# Transition scattering

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The propagation in a medium of waves of various types (longitudinal acoustic, plasma, etc.) involves the simultaneous presence of a dielectric permittivity wave. When such a wave is incident on a fixed charge, electromagnetic radiation is emitted, regardless of whether there is an electromagnetic field in the incident wave. The intensity of this transition scattering is calculated, a more general formulation of the problem is indicated, and it is pointed out that transition scattering may be of interest not only in plasma physics but also for other media.

In propagation in a medium of various perturbations and waves, dielectric permittivity waves also arise in a number of cases. In the simplest conditions, to which we will limit ourselves, the dielectric permittivity is

$$\varepsilon=\varepsilon_0+\varepsilon_1\cos(\mathbf{k}_0\mathbf{r}-\omega_0 t-\varphi_0). \tag{1}$$

Thus, in an isotropic plasma under certain conditions $\epsilon = 1 - 4\pi\epsilon^2 N/m\omega^2$ and for propagation of a longitudinal (plasma) wave in which the field $\mathbf{E} = \mathbf{E}_0\cos(\mathbf{k}_0\mathbf{r} - \omega_0 t)$ varies with a frequency $\omega_0 = \omega_{pe}$, to a first approximation

$$\varepsilon(\omega)=1-\frac{\omega_{pe}^2}{\omega^2}+\frac{ek_0E_0}{m\omega^2}\sin(\mathbf{k}_0\mathbf{r}-\omega_0 t),\qquad \omega_{pe}^2=4\pi e^2N_0/m. \tag{2}$$

This result is obvious if we use the equation $\operatorname{div}\mathbf{E} = 4\pi e(N - N_0)$ and the longitudinal condition $\mathbf{k}_0\mathbf{E}_0 = k_0E_0$. In this case we are taking into account one of the long known mechanisms of plasma nonlinearity (see, for example, refs. 1 and 2). For an arbitrary medium the presence of an electric field in the permittivity wave is not at all obligatory (in case (2) there is a field with amplitude $\mathbf{E}_0$). Thus, in a longitudinal acoustic wave the susceptibility $\epsilon$ changes as the result of a change in the density of the medium, which is essentially not associated with the appearance of the electric field.

When a permittivity wave (1) falls on an electric charge, an additional alternating polarization arises near this charge and, consequently, an electromagnetic wave diverging from the charge is formed. Thus, we can speak of the transformation of the permittivity wave into an electromagnetic wave (more accurately, waves of various types), but we prefer the term transition scattering. The point is that in the presence of a polarization wave and a uniform motion of the charge, the radiation arising usually is considered a variety of transition radiation. For a charge velocity $\mathbf{v} \neq 0$ such radiation is present, of course, even for $\omega_0 = 0$, $\mathbf{k}_0 \neq 0$ or for $\mathbf{k}_0 = 0$ but $\omega_0 \neq 0$ (for a discussion of the similar but less well known transition radiation in a nonstationary medium, see refs. 3 and 4). For a fixed charge (for $v = 0$), radiation of transverse waves arises only when $\omega_0 \neq 0$ and $\mathbf{k}_0 \neq 0$ simultaneously; however, in view of the above this case is also related to transition radiation. Note that in ref. 3, transition radiation for $\mathbf{v} = 0$ was not taken into account, since the longitudinal field of the charge was neglected. On the other hand, Gaĭlitis and Tsytovich[5] (see also ref. 2, Sec. 6.5) realistically took into account transition scattering in the longitudinal field in a plasma, but in application to one of the electrons or ions of the plasma. In this case it is necessary to take into account simultaneously also the ordinary scattering due to motion of the charge under the action of the field of the incident wave. Here transition scattering and its interference with ordinary scattering appear as playing a very large role in some cases. On the other hand, the general nature of transition scattering remains somewhat in the dark. In the present article, on the contrary, attention is concentrated on transition scattering in its simplest form but without specification of the type of medium.

