_{m'}(p'))(p_x - p_x')dp'. \quad (27)$$

Formulas (25)–(27) show that the determination of ρ_k reduces to solution of the equations for the diagonal operators $\hat{\rho}_{x,z}^{(0)} \equiv e\mathcal{E}_{x,z}\hat{C}_{x,z}$ and $\hat{C}_y$. Introducting the notation $b_m^{(x,z)}(p) \equiv v_m^{(x,z)}(p)$, $b_m^{(y)} \equiv Q_m(p)$ and expressing $(\hat{J}_s\hat{C}_k)_{\eta\eta}$ in explicit form, we obtain with the aid of (20), (21), the "optical theorem" for $\hat{t}^{(\pm)}$[13], and the relation $2\pi i\delta(x) = (x - i0)^{-1} - (x + i0)^{-1}$,

$$\frac{2\pi}{\hbar}\sum_{R,\eta'}(c_I|t_{\eta\eta'}^{(I)}(E_\eta, R)|^2 + c_{II}|t_{\eta\eta'}^{(II)}(E_\eta, R)|^2)\delta(E_\eta - E_{\eta'})(\chi_\eta^{(k)} - \chi_{\eta'}^{(k)}) = b_{\eta\eta}^{(k)},$$
$$\hat{C}_k = \hat{f}_0'\chi^k. \quad (28)$$

We now express the sums over R in terms of the amplitudes $a_\alpha(\eta)$ (Sec. 1). To this end we use Eqs. (2) and (24) as well as the independence of t(E, 0) of H. Discarding the terms that oscillate rapidly in Y and neglecting the quantities proportional to a_0/r_H and d_0/r_H, we obtain from (28)

$$\sum_{n'}\int_0^{2\pi} d\mathcal{H}'\int dp_z'\sum_{\alpha,\alpha'=1}^{N}|a_\alpha(\eta)a_{\alpha'}(\eta')|^2\bar{J}_{\alpha\alpha'}(p_z, p_z', E_\eta)(\chi_\eta^{(k)} - \chi_{\eta'}^{(k)}) = \sum_{\alpha=1}^{N}\bar{b}_\alpha^{(k)}(p_z, E_\eta)|a_\alpha(\eta)|^2,$$

$$\bar{J}_{\alpha\alpha'} = \bar{J}_{\alpha\alpha'}^{(I)} + \bar{J}_{\alpha\alpha'}^{(II)}, \quad \bar{J}_{\alpha\alpha'}^{(I,II)}(p_z, p_z', E) = \int_{(\alpha, p_z, E)} v_y^{-1}dp_x \int_{(\alpha', p_z', E)} v_y^{-1}dp_x' J_{mm'}^{(I,II)}(p, p'),$$
$$\bar{b}_\alpha^{(k)} = \int_{(\alpha, p_z, E)} b_m^{(k)}(p)v_y^{-1}dp_x, \quad (29)$$

The symbol (α, p_z, E) denotes that the integration proceeds along the contour of the α-th section of the trajectory $\epsilon_{m(\alpha)}(p) = E$, $p_z = \text{const}$; we used formula (5) to express the right-hand side of (29).

It is very important in what follows that the random

dependence of a_α on η is expressed in terms of E_η. Indeed, it follows from (3) and (4) that

$$a_\alpha(\eta)=\bar{a}_\alpha(S(E_\eta,p_z)\hbar^{-1},\mathscr{H}) \quad (S=\{S_1,\ldots,S_N\}),$$

where $\bar{a}_\alpha(\varphi,\mathscr{H})$ are smooth analytic functions of φ_α and $\mathscr{H}$ and are periodic in φ_α and $\mathscr{H}$ with period 2π. This suggests that the solution of (29) be sought in the form

$$\chi_\eta^{(k)}=\chi^{(k)}(S(E_\eta,p_z)\hbar^{-1},\mathscr{H},E_\eta,p_z), \tag{30}$$

where the function $\chi^{(k)}(\varphi,\mathscr{H},p_z,E)$, which is periodic in φ_α and $\mathscr{H}$, is assumed to depend smoothly on all its arguments.

To obtain an effective kinetic equation that determines $\chi^{(k)}(\varphi,\mathscr{H},p_z,E)$ we start from the following two equations:

$$2\pi\hbar\sum_n\delta(E-E_n(p_z,\mathscr{H}))=\sum_{\alpha=1}^N T_\alpha F_\alpha(S(E,p_z)\hbar^{-1},\mathscr{H}), \tag{31}$$

$$|\bar{a}_\alpha(\varphi,\mathscr{H})|^2=F_\alpha(\varphi,\mathscr{H})\left(\sum_{\alpha=1}^N T_\alpha F_\alpha(\varphi,\mathscr{H})\right)^{-1},$$
$$F_\alpha=2\pi\left|\frac{\partial D}{\partial\varphi}\right|\delta(|D(\varphi,\mathscr{H})|), \tag{32}$$

where $D(\varphi,\mathscr{H})$ is obtained from the second determinant of (4) by the substitution $S_\alpha\hbar^{-1}\to\varphi_\alpha$. Formula (31) follows directly from (4); formula (32) can be proved by using the technique of [4]. Equations (31) and (32) enable us to express (29) in the form

$$-2\pi\sum_{\alpha,\alpha'=1}^N|a_\alpha(\eta)|^2\int_0^{2\pi}d\mathscr{H}'\int dp_z'\,\bar{J}_{\alpha\alpha'}(p_z,p_z',E_\eta)F_{\alpha'}(\varphi(E_\eta,p_z'),\mathscr{H}')$$
$$\times[\chi^{(k)}(\varphi(E_\eta,p_z'),\mathscr{H}',E_\eta,p_z')-\chi_\eta^{(k)}]=\sum_{\alpha=1}^N b_\alpha^{(k)}(E_\eta,p_z)|a_\alpha(\eta)|^2,$$
$$\varphi(E_\eta,p_z)=S(E_\eta,p_z)\hbar^{-1}. \tag{33}$$

