Nonlinear conversion of radiation into plasma waves

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The stationary spectral distribution and the electric field strength of high-frequency plasma waves (plasmons parametrically excited in a homogeneous isotropic plasma by a high-intensity nonpotential pumping wave of frequency close to the double electron Langmuir frequency are found by analyzing and solving the nonlinear integral equation. The induced scattering of the plasma waves by ions and the nonlinear shift of the plasma-wave frequency are the noise-level saturation mechanisms that lead to the stationary turbulent state of the initially parametrically unstable plasma. The coefficient of nonlinear conversion of the pumping radiation into plasma waves reaches a value of the order of unity when the pumping flux only slightly exceeds the threshold value.

A great deal of attention is being paid at present to the determination of the laws governing the nonlinear interaction of high-power radiation beams with plasmas. The phenomenon of fast radiation energy transfer to plasma particles predicted by parametric-resonance theory[1] has been experimentally observed in the microwave band[2] (for greater details, see the review[3]). In[4–6] the authors formulate a number of experiment-related[2] theses for the theory of nonlinear radiation energy dissipation based on the idea that a high-power pumping wave can be converted into two plasma waves, one of which is a high-frequency electronic wave of frequency close to that of the radiation flux, while the other is a low-frequency wave.

At the same time, a number of authors[7–10] have discussed the possibility of parametric resonance in a plasma, a resonance which can be likened to the decay of a pumping radiation wave into two high-frequency electron plasma waves. A quasilinear theory taking account of the influence of the fast electrons has been constructed for such a resonance by Galeev, Oraevskiĭ, and Sagdeev[10], who also point out the necessity for a quantitative description of the nonlinear phase of the instability on the basis of turbulent-plasma theory, as has been done in[4–6] for the other parametric instability. The necessary nonlinear solution was, however, not obtained in[10]. Therefore, our aim in the present paper is to supplement the procedure used in[10], and to obtain the quasi-stationary solution for the plasma-oscillation noise, which, under the conditions of parametric resonance, characterizes the turbulent state.

The contents of the paper are divided into three parts. In the first part we formulate for the spectral energy density of the longitudinal electronic waves a nonlinear integral equation corresponding to the theory of the weakly-turbulent plasma (see, for example,[11]), in which the primary mechanism for the nonlinear transfer across the spectrum is the induced scattering of the high-frequency plasma waves by the ions. In the second part we analyze and solve the nonlinear integral equation for the spectral energy density of parametrically excited plasmons. Expressions are obtained for the intensity of the effective field of the plasma waves and for the coefficient of the nonlinear conversion by the plasma of a high-power radiation into plasma waves. Finally, in the third part we discuss the effect of the nonlinear plasma-wave frequency shift on the level of stationary fluctuations in a parametrically excited turbulent plasma.

It seems to use that any future theory should unify the nonlinear approach and the Galeev-Oraevskiĭ-Sagdeev quasilinear approach[10].

1. PARAMETRIC RESONANCE AND SPECTRAL PUMPING

We shall be interested in the action of electromagnetic radiation on a homogeneous isotropic plasma. Assuming such radiation to be monochromatic with frequency ω_0 and wave vector $\mathbf{k}_0$, we have for the intensity of the electric field of the pump wave

$$\mathbf{E}(\mathbf{r},\, t)=\mathbf{E}_0 \sin(\omega_0 t-\mathbf{k}_0\mathbf{r}). \tag{1.1}$$

We shall assume that the frequency of the pump field is close to twice the electron Langmuir frequency of the plasma

$$\omega_0=2\omega_{Le}(1+\Delta),\quad |\Delta|\ll 1, \tag{1.2}$$

where $\omega_{Le}^2 = 4\pi e^2 n_e/m$. It is known[7–9] that under the conditions of such a resonance, at a sufficiently high level of pump power, the plasma turns out to be parametrically unstable. Then, according to the linear theory of parametric resonance, the plasma waves with the frequencies ω and $\omega - \omega_0$ and wave vectors $\mathbf{k}$ and $\mathbf{k} - \mathbf{k}_0$ grow in time with the increment γ.

Under the assumption that the wavelengths of the parametrically excitable waves are much smaller than the wavelength of the pump wave ($k \gg k_0$), we have for the spectrum $\omega(\mathbf{k})$ of the plasma waves and the growth rate $\gamma(\mathbf{k})$[9,12]:

$$\omega(\mathbf{k})=\omega_{Le}[1+\Delta+{}^3/_2 r_{De}{}^2\mathbf{k}\mathbf{k}_0], \tag{1.3}$$

$$\gamma(\mathbf{k})=-\tilde{\gamma}(\mathbf{k})+\omega_{Le}[(\mathbf{k}\mathbf{r}_E)^2(\mathbf{k}\mathbf{k}_0)^2k^{-4}-(\Delta-{}^3/_2k^2r_{De}{}^2)^2]^{1/2}. \tag{1.4}$$

Here we have neglected the difference between the damping constants $\tilde{\gamma}$ of the plasma waves with the wave vectors $\mathbf{k}$ and $\mathbf{k} - \mathbf{k}_0$, and have used the following notation: $r_{De} = (\kappa T_e/4\pi e^2 n_e)^{1/2}$ is the Debye radius of electrons of temperature T_e (κ is the Boltzmann constant) and $\mathbf{r}_E = e\mathbf{E}_0/m\omega_0^2$ is the amplitude of the oscillations of an electron in the field of the pumping wave. The damping of the plasma waves is determined by the electron-ion collisions (of collision frequency ν_{ei}) and the inverse Cerenkov effect on electrons (the Landau damping):

$$\tilde{\gamma}(\mathbf{k})=\frac{1}{2}\nu_{ei}+\sqrt{\frac{\pi}{8}}\frac{\omega_{Le}}{(kr_{De})^3}\exp\left[-\frac{\omega^2(\mathbf{k})}{2k^2v_{Te}^2}\right], \tag{1.5}$$
$$v_{Te}=r_{De}\omega_{Le}.$$