1. We will consider a charge $q$ moving with constant velocity $\mathbf{v}$ in the medium with permittivity given by Eq. (1), limiting ourselves to phase values $\varphi_0 = 0$ and $\varphi_0 = \pi/2$. Then we have for the Fourier component of the electric induction $\mathbf{D}$ ($\epsilon_1^+ = \epsilon_1$, $\epsilon_1^- = -i\epsilon_1$):

$$\mathbf{D}(\omega,\mathbf{k})=\mathbf{D}_{\omega\mathbf{k}}=\varepsilon_0(\omega)\mathbf{E}_{\omega\mathbf{k}}+{}^1/{}_2\varepsilon_1^{\pm}(\mathbf{E}_{\omega+\omega_0,\mathbf{k}+\mathbf{k}_0}\pm\mathbf{E}_{\omega-\omega_0,\mathbf{k}-\mathbf{k}_0}). \tag{3}$$

It is necessary to keep in mind that in the general case (and particularly in a plasma) the spatial dispersion must be taken into account. In that case instead of Eq. (3) it is often possible to use a relation of the type

$$D_{j\omega\mathbf{k}}=\varepsilon_{0,ij}(\omega,\mathbf{k})E_{j\omega\mathbf{k}}+{}^1/{}_2\varepsilon_{1,ij}^{\pm}(\omega,\mathbf{k};\omega_0\mathbf{k}_0)(E_{j,\omega+\omega_0,\mathbf{k}+\mathbf{k}_0}\pm E_{j,\omega-\omega_0,\mathbf{k}-\mathbf{k}_0}). \tag{3a}$$

In the most general case the coefficients $\epsilon_{1,ij}$ can be found from the expressions for the nonlinear polarizabilities and for $E_{j,\omega-\omega_0,\mathbf{k}-\mathbf{k}_0}$ $\epsilon_{1,ij}$ enters with an argument differing in the sign of $\omega_0$ and $\mathbf{k}_0$ in comparison with $\epsilon_{1,ij}$ for $E_{j,\omega+\omega_0,\mathbf{k}+\mathbf{k}_0}$.

Writing the field equation in the form ($\hat{\epsilon}$ is the permittivity operator)

$$\operatorname{rot}\operatorname{rot}\mathbf{E}+\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\hat{\varepsilon}\mathbf{E}=-\frac{4\pi}{c^2}\frac{\partial\mathbf{j}}{\partial t},\qquad \mathbf{j}=q\mathbf{v}\delta(\mathbf{r}-\mathbf{v}t), \tag{4}$$

we obtain (for the sake of simplicity we limit ourselves to explicit inclusion of the dependence of $\epsilon_0$ only on $\omega$)

$$\left(k^2\delta_{ij}-k_ik_j-\frac{\omega^2}{c^2}\varepsilon_0(\omega)\delta_{ij}\right)E_{j\omega\mathbf{k}}=\frac{4\pi i\omega qv_i}{(2\pi)^3c^2}\delta(\omega-\mathbf{kv})+\frac{\omega^2}{2c^2}\varepsilon_1^{\pm}(E_{i,\omega+\omega_0,\mathbf{k}+\mathbf{k}_0}\pm E_{i,\omega-\omega_0,\mathbf{k}-\mathbf{k}_0}). \tag{5}$$

Below we will assume that

$$|\varepsilon_1|\ll\varepsilon_0 \tag{6}$$

and will use the method of successive approximations.