The functions F_α and $F_\alpha\chi$ can be expanded in a Fourier series in the harmonics $\exp\{il\varphi(E,p_z')\}$ ($l_\alpha=0,\pm1,\ldots$). All the terms of this series with $l\neq0$ oscillate rapidly as functions of p_z', with a period $\sim\varkappa p_0\ll p_0$ and q_0, representing the characteristic intervals of variation of $\bar{J}^{(I)}$ and $\bar{J}^{(II)}$, respectively. In the integration with respect to p_z' they make a small contribution $\lesssim\sqrt{\varkappa p_0/q_0}$ to (33). With this accuracy, we can retain in (33) only the zeroth harmonics F_α and $F_\alpha\chi$, after which only $|a_\alpha(\eta)|^2$ and χ_η remain with an irregular dependence on η. Replacing in them $S_\alpha(E_\eta,p_z)\hbar^{-1}$ by the independent variables φ_α and using the identity (32), we obtain

$$\int_0^{2\pi}\ldots\int_0^{2\pi}d\mathscr{H}'\,d\varphi_1'\ldots d\varphi_N'\int dp_z'W(\varphi,\mathscr{H},p_z|\varphi',\mathscr{H}',p_z')$$
$$\times(\chi^{(k)}(\varphi,p_z,\mathscr{H},E)-\chi^{(k)}(\varphi',\mathscr{H}',p_z',E)=\sum_{\alpha=1}^N\bar{b}_\alpha^{(k)}(E,p_z)F_\alpha(\varphi,\mathscr{H}), \tag{34}$$

$$W(\varphi,\mathscr{H},p_z|\varphi',\mathscr{H}',p_z')=\frac{1}{2\pi^{N+1}}\sum_{\alpha,\beta=1}^N F_\alpha(\varphi,\mathscr{H})$$
$$\times F_\beta(\varphi',\mathscr{H}')\bar{J}_{\alpha\beta}(p_z,p_z',E)>0. \tag{34a}$$

The quantity $W(\varphi,\mathscr{H},p_z|\varphi',\mathscr{H}',p_z')$ in the effective kinetic equation (34) can be interpreted as the probability, per unit time, of a transition with a change of p_z, $\mathscr{H}$, and the "quantum numbers" φ_α. The integral operator in (34) is a degenerate operator of rank N in terms of the variables φ_α and $\mathscr{H}$. It is therefore possible to change from the (N + 2)-dimensional integral equation (34) to the following system of N one-dimensional integral equations:

$$X_\alpha^{(k)}-\sum_{\alpha',\alpha''=1}^N\int M_{\alpha\alpha''}(p_z,E)\bar{J}_{\alpha''\alpha'}(p_z,p_z',E)X_{\alpha'}^{(k)}(p_z',E)dp_z'$$
$$=\sum_{\beta=1}^N M_{\alpha\beta}(p_z,E)\bar{b}_\beta^{(k)}(p_z,E), \tag{35}$$

$$M_{\alpha\alpha'}=2\pi\left\langle\left|\frac{\partial D}{\partial\varphi_\alpha}\frac{\partial D}{\partial\varphi_{\alpha'}}\right|\delta(|D(\varphi,\mathscr{H})|)\left[\sum_{\beta=1}^N\left|\frac{\partial D}{\partial\varphi_\beta}\right|\nu_\beta(p_z,E)\right]^{-1}\right\rangle,$$

$$\nu_\beta=\int_{(\beta,p_z,E)}(v_m^{(I)}(p)+v_{II})\frac{dp_x}{v_y}. \tag{35a}$$

Here $X_\alpha^{(k)}=\langle\chi^{(k)}(\varphi,\mathscr{H},p_z,E)F_\alpha(\varphi,\mathscr{H})\rangle$, the angle brackets $\langle\,\rangle$ denote averaging over φ_α and $\mathscr{H}$ with weight $(2\pi)^{-N-1}$, the quantities $\nu_m^{(I)}(p)$ and ν_{II} are defined in Sec. 3, and $X_\alpha^{(k)}(p_z,E)$ is proportional to the number density of particles located on section α. In the derivation of (35) we used expressions (35) and took into account the fact that $\langle F_\alpha\rangle=1$. In (35), the entire dependence on w(H) is contained in the matrices $M_{\alpha\alpha'}$, which describe the effective magnetic-breakdown relaxation in the plane p_z = const.

To calculate

$$\sigma_{ik}=(V\mathscr{E}_k)^{-1}\mathrm{Sp}\,\hat{\rho}_k\hat{v}_i$$

in terms of χ, we note that

$$\mathrm{Sp}\,\hat{d}\hat{\chi}\hat{f}_0'=\frac{V}{(2\pi\hbar)^3}\sum_{\alpha=1}^N\int dp_z X_\alpha(p_z,\varepsilon_F)\int_{(\alpha,p_z,\varepsilon_F)}d_m(p)\frac{dp_x}{v_y};$$
$$\mathrm{Sp}\,\hat{d}\hat{J}\hat{a}=\mathrm{Sp}\,\hat{a}\hat{J}\hat{d}. \tag{36}$$

Here $\hat{a}$ is an arbitrary Hermitian operator and $\hat{d}=d_m(\hat{p})$ is an arbitrary operator of the type (5). The first relation in (36) can be proved on the basis of (5) and (30)–(32), neglecting the quantities that oscillate rapidly in p_z. The second relation can be verified directly with the aid of (20) and (23); as seen from (26), it enables us to use the correspondence principle (23) to calculate σ_{iy} in the first nonvanishing order in γ. After simple transformations, in which (23), (25)–(27), and (36) are taken into account, we obtain

$$\sigma_{\mu\mu'}=\frac{e^2}{(2\pi\hbar)^3}\sum_{\alpha=1}^N\int\bar{v}_\alpha^{(\mu)}(p_z,\varepsilon_F)X_\alpha^{(\mu')}(p_z,\varepsilon_F)dp_z \quad (\mu,\mu'=x,z), \tag{37}$$

$$\sigma_{\mu y}=\frac{ecH^{-1}}{(2\pi\hbar)^3}\left(\sum_m\int_{\varepsilon_m(p)=\varepsilon_F}v_m^{(\mu)}(p)p_x\frac{dS_p}{|v_m(p)|}\right.$$
$$\left.-\sum_{\alpha=1}^N\int\bar{v}_\alpha^{(\mu)}(p_z,\varepsilon_F)X_\alpha^{(y)}(p_z,\varepsilon_F)dp_z\right), \tag{38}$$

$$\sigma_{yy}=\frac{(c/H)^2}{(2\pi\hbar)^3}\left(\sum_m\int_{\varepsilon_m(p)=\varepsilon_F}Q_m(p)p_x\frac{dS_p}{|v_m(p)|}\right.$$
$$\left.-\sum_{\alpha=1}^N\int\bar{Q}_\alpha(p_z,\varepsilon_F)X_\alpha^{(y)}(p_z,\varepsilon_F)dp_z\right). \tag{39}$$

Here dS_p is the area element on the Fermi surface; the entire dependence on the MB is contained in the "diagonal" quantities $X_\alpha^{(k)}$; $\sigma_{ik}(H)=\sigma_{ki}(-H)$. Formulas (37)–(39), together with (35), describe completely the two coherent situations A and B.

