Structure functions of deep inelastic *ep* scattering

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Results are presented of an analysis of all existing data on deep inelastic *ep* scattering. Various parametrizations of the functions νW_2 and $R=\sigma_S/\sigma_T$ are considered. The region of scale invariance of the function νW_2 is established. It is shown that the experimetal data are satisfactorily described by various parametrizations of R.

1. INTRODUCTION

Experiments on the deep inelastic scattering of high energy electrons by protons[1,2] have led to substantial progress in understanding of photon-hadron interactions. Scale invariance (scaling) was discovered and it was shown that in the deep inelastic region the structure functions are of the order of unity.[1)] The discovery of scale invariance stimulated a large number of new directions in theoretical research: the parton model,[4] self-similar asymptotic behavior in local field theory,[5] and so forth.

In the present article we report[2)] the results of analysis of all experimental data on deep inelastic ep scattering.[1,2,6,7] The method of analysis used was that previously used in analysis of data on elastic ep scattering[8] and elastic scattering of electrons by deuterons.[9] To determine the structure functions by this method it is necessary to parametrize these functions from the beginning. The parameters introduced here are determined directly from the experimental data by minimization of the χ^2 functional. In a combined analysis of the data of different groups, norms are introduced—variable parameters taking into account possible errors in normalization of the cross sections. This method, in contrast to that used previously,[2] permits use in determination of the structure functions of the entire set of experimental data.

The analysis showed that scale invariance of νW_2 occurs in the region W ≥ 2.3 BeV (W is the mass of the final hadrons). Various parametrizations were considered for the ratio of the total cross sections for absorption of virtual photons with longitudinal and transverse polarizations

$$R=\sigma_S/\sigma_T. \tag{1}$$

From the results of the analysis it follows that the existing experimental data do not permit exclusion of those expressions for R which signify violation of scale invariance of the function $2MW_1$ in the region of variables where scale invariance of νW_2 exists. If we assume that νW_2 depends on the variable ω' proposed by Bloom and Gilman,[10] the experimental data can be described for W ≥ 1.8 BeV.

2. CROSS SECTION OF THE PROCESS. METHOD OF DATA ANALYSIS

It is well known that the differential cross section of the process

$$e+p\to e+\dots \tag{2}$$

for unpolarized initial particles has the following general form in the one-photon approximation (laboratory system):

$$\frac{d^2\sigma}{d\Omega\, dE'}=\left(\frac{d\sigma}{d\Omega}\right)_M\left[W_2+2W_1\,\mathrm{tg}^2\frac{\theta}{2}\right], \tag{3}$$

where

$$\left(\frac{d\sigma}{d\Omega}\right)_M=\frac{\alpha^2\cos^2(\theta/2)}{4E^2\sin^4(\theta/2)}. \tag{4}$$

Here E and E′ are the initial and final electron energies, θ is the scattering angle, and W_1 and W_2 are functions of the scalars

$$\nu=-\frac{1}{M}pq=E-E',\quad q^2, \tag{5}$$

where q and p are the 4-momenta of the photon and proton and M is the proton mass. The functions W_1 and W_2 are determined by the hadron part of the matrix element. We have

$$\sum\int\langle p'|J_\mu|p\rangle\langle p|J_\nu|p'\rangle\delta(p'-p-q)\,d\Gamma=\frac{e^2}{(2\pi)^6}\frac{M}{p_0}\left[W_1\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)+\frac{1}{M^2}W_2\left(p_\mu-\frac{pq}{q^2}q_\mu\right)\left(p_\nu-\frac{pq}{q^2}q_\nu\right)\right], \tag{6}$$

where J_μ is the hadron electromagnetic current operator and p′ is the total 4-momentum of the final hadrons.

The total cross sections σ_T and σ_S are connected with W_1 and W_2 by the relations[11]

$$\sigma_T=(2\pi)^2\alpha\frac{1}{k}W_1,\qquad \sigma_S=(2\pi)^2\alpha\frac{1}{k}\left(-W_1+\frac{q^2+\nu^2}{q^2}W_2\right). \tag{7}$$

Here

$$k=\nu-q^2/2M,\quad \alpha=e^2/4\pi. \tag{8}$$

For $q^2\to 0$ we have

$$\sigma_T\to\sigma_\gamma,\quad \sigma_S\to 0, \tag{9}$$

where σ_γ is the total cross section for absorption of protons by real photons with energy ν.