The threshold pump-field value $E_{thr}(\mathbf{k})$ for parametric instability is determined by equating the growth rate (1.4) to zero. The minimum value E_{min} of the threshold field is attained for the oscillations excitable with the wave vector equal in magnitude to

$$k=k_m=(2\Delta/3r_{De}^2)^{1/2}. \quad (1.6)$$

in the plane of polarization of the pumping wave at angles $\pi/4$ and $3\pi/4$ to the direction of propagation of the wave. The two excitable plasma waves with the wave vectors **k** and **k** − **k**₀ grow with the same growth rate, and their spectral energy densities W(**k**) and W(**k** − **k**₀) are connected by the relation

$$W(\mathbf{k}-\mathbf{k}_0)=W(\mathbf{k}). \quad (1.7)$$

The growth of the intensity of the plasma oscillations makes it necessary to take their nonlinear interaction into account. The strongest nonlinear interaction is the induced scattering of the plasma waves by the ions (for details, see[11]). Allowance for such an interaction allows us to write down the following equation for the spectral energy density of, for example, the plasma wave with the wave vector **k**:

$$\frac{\partial W(\mathbf{k})}{\partial t}+\frac{\partial\omega}{\partial\mathbf{k}}\frac{\partial W(\mathbf{k})}{\partial\mathbf{r}}-\frac{\partial\omega}{\partial\mathbf{r}}\frac{\partial W(\mathbf{k})}{\partial\mathbf{k}} =A(\mathbf{k})+W(\mathbf{k})\left[2\gamma(\mathbf{k})-\int d\mathbf{k}'Q(\mathbf{k},\mathbf{k}')W(\mathbf{k}')\right]. \quad (1.8)$$

Here γ is given by the expression (1.4), while the kernel Q has the form

$$Q(\mathbf{k},\mathbf{k}')=\frac{\omega_{Le}}{4(2\pi)^{5/2}n_e\varkappa T_e}\frac{(\mathbf{k}\mathbf{k}')^2}{(kk')^2}\frac{r_{De}^2r_{Di}^2}{(r_{De}^2+r_{Di}^2)^2}\frac{\omega-\omega'}{|\mathbf{k}-\mathbf{k}'|v_{Ti}} \times\exp\left[-\frac{(\omega-\omega')^2}{2(\mathbf{k}-\mathbf{k}')^2v_{Ti}^2}\right], \quad (1.9)$$

where ω and ω' are the frequencies (1.3) of the interacting plasma waves with the wave vectors **k** and **k**′, $v_{Ti} = (\kappa T_i/M)^{1/2}$ is the thermal velocity of ions with temperature T_i and mass M, $r_{Di} = v_{Ti}/\omega_{Li}$ is the ion Debye radius, and $\omega_{Li} = (4\pi e_i^2 n_i/M)^{1/2}$ is the ion Langmuir frequency. Finally, the first term of the right-hand side of Eq. (1.8) characterizes the spontaneous emission by the electrons of plasma oscillations modified by the field of the pump wave:

$$A(\mathbf{k})=2\tilde{\gamma}(\mathbf{k})\varkappa T_e(1+E_0^2/E_{thr}^2). \quad (1.10)$$

The induced scattering of the plasma waves by the ions leads to the plasma-wave energy transfer across the spectrum in k space from the parametric-excitation region, where $\gamma(\mathbf{k}) > 0$, to the region of wave absorption by the plasma particles, where $\gamma(\mathbf{k}) < 0$. As a result of such spectral pumping, it becomes possible to establish a stationary plasma-oscillation distribution. According to (1.8), we have in the stationary and spatially homogeneous case

$$W(\mathbf{k})\left[-2\gamma(\mathbf{k})+\int d\mathbf{k}'Q(\mathbf{k},\mathbf{k}')W(\mathbf{k}')\right]=A(\mathbf{k}). \quad (1.11)$$

In the following section we shall consider the consequences ensuing from Eq. (1.11). Notice that in the turbulent state the high plasma-wave fluctuation level—in comparison with the thermal fluctuation level—corresponds to the asymptotic relation (cf.[4])

$$\left[-2\gamma(\mathbf{k})+\int d\mathbf{k}'Q(\mathbf{k},\mathbf{k}')W(\mathbf{k}')\right]\to+0. \quad (1.12)$$

The spectral energy density allows us to determine the effective intensity E of the electric field of the plasma waves:

$$\frac{E^2}{8\pi}=\int\frac{d\mathbf{k}}{(2\pi)^3}[W(\mathbf{k})+W(\mathbf{k}-\mathbf{k}_0)]. \quad (1.13)$$

Accordingly, below we shall use the conversion coefficient

$$K=(E/E_0)^2, \quad (1.14)$$

characterizing the transformation of a high-power pumping radiation into plasma waves.