In the zeroth approximation (i.e., for $\epsilon_1 = 0$)

$$\mathbf{E}_{\omega\mathbf{k}}^{(0)}=\mathbf{E}_{\omega\mathbf{k}}^{(0)l}+\mathbf{E}_{\omega\mathbf{k}}^{(0)t}=\frac{4\pi i\omega q}{(2\pi)^3c^2}\left(\mathbf{v}-\frac{\mathbf{k}c^2}{\omega\varepsilon_0(\omega)}\right)\frac{\delta(\omega-\mathbf{kv})}{k^2-\omega^2\varepsilon_0(\omega)/c^2}, \tag{7}$$

where for the longitudinal part (the index $l$) and the transverse part (index t) of the field

$$\mathbf{E}_{\omega\mathbf{k}}^{(0)l}=-\frac{4\pi iq\mathbf{k}}{(2\pi)^3k^2\varepsilon_0(\omega)}\delta(\omega-\mathbf{kv}),\qquad \mathbf{E}_{\omega\mathbf{k}}^{(0)t}=\frac{4\pi i\omega q(\mathbf{v}-\mathbf{k}(\mathbf{kv})/k^2)}{(2\pi)^3c^2(k^2-\omega^2\varepsilon_0(\omega)/c^2)}\delta(\omega-\mathbf{kv}). \tag{8}$$

In the coordinate representation

$$\mathbf{E}^{(0)l}(\mathbf{r},t)=\frac{q(\mathbf{r}-\mathbf{v}t)}{\varepsilon_0|\mathbf{r}-\mathbf{v}t|^3},\qquad (9)$$
$$\operatorname{div}\mathbf{E}^{(0)l}=4\pi\rho=4\pi q\delta(\mathbf{r}-\mathbf{v}t).$$

It is evident that for a fixed charge (for $\mathbf{v}=0$) the field is

$$\mathbf{E}^{(0)}=\mathbf{E}^{(0)l},\quad \mathbf{E}^{(0)t}=0.$$

In the first approximation for the radiated (scattered) transverse waves to which we have limited our discussion, we obtain

$$\left(k^2-\frac{\omega^2}{c^2}\varepsilon_0(\omega)\right)\mathbf{E}_{\omega\mathbf{k}}^{(1)t}=\frac{\omega^2}{2c^2}\varepsilon_1^{\pm}\left[\mathbf{E}^{(0)}_{\omega+\omega_0,\mathbf{k}+\mathbf{k}_0}\pm\mathbf{E}^{(0)}_{\omega-\omega_0,\mathbf{k}-\mathbf{k}_0}-\frac{\mathbf{k}(\mathbf{k}\mathbf{E}^{0}_{\omega+\omega_0,\mathbf{k}+\mathbf{k}_0})}{k^2}\mp\frac{\mathbf{k}(\mathbf{k}\mathbf{E}^{0}_{\omega-\omega_0,\mathbf{k}-\mathbf{k}_0})}{k^2}\right],\qquad (10)$$
$$\mathbf{H}_{\omega\mathbf{k}}^{(1)}=\frac{c}{\omega}[\mathbf{k}\mathbf{E}_{\omega\mathbf{k}}^{(1)t}].$$

If we represent the field $\mathbf{E}_{\omega\mathbf{k}}^{(0)}$ only as the field $\mathbf{E}_{\omega\mathbf{k}}^{(0)}t$, the result is identical to that obtained in ref. 3, but here no allowance is made for the effect which is the main effect, roughly speaking, for $v\omega/k \ll c^2/\epsilon_0$ and the only effect for $v=0$. In the case of a fixed charge, considered below,