Closed configurations differ from the open ones only in that E_η and $a_\alpha(\eta)$ do not depend on $\mathscr{H}$ and in that $v_{\eta\eta}^{(x)}=0$. In this case σ_{iy} and σ_{zz} can be calculated from formulas (37)–(39) and (35), by regarding $D(\varphi)$ as the first of the determinants of (4) ($S\hbar^{-1}\to\varphi$); σ_{ix}

for closed configurations is obtained from σ_{iy} by making the substitution $p_x \to -p_y$ in (27), (38), and (39).

2. Let us examine the transition to the usual classical description[6] as $w \to 0$ and $w \to 1$, when the magnetic breakdown configuration breaks up into L_0 and L_1 independent classical trajectories made up of one or several sections α. For the sake of argument, these trajectories can be regarded as closed. It follows therefore from (1), (4), and (32) that

$$\lim_{w\to 0,1} |D(\varphi)| = 2^L \prod_{l=1}^{L} \left| \cos\left(\sum_{(l)} \varphi_\alpha/2 \right) \right|, \quad L = L_0, L_1, \tag{40}$$

where l numbers the independent orbits and the symbol (l) means that the summation is over all the sections α belonging to the orbit (if the magnetic-breakdown configuration is open, then a phase shift equal to $\mathcal{K}/2$ appears in the arguments of several of the cosine functions). Substituting (40) in (35), we find that

$$\lim_{w\to 0,1} M_{\alpha\alpha'} = \nu_l^{-1} = \left(\sum_{(l)} \nu_\alpha \right)^{-1},$$

if α and α' pertain to the same l; otherwise, $M_{\alpha\alpha'} = 0$. From this fact and from (35) it follows that all $X_\alpha = X_l$ on the orbit l, where X_l satisfy, accurate to $\sim t_{imp}/t_{dif}$, the classical electron-impurity kinetic equation averaged over the time of revolution along the orbits l:

$$\sum_{l'=1}^{L} \int \bar{J}_{ll'}^{(I)}(p_z, p_z') \left(X_l^{(k)}(p_z) - X_{l'}^{(k)}(p_z') \right) dp_z' = \bar{b}_l^{(k)}(p_z),$$

$$\bar{J}_{ll'}^{(I)} = \sum_{(l)} \sum_{(l')} \bar{J}_{\alpha\alpha'}^{(I)}, \quad \bar{b}_l^{(k)} = \sum_{(l)} \bar{b}_\alpha^{(k)}. \tag{41}$$

The quantity $X_l^{(k)}(p_z)$ has the meaning of a classical distribution function that is homogeneous along each of the orbits.

We consider next the Hall mobility σ_{xy} for metals with closed magnetic-breakdown configurations. In this case the second term, which results from Tr $\hat{v}_x \hat{\chi}(y)$ vanishes together with $v_{\eta\eta}^{(x)}$, and at $n_- \neq n_+$ (n_- and n_+ are the number densities of the electrons and holes) we arrive at the well-known formula

$$\sigma_{xy} = ec(n_- - n_+) H^{-1}. \tag{42}$$

The form of the remaining σ_{ik} (including σ_{xy} for open magnetic-breakdown configurations) depends essentially on the ratio of t_{imp} and $\bar{t}$.

3. In case A it is in general impossible to establish an explicit dependence of σ_{ik} on w, owing to the complicated structure of the integral kernels $\bar{J}_{\alpha\alpha'}(p_z, p_z') \approx \bar{J}_{\alpha\alpha'}^{(I)}(p_z, p_z')$. A qualitative investigation of this situation can be carried out in the t_0 approximation, putting $t_0 = t_{imp}$[4,14,15]. We consider here rigorously a special but frequently encountered (Be, Mg, Zn, etc.) case, when a narrow layer of open MB configurations, of width $\delta p \ll p_0$, exists in p-space, and the remaining configurations are closed. If γ_{imp} is small enough, then the main contribution to $X_\alpha^{(x)}$ is made by electrons moving with average transverse velocity $|\partial E_n/\partial P_x| \sim v_0$ (see Sec. 1), i.e., electrons from a narrow layer. Consequently, at k = x the integral part of Eqs. (35) is proportional to $(\delta p/p_0) X_\alpha^{(y)}$. Discarding it in the zeroth order in $\delta p/p_0$ and calculating the integrals $M_{\alpha\beta}$ in the right-hand side of (35), we easily obtain σ_{xx} and σ_{zx} (37) for an arbitrary system of magnetic-breakdown orbits.