From Eq. (7) it follows that

$$2MW_1=\omega\nu W_2\frac{1+q^2/\nu^2}{1+R}, \tag{10}$$

where the variable ω is defined as follows:

$$\omega=2M\nu/q^2. \tag{11}$$

Since R ≥ 0, we find from Eq. (10)

$$(1+q^2/\nu^2)\,\omega\nu W_2\geq 2MW_1. \tag{12}$$

To determine νW_2 and R (or νW_2 and $2MW_1$) by the method used, it is necessary to parametrize these functions. The corresponding parameters are then found from all existing data by minimization of the functional

$$\chi^2=\sum_k\sum_i\frac{1}{\Delta_{ik}^2}(\sigma_{i,k}^{\text{exp}}-N_k\sigma_i^{\text{theo}})^2. \qquad (13)$$

Here $\sigma_{i,k}^{\text{exp}}$ is the differential cross section for the process at the i-th point measured in the k-th experiment, Δ_{ik} is the error in $\sigma_{i,k}^{\text{exp}}$, σ_i^{theo} is the cross section at the i-th point calculated with Eq. (3), and N_k are normalization factors. Minimization of the χ^2 functional is carried out by the linearization method.[12] The method of analysis has been described in more detail in an earlier article.[13]

Bjorken[13] showed on the basis of current algebra that for $q^2 \to \infty$, $\nu \to \infty$ and fixed $\omega = 2M\nu/q^2$ the dimensionless functions νW_2 and $2MW_1$ depend only on the variable ω (scale invariance). It follows from experiments on deep inelastic ep scattering that scale invariance is consistent with the data even for $\sqrt{q^2}$ and ν of the order of 1 BeV. To start with we will assume scale invariance of νW_2.

In parametrization of this function we will take into account the relation obtained in refs. 10 and 15 connecting the behavior of the function νW_2 for $\omega \to 1$ with the behavior of the proton magnetic form factor $G_M(q^2)$ for $q^2 \to \infty$. If we write

$$\nu W_2 \underset{\omega\to 1}{\infty} \left(1-\frac{1}{\omega}\right)^p, \quad G_M(q^2) \underset{q^2\to\infty}{\infty} \frac{1}{(q^2)^n}, \qquad (14)$$

then, as shown by Drell and Yan[15] and Bloom and Gilman[10],

$$p=n+1. \qquad (15)$$

From analysis of the data on elastic ep scattering it follows that n = 2. Thus, if we assume Eq. (15),

$$p=3. \qquad (16)$$

In addition, it follows from the theory of complex angular momenta[16] that

$$\nu W_2 \underset{\omega\to\infty}{\longrightarrow} \text{const}, \quad 2MW_1/\omega \underset{\omega\to\infty}{\longrightarrow} \text{const}. \qquad (17)$$

We will consider first for the function νW_2 the following expression which satisfies conditions (14), (16), and (17):

$$\nu W_2=\sum_i a_i\left(1-\frac{1}{\omega}\right)^{i+3}. \qquad (18)$$

The function $2MW_1$ is related to νW_2 and R by Eq. (10). We will consider various expressions for the ratio R.

3. RESULTS OF THE ANALYSIS

1. We will assume that

$$R=q^2/\nu^2. \qquad (19)$$

Then, as can be seen from Eq. (10), the functions $2MW_1$ and νW_2 are related by the equation

$$2MW_1=\omega\nu W_2. \qquad (20)$$

It should be noted that this relation arises in the parton model for the case of partons with spin 1/2.[17] It occurs if the squares of the moduli of the matrix elements for photon absorption, summed over the final states, are equal for transverse and longitudinal polarization of the photons.

The results of the analysis of the data obtained by the SLAC-MIT group[1,2] show that a satisfactory description of the experimental data is achieved in this case if the mass of the final hadrons $W \geq 2.5$ BeV. Note that the variable W is related to q^2 and ν by the equation

$$W^2=2M\nu+M^2-q^2. \qquad (21)$$

For this case the following three parameters are different from zero in Eq. (18):

$$a_0=1.12\pm0.07, \quad a_1=1.00\pm0.20, \quad a_2=-1.98\pm0.14 \qquad (22)$$

($\chi^2/\overline{\chi}^2$ = 162/135; the confidence level is 5.5%).

