Anistropy of stimulated Mandel'shtam–Brillouin scattering

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It is shown that in an isotropic liquid medium the angular distribution of SMBS depends on the intensity of the pumping wave. At low intensities the scattered radiation is directed forward direction and at high intensities backward.

1. INTRODUCTION

The stimulated Brillouin scattering is known to occur because the state of a medium in an electromagnetic field is unstable when the density of this field exceeds a certain threshold value. Beyond this threshold the intensity of the scattered radiation increases and acoustic waves appear and grow in amplitude. Since the rate of growth of these waves as well as the threshold field vary with the direction, we can expect an anisotropy in the intensity of the stimulated Brillouin scattering and this anisotropy should depend on the pumping wave amplitude.

We shall show that in a homogeneous isotropic liquid medium traversed by a plane linearly polarized pumping wave of constant amplitude the minimum threshold of the stimulated Brillouin scattering corresponds to the forward direction, i.e., this threshold corresponds to the scattering in the direction of the pumping wave. However, in the case of small scattering angles, the rate of development of instabilities is slow. As the intensity of the pumping wave is increased, the range of angles in which the stimulated effect is observed expands and the angle at which the instabilities develop most rapidly increases. Finally, at sufficiently high pumping wave intensities, the instability corresponding to the backward direction grows most rapidly.

We shall conclude by considering the possibilities of measuring the predicted features of the stimulated Brillouin scattering and we shall consider some experiments in which these features have probably been manifested.

2. INITIAL RELATIONSHIPS

Let us consider a homogeneous liquid medium traversed by a linearly polarized pumping wave

$$\mathbf{E}=\mathbf{E}_0\cos(\omega_0 t-\mathbf{k}_0\mathbf{r}).$$

In the linear approximation the equations for parametrically coupled, via the pumping wave, perturbations of the density ρ_1 and electric field **E** are (see, for example,[1]):

$$\left(v^2\Delta+\Gamma\Delta\frac{\partial}{\partial t}-\frac{\partial^2}{\partial t^2}\right)\rho_1=\frac{1}{4\pi}\rho\left(\frac{\partial\varepsilon'}{\partial\rho}\right)_s\Delta\langle\mathbf{E}\mathbf{E}_1\rangle, \tag{2.1}$$

$$c^2\Delta\mathbf{E}_1-\frac{\partial^2}{\partial t^2}\int_{-\infty}^{t}dt'\varepsilon(t-t')\mathbf{E}_1(t',\mathbf{r})=\left(\frac{\partial\varepsilon'}{\partial\rho}\right)_s\left[\frac{\partial^2}{\partial t^2}(\mathbf{E}\rho_1)-\frac{c^2}{\varepsilon'}\nabla(\mathbf{E}\nabla\rho_1)\right] \tag{2.2}$$

where $v\approx(\partial p/\partial\rho)_S^{1/2}$ is the adiabatic velocity of sound;[1)] ρ is the unperturbed density of the liquid; $\epsilon'=\mathrm{Re}\,\epsilon(\omega_0)$ is the real part of the permittivity, where[2]

$$\varepsilon(\omega)=\int_0^\infty dt\,e^{i\omega t}\varepsilon(t).$$

The quantity Γ represents the absorption of sound and can be expressed in terms of the viscosities η and ζ and in terms of the thermal conductivity κ:[3]

$$\Gamma=\frac{1}{\rho}\left[\left(\frac{4}{3}\eta+\zeta\right)+\varkappa\left(\frac{1}{c_v}-\frac{1}{c_p}\right)\right],$$

where c_V and c_P are, respectively, the specific heat at constant volume and constant pressure.