$$\begin{aligned}\mathbf{E}_{\omega\mathbf{k}}^{(1)t}=&-\frac{\omega^2\varepsilon_1^{\pm}}{2c^2(k^2-\omega^2\varepsilon_0(\omega)/c^2-i\Delta\omega/|\omega|)}\frac{4\pi iq}{(2\pi)^3\varepsilon_0(0)}\\&\times\left\{\left[\frac{\mathbf{k}+\mathbf{k}_0}{(\mathbf{k}+\mathbf{k}_0)^2}-\frac{\mathbf{k}(k^2+\mathbf{k}\mathbf{k}_0)}{k^2(\mathbf{k}+\mathbf{k}_0)^2}\right]\delta(\omega+\omega_0)\pm\left[\frac{\mathbf{k}-\mathbf{k}_0}{(\mathbf{k}-\mathbf{k}_0)^2}-\frac{\mathbf{k}(k^2-\mathbf{k}\mathbf{k}_0)}{k^2(\mathbf{k}-\mathbf{k}_0)^2}\right]\delta(\omega-\omega_0)\right\}=-\frac{iq\omega_0^2\varepsilon_1^{\pm}}{4\pi^2c^2\varepsilon_0(0)}\left(\mathbf{k}_0-\frac{\mathbf{k}(\mathbf{k}\mathbf{k}_0)}{k^2}\right)\\&\cdot\left\{\frac{\delta(\omega+\omega_0)}{(\mathbf{k}+\mathbf{k}_0)^2(k^2-\omega_0^2\varepsilon_0(-\omega_0)/c^2+i\Delta)}\mp\frac{\delta(\omega-\omega_0)}{(\mathbf{k}-\mathbf{k}_0)^2(k^2-\omega_0^2\varepsilon_0(\omega_0)/c^2-i\Delta)}\right\}\end{aligned}\qquad (11)$$

where in the absence of absorption at a frequency $\omega_0$ the quantity $\Delta\to 0$.

Let us turn to the components

$$\mathbf{E}_{\mathbf{k}}(t)=\int\mathbf{E}_{\omega\mathbf{k}}e^{-i\omega t}d\omega.$$

Then, according to Eq. (11) and omitting the index t, we have

$$\begin{aligned}\mathbf{E}_{\mathbf{k}}^{(1)}(t)=&-\frac{iq\omega_0^2\varepsilon_1^{\pm}(\mathbf{k}_0-\mathbf{k}(\mathbf{k}\mathbf{k}_0)/k^2)}{4\pi^2c^2\varepsilon_0(0)}\\&\times\left\{\frac{\exp(i\omega_0t)}{(\mathbf{k}+\mathbf{k}_0)^2}\left(k^2-\frac{\omega_0^2}{c^2}\varepsilon_0(-\omega_0)+i\Delta\right)^{-1}\right.\\&\left.\mp\frac{\exp(-i\omega_0t)}{(\mathbf{k}-\mathbf{k}_0)^2}\left(k^2-\frac{\omega_0^2}{c^2}\varepsilon_0(\omega_0)-i\Delta\right)^{-1}\right\}.\end{aligned}\qquad (12)$$

The variation of the field energy in the first approximation is

$$\begin{aligned}\frac{\partial W}{\partial t}&=\frac{1}{4\pi}\int\left(\mathbf{E}\hat{\varepsilon}_0\frac{\partial\mathbf{E}}{\partial t}+\mathbf{H}\frac{\partial\mathbf{H}}{\partial t}\right)d\mathbf{r}\\&=2\pi^2\int\left(\mathbf{E}_{-\mathbf{k}}(t)\hat{\varepsilon}_0\frac{\partial\mathbf{E}_{\mathbf{k}}(t)}{\partial t}+\mathbf{H}_{-\mathbf{k}}(t)\frac{\partial\mathbf{H}_{\mathbf{k}}(t)}{\partial t}\right)d\mathbf{k}.\end{aligned}\qquad (13)$$

Substituting the field (12) into (13) and discarding terms which oscillate in time, we obtain an expression for the average power of the radiation (scattering):

$$\begin{aligned}S=\frac{\overline{\partial W}}{\partial t}=&\frac{iq^2\omega_0^3}{8\pi^2c^2\varepsilon_0^2(0)}\int|\varepsilon_1|^2\frac{[\mathbf{k}\mathbf{k}_0]^2}{k^2}d\mathbf{k}\\&\times\left\{\frac{1}{(\mathbf{k}+\mathbf{k}_0)^4(k^2-\omega_0^2\varepsilon_0(-\omega_0)/c^2+i\Delta)}-\frac{1}{(\mathbf{k}-\mathbf{k}_0)^4(k^2-\omega_0^2\varepsilon_0(\omega_0)/c^2-i\Delta)}\right\}\\&=\frac{q^2\omega_0^3}{4\pi c^2\varepsilon_0^2(0)}\int|\varepsilon_1|^2\frac{[\mathbf{k}\mathbf{k}_0]^2\delta(k^2-\omega_0^2\varepsilon_0(\omega_0)/c^2)d\mathbf{k}}{k^2(\mathbf{k}+\mathbf{k}_0)^4}.\end{aligned}\qquad (14)$$