2. It is possible to obtain a relation between $2MW_1$ and νW_2 which agrees with experiment over a wider range of the variable W. Let us assume that scale invariance occurs both for νW_2 and for $2MW_1$. This assumption means that the ratio $(1 + R)/(1 + q^2/\nu^2)$ depends only on ω. We will expand this ratio in a series in $1/\omega$. We obtain

$$\frac{\omega\nu W_2}{2MW_1}=1+\frac{d}{\omega}+\frac{d_1}{\omega^2}+\ldots \qquad (23)$$

Analysis of the data showed that in this expansion it is sufficient to take into account the term d/ω. We have

$$2MW_1=\omega\nu W_2(1+d/\omega)^{-1}. \qquad (24)$$

The experimental data obtained in refs. 1 and 2 for $W \geq 2.3$ BeV can be described by means of Eq. (24). In the expansion (18) it is sufficient to assume that a_0 and a_2 are nonzero. We obtain the following parameter values:

$$a_0=1{,}64\pm0{,}02, \quad a_2=-1.50\pm0.02, \quad d=0.63\pm0.09. \qquad (25)$$

The ratio $\chi^2/\overline{\chi}^2$ = 162/150; the confidence level is 23.9%.

3. As we have seen, the data of refs. 1 and 2 in the region $W \geq 2.3$ BeV are satisfactorily described by means of three parameters if we assume that scale invariance exists both of νW_2 and of $2MW_1$. We can convince ourselves, however, that the existing data also are satisfactorily described for those parametrizations of R which signify violation of scale invariance of the functions $2MW_1$. We will consider the following expressions for R:

$$R=c_1q^2/M^2, \qquad (26.1)$$

$$R=c_2q^2/W^2, \qquad (26.2)$$

$$R=c_3/\omega, \qquad (26.3)$$

$$R=\text{const}. \qquad (26.4)$$

The first three expressions satisfy the condition R = 0 for $q^2 = 0$. Note that in the Bjorken limit ($q^2 \to \infty$, $\nu \to \infty$, ω fixed) $R = c_1q^2/M^2 \to \infty$, $R = c_2q^2/W^2 \to c_2(\omega - 1)^{-1}$. Equations (26.1) and (26.4) have been discussed previously in ref. 2.

The results of analysis of the data of Bloom et al.[1] and Miller et al.[2] in the region $W \geq 2.3$ BeV are presented in Table I. It is evident from the table that for

TABLE I. Results of analysis of the SLAC-MIT data [1,2] with parametrization (18)

R	a_0	a_1	a_2		$\chi^2/\overline{\chi}^2$
From (26.1)	1.59±0.02	—	−1.44±0.02	$c_1 = 0.035\pm0.004$	124/150
From (26.2)	1.61±0.02	—	−1.47±0.02	$c_2 = 0.41\pm0.05$	145/150
From (26.3)	1.63±0.02	—	−1.48±0.02	$c_3 = 0.82\pm0.09$	142/150
R = const	1.57±0.02	—	−1.38±0.02	$R = 0.27\pm0.03$	181/150
R = const	1.29±0.06	0.78±0.17	−1.90±0.12	$R = 0.23\pm0.03$	162/149

satisfactory description of the experimental data it is sufficient to assume the coefficients a_0 and a_2 to be nonzero in expansion (18). The values of these coefficients for various parametrizations of R agree within experimental error. Note also that the coefficients a_0 and a_2 are nearly of the same magnitude and opposite in sign. The value found by us for the coefficient c_1 agrees with the value determined in ref. 2 by another method.

For R = const the description of the experimental data is substantially improved if we assume, in addition to a_0 and a_2, that the parameter a_1 is also nonzero.

4. We also analyzed that data of refs. 1 and 2, parametrizing the functions νW_2 and $2MW_1$. For νW_2 we assumed Eq. (18). For the function $2MW_1$ we assumed the expression

$$2MW_1=\omega\sum_i b_i(1-\omega^{-1})^{i+3}. \tag{27}$$

It is evident that Eq. (27) satisfies Eq. (17). It turned out that the data can be described by means of Eqs. (18) and (27). In the expansion (18) it is sufficient in this case to retain two terms, and in the expansion (27)—three terms. As a result of analysis of the data in the region $W \geq 2.3$ BeV we find

$$a_0=1.68\pm0.02,\quad a_2=-1.57\pm0.04,$$
$$b_0=0.72\pm0.14,\quad b_1=1.35\pm0.42,\quad b_2=-1.84\pm0.31, \tag{28}$$

($\chi^2/\overline{\chi}^2$ = 154/148; the confidence level is 35.6%).