2. THE STATIONARY LEVEL OF PARAMETRICALLY EXCITED PLASMA OSCILLATIONS THAT RESULTS FROM SPECTRAL PUMPING

Let us consider Eq. (1.11) in the case when the pumping wave is spontaneously polarized and its polarization tensor has the form

$$E_{0i}E_{0j}={}^1/_2E_0^2(\delta_{ij}-k_{0i}k_{0j}/k_0^2).$$

Then the growth rate (1.4) turns out to be dependent only on the modulus k of the wave vector and the polar angle θ between the vectors **k** and **k**₀:

$$\gamma(k,\theta)=\tilde{\gamma}(k)\left\{-1+\left[p^2\sin^2 2\theta-\left(\frac{\omega_{Le}\Delta}{\tilde{\gamma}}\right)^2\left(1-\frac{k^2}{k_m^2}\right)^2\right]^{1/2}\right\}. \quad (2.1)$$

Here p determines the excess of the pump field over the minimum threshold value at which parametric instability develops:

$$p=\frac{E_0}{E_{min}},\quad \frac{E_{min}^2}{32\pi n_e\varkappa T_e}=\frac{\tilde{\gamma}(k_m)\omega_0^4}{k_0^2v_{Te}^2\omega_{Le}^4}. \quad (2.2)$$

In accord with the expression (2.1), the solution of Eq. (1.11) can be sought in the form W(**k**) = W(k, θ). Notice that the results obtainable in this case are qualitatively valid for any other—e.g., linear—polarization, since the dependence of the growth rate on the azimuthal angle φ of the vector **k** is relatively weak.

Thus, for the stationary spectral energy density of the plasma waves, we have the following nonlinear double-integral equation:

$$W(k,\theta)\left\{1-\left[p^2\sin^2 2\theta-\left(\frac{\omega_{Le}\Delta}{\tilde{\gamma}}\right)^2\left(1-\frac{k^2}{k_m^2}\right)^2\right]^{1/2} +\int_0^\infty k'^2\,dk'\int_0^\pi\sin\theta'\,d\theta' Q(k,\theta;k',\theta')W(k',\theta')\right\} =\varkappa T_e\left\{1+p^2\sin^2 2\theta\left[1+\left(\frac{\omega_{Le}\Delta}{\tilde{\gamma}}\right)^2\left(1-\frac{k^2}{k_m^2}\right)^2\right]^{-1}\right\}. \quad (2.3)$$

Here

$$Q(k,\theta;k',\theta')=\left(\frac{3}{2\pi}\right)^{3/2}\frac{1}{4n_e\varkappa T_e}\frac{v_{Te}^4}{cv_{Ti}\tilde{\gamma}\omega_{Le}}\frac{1}{(r_{De}^2+r_{Di}^2)^2} \times\int_0^{2\pi}\frac{d\varphi'}{2\pi}(\boldsymbol{\varkappa}\boldsymbol{\varkappa}')^2(\boldsymbol{\varkappa}_0\boldsymbol{\varkappa}'')\exp[-\beta^2(\boldsymbol{\varkappa}_0\boldsymbol{\varkappa}'')^2], \quad (2.4)$$

where $\beta = (3/2)^{3/2}(v_{Te}^2/cv_{Ti})$, while

$$\boldsymbol{\varkappa}=\frac{\mathbf{k}}{k},\quad \boldsymbol{\varkappa}'=\frac{\mathbf{k}'}{k'}\quad \boldsymbol{\varkappa}_0=\frac{\mathbf{k}_0}{k_0},\quad \boldsymbol{\varkappa}''=\frac{\mathbf{k}-\mathbf{k}'}{|\mathbf{k}-\mathbf{k}'|};$$

are unit vectors.

Let us further use the condition

$$1\gg\Delta\gg\nu_{ei}/\omega_{Le}, \quad (2.5)$$

owing to which for not too strong pump fields, when $p \ll \omega_{Le}\Delta/\tilde{\gamma}$, the growth rate (2.1) of the parametric instability turns out to be strongly dependent on the wave-vector detuning $k^2 - k_m^2$. This same strong dependence, it turns out, determines the dependence of the spectral energy density on the magnitude of the wave vector. As a result, it is possible to integrate Eq. (2.3) and obtain an equation for a single-variable function

$s(\theta)$. Then in the approximation of a high turbulent-fluctuation level, as compared to the thermal-fluctuation level ($s \gg s_0$), we obtain

$$s^2(\theta)\left[1-p|\sin 2\theta|+\alpha\int d\theta' \sin\theta' s(\theta') Q(\theta,\theta')\right] = s_0^2 p|\sin 2\theta|(1+p^2\sin^2 2\theta)^2. \tag{2.6}$$

Here

$$s_0=\sqrt{\frac{\Delta}{3}}\frac{\tilde{\gamma}(k_m)}{6\pi\omega_{Le}n_e r_{De}^3},$$

$$\alpha=\frac{\pi}{108}\frac{c^2}{v_{Te}^2}\frac{v_{Ti}^2 r_{Di}^2}{\omega_{Le}\tilde{\gamma}(k_m)}\frac{1}{(r_{De}^2+r_{Di}^2)^2}, \tag{2.7}$$

$$Q(\theta,\theta')=\frac{8\beta^3}{\sqrt{\pi}}\int_0^{2\pi}\frac{d\varphi'}{2\pi}(\varkappa\varkappa')^2(\varkappa_0\varkappa'')\exp[-\beta^2(\varkappa_0\varkappa'')^2],$$

where we have taken into account in Q, in contrast to (2.4), the fact that $k \approx k' \approx k_m$. It should be noted that Eq. (2.6) corresponds to a relatively narrow (in terms of the magnitude k of the wave vector) plasma-wave line:

$$W(k,\theta)\approx 4\pi^2 k^{-2} n_e \varkappa T_e s(\theta)\delta(k-k_m). \tag{2.8}$$

Therefore, the spectral pumping of the plasma waves involves only changes in the directions of the wave vector without any change in its magnitude near the threshold.