Here we have taken into account that in the absence of absorption $\epsilon(\omega_0)=\epsilon(-\omega_0)$, $\Delta\to+0$. If $\epsilon_0(0)$ is a complex quantity, it is necessary to replace $\epsilon_0^2(0)$ by $|\epsilon_0(0)|^2$. This same result can be obtained by considering the expression

$$\frac{4\pi i\omega}{c^2}\mathbf{j}_{\omega\mathbf{k}}=\frac{\omega^2\varepsilon_1^{\pm}}{2c^2}(\mathbf{E}_{\omega+\omega_0,\mathbf{k}+\mathbf{k}_0}\pm\mathbf{E}_{\omega-\omega_0,\mathbf{k}-\mathbf{k}_0}),$$

which enters into Eq. (5), converting to the quantities $\mathbf{j}_{\mathbf{k}}(t)$, and then calculating the work of the field

$$-\int\mathbf{E}\mathbf{j}\,d\mathbf{r}=-(2\pi)^3\int\mathbf{E}_{-\mathbf{k}}\mathbf{j}_{\mathbf{k}}\,d\mathbf{k}=\frac{\partial W}{\partial t}.$$

2. Carrying out the integration over $k^2=\omega_0^2\epsilon_0(\omega_0)/c^2$ in Eq. (14), we obtain

$$S=\frac{q^2\omega_0^2k_0^2k^2}{4c\varepsilon_0^2(0)\sqrt{\varepsilon_0(\omega_0)}\,(k^2+k_0^2)^2}\int_0^{\pi/2}\left(1-\frac{2kk_0}{(k^2+k_0^2)}\cos\theta\right)^{-2}\sin^3\theta\,d\theta|\varepsilon_1^{\pm}|^2,\qquad (15)$$

where $\theta$ is the angle between $\mathbf{k}$ and $\mathbf{k}_0$.

The further integration over $\theta$ depends on whether or not spatial dispersion effects in $\epsilon_1^{\pm}$ are taken into account (see Eq. (3a)), since with inclusion of the spatial dispersion, $\epsilon_1^{\pm}$ depends on $\theta$. An example of a medium with spatial dispersion will be given below. Here we neglect the spatial dispersion of $\epsilon_1^{\pm}$. In this case $\epsilon_1$ depends, generally speaking, on the frequency $\omega_0$. Then

$$S=\frac{q^2\omega_0^2|\varepsilon_1|^2}{8c\varepsilon_0^2(0)\sqrt{\varepsilon_0(\omega_0)}}\left(\frac{1}{\Lambda}\ln\frac{1+\Lambda}{1-\Lambda}-2\right),\qquad (15a)$$
$$\Lambda=\frac{2}{k/k_0+k_0/k}.$$

In the limiting cases

$$k\gg k_0\ \text{and}\ k\ll k_0;\quad \frac{1}{\Lambda}\ln\frac{1+\Lambda}{1-\Lambda}-2\approx\frac{2}{3}\Lambda^2,$$
$$S\approx\frac{q^2|\varepsilon_1|^2\omega_0^2}{3c\varepsilon_0^2(0)\sqrt{\varepsilon(\omega_0)}\,(k/k_0+k_0/k)^2}\qquad (16)$$

or

$$S=\frac{q^2\omega_0^4|\varepsilon_1|^2\sqrt{\varepsilon_0(\omega_0)}}{3c^3k_0^2\varepsilon_0^2(0)},\quad k\ll k_0,\qquad (17)$$

$$S=\frac{q^2ck_0^2|\varepsilon_1|^2}{3\varepsilon_0^2(0)\varepsilon_0^{3/2}(\omega_0)};\quad k\gg k_0,\qquad (18)$$
$$k^2=\omega_0^2\varepsilon_0(\omega_0)/c^2.$$