The functions νW_2 and $2MW_1$ should satisfy inequality (12), which is equivalent to the condition $\sigma_S \geq 0$. We note that in the last of the cases discussed this inequality at some points is satisfied only within two standard deviations.

5. Let us turn to discussion of the results presented in Table II ($W \geq 2.3$ BeV) of a combined analysis of all published data on deep inelastic ep scattering. In the last three columns we have shown the norm values obtained: N_1 is the norm for the data of the SLAC-MIT group[1,2] (153 points), N_2 and N_3 are the respective norms for the DESY 1971 data[7] (83 points) and the DESY 1969 data[6] (31 points).

From Table II we can conclude that only introduction of norms permits description of all existing data on deep inelastic ep scattering. The norm values for different parametrizations of R agree within experimental error. A combined description of all data in this case requires a significant renormalization of the results of Moritz et al.[7] Comparison of Tables I and II shows that the parameter values obtained in analysis of the data of refs. 1 and 2 and of the world set of data agree within experimental error.

6. Up to this time we have used Eq. (18) for νW_2. We can convince ourselves that the experimental data on deep inelastic ep scattering can be satisfactorily described also for other parametrizations of the structure function νW_2. Let us consider the expression

$$\nu W_2=\left(\frac{\omega^2-1}{\omega^2+\omega_0^2}\right)^3\left(r_1+r_2\frac{1}{\sqrt{\omega}}\right), \tag{29}$$

where r_1, r_2, and ω_0^2 are the parameters which can be varied. This expression was proposed by Moffat and Snell[18] and is based on the theory of complex angular momenta. It is evident that Eq. (29) satisfies both the threshold relation (16) and the asymptotic condition (17). For the ratio R the same expressions were considered as before. The results of analysis of the data of refs. 1 and 2 are given in Table III ($W \geq 2.3$ BeV). As can be seen, if Eq. (29) is used for νW_2, an acceptable description of the data is obtained for all five of the parametrizations of the ratio R considered by us.

From comparison of the results given in Tables I and III, it follows that the parameters entering into the expression for R agree within experimental error for the two parametrizations of νW_2. In Table IV we have given the values of the functions νW_2 calculated from Eqs. (18) and (29) for $R = c_1q^2/M^2$ (columns 2 and 4) and for R = const (columns 3 and 5). Figures 1 and 2 show the function νW_2 as a function of ω, as obtained respectively from Eqs. (18) and (29) for the various parametrizations of the ratio R in the region of variation of ω from 1 to 100. In all cases we used the parameter values obtained as the result of analysis of the data of refs. 1 and 2.

By analyzing the SLAC results,[1,2] we became convinced that for $W \geq 2.3$ BeV it is possible to describe the data with three parameters over the entire experimentally investigated interval of q^2 ($0.25\ (\text{BeV}/c)^2 \leq q^2 \leq 19.2\ (\text{BeV}/c)^2$). The variable ν varies in this case over the range $1.87\ \text{BeV} \leq \nu \leq 16.28$ BeV, and the variable ω—over the range $1.25 \leq \omega \leq 37.8$. Values of the variable ω greater than 20 correspond to $q^2 \leq 1\ (\text{BeV}/c)^2$. The question of the behavior of νW_2 at large ω presents considerable interest for the theory. We made a separate analysis of the SLAC results, selecting data with $q^2 \geq 0.5\ (\text{BeV}/c)^2$, $q^2 \geq 1\ (\text{BeV}/c)^2$, and $q^2 \geq 1.5\ (\text{BeV}/c)^2$. The parameter values obtained as the result of this analysis for $W \geq 2.3$ BeV are given in Tables V and VI.

7. Determination of the structure function νW_2 permits calculation of the integrals entering into the sum rules for this function. Let us first consider the Callan-Gross sum rule[19]:

$$\int_1^\infty \nu W_2\frac{d\omega}{\omega^2}=\sum_N\sum_{i=1}^N Q_i^2\frac{1}{N}P_N. \tag{30}$$