The growth rate (2.1) attains its maximum at two values of the angle: $\theta = \pi/4$ and $\theta = 3\pi/4$. Therefore, the turbulent region is a doubly connected region, and consists of two cones, one of which is situated in the range $0 < \theta < \pi/2$ and the other in the range $\pi/2 < \theta < \pi$. This allows us to write Eq. (2.6) for the two functions

$$s_1(x)=s(\theta-\pi/4),\quad 0<\theta<\pi/2,$$
$$s_2(x)=s(\theta-3\pi/4),\quad \pi/2<\theta<\pi$$

in the form of two equations having near the parametric-instability threshold ($p - 1 \ll 1$) the following form:

$$s_1^2(x)\left[2px^2-(p-1)+\alpha\int dx' s_1(x')Q(x'-x)+\alpha R\int dx' s_2(x')\right]=4s_0^2.$$
$$s_2^2(x)\left[2px^2-(p-1)+\alpha\int dx' s_2(x')Q(x'-x)-\alpha R\int dx' s_1(x')\right]=4s_0^2. \tag{2.9}$$

The kernel

$$Q(x)=\frac{4\beta^3}{\pi^{3/2}}x\int_0^{\pi} d\varphi\frac{\cos^4\varphi}{(\sin^2\varphi+{}^1/_2x^2\cos^2\varphi)^{1/2}}\exp\left[-\frac{\beta^2}{4}\frac{x^2}{\sin^2\varphi+{}^1/_2x^2\cos^2\varphi}\right] \tag{2.10}$$

determines the nonlinear interaction of the waves inside each of the two cones of the turbulent region. The cross interaction between waves of different cones is determined by the transfer constant

$$R=\frac{8\beta^3}{\pi^{3/2}}\int_0^{\pi} d\varphi\frac{\sin^4\varphi}{(1+\sin^2\varphi)^{1/2}}\exp\left(-\frac{\beta^2}{1+\sin^2\varphi}\right). \tag{2.11}$$

Let us further consider two limiting cases of the nonlinear wave interaction described by Eqs. (2.9), cases which we shall respectively call differential and integral spectral pumping approximations. To the first case of differential pumping corresponds a sufficiently hot and nonisothermal plasma ($\beta \gg 1$), when the kernel Q(x) can be approximated by the derivative of the δ-function (cf.[4]):

$$Q(x)=-\delta'(x). \tag{2.12}$$

In this case ($\beta \gg 1$) the exponential smallness of the transfer constant allows us to consider the two cones of the turbulent region independently of each other. Assuming $s_1(x) = s_2(x) = s(x)$, we have

$$\varepsilon\frac{dy}{dx}+x^2-a^2=\frac{1}{y^2}$$
$$a^2=\frac{p-1}{2p},\quad \varepsilon=\frac{\alpha s_0}{\sqrt{2}\,p^{3/2}},\quad y(x)=\frac{s(x)}{s_0}\sqrt{\frac{p}{2}} \tag{2.13}$$

An approximate solution to this equation can be obtained in two steps. First of all, notice that the right-hand side y^{-2} of the equation is determined by the contribution of the spontaneous term, and is small at the high (turbulent) noise level of interest to us. Therefore, in almost the entire turbulent region the angular distribution of the noise is given by the solution to the linear equation

$$\varepsilon\frac{dy}{dx}+x^2-a^2=0,$$

a solution which can be obtained up to a constant C of integration:

$$y(x)=\frac{a^3}{\varepsilon}\left(C+\frac{x}{a}-\frac{x^3}{3a^3}\right). \tag{2.14}$$

The constant C is found with the aid of arguments following directly from the qualitative properties of the basic integral equation (2.3). It is precisely in view of the fact that, as a result of the nonlinear interaction (the scattering by the ions), turbulent noise is pumped over from left to right into the region of larger angles θ (of larger x) that it is natural to expect that in the stationary turbulent state, when the contributions of the linear and nonlinear terms in the integral equation (2.4) almost completely cancel each other out, the noise y(x) at the left boundary ($x = -a$) of the instability region will be close to the spontaneous noise. Such closeness corresponds, in the framework of the above neglect of spontaneous noise, to the equality $y(x) = 0$ for $x = -a$, which yields $C = 2/3$ in (2.14). The right boundary $x = 2a$ of the turbulent-noise region is found from the condition that the spectral energy density be positive, i.e., $y(x) \geq 0$:

$$y(x)=\frac{a^3}{\varepsilon}\left(\frac{2}{3}+\frac{x}{a}-\frac{x^3}{3a^3}\right),\quad -a\leqslant x\leqslant 2a. \tag{2.15}$$

Thus, in the approximation of zero spontaneous noise, the spectral energy density of the plasma waves is distributed over the angles according to the formula (2.15).