The inequality

$$k/k_0=\omega_0\sqrt{\varepsilon_0(\omega_0)}/ck_0=v_{ph}/c_{ph}\ll 1\qquad (19)$$

indicates that the phase velocity of the permittivity waves $v_{ph}=\omega_0/k_0$ is small in comparison with the phase velocity of the transverse (light) waves

$$c_{ph}=c/n,\quad n=\sqrt{\varepsilon_0(\omega_0)}.$$

For acoustic and a number of other perturbations, inequality (19) is satisfied very closely.

When condition (19) is satisfied, the scattered radiation, as is clear from Eq. (12) or (15), has a dipole nature, the dipole being oriented along $\mathbf{k}_0$. For comparison we present the well known expression for the average power of electromagnetic waves in the case of an electric dipole $\mathbf{p}=q\mathbf{r}=q\mathbf{r}_0\cos\omega_0t$ located in a medium with refractive index n,

$$S_p=q^2\omega_0^4r_0^2n/3c^3.\qquad (20)$$

Comparing Eqs. (17) and (20), we see that

$$\frac{S}{S_p}=\frac{|\varepsilon_1|^2}{\varepsilon_0^2(0)k_0^2r_0^2}=\frac{|\varepsilon_1|^2}{(\varepsilon_0(0))^2}\left(\frac{\lambda_0}{2\pi r_0}\right)^2,\quad \lambda_0=\frac{2\pi}{k_0}.\qquad (21)$$

Here, of course, it is assumed that in Eqs. (17) and (20) the charges q and other identically designated quantities are actually identical, although this is in no way obligatory.

The oscillation amplitude $\mathbf{r}_0$ of a charge q forming a dipole $\mathbf{p} = q\mathbf{r}$ was assumed above to be given. Let us assume now that we are discussing scattering of some field $\mathbf{E} = \mathbf{E}_0 \cos(\mathbf{k}_0\mathbf{r} - \omega_0 t)$ by a charge q with mass M. Then in the nonrelativistic case

$$M\ddot{\mathbf{r}}=q\mathbf{E},\quad \mathbf{r}=\mathbf{r}_0\cos(\mathbf{k}_0\mathbf{r}-\omega_0 t),\quad \mathbf{r}_0=-q\mathbf{E}_0/M\omega_0^2 \tag{22}$$

and the power (intensity) of the scattered waves (see Eq. (20)) is

$$S_p=q^4E_0^2n/3M^2c^3. \tag{23}$$

If E is the field in a plasma wave, then, according to Eqs. (2) and (17),

$$|\varepsilon_1|=ek_0E_0/m\omega_0^2,\quad \varepsilon_0(\omega_0)=1-\omega_{pe}^2/\omega_0^2,\quad S=q^2e^2E_0^2n/3c^3m^2(\varepsilon_0(0))^2. \tag{24}$$

In a plasma, however, it is necessary to take into account the spatial dispersion, and Eq. (24) is unsuitable or, more accurately, valid only for $\epsilon_0(0)\approx 1$. With inclusion of spatial dispersion[2,5] we obtain instead of Eq. (2)

$$|\varepsilon_1|=\frac{eE_0(\mathbf{k}+\mathbf{k}_0)^2}{m\omega_0^2k_0}(\varepsilon_e(0)-1), \tag{25}$$

where $\epsilon_e(0)$ is the electronic part of the dielectric permittivity $\epsilon_0$ for $\omega = 0$, $\mathbf{k}+\mathbf{k}_0 \neq 0$. If $\epsilon_e(0) \gg 1$, then

$$\varepsilon_e(0)\approx\omega_{pe}^2/(\mathbf{k}+\mathbf{k}_0)^2v_{Te}^2.$$

Substitution of Eq. (25) into (14) leads to the result

$$S=\frac{q^2e^2E_0^2n}{3c^3m^2}\left|\frac{\varepsilon_e(0)-1}{\varepsilon_0(0)}\right|^2. \tag{26}$$