If we seek the solution of Eqs. (2.1) and (2.2) in the form $\rho_1=\rho_0 e^{-i\omega t+i\mathbf{k}\cdot\mathbf{r}}$, $\mathbf{E}_1=e^{-i\omega t+i\mathbf{k}\cdot\mathbf{r}}(\mathbf{E}_{1+}e^{-i\omega_0 t+i\mathbf{k}_0\mathbf{r}}+\mathbf{E}_{1-}e^{i\omega_0 t-i\mathbf{k}_0\mathbf{r}})$, the relationship between the frequency ω and the wave vector **k** is given by the dispersion equation

$$(\omega^2+i\omega k^2\Gamma-k^2v^2)=-\frac{k^2}{16\pi}\rho\left(\frac{\partial\varepsilon'}{\partial\rho}\right)_s^2\left[\frac{c}{\varepsilon'}(\mathbf{k}\mathbf{E}_0)^2-\omega_0^2E_0^2\right]$$
$$\times\left\{\frac{1}{(\omega+\omega_0)^2\varepsilon(\omega+\omega_0)-c^2(\mathbf{k}+\mathbf{k}_0)^2}+\frac{1}{(\omega-\omega_0)^2\varepsilon(\omega-\omega_0)-c^2(\mathbf{k}-\mathbf{k}_0)^2}\right\}. \tag{2.3}$$

Small terms of the order of ω/ω_0 are omitted from the right-hand side of Eq. (2.3).

We shall assume that the wave vector **k** is real and solve Eq. (2.3) for ω. This formulation corresponds to the traditional problem of the time dependence of the initial perturbations[4] and is usually considered in connection with the stimulated Brillouin scattering in a resonator.[1] [2)] If we then find that $\mathrm{Im}\,\omega>0$, the initial perturbations grow in time and the system is unstable. The conditions corresponding to the onset of the instability follow from $\mathrm{Im}\,\omega=0$.

We shall consider only low pumping fields when the approximation of a weak parametric coupling is valid[5] and the change in the dispersion law of the acoustic waves is slight:

$$\omega=kv+\Delta,\quad|\Delta|\ll kv.$$

Using the dispersion law for the pumping wave $\omega_0^2\epsilon'=k_0^2c^2$, we obtain the following equation for Δ from Eq. (2.3):

$$(2\Delta+ik^2\Gamma)(2\Delta+i\gamma_0)=\mu\frac{kv\omega_0^2}{\partial(\omega_0^2\varepsilon')/\partial\omega_0}\left(\frac{k^2}{k_0^2}\cos^2\varphi-1\right), \tag{2.4}$$

where

$$\cos\varphi=\frac{\mathbf{k}\mathbf{E}_0}{kE_0},\quad\gamma_0=\frac{2\omega_0^2\varepsilon''}{\partial(\omega_0^2\varepsilon')/\partial\omega_0}, \tag{2.5}$$
$$\varepsilon''=\mathrm{Im}\,\varepsilon(\omega_0),\quad\mu=\frac{E_0^2}{8\pi\rho v^2}\left[\rho\left(\frac{\partial\varepsilon'}{\partial\rho}\right)_s\right]^2.$$

The quantity k in Eq. (2.4) is governed by the laws of conservation $\mathbf{k}_0-\mathbf{k}=\mathbf{k}'$, $\omega_0-\omega=\omega'$ ($\mathbf{k}'$ and ω' are the wave vector and frequency of the scattered wave) and if small terms of the order of v/c are ignored, we find that

$$k\approx2k_0\cos\chi, \tag{2.6}$$

where $\cos\chi=\mathbf{k}\cdot\mathbf{k}_0/kk_0$. If we introduce the scattering angle ϑ, which is the angle between the vectors $\mathbf{k}_0$ and $\mathbf{k}'$, we find that $k=2k_0\sin(\vartheta/2)$.[6]

3. THRESHOLD FIELDS

In this section we shall consider the change in the threshold fields of a pumping wave as a result of changes in the angles φ and χ.

The solution of Eq. (2.4) is

$$\Delta_{\mp}=-\frac{i}{4}\left(k^2\Gamma+\gamma_0\mp\left\{(k^2\Gamma+\gamma_0)^2-4\left[k^2\Gamma\gamma_0-\frac{kv\omega_0^2}{\partial(\omega_0^2\varepsilon')/\partial\omega_0}\mu\left(1-\frac{k^2}{k_0^2}\cos^2\varphi\right)\right]\right\}^{1/2}\right) \quad (3.1)$$

and it follows from this solution that one of the roots describes growing perturbations ($\mathrm{Im}\,\Delta_- > 0$) if

$$\mu>\mu_{\mathrm{th}}=\frac{k\Gamma\gamma_0\partial(\omega_0^2\varepsilon')/\partial\omega_0}{v\omega_0^2(1-k^2k_0^{-2}\cos^2\varphi)}. \quad (3.2)$$