Here Q_i is the charge of i-th parton, P_N is the probability of observing N partons in a proton. Using the values of a_0 and a_2 given in Table I for R = const, we obtain

$$\int_1^\infty \nu W_2\frac{d\omega}{\omega^2}=0.162\pm0.001. \tag{31}$$

If we use the values of a_0 and a_2 obtained for $R = c_1q^2/M^2$,

TABLE II. Results of analysis of the data of refs. 1, 2, 6, and 7 with parametrization (18)

	a_0	a_2		N_1	N_2	N_3	$\chi^2/\overline{\chi}^2$
R from (26.1)	1.55±0.02	−1.42±0.02	c_1 = 0.038±0.004	1.041±0.005	0.755±0.010	0.997±0.053	249/261
R from (26.2)	1.58±0.02	−1.45±0.02	c_2 = 0.47±0.05	1.039±0.005	0.763±0.010	1.027±0.054	267/261
R from (26.3)	1.59±0.02	−1.46±0.02	c_3 = 0.91±0.09	1.040±0.005	0.761±0.010	1.074±0.057	266/261
R = const	1.54±0.02	−1.37±0.02	R = 0.31±0.03	1.039±0.005	0.759±0.010	1.147±0.062	303/261 *
(24)	1.61±0.02	−1.48±0.02	d = 0.70±0.08	1.038±0.005	0.765±0.010	1.121±0.060	286/261

*The confidence level is 3.7%.

then

$$\int_1^\infty \nu W_2 \frac{d\omega}{\omega^2} = 0.158 \pm 0.001. \tag{32}$$

Finally, from the results of analysis of the SLAC data for $q^2 \geq 1.5$ $(BeV/c)^2$ for $R = 0.26 \pm 0.04$ we obtain

TABLE III. Results of analysis of the SLAC-MIT data [1,2] with parametrization (29) ($W \geqslant 2.3$ BeV)

	r_1	r_2	ω_0^2		$\chi^2/\overline{\chi}^2$
R from (26.1)	0.11±0.01	0.69±0.03	0.68±0.03	$c_1 = 0.036 \pm 0.004$	131/149
R from (26.2)	0.11±0.01	0.67±0.03	0.59±0.03	$c_2 = 0.56 \pm 0.06$	148/149
R from (26.3)	0.10±0.01	0.72±0.03	0.69±0.03	$c_3 = 0.88 \pm 0.10$	144/149
(24)	0.10±0.01	0.74±0.03	0.69±0.03	$d = 0.67 \pm 0.10$	161/149
R = const	0.12±0.01	0.71±0.03	0.78±0.03	$R = 0.26 \pm 0.03$	164/149

TABLE IV

ω	The function νW_2 according to (18)		The function νW_2 according to (29)	
	$R = 0{,}035q^2/M^2$	$R = 0{,}27$	$R = 0{,}036\, q^2/M^2$	$R = 0{,}26$
1.334	0.022±0.0005	0.021±0.0004	0.024±0.0002	0.023±0,0002
2.024	0.161±0.002	0.157±0.002	0.159±0.001	0.157±0.002
3.213	0.299±0.002	0.303±0.003	0.297±0.002	0.298±0.003
4.462	0.336±0.002	0.345±0.003	0.339±0.002	0.344±0.003
4.680	0.338±0.002	0.348±0.003	0.342±0.002	0.347±0.003
5.294	0.340±0.002	0.351±0.003	0.352±0.003	0.344±0.002
6.637	0.335±0.002	0.347±0.003	0.350±0.003	0.340±0.002
7.805	0.327±0.003	0.339±0.003	0.343±0.003	0.330±0.003
10.101	0.309±0.003	0.322±0.004	0.327±0.004	0.310±0.003
21.206	0.255±0.005	0.267±0.006	0.272±0.005	0.247±0.005
33.443	0.226±0.006	0.238±0.007	0.245±0.005	0.217±0.005

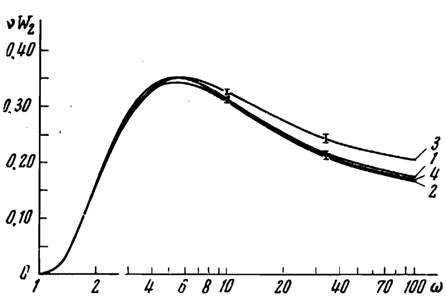

FIG. 1. The function νW_2 as a function of ω according to Eq. (18) for various parametrizations: 1—$R = 0.035q^2/M^2$, 2—$R = 0.41q^2/W^2$; 3—$R = 0.27$; 4—Eq. (24), $d = 0.63$.

