Nonlinear dissipation of electromagnetic waves in a plasma

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Absorption of an intense transverse electromagnetic wave in a plasma is studied within the framework of the theory of nonlinear wave interaction. The dissipation mechanism consists in nonlinear transformation of a transverse wave into an electron Langmuir oscillation (due to induced scattering by the plasma particles), which is absorbed by the electrons with a Landau decay decrement. The effective nonlinear dissipation frequency is found to be a transcendental function of the time-dependent intensity of the absorbed transverse wave. The results are discussed in connection with the problem of interaction between intense light beams and matter. In a laser plasma with a temperature $\gtrsim$ 1 keV the effective nonlinear dissipation frequency of the light beams $\gtrsim 3 \times 10^{14}$ W/cm^2 exceeds the Coulomb frequency of the electron-ion collisions by at least an order of magnitude.

INTRODUCTION

Much attention is being paid of late to nonlinear absorption (emission), not connected directly with Coulomb collisions of the particles, of electromagnetic waves in a plasma. The foundations for the study of such absorption are the theory of parametric resonance[1–3] and the theory of nonlinear interaction of waves in a plasma[4,5]. The action of powerful radiation in the optical[6,7], microwave[8–10], and HF[11] bands on a plasma is accompanied by experimentally observable nonlinear effects of generation of harmonics of the fundamental frequency of the emission[12,13], and anomalies in the reflection[14] and absorption[8,9,15] of the radiation. The stationary picture of the absorption of an intensive electromagnetic wave can be obtained within the framework of the theory of amplitude saturation by the potential plasma oscillations that are parametrically excited by the wave and are subject to induced scattering by ions[16–19]. The pulsed character of the action of the powerful radiation on a plasma (for example, in the case of picosecond-pulse lasers) and the need for determining the characteristic transient time in the absorption make it urgent to determine the temporal evolution of the nonlinear absorption.

In this communication we describe analytically the process of dissipation of a sufficiently powerful transverse (non-potential) electromagnetic wave. The absorption proceeds, roughly speaking, through two simultaneous stages.

In the first stage, the electromagnetic wave is transformed into an electron Langmuir oscillation. It is this stage which ensures the entry of the electromagnetic energy into the plasma. Then, in the second stage, the electric energy of the Langmuir oscillation, which is pumped into the plasma, is absorbed by the electrons at a rate determined by the linear Landau damping decrement. If the intensity of the electromagnetic wave is low and the plasma is inhomogeneous, then the transformation into electron Langmuir oscillations can, if the conditions are suitably chosen, be also linear and due to the inhomogeneity of the plasma[20,21]. In particular, the stationary dissipation of the p-polarized light wave that becomes linearly transformed, as a result of the inhomogeneity of the laser plasma, into a Langmuir oscillation absorbed by electrons in accordance with the inverse Cerenkov effect, was investigated in[22]. On the other hand, if the intensity of the electromagnetic wave is high, then its transformation into electron Langmuir oscillations occur also in a homogeneous plasma as a result of the nonlinear interaction (induced scattering by the plasma particles with change of the type of wave polarization in the scattering process). The analysis proposed below makes it possible to trace such a nonlinear nonstationary dissipation of the transverse wave in a homogeneous plasma, which turns out to be more effective than the linear dissipation in the inhomogeneous plasma at sufficiently high intensity of the absorbed wave.