Let us examine by way of example the configuration of Fig. b, which is symmetrical about the p_y axis, assuming for simplicity that $T_2 \ll T_1$ ($T_{1,2}$ are the periods of revolution along the orbits 1 and 2). In view of the symmetry of the orbits, the integration variables in (35a) are $\varphi_1 = \varphi_{1,+} = \varphi_{1,-}$, $\varphi_2 = \varphi_{2,+} = \varphi_{2,-}$ ($\alpha \equiv (m, \pm)$, where "+" and "–" denote the upper and lower sections of the orbit m). It also follows from symmetry considerations that $X_\alpha^{(y)} = 0$ outside the layer δp. The integral part of (20) is therefore small also at k = y. Taking the foregoing into account, we obtain with the aid of (37), (38), (35), and (32) in the zeroth-order approximation in T_2/T_1:

$$\sigma_{xx} = \sigma_{xx}^{(0)} + \sigma_{xx}^{(MB)}, \quad \sigma_{xx}^{(MB)} = \frac{8e^2 b_y^2}{\pi (2\pi\hbar)^3} \int \arcsin \frac{w(H, p_z, \varepsilon_F)}{2 - w(H, p_z, \varepsilon_F)} \frac{dp_z}{\bar{v}(p_z, \varepsilon_F)}, \tag{43}$$

$$\sigma_{xy} = \sigma_{xy}^{(0)} + \sigma_{xy}^{(MB)}, \quad \sigma_{xy}^{(MB)} = -\frac{8ecb_y H^{-1}}{(2\pi\hbar)^3 \pi} \int \frac{\bar{Q}_+(p_z, \varepsilon_F)}{\bar{v}(p_z, \varepsilon_F)} \times \arcsin \frac{w(H, p_z, \varepsilon_F)}{2 - w(H, p_z, \varepsilon_F)} dp_z. \tag{44}$$

Here $\sigma_{ik}^{(0)} = \sigma_{ik}|_{w=0}$, $\bar{\nu} = \bar{\nu}_{1,+} = \bar{\nu}_{1,-}$, $\bar{Q}_+ = \bar{Q}_{1,+} = -\bar{Q}_{1,-}$. Since $\sigma_{xx}^{(MB)} \sim w(\delta p/p_0)\sigma_0$ (σ_0 is the electric conductivity at H = 0), we get $\sigma_{xx}^{(MB)} \gg \sigma_{xx}^{(0)} \sim \gamma_{imp}^2 \sigma_0$ at $w \gg \gamma_{imp}^2 (p_0/\delta p)$. The effect of the magnetic breakdown on σ_{xy} becomes manifest only in metals with $n_- = n_+$, where $\sigma_{xy}^{(0)} \sim \gamma_{imp}^2 \sigma_0$. According to (44), the "uncompensation" produced by the MB becomes significant when $w \gg \gamma_{imp}(p_0/\delta p)$.

Even at small angles ϑ between H and the chosen symmetry directions, all the variables $\varphi_{m,\pm}$ in (35a) become independent. This changes radically the dependence of $M_{\alpha\beta}$, σ_{xx}, and σ_{xy} on w. According to the scheme for the derivation of (34), this change occurs when $|S_{m,+} - S_{m,-}|\hbar^{-1}$ becomes much larger than unity, corresponding to $\vartheta \gtrsim \sqrt{\kappa}$. Such a strong anisotropy of σ_{xx} and σ_{xy} is due to coherent interference effects that are typical of MB.

4. Case B is easiest to investigate by making direct use of (34), which can be rewritten in the form

$$(\hat{W}_I + \hat{W}_{II}) \chi^{(k)}(\varphi, k, p_z) = B^{(k)}(\varphi, \mathcal{K}, p_z), \quad B^{(k)} \equiv \sum_{\alpha=1}^{N} b_\alpha^{(k)}(p_z, \varepsilon_F) F_\alpha(\varphi, k). \tag{45}$$

The operators $\hat{W}_I$ and $\hat{W}_{II}$ are obtained from "collision integral" (34) by replacing $\bar{J}_{\alpha\beta}$ with $\bar{J}_{\alpha\beta}^{(I)}$ and $\bar{J}_{\alpha\beta}^{(II)}$, respectively. Since the function $\chi^{(k)}$ changes significantly together with $\bar{b}_\alpha^{(k)}(p_z)$ over an interval $\sim p_0 \gg q_0$, it follows that $\bar{J}_{\alpha\alpha'}^{(II)}(p_z, p_z')$ in (45) can be represented, accurate to quantities $\sim q_0/p_0$, in the following form:

$$\bar{J}_{\alpha\alpha'}^{(II)}(p_z, p_z') = m_\alpha^*(p_z) \nu_{II} \delta_{\alpha\alpha'} \delta(p_z - p_z'), \tag{46}$$

$$m_\alpha^* = \int_{(\alpha, p_z, \varepsilon_F)} dp_x/v_y.$$

According to this expression, the integral kernel of the operator $\hat{W}_{II}$ depends parametrically on p_z, and consequently the transitions with change of p_z in (45) are determined only by the operators $\hat{W}_I$, i.e., by the impurities. On the other hand, the inequality $\bar{t} \ll t_{imp}$ makes it possible to solve (45) by iteration with respect to the parameter $\bar{t}/t_{imp}$, regarding $\hat{W}_I$ as a small per-

turbation. It must be recognized here that the homogeneous equation $\hat{W}_{II}\chi = 0$ has a nontrivial solution $\chi_0(\varphi, \mathscr{K}, p_z) \equiv \chi(p_z)$, where $\chi(p_z)$ is an arbitrary function of p_z. This suggests that the solution of (45) be sought in the form

$$\chi^{(k)}=\bar{\chi}^{(k)}(p_z)+\chi_1^{(k)}(\varphi,\mathscr{K},p_z)+O((\bar{t}/t_{imp})^2),$$

where $\chi_1^{(k)}$ satisfies the equation

$$\hat{W}_{II}\chi_1^{(k)}=B^{(k)}(\varphi,\mathscr{K},p_z)-\hat{W}_I\bar{\chi}^{(k)}(p_z), \qquad (47)$$

and the function $\bar{\chi}^{(k)}$ is determined from the following condition of solvability of Eq. (47): $\langle(W_I\bar{\chi}^{(k)} - B^{(k)})\rangle = 0$ (the operation $\langle\ldots\rangle$ is defined in (35)). Recognizing that $\langle F_\alpha\rangle = 1$, this relation can be rewritten in the form

$$\int\Big(\sum_{\alpha,\alpha'=1}^{N}\bar{J}_{\alpha\alpha'}^{(I)}(p_z,p_z')\Big)(\bar{\chi}^{(k)}(p_z)-\bar{\chi}^{(k)}(p_z'))\,dp_z'=\bar{B}^{(k)}(p_z), \qquad (48)$$