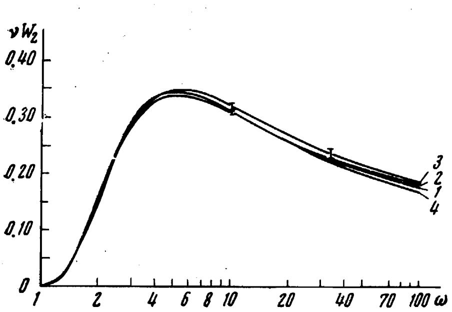

FIG. 2. The function νW_2 as a function of ω according to Eq. (29) with various parametrizations: 1—$R = 0.036q^2/M^2$; 2—$R = 0.056q^2/W^2$; 3—$R = 0.26$; 4—Eq. (24), $d = 0.67$.

TABLE V. Results of analysis of the SLAC-MIT data [1,2] with parametrization (18) ($W \geqslant 2.3$ BeV)

	q^2 [(BeV/c)^2] ⩾				q^2 [(BeV/c)^2] ⩾		
	0.5	1.0	1.5		0.5	1.0	1.5
a_0	1.54±0.02	1.54±0.02	1.53±0.02	a_0	1.59±0.02	1.59±0.02	1.59±0.03
a_2	−1.34±0.02	−1.32±0.03	−1.30±0.03	a_2	−1.42±0.03	−1.43±0.03	−1.44±0.04
R	0.26±0.03	0,26±0.03	0.26±0.04	c_2	0.37±0.05	0,38±0.05	0.37±0.05
$\chi^2/\overline{\chi}^2$	145/148	128/141	119/132	$\chi^2/\overline{\chi}^2$	125/148	117/141	107/132

$$\int_1^\infty \nu W_2 \frac{d\omega}{\omega^2} = 0.166 \pm 0.002. \tag{33}$$

Note that in the ordinary quark model the right-hand side of Eq. (30) is equal to 1/3.

For the integral entering into the Gottfried sum rule[20] we have for R = const and for all q^2

$$\int_1^{30} \nu W_2 \frac{d\omega}{\omega} = 0.97 \pm 0.01. \tag{34}$$

8. Up to this time we have assumed that νW_2 depends only on ω. We convinced ourselves that for very different assumptions about R in this case it is possible to describe the entire set of data on deep inelastic ep scattering. In order to estimate the magnitude of the possible violation of scale invariance, we analyzed the SLAC data with the assumption that the function νW_2 has the form

$$\nu W_2 = \sum_i a_i \left(1 - \frac{1}{\omega}\right)^{i+3} + \alpha \frac{M^2}{q^2}. \tag{35}$$

It is evident that the second term of this expression disappears in the Bjorken limit. We will assume that the functions $2MW_1$ and νW_2 are related by Eq. (24). As the result of analysis of the data on refs. 1 and 2 we find that for $\chi^2/\overline{\chi}^2 = 150/149$:

$$a_0 = 1.65 \pm 0.02, \quad a_2 = -1.47 \pm 0.02, \\ d = 0.61 \pm 0.08, \quad \alpha = -0.02 \pm 0.01. \tag{36}$$

Comparing Eqs. (36) and (25), we see that inclusion in νW_2 of a term violating scale invariance does not change the values of the parameters a_0, a_2, and d within experimental error. The parameter α which characterizes the violation of scale invariance is equal to zero within two standard deviations.

9. Bloom and Gilman[10] proposed the scale variable

$$\omega' = (2M\nu + M^2)/q^2, \tag{37}$$

which for $2M\nu > M^2$ is identical with ω. It was shown that νW_2 is a function of ω' over a wider range of the variable W. We analyzed the data of refs. 1 and 2 with parametrization of νW_2 as follows:

$$\nu W_2 = \sum_{i=0} A_i \left(1 - \frac{1}{\omega'}\right)^{i+3}, \tag{38}$$

where

$$\omega' = (2M\nu + \beta)/q^2. \tag{39}$$

Here it turned out that in the region directly adjoining the resonance ($W \geq 1.8$ BeV), the existing data can be described (see Table VII). In the expansion (38) it is sufficient to take as nonzero three parameters: A_0, A_1, and A_2. If we assume that R = const, then we obtain for the parameters being varied

$$A_0 = 0.60 \pm 0.08, \; A_1 = 1.96 \pm 0.17, \; A_2 = -2.37 \pm 0.10, \\ \beta = 0.91 \pm 0.07, \; R = 0.20 \pm 0.02, \tag{40}$$

for $\chi^2/\overline{\chi}^2 = 169/187$.