1. INITIAL RELATIONS

In a homogeneous isotropic plasma, the joint evolution of the spectral densities of the energy of the electron Langmuir oscillation $W_l(\mathbf{k})$ and the transverse electromagnetic wave $W_{tr}(\mathbf{k})$ is described by the equations[4,5]

$$\frac{\partial W_l(\mathbf{k})}{\partial t} + 3v_{Te}(kr_{De})\varkappa\frac{\partial W_l(\mathbf{k})}{\partial \mathbf{r}} = -2\tilde{\gamma}(\mathbf{k})W_l(\mathbf{k}) + W_l(\mathbf{k})\cdot$$
$$\times\int d\mathbf{k}'\{(\varkappa\varkappa')^2Q_{ll}(\mathbf{k},\mathbf{k}')W_l(\mathbf{k}') + {}^1/_2[\varkappa\times\varkappa']^2Q_{lt}(\mathbf{k},\mathbf{k}')W_{tr}(\mathbf{k}')\},$$
$$\frac{\partial W_{tr}(\mathbf{k})}{\partial t} + \frac{c^2k}{\omega_{Le}}\varkappa\frac{\partial W_{tr}(\mathbf{k})}{\partial \mathbf{r}} = -\nu_{ei}W_{tr}(\mathbf{k}) + W_{tr}(\mathbf{k}) \tag{1.1}$$
$$\times\int d\mathbf{k}'\{{}^1/_4[1+(\varkappa\varkappa')^2]Q_{tt}(\mathbf{k},\mathbf{k}')W_{tr}(\mathbf{k}') + {}^1/_2[\varkappa\times\varkappa']^2Q_{tl}(\mathbf{k},\mathbf{k}')W_l(\mathbf{k}')\}. \tag{1.2}$$

The left-hand sides of (1.1) and (1.2) correspond to the change of the spectral energy densities with the time t and with the coordinate $\mathbf{r}$, $v_{Te} = (\kappa T_e/m)^{1/2}$ is the ther thermal velocity of the electrons with temperature T_e and mass m (κ is the Boltzmann constant), and $\omega_{Le} = (4\pi N_e e^2/m)^{1/2}$ is their Langmuir frequency (e is the electron charge, N_e is the number of plasma electrons per unit volume); $r_{De} = (v_{Te}/\omega_{Le})$ is the Debye radius of the electrons, and c is the speed of light in vacuum. The group velocities of the interacting waves (see the second terms in the left-hand sides of (1.1) and (1.2)) are directed along the wave vectors $\mathbf{k} = \mathbf{k}\boldsymbol{\kappa}$ with unit vector $\boldsymbol{\kappa}$. The terms linear in the energy density in the right-hand sides of (1.1) and (1.2) describe the linear damping of the waves because of their interaction with the plasma particles. The Langmuir oscillation with spectrum

$$\omega_l(\mathbf{k}) = \omega_{Le} + {}^3/_2k^2r_{De}v_{Te} \tag{1.3}$$

is characterized by a damping decrement $\tilde{\gamma}$:

$$\tilde{\gamma}(\mathbf{k})=\frac{\nu_{ei}}{2}+\sqrt{\frac{\pi}{8}}\frac{\omega_{Le}}{(kr_{De})^3}\exp\left\{-\frac{1}{2}\frac{\omega_l^2(\mathbf{k})}{k^2v_{Te}^2}\right\}, \tag{1.4}$$

which takes into account the dissipation due to the collisions and the Cerenkov effect (Landau damping). The transverse electromagnetic wave with spectrum

$$\omega_t(\mathbf{k})=\omega_{Le}+{}^1/_2c^2k^2\omega_{Le}^{-1} \tag{1.5}$$

attenuates in the linear approximation only as a result of collisions of the electrons with ions of frequency ν_{ei}. The kernels Q in the nonlinear integral terms of (1.1) and (1.2) are determined by the corresponding scattering cross section[4,5]:

$$Q_{tt}(\mathbf{k},\mathbf{k}')=Q(\omega_t(\mathbf{k})-\omega_t(\mathbf{k}'),\ \mathbf{k}-\mathbf{k}'), \tag{1.6}$$

$$Q_{ll}(\mathbf{k},\mathbf{k}')=Q(\omega_l(\mathbf{k})-\omega_l(\mathbf{k}'),\ \mathbf{k}-\mathbf{k}'),$$