$$\bar{B}^{(k)}=\sum_{\alpha=1}^{N}\bar{b}_\alpha^{(k)}(p_z).$$

If $\bar{B}^{(k)} \neq 0$, then $\chi_1^{(k)} \sim (\bar{t}/t_{imp})\tilde{\chi}^{(k)}$, and we have $X_\alpha^{(k)} = \bar{\chi}^{(k)}(p_z) + O(\bar{t}/t_{imp})$ in (37) and (38). It follows therefore that the quantity $\bar{\chi}^{(k)}(p_z)$ can be regarded as a sort of "ergodic" distribution function that is general for the entire magnetic-breakdown configuration. As is seen from (48), this quantity is independent of w and ν_{II}. We emphasize that the ergodic distribution $X_\alpha^{(k)}(p_z) = \bar{\chi}^{(k)}(p_z)$ does not coincide, generally speaking, with the classical distributions $X_l^{(k)}$ (see (41)) that are obtained in the limit as w → 0 or 1. The point is that at w = 0 or 1 the homogeneous equation $\hat{W}_{II}\chi = 0$ has L_0 or L_1 independent solutions (as many as there are independent classical trajectories). In the general case, $L_{0,1} > 1$ and (48) is not a sufficient condition for the solvability of (47). A simple analysis shows that the transition from (47) to the classical equations (41)[4] occurs at $w(1-w) \sim \bar{t}/t_{imp} \ll 1$.

In a number of situations of physical interest we have $\bar{B}^{(k)} = 0$ (see below). In this case $\bar{\chi}^{(k)} = 0$, $X_\alpha^{(k)} = \langle\chi_1^{(k)}F_\alpha\rangle$. As a result of (46) and of the degeneracy of the integral kernel $\hat{\bar{W}}_{II}$ with respect to φ_α and k, Eq. (47) is equivalent to the following system of linear algebraic equations for the quantities $X_\alpha^{(k)}$:

$$X_\alpha^{(k)}(p_z)-\sum_{\beta=1}^{N}M_{\alpha\beta}'m_\beta^*X_\beta^{(k)}(p_z)=\nu_{II}^{-1}\sum_{\beta=1}^{N}M_{\alpha\beta}'\,b_\beta^{(k)}(p_z), \qquad (49)$$

where $M'_{\alpha\beta}$ is obtained from $M_{\alpha\beta}$ of the system (35) by the substitution $\bar{\nu}_\beta \to m_\beta^*$. As seen from (49), the quantities $X_\alpha^{(k)}$, which are proportional to $\bar{\nu}_{II}^{-1}$, can be expressed in elementary fashion via the integrals $M'_{\alpha\beta}(w)$, which are determined uniquely by the topology of the magnetic-breakdown configuration and by the values of the masses m_β^*.

The results can be interpreted in the following manner. If $\bar{B}^{(k)} = 0$, then the stationary current in the system is established mainly by the rapid relaxation, due to the d-centers, of the "coherent" variables $\mathscr{K}$ and φ_α. This process is described by Eq. (49). At $\bar{B}^{(k)} \neq 0$, the presence of the d-centers leads only to "ergodicity" of the electron distribution in the plane p_z = const, and stationarity is ensured by the electron-impurity interaction, which does not conserve p_z.

We now apply the calculation scheme described above to open configurations, which break up into closed orbits as w → 0 or w → 1 (see Figs. b and c). In this situation, which is typical of MB, we have:

$$\bar{B}^{(x)}=\sum_{\alpha=1}^{N}\Delta_\alpha\equiv 0,\quad \Delta_\alpha=\int\limits_{(\alpha,p_z,E)}dp_y;\quad \bar{B}^{(y)}=\sum_{\alpha=1}^{N}\bar{Q}_\alpha;$$

$$\bar{B}^{(z)}=\frac{\partial}{\partial p_z}\sum_{\alpha=1}^{N}\int\limits_{(\alpha,p_z,E)}p_x\,dp_y\neq 0. \qquad (50)$$

It follows therefore that σ_{xx} and σ_{zx} are determined in closed form on the basis of (49) ($b_\alpha^{(x)} \equiv \Delta_\alpha$) and (37). In particular, for the symmetrical configuration b in the figure ($T_2 \ll T_1$) we have

$$\sigma_{xx}=\frac{16e^2b_y^2}{(2\pi\hbar)^3\nu_{II}}\int\frac{\Lambda(w)\,dp_z}{m_1^*(p_z,\varepsilon_F)\,(1-\Lambda(w))}\sim\frac{w}{1-w}\frac{\bar{t}}{t_{imp}}\sigma_0,$$

$$\Lambda(w)\equiv\frac{1}{2\pi}\arcsin\frac{w}{2-w}. \qquad (51)$$

Here m_1^* is the effective mass of the orbit 1. Formula (51) holds true for $w(1-w) \gg \bar{t}/t_{imp}$; in its derivation, the width of the layer of open configurations, unlike in (43), was assumed to be arbitrary.

The remaining components, namely σ_{iy} and σ_{zz} ((37)–(39)) and, in analogy, the entire tensor σ_{ik}, are determined for closed configurations, accurate to $\sim \bar{t}/t_{imp}$, by the ergodic distributions $X^{(y,z)} = \bar{\chi}^{(y,z)}(p_z, \epsilon_F)$, which satisfy Eq. (48). It is of interest to note that the second term in (38) vanishes, owing to the ergodicity of $X_\alpha^{(y)}$ and therefore there is no magnetic-breakdown "uncompensation" of σ_{xy} in the zeroth order approximation in $\bar{t}/t_{imp}$ (cf. (44)).

5. CALCULATION OF THE ELECTRIC CONDUCTIVITY (CASE C)

As is clear from the foregoing, the density matrices $\hat{\rho}_k$ describing the coherent situations A and B must satisfy the relation $|(\hat{J}_{II}\hat{\rho})_{\eta\eta'}| \sim \nu_{II}|\rho_{\eta\eta'}|$. For the case C ($\bar{\gamma} \gg 1$) this estimate is incorrect. To verify this, we assume the opposite. Then, accurate to terms $\sim \bar{\gamma}^{-1}$, we have $\hat{\bar{J}}_{II}\hat{\rho} = -e\mathscr{E}_k\hat{v}_k\hat{f}_0'$. According to the correspondence principle (23), this equation has the quasiclassical solution $\hat{\rho} = \bar{\rho}_m(\hat{p})$ and $\hat{J}_{II}^{(c)}\bar{\rho} = -e\mathscr{E}_k\hat{v}_kf_0'$, where $\hat{J}_{II}^{(cl)})$ is determined by formula (23). Since

$$\hat{J}_{II}^{(cl)}\bar{\rho}\sim t_{dif}^{-1}\bar{\rho}\ll\Omega_0\bar{\rho},$$

we arrive at a contradiction.