TABLE VI. Results of analysis of the SLAC-MIT data [1,2] with parametrization (29) ($W \geqslant 2.3$ BeV)

	q^2 [(BeV/c)^2] ⩾				q^2 [(BeV/c)^2] ⩾		
	0.5	1.0	1.5		0.5	1,0	1.5
r_1	0.15±0.01	0.18±0.02	0.21±0.03	r_1	0.15±0.01	0.18±0.02	0.20±0.02
r_2	0.62±0.03	0.56±0.04	0.50±0.06	r_2	0.58±0.03	0.52±0.04	0.46±0.05
ω_0^2	0.72±0.03	0.67±0.04	0.64±0.04	ω_0^2	0.54±0.03	0.50±0.04	0.46±0,04
R	0,25±0.03	0.26±0.03	0.26±0.04	c_2	0.55±0.06	0,54±0.06	0.50±0.06
$\chi^2/\overline{\chi}^2$	130/147	116/140	107/131	$\chi^2/\overline{\chi}^2$	112/147	100/140	88/131

TABLE VII. Results of analysis of the SLAC-MIT data [1,2] with parametrization (38) and (39)

	A_0	A_1	A_2	$c_{1,2,3}$	β	$\chi^2/\overline{\chi^2}$	Confidence level
R from (26.1)	0.93 ± 0.11	1.38 ± 0.23	-2.18 ± 0.13	0.028 ± 0.004	0.63 ± 0.07	246/191	0.4%
R from (26.2)	0.98 ± 0.11	1.22 ± 0.23	-2.06 ± 0.13	0.37 ± 0.04	0.76 ± 0.07	230/191	2.68%
R from (26.3)	0.70 ± 0.09	1.91 ± 0.19	-2.48 ± 0.11	0.61 ± 0.07	0.85 ± 0.07	223/191	5.48%

In this analysis we chose data with $q^2 \geq 0.5$ $(\text{BeV/c})^2$. If this limitation is not imposed, then, as can be seen from Table VII, for $W \geq 1.8$ BeV Eqs. (38) and (39) can satisfactorily describe the data in the case where Eqs. (26.2) and (26.3) are used for R.

The values of A_i and β obtained by us are in agreement with the values found by Miller et al.[2]

If we assume that $2MW_1$ and νW_2 are related by the equation

$$2MW_1=\omega'\nu W_2(1+D/\omega')^{-1} \tag{41}$$

and νW_2 is given by Eq. (38), then for the parameters being varied we find ($W \geq 1.8$ BeV):

$$A_0=0.50\pm0.08,\ A_1=2.35\pm0.17,\ A_2=-2.71\pm0.10.$$
$$\beta=1.03\pm0.09,\ D=0.33\pm0.07$$

($\chi^2/\overline{\chi^2} = 215/191$; the confidence level is 11.5%).

In conclusion we would like to mention that there are seven points of refs. 1 and 2 which give a large (greater than 9) contribution to χ^2 for all of the parametrizations considered by us. If these points are not included in the analysis of the data, the quality of the description is substantially improved in all cases.

4. CONCLUSION

We have analyzed the existing data on deep inelastic ep scattering. We have separately analyzed the data of the SLAC-MIT group[1,2] and the world data.[1,2,6,7] We have shown that for $W \geq 2.3$ BeV the data are satisfactorily described on the assumption that the function νW_2 depends only on the scale variable ω. Various parametrizations of this function were considered. Violation of scale invariance of νW_2 was introduced. It was shown that the parameter characterizing the violation is nonzero only by two standard deviations.

Various expressions have been considered for the ratio R. The existing data do not permit definite conclusions to be drawn about R and the scale invariance of $2MW_1$. For unique determination of the ratio R, data are necessary on the cross sections for deep inelastic scattering of leptons by protons over a wider range of the kinematic variables.

We have checked in detail the validity of the relation $2MW_1 = \omega\nu W_2$ which follows from the parton model in the case in which the spin of the partons is 1/2. If this relation is modified by introduction of an additional term whose contribution disappears asymptotically, then, as shown by analysis of the data, all data can be described satisfactorily. If we assume that νW_2 is a function of ω', then in this case all data for $W \geq 1.8$ BeV can be described.

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1)The possibility of this behavior of the structure functions was discussed many years ago by Markov. [3]

2)Preliminary results were presented at the XVI Conference on High-Energy Physics at Chicago and Batavia in September 1972.

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