$$Q_{lt}(\mathbf{k},\mathbf{k}')=Q(\omega_l(\mathbf{k})-\omega_t(\mathbf{k}'),\mathbf{k}-\mathbf{k}')=-Q_{tl}(\mathbf{k}',\mathbf{k}), \tag{1.7}$$

in which the function $Q(\omega'', \mathbf{k}'')$ takes the form ($\omega'' \equiv \omega - \omega'$, $\mathbf{k}'' \equiv \mathbf{k}-\mathbf{k}'$)

$$Q(\omega'',\mathbf{k}'')=-\frac{\omega_{Le}}{16\pi^3}\frac{(k''r_{De})^2}{N_e\varkappa T_e}\left\{\delta\varepsilon_e''(\omega'',\mathbf{k}'')\times\left|\frac{1+\delta\varepsilon_i(\omega'',\mathbf{k}'')}{\varepsilon(\omega'',\mathbf{k}'')}\right|^2+\delta\varepsilon_i''(\omega'',\mathbf{k}'')\left|\frac{\delta\varepsilon_e(\omega'',\mathbf{k}'')}{\varepsilon(\omega'',\mathbf{k}'')}\right|^2\right\}. \tag{1.8}$$

Here $\delta\epsilon_i$ and $\delta\epsilon_e$ are the longitudinal partial dielectric constants of the ionic and electronic components of the plasma at the beat frequency (ω'', $\mathbf{k}''$) of the scattered waves, $\delta\epsilon_i''$ and $\delta\epsilon_e''$ are their imaginary parts, and $\epsilon \equiv 1 + \delta\epsilon_i + \delta\epsilon_e$ (see[23]). We note that the concrete expression (1.8) for Q takes into account scattering only via a virtual potential oscillation, and we confine ourselves to this scattering.

The physical meaning of the individual nonlinear terms in equations (1.1) and (1.2) is simple. Namely, the first nonlinear term in the right-hand side of (1.1) describes the induced scattering of the electron Langmuir oscillation with frequency (1.3) and wave vector $\mathbf{k}$, by the plasma particles, into a Langmuir oscillation with frequency $\omega_l(\mathbf{k}')$ and wave vector $\mathbf{k}' = k'\boldsymbol{\kappa}'$. Such a scattering of the Langmuir oscillations by ions was used in[17,18] as a nonlinear mechanism for saturating the levels of the Langmuir noise parametrically excited in the plasma by the electric field of a powerful transverse pump wave of given amplitude. Accordingly, the first nonlinear term in (1.2) gives the induced scattering of an electromagnetic wave of frequency $\omega_t(\mathbf{k})$ by particles into a wave of frequency $\omega_t(\mathbf{k}')$. The angle factor $(1/4)[1 + (\boldsymbol{\kappa}\cdot\boldsymbol{\kappa}')^2]$ corresponds to the natural polarization of the transverse waves. For scattering of a transverse wave linearly polarized along a unit vector $\mathbf{e}$ into a linearly polarized transverse wave $\mathbf{e}'$, such a factor should be replaced by the square of the scalar product $(\mathbf{e}\cdot\mathbf{e}')^2$ of the polarization unit vectors. The two aforementioned terms determine the redistribution of the wave energy over the spectrum, from the higher frequencies to the lower ones, within the spectral line of the scattering waves. Their contribution to the evolution of the spectral energy densities $W_l(\mathbf{k})$ and $W_{tr}(\mathbf{k})$ decreases when the line becomes narrower, so that in the limit of the monochromatic waves they can be neglected. To the contrary, the nonlinear terms of (1.1) and (1.2) with kernels (1.7) characterize the nonlinear interaction of the transverse wave and of the electron Langmuir oscillations with arbitrary line width (under conditions of satisfaction of the inequality $\tilde{\gamma} \ll \omega_{Le}$). They describe the mutual nonlinear transformation of Langmuir oscillation and of the transverse wave, in particular, also in the case of a very narrow line of the scattered waves:

$$W_l(\mathbf{k})=I_1(\mathbf{r},t)\delta(\mathbf{k}-\mathbf{k}_1),\quad W_{tr}(\mathbf{k})=I_2(\mathbf{r},t)\delta(\mathbf{k}-\mathbf{k}_2). \tag{1.9}$$