The foregoing reasoning suggests that we seek the solution of the QKE (21) in the form $\hat{\rho} = \rho_m^{(0)}(\hat{p}) + \hat{\rho}^{(1)}$, where $\rho_m^{(0)}(p)$ satisfies the classical Boltzmann equation with an electron-impurity collision integral $\hat{J}_I^{(cl)}$, and $\hat{\rho}^{(1)}$ is a small increment to $\hat{\rho}^{(0)}$ and has no classical analog. The influence of MB becomes manifest in the fact that $\rho_m^{(0)}(p)$ experiences discontinuities on going through the magnetic-breakdown nodes. With allowance for this circumstance, the equation for $\rho_m^{(0)}(p)$ takes the form

$$\partial\rho_\alpha/\partial\tau_\alpha+\hat{J}_I^{(cl)}\rho_m^{(0)}(p)=-e\mathscr{E}_k\hat{v}_kf_0'(E), \qquad (52)$$

$$\rho_\alpha(\tau_\alpha,E,p_z)\equiv\rho_{m(\alpha)}^{(0)}(p(\tau_\alpha,E,p_z)).$$

The variable τ_α was introduced in Sec. 1 (formula 5)); the functions $\rho_\alpha(\tau_\alpha, E, p_z)$ are analytic on the intervals

$[0, T_\alpha]$. For an unambiguous determination of ρ_α, Eq. (52) should be supplemented by boundary conditions, the form of which is determined by the requirement $|\rho^{(1)}_{\eta\eta'}| \ll |\rho^{(0)}_{\eta\eta'}|$.

Substituting $\hat\rho = \hat\rho^{(0)} + \hat\rho^{(1)}$ in the QKE (21) and applying formulas (5), (23), and (52) to $\rho^{(0)}$, we obtain after simple transformations, carried out in the zero-order approximation in $\Omega_0\tau_0 \ll 1$, taking into account the discontinuous character of $\rho^{(0)}_m(p)$,

$$(\tilde{\mathscr{H}}+J_*)\hat\rho^{(1)} = \sum_{\alpha=1}^{N}(\hat f_\alpha^{(+)} - \hat f_\alpha^{(-)}), \quad \hat f_\alpha^{(\pm)} = \rho_\alpha^{(\pm)}\,(\hat{\mathscr{H}}, \hat p_z)\hat\delta_\alpha(\tau_\alpha^{(\pm)}), \qquad (53)$$

$$\rho_\alpha^{(\pm)}(E, p_z) \equiv \rho_\alpha(\tau_\alpha^{(\pm)}, E, p_z), \quad \tau_\alpha^{(\pm)} = T_\alpha - \varepsilon, \quad \tau_\alpha^{(-)} = \varepsilon, \quad \varepsilon \sim \tau_0,$$

where the matrix elements of the operator $\hat\delta_\gamma(t)$ are obtained in the η representation by substituting the δ functions $\delta_{\alpha\gamma}\delta(\tau_\alpha - t)$ in the quasiclassical expression (5)[5]. It is expedient in what follows to express $\hat\rho^{(1)}$ in terms of the solutions $\Phi_\alpha^{(\pm)}(t)$ of the nonstationary QKE, which satisfy the initial conditions $(\hat\Phi_\alpha^{(\pm)}(0) = \hat f_\alpha^{(\pm)})$;

$$\hat\rho^{(1)} = \sum_{\alpha=1}^{N}\int_0^\infty (\hat\Phi_\alpha^{(+)}(t) - \hat\Phi_\alpha^{(-)}(t))\,dt, \quad \hat\Phi_\alpha^{(\pm)}(t) = \hat L(t)\hat f_\alpha^{(\pm)}, \qquad (54)$$

$$\hat L \equiv \exp\{-(\tilde{\mathscr{H}}+J_*)t\}.$$

We now obtain the form of $\hat\Phi_\alpha^{(\pm)}(t)$ for $t = t'$, where t' is an arbitrary instant of time satisfying the inequality $\tau_0 \ll t' \ll \nu_{II}^{-1} \ll \Omega_0^{-1}$. In the zeroth approximation in $\nu_{II}\tau_0$ we obtain

$$\hat\Phi_\alpha^{(\pm)}(t') = \exp\{-\tilde{\mathscr{H}}t'\}\,\hat f_\alpha^{(\pm)}.$$

Since the operators $\hat f_\alpha^{(+)}$ and $\hat f_\alpha^{(-)}$ are "localized" on opposite sides of the MB region, it follows that $\hat\Phi^{(+)}(t')$ and $\hat\Phi^{(-)}(t')$ have entirely different structures. Whereas $\Phi^{(-)}(t')$ is completely determined by quasiclassical dynamics

$$\hat\Phi_\alpha^{(-)}(t') = \hat\rho_\alpha^{(-)}\hat\delta_\alpha(t') \quad (\hat\Phi_\alpha^{(-)}(t) = \hat L(t-t')\hat\Phi_\alpha^{(-)}(t')), \qquad (55)$$

the operators $\hat\Phi_\alpha^{(+)}(t')$ are strongly influenced by the magnetic-breakdown scattering. Their structure becomes clear if we express $\hat\Phi^{(+)}(t')$ in the (p, m) representation with allowance for the inequality $kT \gg h\Omega_0$, $\tau_0 \ll t'$. Omitting the intermediate steps, which are based on the technique used in[4], we present the final result directly:

$$\hat\Phi_\alpha^{(+)}(t') = \hat\rho_\alpha^{(+)}((1-w)\hat\delta_{\alpha'}(t') + w\hat\delta_{\beta'}(t')) + \hat A_\alpha(t'),$$

$$\hat A_\alpha(t') = \sum_{p,p'} A_\alpha(p_y, p_y')\,\delta_{p_z p_z'}\,\delta_{p_z p_z'}\,|m(\alpha'), \mathbf{p}\rangle\langle m(\beta'), \mathbf{p}|$$

$$(\hat\Phi_\alpha^{(+)}(t) = \hat L(t-t')\hat\Phi_\alpha^{(+)}(t')). \qquad (56)$$

Here α' and β' label the sections that go out of that magnetic-breakdown node into which α enters (cf. Sec. 1) and w is the MB probability for the same node; the function $A(p_y, p_y')$ differs from zero with respect to both variables in a small vicinity of the point $p_y(\tau_{\alpha'} = t') \approx p_y(\tau_{\beta'} = t')$, and we are not interested in its detailed form.