In a plasma for $\epsilon_0(0) \gg 1$

$$\varepsilon_0(0)=\varepsilon_e(0)(1+T_e/T_i), \tag{27}$$

i.e., even for $T_e = T_i$

$$S=q^2e^2E_0^2n/12c^3m^2. \tag{28}$$

Actually, however, it is necessary to take into account both effects (namely, ordinary Thomson scattering (23) and transition scattering (26)), in which case interference occurs. As a result the total cross section for scattering of a plasma wave by an electron is

$$S_{eff}=\frac{e^4E_0^2n}{12c^3m^2}=\frac{1}{4}S_p(M=m). \tag{29}$$

It is clear from the above that inclusion of transition scattering in a plasma is necessary even in consideration of scattering by electrons. Furthermore, on taking into account the motion of the charge the power $S_{eff}$ may be strongly reduced in comparison with the power given by Eq. (29). In scattering by a positron located in a plasma ($q = e$, $M = m$) interference, on the contrary, leads to addition of the amplitudes and therefore the scattering power for $T_e = T_i$ increases by a factor of $9/4$ in comparison with the power given by Eq. (23) for Thomson scattering $S_p(M = m)$. In scattering of plasma waves by ions with radiation of transverse waves, transition radiation is the principal effect. The case of a plasma has been discussed in more detail in refs. 2 and 5. However, even for a plasma a number of problems have not yet been solved (scattering of strongly polarized waves, where $|\epsilon_1|$ is no longer small in comparison with $\epsilon_0$, analysis of a number of special cases in magnetoactive and nonuniform plasmas).

In an arbitrary anisotropic medium (in particular, in noncubic crystals), electromagnetic waves generally have a longitudinal component. As a result, in propagation of such waves there arise polarization waves which are related to the wave described by Eq. (2). In this case usually $k \sim k_0$ and in calculation of the transition radiation power it is necessary to use Eq. (15).

In propagation in a medium of acoustic waves, shock waves, and other comparatively slow perturbations with a phase velocity $v_{ph} = \omega_0/k_0 \ll c_{ph} = c/n$ (see condition (19)), case (17) always occurs, i.e.,

$$S=\frac{q^2\omega_0^4|\varepsilon_1|^2n}{3c^3k_0^2|\varepsilon_0(0)|^2}=\frac{q^2\omega_0^2(v_{ph}^2/c^2)|\varepsilon_1|^2n}{3c|\varepsilon_0(0)|^2}. \tag{30}$$

This result, it is true, also needs generalization for an anisotropic medium and for sufficiently strong permittivity waves where condition (6) is violated. Nevertheless, Eq. (30) can probably be used as a guide even for $|\epsilon_1| \lesssim \epsilon_0$ and $v_{ph} \lesssim c$. As far as the use of transition scattering is concerned, we can consider measurement by this means (from the intensity of electromagnetic radiation), for example, of the amplitude of variation of the density $\rho_1$ in nonlinear and shock waves (we assume that $\epsilon_1 = \rho_1 \partial\epsilon/\partial\rho$). For this purpose there should be contained in the medium, of course, some quantity of charges (say, ions with charge eZ). Then, under coherent scattering conditions the intensity S is proportional to $e^2Z^2N_iV$, where $N_i$ is the concentration of ions and V is the scattering volume. It is not clear to us whether the corresponding possibilities have a real significance; however, even independently of this, the effect of transition scattering deserves attention not only in plasma physics. We note also that transition scattering may have analogs in the case of transformation of waves of other types (for example, a longitudinal acoustic wave in a deformed region may produce a transverse acoustic wave).


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186