The quantity μ_{th} represents, in accordance with Eq. (2.5), the threshold fields in which the instability appears and the stimulated Brillouin scattering becomes possible [$\mathrm{Im}\Delta(\mu_{\mathrm{th}}) = 0$]. Using Eq. (2.6), we find from Eq. (3.2) that

$$\frac{\mu_{\mathrm{th}}(\chi,\varphi)}{\mu_0}=\frac{\cos\chi}{1-4\cos^2\chi\cos^2\varphi},\quad \mu_0=2k_0\Gamma\gamma_0\frac{\partial(\omega_0^2\varepsilon')}{\partial\omega_0}v^{-1}\omega_0^{-2}. \quad (3.3)$$

If $\varphi = 90°$, the well-known result given in [1] is obtained from Eq. (3.3).

Figure 1 shows the surface $\mu_{\mathrm{th}}(\chi, \varphi)/\mu_0$. The length of the vector drawn from the origin to the point of intersection with the surface represents the threshold field for the excitation of acoustic waves along a given direction.[3)] In particular, if $\varphi = \chi = \pi/4$, the threshold field is anomalously high. This is due to the fact that in this case the pumping and scattered waves are mutually perpendicular so that the high-frequency pressure in Eq. (2.1) is zero.

It is clear from Fig. 1 that the lowest fields for the excitation of acoustic waves correspond to a direction normal to k_0, i.e., to the case when the radiation is scattered along k_0 ($\vartheta = 0$). This result can easily be understood on the basis of simple physical considerations. An instability and, consequently, the stimulated Brillouin scattering appear if the energy transferred from the pumping wave to the acoustic and scattered electromagnetic waves (this energy is proportional to the intensity of the pumping wave) exceeds the energy lost by the latter waves due to the usual dissipative mechanisms. Since the absorption of the acoustic waves because of the viscosity and heat conduction decreases rapidly with increasing wavelength, it follows that weaker pumping waves are sufficient for the excitation of longer acoustic waves. However, it is clear from the condition (2.6) that the longer acoustic waves (small k) correspond to the scattering at small angles ϑ, i.e., to the forward scattering.[4)]

4. MAXIMUM GROWTH INCREMENT

In this section we shall determine the direction along which the acoustic and scattered waves grow most rapidly for a given amplitude of the pumping wave.

Using Eq. (2.6), we can rewrite Eq. (3.1) in the form

$$\mathrm{Im}\,\Delta_-=\gamma(\chi,\varphi)=\frac{\gamma_0}{4}\left(\left\{[a\cos^2\chi-1]^2+4\frac{\mu}{\mu_0}a\cos\chi(1-4\cos^2\chi\cos^2\varphi)\right\}^{1/2}-(a\cos^2\chi+1)\right), \quad (4.1)$$

where $a = (2k_0)^2\Gamma/\gamma_0$. For a pumping wave of a given intensity (i.e., for a given value of μ) there are angles χ_0 and φ_0 for which this intensity is equal to the threshold value so that $\gamma(\chi_0, \varphi_0) = 0$. According to Eq. (3.3), these angles satisfy the condition

$$\cos\chi_0/(1-4\cos^2\chi_0\cos^2\varphi_0)=\mu/\mu_0. \quad (4.2)$$

It also follows from Eq. (4.1) that irrespective of the value of μ we find that $\gamma \to 0$ if $\chi \to \pi/2$. This reflects

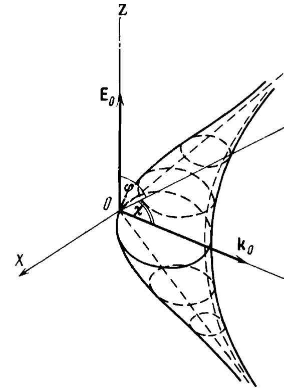

FIG. 1

the circumstance that as the acoustic wavelength increases, the dissipation and the high-frequency pressure both become weaker [see Eq. (2.1)]. Therefore, if $\chi_0 \sim \pi/2$, an instability appears at a low pumping wave intensity but the growth of the acoustic and scattered waves is slow.