The second stage in the solution of Eq. (2.13) consists in allowing for a finite spontaneous-noise level at the boundaries of the turbulent region. If the noise is close to the spontaneous-noise level, then the first term on the left-hand side of Eq. (2.13), describing the nonlinear interaction, is small compared to the linear terms. Therefore,

$$y(x)\approx(x^2-a^2)^{-1/2},\quad |x|>a. \tag{2.16}$$

The spontaneous solution (2.16) is useless deep inside the turbulent region, but it must be matched with the turbulent solution (2.14) at the boundaries of the region. For $a^3 \gg \epsilon$, the turbulent noise varies rapidly as the angle x/a varies. Therefore, it is more natural to match the distributions (2.14) and (2.16) at the turbulent zone's right-hand boundary, $x = 2a$, where both (2.14) and (2.16) are close to the spontaneous noise. Such a matching yields a more accurate value for the constant C in the form

$$C={}^2/_3+\varepsilon/a^4\sqrt{3},\quad a^4\gg\varepsilon. \tag{2.17}$$

In consequence, the approximate solution to the nonlinear differential equation (2.13) determining the angular distribution of the plasma-oscillation energy with allowance for the finite spontaneous-noise level can be represented by the formulas (2.14) and (2.17) for $-a < x < 2a$ and by the formula (2.16) for $-1 \ll x < -a$ and $2a < x \ll 1$. The validity of these arguments about the approximate analytic solution to Eq. (2.13) is illustrated by Fig. 1, which shows the exact numerical solution to Eq. (2.13). Notice that the high turbulent-noise level is due precisely to the large value of the parameter $a^4/\epsilon \gg 1$. This same parameter, according to the foregoing, determines the accuracy in the choice of the point where the solutions are matched. In the opposite limit $a^4 \ll \epsilon$, the noise differs only slightly from the spontaneous noise.

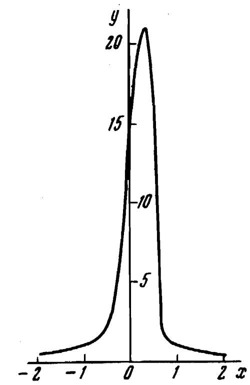

FIG. 1. The angular distribution of the turbulent spectral energy density $W(k, \theta)$ of parametrically excited plasma waves in the approximation of differential energy pumping across the spectrum (see the formulas (2.8) and (2.13); cf. (2.19)). The curve $y(x)$ illustrates the numerical solution of Eq. (2.13) for $\epsilon = 3 \times 10^{-3}$ and $a^2 = 0.1$ in the two turbulent regions for $x = \theta - \pi/4$ and $x = \theta - 3\pi/4$.

The refinement of the angular distribution arising from the allowance for the finite spontaneous-noise level is unimportant in view of the smallness of the parameter $\epsilon/a^4 \ll 1$ used to determine the total (integrated over the angles) turbulent noise, which can be obtained with the aid of (2.15) by integrating $y(x)$ over the turbulent region[1)]:

$$\frac{E^2}{8\pi} = \frac{243}{\pi p\sqrt{2}}(p-1)^2 \frac{\tilde{\gamma}\omega_{Le}}{\omega_{Li}^2}\frac{v_{Te}^2}{c^2}\left(1+\frac{r_{De}^2}{r_{Di}^2}\right)^2 n_e \varkappa T_e. \quad (2.18)$$

The spectral distribution of the energy density of parametrically excited plasma waves is given with the same accuracy by the expression

$$\frac{W(k,\theta)}{n_e\varkappa T_e} = \frac{648\pi}{\Delta}\frac{r_{De}^2 v_{Te}^2}{c^2}\frac{\tilde{\gamma}\omega_{Le}}{\omega_{Li}^2}\left(1+\frac{r_{De}^2}{r_{Di}^2}\right)^2 \delta\left(k-\left(\frac{2\Delta}{3r_{De}^2}\right)^{1/2}\right)$$
$$\times\left[\frac{2}{3}+\left(\theta-\frac{\pi}{4}\right)\left(\frac{2p}{p-1}\right)^{1/2}-\frac{1}{3}\left(\theta-\frac{\pi}{4}\right)^3\left(\frac{2p}{p-1}\right)^{3/2}\right], \quad (2.19)$$
$$-\left(\frac{p-1}{2p}\right)^{1/2} \leqslant \theta-\frac{\pi}{4} \leqslant \left(2\frac{p-1}{p}\right)^{1/2}.$$

A relation analogous to (2.19) is valid in the region $\theta \approx 3\pi/4$. The angular width $\Delta\theta$ of the distribution (2.19) is determined by the angular dimensions of the buildup region:

$$\Delta\theta \sim ((p-1)/p)^{1/2}.$$

The substitution of the distribution (2.19) in the basic equations (2.10) yields the following criteria for the applicability of the differential approximation:

$$p-1 \gg \frac{16c^2 v_{Ti}^2}{27 v_{Te}^4} = \frac{4}{\beta^2}. \quad (2.20)$$

At the same time, the weak-turbulence condition gives an upper bound for the excess over the threshold:

$$(p-1)^2 \ll \frac{\pi p\sqrt{2}}{243}\frac{c^2 v_{Ti}^2 r_{Di}^2}{\tilde{\gamma}\omega_{Le} v_{Te}^2 (r_{De}^2+r_{Di}^2)^2}. \quad (2.21)$$

Comparison of these two inequalities allows us to speak of quite a wide range of admissible excesses over the threshold. In this range, we can, according to the formula (2.18), write the following expression for the coefficient of nonlinear conversion of pumping radiation into plasma waves:

$$K = \frac{(3/2)^6}{\pi\sqrt{2}}\frac{(p-1)^2}{p^3}\frac{\omega_{Le}^3}{\tilde{\gamma}\omega_{Li}^2}\frac{v_{Te}^4}{c^4}\left(1+\frac{r_{De}^2}{r_{Di}^2}\right)^2. \quad (2.22)$$

This expression can attain values comparable to unity.