A decisive factor in what follows is that $\hat A_\alpha(t')$ is antidiagonal with respect to m in the (p, m) representation ($m(\alpha') \neq m(\beta')$), and consequently has no classical analog. Besides $\hat A_\alpha(t')$, the same property is possessed by the operator $\hat A_\alpha(t) = \hat L(t-t')\hat A_\alpha(t')$, which remains antidiagonal for arbitrary $t \ll \Omega_0^{-1}$.[6] For this operator we have

$$\mathrm{Sp}\,\hat A_\alpha J_* \hat A_\alpha \sim \nu_{II}\,\mathrm{Sp}\,\hat A_\alpha^2 \qquad (\mathrm{Sp}\,\hat A_\alpha J_* \hat A_\alpha > 0).$$

This estimate, together with the identity

$$(d/dt)\,\mathrm{Sp}\,\hat A_\alpha^2 + \mathrm{Sp}\,\hat A_\alpha J_* \hat A_\alpha = 0,$$

which follows from the nonstationary QKE and the relation $(\tilde J\hat A)^+ = \tilde J\hat A^+$ (it follows from (6) with allowance for the fact that $\hat{\mathscr{H}}$ and $\hat u$ are Hermitian) shows that $\hat A_\alpha(t) \to 0$ like $\exp[-\nu_{II}t]$. Consequently, $\hat A_\alpha(t)$ makes a small contribution, $\lesssim \overline{\gamma}^{-1}$, to the integral (54). (The vanishing of $\hat A_\alpha(t)$ does not contradict the particle-number conservation law $(d/dt)\mathrm{Tr}\,A_\alpha(t) = 0$, since $\mathrm{Tr}\,\hat A_\alpha(t') = 0$). To the contrary, the operators $\hat L(t-t')\hat\rho^{(\pm)}\hat\delta_\alpha(t')$ (see (55) and (56)) remain finite as $t \to \infty$ ($\mathrm{Tr}\hat\rho^{(\pm)}\hat\delta_\alpha(t') \neq 0$), and therefore makes an infinite contribution to (56). These infinities must be strictly cancelled out. As seen from (55) and (56), this compensation arises if

$$\rho_{\alpha'}^{(-)}(E, p_z) = (1-w)\rho_\alpha^{(+)}(E, p_z) + w\rho_\beta^{(+)}(E, p_z),$$

$$\rho_{\beta'}^{(-)}(E, p_z) = w\rho_\alpha^{(+)}(E, p_z) + (1-w)\rho_\beta^{(+)}(E, p_z), \qquad (57)$$

where α, β, α', β', and w are defined in the same manner as in (1). Equations (57) ensure the smallness of the increment ($|\rho^{(1)}_{\eta\eta'}| \sim \nu_{II}\tau_0|\rho^{(0)}_{\eta\eta'}|$ according to (54)–(57)) and are simultaneously the sought boundary conditions needed for a unique determination of $\rho_\alpha(\tau_\alpha, E, p_z)$ (see (52)) and of the current density

$$j = \frac{e^2H}{c(2\pi\hbar)^3}\sum_{\alpha=1}^{N}\int dp_z\int_0^{\tau_\alpha} v(\tau_\alpha, \varepsilon_F, p_z)\rho_\alpha(\tau_\alpha, \varepsilon_F, p_z)\,d\tau_\alpha. \qquad (58)$$

The boundary conditions (57) express mathematically the fact that w(H) acquires a stochastic meaning under the conditions C (see Sec. 1). In the special case $\hat J_I^{(cl)} \equiv t_0^{-1}$ (t_0 approximation) the approach developed here is equivalent to the method postulated by FS[2]

If $\gamma_{imp} \ll 1$, then formulas (52), (57), and (58) enable us to obtain the dependence of σ_{ik} on w in closed form for arbitrary $\hat J_I^{(cl)}$ and for arbitrary magnetic-breakdown configurations. Without dwelling on the details of the rather simple calculations, we indicate that the components σ_{iy} and σ_{zz} for open systems of magnetic-breakdown orbits (x is the direction in which the orbit is open in coordinate space), and also the entire tensor σ_{ik} in the case of closed configurations, do not depend on w in the first nonvanishing order in $\gamma_{imp}/w(1-w)$. These components σ_{ik} are identical to the σ_{ik} calculated on the basis of ergodic distributions of the coherent situation B (Sec. 4, subsection 4). The transition to the limit w = 0, 1 in case C occurs on intervals $w \lesssim \gamma_{imp}$, $1-w \lesssim \gamma_{imp}$, i.e., more abruptly than under coherent conditions.

Under the conditions C, the component σ_{xx} for open configurations remains finite even at $\hat J_I^{cl} = 0$. In the limit as $\gamma_{imp} \to 0$ we have

$$\rho_\alpha(\tau_\alpha) = \rho_\alpha^{(-)} - c\mathscr{E}_x H^{-1} f_0' p_y(\tau_\alpha),$$

$$\rho_\alpha^{(+)} - \rho_\alpha^{(-)} = -c\mathscr{E}_x H^{-1}\int_{(\alpha, p_z, E)} dp_y. \qquad (59)$$

Solving simultaneously the systems of linear algebraic equations (57) and (59) and recognizing that $\rho_m^{(0)}(p_y) = \rho_m^{(0)}(p_y + b_y)$, we can easily obtain the dependence of σ_{xx} and σ_{xx} on w for each concrete magnetic-breakdown configuration. At $w(1-w) \sim 1$, this yields $\sigma_{xx} \sim (n_- + n_+)ecH^{-1}$. The appearance of dissipative

effects, the values of which do not depend on γ_{imp} and $\bar{\gamma}$, qualitatively distinguishes case C from the coherent situations A (43) and B (51).