Thus, for χ_0 and φ_0 as well as for $\chi = \pi/2$ the growth increment vanishes. Obviously, there are angles $\chi_{\max}$ and $\varphi_{\max}$ for which the function $\gamma(\chi, \varphi)$ has a maximum. It follows from $\partial\gamma/\partial\varphi = 0$ that $\varphi_{\max} = \pi/2$. We thus find from Eq. (4.1) that

$$\gamma\left(\chi,\frac{\pi}{2}\right)=\frac{\gamma_0}{4}\left(\left\{[a\cos^2\chi-1]^2+4\frac{\mu}{\mu_0}a\cos\chi\right\}^{1/2}-(a\cos^2\chi+1)\right). \quad (4.3)$$

In this case, it follows from Eq. (4.2) that the increment vanishes for the angle defined by $\cos\chi_0 = \mu/\mu_0$.

We shall consider Eq. (4.3) in two cases.

1. Low pumping wave intensities ($\mu/\mu_0 < 1$)

The instability region is bounded by the angle $\chi < \chi_0$, where $\cos\chi_0 = \mu/\mu_0 < 1$ and the backward stimulated Brillouin scattering is impossible. Equation (4.3) and the condition $\partial\gamma/\partial\chi = 0$ lead to a cubic equation for $\cos\chi_{\max}$, whose real positive solution is

$$\cos\chi_{max}=\left(\frac{\mu}{4\mu_0 a}\right)^{1/3}\left\{\left(1+\sqrt{1+\frac{16}{27}\frac{\mu_0^2}{a\mu^2}}\right)^{1/3}+\left(1-\sqrt{1+\frac{16}{27}\frac{\mu_0^2}{a\mu^2}}\right)^{1/3}\right\}. \quad (4.4)$$

If the amplitude of the pumping wave is so low that $16/27 > a\mu^2/\mu_0^2$, we find that

$$\cos\chi_{max}=\mu/2\mu_0\quad \gamma_{max}\approx{}^1/_8 a\gamma_0(\mu/\mu_0)^2. \quad (4.5)$$

However, if $16/27 < a\mu^2/\mu_0^2$ and $a > 16/27$, we find from Eq. (4.4) that

$$\cos\chi_{max}\approx(\mu/2a\mu_0)^{1/3},\quad \gamma_{max}\approx{}^1/_2\gamma_0 a^{1/3}(\mu/2\mu_0)^{2/3}. \quad (4.6)$$

2. High pumping wave intensities ($\mu/\mu_0 > 1$)

In this case, the acoustic waves are unstable for any value of the angle χ in the $\varphi = \pi/2$ plane and this applies to the waves corresponding to the backward scattering. However, in order that the maximum growth increment should correspond to the angle $\chi_{\max} = 0$ (backward scattering), we need pumping wave intensities given—according to Eq. (4.3)—by the relationship $\mu/\mu_0 \geq 2(a + 1)$, i.e.,

$$\frac{E_0^2}{8\pi}\geqslant(2k_0)^2\Gamma\gamma_0\left(\frac{(2k_0)^2\Gamma}{\gamma_0}+1\right)\frac{v}{k_0\omega_0^2}\times\frac{\partial(\omega_0^2\varepsilon')/\partial\omega_0}{\rho(\partial\varepsilon'/\partial\rho)_s^2}. \quad (4.7)$$

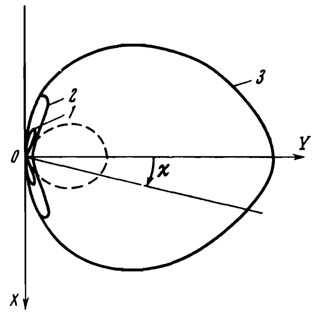

FIG. 2. Function $\gamma(\chi)/\gamma_0$ for different values of the parameter μ/μ_0: $1-\mu/\mu_0 < 1$; $2-1 < \mu/\mu_0 < a$; $3-\mu/\mu_0 > a$. The dashed curve represents the dependence $\mu_{th}(\chi)/\mu_0$.

The stronger the damping of acoustic waves, the greater is the intensity of the pumping wave required to ensure that the stimulated Brillouin scattering grows fastest along the backward direction.