The above-considered differential-pumping approximation is inapplicable when the inequality (2.20) is violated, and in the opposite limit

$$p-1 \ll \frac{16c^2 v_{Ti}^2}{27 v_{Te}^4} \quad (2.23)$$

there obtains integral pumping of noise across the spectrum. Two different cases are possible here. Let us first consider the case of a sufficiently hot plasma, when $\beta \gg 1$. Then, as in the case of differential pumping, the transfer constant is exponentially small, and the regions of the two cones can be considered independently. Therefore, for example, we have for the function $s_1(x)$ the relation

$$s_1^2(x)\left[2px^2-(p-1)+\alpha\int dx' s_1(x')Q(x'-x)\right] = 4s_0^2, \quad (2.24)$$

where

$$Q(x) = \frac{4\beta^3}{\pi^{3/2}}x\int_0^\pi d\varphi\frac{\cos^4\varphi}{(\sin^2\varphi+{}^1/_2 x^2\cos^2\varphi)^{1/2}} \approx \frac{8\beta^3}{\pi^{3/2}}x\ln\frac{1}{|x|}. \quad (2.25)$$

To find the total intensity of the turbulent fluctuations, it is sufficient to limit ourselves to the comparatively simple estimate resulting from Eq. (2.24) and not requiring knowledge of the exact solution. This equation allows us to assert that the characteristic width of the angular distribution of the turbulent noise is given by the relation

$$x^2 \sim (p-1)/2p. \quad (2.26)$$

Then bearing the asymptotic equation (1.12) and the expression (2.25) in mind, we can write down the following estimate:

$$\int dx s_1(x) = C\frac{\pi^{3/2}}{8\alpha\beta^3}[2p(p-1)]^{1/2}, \quad (2.27)$$

where C is a constant of the order of unity. Hence, for the effective plasma-wave field strength, allowing for the analogous contribution from the second cone, we have:

$$\frac{E^2}{8\pi} = \frac{64}{3}C\sqrt{\frac{\pi}{3}}\frac{\tilde{\gamma}\omega_{Le}}{\omega_{Li}^2}\frac{c v_{Ti}^3}{v_{Te}^4}\left(1+\frac{r_{De}^2}{r_{Di}^2}\right)^2[p(p-1)]^{1/2} n_e\varkappa T_e. \quad (2.28)$$

The coefficient of conversion of high-power pumping radiation into plasma waves corresponding to this expression has the form

$$K = C\sqrt{\frac{\pi}{3}}\frac{(p-1)^{1/2}}{p^{3/2}}\frac{\omega_{Li} v_{Te} r_{De}}{\tilde{\gamma} c r_{Di}}\left(1+\frac{r_{Di}^2}{r_{De}^2}\right)^2. \quad (2.29)$$

The second case of integral pumping, $\beta \ll 1$, is distinguished by a large transfer coefficient:

$$R = \frac{8\beta^3}{\pi^{3/2}}\int_0^\pi d\varphi\frac{\sin^4\varphi}{(1+\sin^2\varphi)^{1/2}} = \frac{8\beta^3\delta}{\pi^{3/2}}, \quad \delta = 0.87. \quad (2.30)$$

In conformity with this, upon the fulfillment of the inequality

$$p-1 \ll \alpha R\int dx\, s_2(x), \quad (2.31)$$

the transfer of oscillations from the region $\theta \sim \pi/4$ into the region $\theta \sim 3\pi/4$ turns out, according to Eqs. (2.9), to be so substantial that the instability in the zone $\theta \sim \pi/4$ is totally suppressed. In this angle region the turbulent fluctuations remain at the spontaneous-noise level, i.e., $s_1 \sim s_0$. For

$$p-1 \gg \alpha s_0 R \tag{2.32}$$

this circumstance, in its turn, makes the influence of the transfer on the turbulent-fluctuation level in the $\theta \sim 3\pi/4$ region insignificant. Therefore, for the function s_2 we can write down the following equation:

$$s_2^2(x)\left[2px^2-(p-1)+\alpha\int dx'\, s_2(x')Q(x'-x)\right]=4s_0^2. \tag{2.33}$$

Since this equation is similar to Eq. (2.24), the estimate (2.27) and, consequently, the formulas (2.28) and (2.29) are valid for it also, while the inequality (2.31) reduces to $p-1 \ll 1$. If the latter is violated, then the transfer process ceases to stabilize the instability in the cone $\theta \sim \pi/4$, and an estimate of the total noise yields

$$\int dx(s_1+s_2) \sim p/\alpha\beta^3,$$

which again leads to the formulas (2.28) and (2.29).

Figure 2 shows the dependence of the conversion coefficient K on the value of the excess over the threshold. As a function of the detuning, the conversion coefficient turns out to be largest in the region corresponding to the damping of the plasma waves as a result of Coulomb collisions, i.e., when

$${}^3/_4 k_0^2 r_{De}^2 \ll \Delta < {}^3/_2 k_{st}^2 r_{De}^2, \tag{2.34}$$

where $k_{st} \approx r_{De}^{-1}[\ln(\omega_{Le}^2/\nu_{ei}^2)]^{-1/2}$ is the wave number at which the contributions to the logarithmic decrement of the plasma wave due to collisions and the Cerenkov effect are equal.