The method described above can easily be generalized to include the case of magnetic-breakdown configurations that contain very small orbits with Larmor frequencies $\Omega' \gg \nu_{II} \gg \Omega_0$ (we consider below, for simplicity, small "two-corner" orbits, such as orbits 2 in the figure). Eliminating the amplitudes of these orbits from the system (3), we can set each of these orbits in correspondence with an effective s matrix that connects the amplitudes of the four "large" sections directly. The effective MB probability (w') then depends periodically on $\varphi' \equiv cS'(E, p_z)/ehH$ (S' is the area of the two-corner orbit) and is equal to

$$w' = w^2[w^2 + 4(1-w)\sin^2(\varphi'/2+\omega)]^{-1}$$

(the MB probability w and the phase ω, see (1), are assumed here to be the same for both corners of the orbit). If $\hbar\Omega' \gg kT \gg h\Omega_0$, then w in (56) and (57) should be replaced by $w'(\varphi'(\epsilon_F, p_z))$, taking α, β, α' and β' to be the indices of the large sections joined by the two-corner orbit, and consequently $\rho_\alpha(\tau_\alpha, \epsilon_F, p_z)$ in (58) now oscillates as a function of 1/H with a period $2\pi eh/cS'$ (ϵ_F, p_z). On the other hand, if $\hbar\Omega' \ll kT$, then we must make in (56) and (57) the substitution

$$w \to (2\pi)^{-1}\int_0^{2\pi} w'(\varphi')\,d\varphi' = w(2-w)^{-1}.$$

The procedure described here is valid in the zeroth-order approximation in $\Omega_0/\Omega' \ll 1$. The foregoing is a rigorous justification and development of the procedure postulated by Pippard[16] in the t_0 approximation for the calculation of electric-conductivity oscillations that develop against a stochastic background.

As is clear from the foregoing, the limits of applicability of the results obtained in Sec. 5 are determined by the inequalities $\bar{\gamma} \gg 1$ and $\nu_{II}\tau_0 \ll 1$, which can be satisfied for all values of the parameter β_0 (see Sec. 1). If $\beta_0 \lesssim 1$, then the stochastization occurs at $\beta_0 u_0 \gtrsim \hbar\Omega_0$, as follows from the estimate given in Sec. 3 for $\bar{t}$. The values $\beta_0 \gg 1$ ($d_0 \lesssim r_H$) can be shown to correspond to $\bar{\gamma} \sim (r_H/d_0)\sqrt{\beta_0} \gg 1$, i.e., the case of large β_0 and $d_0 \lesssim r_H$ is inevitably stochastic. This is in full agreement with[4].

6. SOME ESTIMATES

Under the experimental conditions, small large-scale perturbations can be produced by dislocations. The parameters of the dislocation field are $u_0 \sim \epsilon_F a_0 d_0^{-1}$, $d_0^{-2} \sim c_{dis}$, $\bar{t} \sim d_0/v_0$, where c_{dis} is the dislocation concentration. We see therefore that the stochastic situation C is realized at $d_0 \ll r_H$, corresponding to $c_{dis} \sim 10^8 - 10^9$ cm^{-2} (H $\sim 10^4 - 10^5$ Oe). The case $d_0 \sim r_H$ is qualitatively equivalent to the situation B. It appears that a rigorous analysis of this case is impossible, since the parameter value for the dislocations introduced in Sec. 2 is $\beta_0 \sim 1$. If $r_H\sqrt{c_{dis}} \ll 1$, then the quantum scale is $\kappa p_0 \gg q_0 \sim hd_0^{-1}$. The situation A is then realized for closed configurations. In the case of open magnetic-breakdown configurations, a diffusion variation of p_z and p_x (along the lines $E_n(p_z, P_x) = \text{const}$, see Sec. 1, 2) is possible, with a characteristic time $t_d \sim \bar{t}(\kappa p_0/q_0)^2$. If $t_d \ll t_{imp}$, then $\sigma_{xx} \sim (d_0^3/r_H^2 v_0 t_{imp})\sigma_0$. At $t_d \gg t_{imp}$, the situation A also takes place for open magnetic-breakdown configurations.

We note also that under MB conditions at temperatures $T \gg \kappa\Theta \sim 10^{-1} - 10^{-2}$°K ($\Theta$ is the Debye temperature) the efficiency of the electron-phonon interaction is increased by $(\Theta/T)^2$ times. Estimates obtained by replacing $\bar{t}$ with the electron-phonon lifetime $t_{c-ph}(h/k\Theta)(\Theta/T)^3$, show that situations B and C arise for this type of interaction at $T \gtrsim 20$°K.

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[1)] Other methods proposed in [7-10] for the calculation of σ_{ik} are in our case either inconvenient or not valid in principle.

[2)] The formalism developed below can easily be generalized to include also the case $\hbar\Omega_0/kT \gtrsim 1$.

[3)] This estimate is incorrect in the absence of MB, for in this case $\hat{\rho}$ is a "quasiclassical" operator of the type (5) and $\hat{u}_R\hat{\rho}$ is replaced approximately by the classical Poisson brackets, which do not depend on $\hbar$.

[4)] If L = 1, then (48) also remains valid when the limit is taken. Of course, this situation is possible only under the strong-breakdown condition $w \to 1$.

[5)] Strictly speaking, the δ functions in (53) should be smeared out by a small amount $\sim\tau_0$. For this reason the $\langle\eta|\hat{\delta}_\alpha|\eta'\rangle \to 0$ as $|E_\eta - E_{\eta'}| \to \infty$.

[6)] The latter can be obtained from the general formula (6) after making the approximate substitution $\tilde{J}_S \to \tilde{J}_{II}$ in $\hat{A}_\alpha(t)$, if it is recognized that the operator $\hat{u}_{II}$ is diagonal in the band number m as a result of the smallness of q_0/p_0.

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219