It follows from Eq. (4.3) that if $\mu/\mu_0 > a$ the dissipative effects are unimportant and the growth increment corresponding to the backward scattering is[1]

$$\gamma=\omega_0\rho\left(\frac{\partial\varepsilon'}{\partial\rho}\right)_s\frac{E_0}{4}\left(\frac{\omega_0\sqrt{\varepsilon'}}{\pi\rho v c\partial(\omega_0^2\varepsilon')/\partial\omega_0}\right)^{1/2}. \quad (4.8)$$

The results obtained in this section are illustrated in Fig. 2. It is clear from this figure that as the intensity of the pumping wave increases, the range of instability becomes wider and the maximum of the growth increment shifts in the direction of the angles corresponding to the backward scattering.

5. CONCLUSIONS

We have ignored the escape of the scattered radiation from the scattering volume, which corresponds best to the stimulated Brillouin scattering in a resonator.[1] It is reported in[8] that the stimulated Brillouin scattering occurs at a small angle ($\vartheta \sim 2.5°$) in a nonaxial resonator and the forward scattering threshold is lower than the backward threshold, in agreement with our conclusions. However, the opposite result is reported in a later paper[9] and it is found that the stimulated Raman scattering has a strong influence on the stimulated Brillouin scattering.

A possible cause of this disagreement with[9] is the requirement of long light pulses for the reliable observation of the stimulated Brillouin scattering.5)

Our relationships allow us to estimate the durations and intensities of the pumping waves required for the observation of the stimulated Brillouin scattering at various angles. We shall consider the following parameters typical of liquids:[1,6] $\rho = 1$ g/cm^3, $\rho(\partial\epsilon'/\partial\rho)_s = 1.5$; $v = 10^5$ cm/sec; $n = \sqrt{\epsilon'} = 1.4$; $\gamma_0 = 10^9$ sec^{-1}; $\gamma_s = (2k_0)^2\Gamma = 10^9$ sec^{-1}, $\omega_0 = 2 \times 10^{15}$ sec^{-1}. According to Eq. (3.3), we have $\mu_0 \sim 10^{-7}$ and $(\mu/\mu_0) = 1$ for $E_0 \approx 3 \times 10^4$ V/cm ($W \approx 3 \times 10^7$ W/cm^2). The quantity a in Eq. (4.1) is unity.6) Equations (4.5) and (4.6) yield the values listed in Table I. It is clear from Table I

TABLE 1

μ/μ_0	W, W/cm²	$\cos\chi_{max}$	ϑ_{max}, deg	γ_{max}, sec^{-1}
0.1	$3\cdot10^6$	0.05	6	10^6
0.5	$1.5\cdot10^7$	0.25	30	$3\cdot10^7$
1	$3\cdot10^7$	0.8	110	$2\cdot10^8$
3	10^8	1	180	10^9

that if experiments in resonators are carried out using relatively weak ($\vartheta \sim 6°$, $W \sim 3 \times 10^6$ W/cm^2) but sufficiently long ($\tau = 1/\gamma_{max} > 10^{-6}$ sec) pumping light pulses, it should be possible to determine the anisotropy of the stimulated Brillouin scattering.

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1)Equation (2.1) actually contains the quantity $(\partial p/\partial\rho)_s - (E_0^2\rho/16\pi)\times(\partial^2\epsilon'/\partial\rho^2)_s$. However, in all practical cases, the second term in this equation is negligible.

2)In this case, the polarization of the scattered wave is naturally determined entirely by the right-hand side of Eq. (2.2), which corresponds to the minimum threshold of the stimulated Brillouin scattering.

3)It should be noted that if the rejected small terms are included, we obtain

$$\mu_{th}(\varphi,\chi=\pi/2)/\mu_0\sim\frac{v}{c}\ll1,\quad \mu_{th}(\chi=\varphi=\pi/4)/\mu_0\sim\frac{c}{v}\gg1.$$

4)It should be noted that for other sound absorption mechanisms, which may occur in a plasma, [7] the angular dependence of the threshold of the stimulated Brillouin scattering is quite different.

5)In the determination of the stimulated Brillouin scattering at small angles in the liquid used in [8] it might be necessary to allow for the relaxation of the viscosity.

6)We should bear in mind that the quantities γ_0 and γ_s vary in a wide range (from 10^8 to 10^{10} sec^{-1}) for different liquids.

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