3. THE STATIONARY LEVEL OF PARAMETRICALLY EXCITED PLASMA OSCILLATIONS THAT RESULTS FROM A NONLINEAR SHIFT IN THEIR FREQUENCY

Besides the above-considered effect whereby the parametric instability is saturated by spectral pumping of the plasma waves, the stationary state of a turbulent, parametrically excited plasma can be attained owing to the nonlinear shift in the frequency of the interacting plasma waves. As applied to the aperiodic parametric instability, the stabilizing role of the nonlinear correction to the plasma-wave spectrum was discovered in[13]. In this section of our treatment of the stationary turbulence of a parametrically unstable plasma, we shall compute the effective electric-field strength of the plasma waves and the coefficient of nonlinear conversion of high-power radiation into these waves in the framework of the mechanism whereby the instability under discussion here is stabilized by the nonlinear frequency shift.

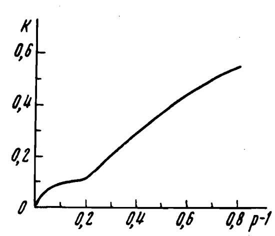

FIG. 2. The coefficient K of conversion of high-power neodymium laser radiation ($\omega_0 = 1.78 \times 10^{15}$ sec^{-1}) into plasma waves as a function of the light-wave electric-field strength (see the notations in (1.14) and (2.2) and the formulas (2.22) and (2.29)). The graph was constructed from the formulas (2.22) and (2.29) as applied to a laser plasma with an electron density $n_e = 2.5 \times 10^{20}$ cm^{-3} and temperature $T_e = T_i = 1$ keV for a detuning $\Delta = 0.2$.

Owing to the negative value of the correction $\delta\omega$ to the frequency of the longitudinal plasma oscillation (see, for example, Sec. 13 in[11]):

$$\delta\omega = -\frac{\omega_{Le}}{4n_e\varkappa T_e}\frac{r_{De}^2}{r_{De}^2+r_{Di}^2}\int\frac{d\mathbf{k}'}{(2\pi)^3}(\varkappa\varkappa')^2 W(\mathbf{k}'), \tag{3.1}$$

its effect amounts to the decrease of the detuning Δ in the growth rate γ of the parametric buildup. In conformity with the results of the preceding section discussed above, the maximum growth rate for detunings satisfying the inequality (2.34) does not depend on Δ. For smaller detunings $\Delta \leq 3/4 k_0^2 r_{De}^2$, a decrease in the detuning lowers the maximum growth rate, which ultimately becomes zero for a definite nonlinear frequency shift, which means for a definite stationary turbulent-noise level. This is the simple physical essence of the stabilization of the instability under consideration by the frequency shift.

Since the upper limit $(3/4)\, k_0^2 r_{De}^2$ of the detunings of interest to us is determined by the wave number k_0 of the pumping radiation, the nonlinear correction (3.1) to the frequency should be allowed for at the same time as the finiteness of the wavelength of the pump field, as compared to that of the plasma wave ($k \sim k_0$), is taken into account. Then for the frequency of the excited waves and the effective growth rate, we have (cf. (1.3) and (1.4)):

$$\omega(\mathbf{k}) = \omega_{Le}[1+\Delta_* + {}^3/_2 k k_0 r_{De}^2 \cos\theta], \tag{3.2}$$

$$\gamma(k,\theta) = -\frac{\nu_{ei}}{2} + \omega_{Le}\left\{\frac{k_0^2 r_E^2}{8}\frac{\sin^2\theta\,(k_0-2k\cos\theta)^2}{k_0^2+k^2-2kk_0\cos\theta} - (\Delta_* + {}^3/_2 k k_0 r_{De}^2\cos\theta - {}^3/_2 k^2 r_{De}^2)^2\right\}^{1/2}. \tag{3.3}$$

Here we have introduced for the effective detuning the notation

$$\Delta_* \equiv \Delta - {}^3/_4 k_0^2 r_{De}^2 + \delta\omega/\omega_{Le}. \tag{3.4}$$

Upon the neglect of pumping across the spectrum, the level of the stationary fluctuations in a turbulent, parametrically excited plasma should be found from the condition that the growth rate be nonpositive: $\gamma(\mathbf{k}) \leq 0$. Investigation of the growth rate (3.3) shows that the decrease of the growth rate with decreasing effective detuning Δ_* is possible only for $\Delta_* < 0$, when the maximum value of the growth rate is attained for two vectors: 1) $k = 0$, $\theta = \pi/2$; 2) $k = k_0$, $\theta = 0$. Since the maximum values of the growth rate for such wave vectors $\mathbf{k}$ are equal, the equation for the spectral noise density $W(\mathbf{k})$ can be written in the form (see also (3.1) and (3.4))

$$-{}^1/_2 \nu_{ei} + \omega_{Le}({}^1/_8 k_0^2 r_E^2 - \Delta_*^2)^{1/2} = 0. \tag{3.5}$$

For a small excess over the threshold ($p-1 \ll 1$), when the turbulent-noise level is high only in a very narrow angle range, Eq. (3.5) determines at once the effective electric-field strength of the plasma waves (1.13):

$$\frac{E^2}{8\pi} = 8n_e\varkappa T_e\left(1+\frac{r_{Di}^2}{r_{De}^2}\right)\left[\Delta - \frac{3}{4}k_0^2 r_{De}^2 + \left(\frac{1}{8}k_0^2 r_E^2 - \frac{\nu_{ei}^2}{4\omega_{Le}^2}\right)^{1/2}\right]. \tag{3.6}$$

This expression is naturally valid only for pump fields above the threshold. For a detuning $\Delta \geq (3/4)\, k_0^2 r_{De}^2$, the minimum threshold value of the pump field is given by the formula (2.2), which allows us to write the stationary turbulent-noise level in the form

$$\frac{E^2}{8\pi} = 8n_e\varkappa T_e\left(1+\frac{r_{Di}^2}{r_{De}^2}\right)\left[\Delta - \frac{9}{4}\frac{v_{Te}^2}{c^2} + \frac{1}{2}\frac{\nu_{ei}}{\omega_{Le}}(p^2-1)^{1/2}\right], \tag{3.7}$$
$$\Delta \geq {}^9/_4 v_{Te}^2/c^2.$$

Comparison of the field (3.7) with the estimates (2.18) and (2.28) for the stationary plasma-wave-turbulence level due to pumping across the spectrum shows that for large detunings $\Delta \gg (v_{Te}/c)^2$, pumping across the spectrum is a more effective mechanism for stabilizing the instability under consideration (it leads, for the same excess over the threshold, to a lower stationary-noise level) than the nonlinear frequency-shift mechanism. This conclusion is in complete accord with the above-presented elucidation of the frequency-shift stabilization mechanism as important for smaller detunings, $\Delta \lesssim (v_{Te}/c)^2$, when the minimum thershold for the parametric instability corresponding to the decay of the pumping wave into two plasmons strongly depends on the detuning:

$$\frac{E^2_{min}}{8\pi n_e \varkappa T_e} = \frac{16}{3}\frac{c^2}{v_{Te}^2}\left[\frac{\nu_{ei}^2}{\omega_{Le}^2} + 4\left(\frac{9}{4}\frac{v_{Te}^2}{c^2} - \Delta\right)^2\right], \quad \Delta < {}^9/_4 v_{Te}^2/c^2. \tag{3.8}$$

For such a threshold, a small decrease in the detuning is sufficient for a complete suppression of the instability. In this case for not too large excess over the threshold, i.e., for

$$p - 1 \ll \min\left(1, \frac{81}{16}\frac{v_{Te}^4}{c^4}\frac{\omega_{Le}^2}{\nu_{ei}^2}\right),$$

the formula (3.6) allows us to write down the following expression for the effective electric-field strength of the plasma waves:

$$\frac{E^2}{8\pi} = 8 n_e \varkappa T_e \left(1 + \frac{r_{Di}^2}{r_{De}^2}\right)(p-1)\left(\frac{9}{4}\frac{v_{Te}^2}{c^2} + \frac{1}{9}\frac{\nu_{ei}^2}{\omega_{Le}^2}\frac{c^2}{v_{Te}^2}\right), \tag{3.9}$$

which is applicable for $|\Delta| \ll 9/4(v_{Te}/c)^2$. The quantity p is to be understood as the ratio of the amplitude of the pump field to the minimum threshold value given by (3.8).

To such a stationary-noise level corresponds a sufficiently large coefficient of conversion of high-power radiation into plasma waves:

$$K = {}^1/_8 (p-1)(1 + r_{Di}^2/r_{De}^2). \tag{3.10}$$

It can be seen from this expression that in a very narrow resonance region ($|\Delta| \ll (v_{Te}/c)^2$), at a low ion temperature as compared to the electron temperature, the conversion coefficient depends only on the excess over the threshold, and attains a magnitude of the order of unity at $p - 1 \sim 1$.

CONCLUSION

The above-developed nonlinear theory of the turbulent state of a parametrically unstable plasma determines the quasi-stationary level of the plasma fluctuations during a parametric resonance in the vicinity of twice the frequency of the electron Langmuir oscillations. As a result of the analysis, we can assert that the action of external radiation on a plasma can lead to the establishment inside the plasma of a high level of plasma oscillations of intensity comparable to that of the pumping wave. Such results arise when we take into account the effect, due to induced scattering by the ions, of the spectral pumping of the plasma waves, as well as when the effect of the nonlinear frequency shift is taken into account.

In the second section of our exposition, we found an analytic expression for the spectral energy density of the plasma waves in the case of natural polarization of the pumping wave. We should emphasize that the expressions found there for the electric-field strength of the plasma waves, (2.18) and (2.28), and for the coefficient of nonlinear conversion of high-power radiation into plasma waves, (2.22) and (2.29), practically remain the same for any other polarization of the pumping wave.

The coefficient (2.22) of nonlinear conversion of neodymium laser radiation into the high-frequency electron oscillations of a laser plasma of temperature ~ 1 keV and density $\sim 2.5 \times 10^{25}$ cm^{-3} attains a value of the order of unity (see Fig. 2) for luminous fluxes $q \sim 10^{12}$ W/cm^2 slightly exceeding the minimum parametric-pumping threshold (2.2) (z is the ionization multiplicity of the ions, n_e is the electron density in cm^{-3}, and T_e is the electron temperature in keV):

$$q_{min} = \frac{\sqrt{3}}{2} c \frac{E^2_{min}}{4\pi} = \frac{4}{\pi\sqrt{3}}\frac{m^2c^3}{e^2}\nu_{ei}^2 \approx 10^{-29} z^2 n_e^2 T_e^{-3}\ [\mathrm{W/cm^2}].$$

Although the results of our analysis is somewhat restricted, since, for example, the wave vectors of the interacting waves are flipped from one cone of the doubly-connected instability region into the other, on the whole the approach employed here is quite general, and can be used to describe turbulent states under a number of other conditions of parametric action of high-power radiation on a plasma.

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[1]In (2.18) we have taken into account the equality of the amplitudes of the two plasma waves excitable by high-power radiation (see (1.7)).

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