Such an approximation of a narrow line makes it possible to demonstrate the physical picture of nonlinear transformation and absorption of a transverse wave. We shall therefore confine ourselves to this approximation. If the transverse wave is linearly polarized, then the angle factor $(1/2)[\boldsymbol{\kappa}\times\boldsymbol{\kappa}']^2$ should be replaced in Eq. (1.1) by $(\boldsymbol{\kappa}\cdot\mathbf{e}')^2$ and in (1.2) by $(\boldsymbol{\kappa}'\cdot\mathbf{e})^2$.

In the narrow-line approximation (1.9), the system of nonlinear integro-differential equations (1.1) and (1.2) reduces to the differential equations

$$\frac{\partial I_1}{\partial t}+3v_{Te}(\mathbf{k}_1r_{De})\frac{\partial I_1}{\partial \mathbf{r}}=-2\tilde{\gamma}(\mathbf{k}_1)I_1+(\boldsymbol{\varkappa}_1\mathbf{e})^2Q_{lt}(\mathbf{k}_1,\mathbf{k}_2)I_1I_2, \tag{1.10}$$

$$\frac{\partial I_2}{\partial t}+\frac{c^2\mathbf{k}_2}{\omega_{Le}}\frac{\partial I_2}{\partial \mathbf{r}}=-\nu_{ei}I_2+(\boldsymbol{\varkappa}_1\mathbf{e})^2Q_{tl}(\mathbf{k}_2,\mathbf{k}_1)I_1I_2, \tag{1.11}$$

in which $\boldsymbol{\kappa}_1$ is a unit vector along the wave vector $\mathbf{k}_1 = k_1\boldsymbol{\kappa}_1$ of the Langmuir oscillation, and $\mathbf{e}$ is the unit vector of the polarization of the transverse wave. The direction of the transformation from the transverse wave to the longitudinal one (Langmuir oscillation) or vice versa is determined by the sign of the kernels Q_{lt} and Q_{tl}, which in turn depends, in accordance with (1.8), on the sign of the frequency difference (1.3) and (1.5) of the transformed waves with wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ ($\delta\epsilon'' \propto \omega''$). If the frequency $\omega_2 \equiv \omega_t(\mathbf{k}_2)$ of the transverse wave exceeds the frequency $\omega_1 \equiv \omega_l(\mathbf{k}_1)$ of the electron Langmuir oscillation

$$\omega_2-\omega_1=\frac{c^2}{2\omega_{Le}}\left(k_2^2-3\frac{v_{Te}^2}{c^2}k_1^2\right)>0, \tag{1.12}$$

then the electromagnetic energy, in accordance with (1.10) and (1.11) flows out from the transverse wave into the longitudinal ones. We confine ourselves to the case (1.12), since it corresponds, in our opinion, to perfectly realizable conditions on the wavelength of the interacting waves. The kernel Q_{lt} is in this case positive:

$$Q_{lt}(\mathbf{k}_1,\mathbf{k}_2)=-Q_{tl}(\mathbf{k}_2,\mathbf{k}_1)\equiv Q>0, \tag{1.13}$$

and the decrease $dI_2/dt < 0$ of the energy of the transverse wave as a result of the second term in the right-hand side of (1.11) describes its nonlinear absorption.

2. NONLINEAR NONSTATIONARY DISSIPATION

Certain properties of nonlinear transformation of a transverse wave into an electron Langmuir wave are quite close to the properties of the linear transformation of the transverse wave into a longitudinal one, as a result of the inhomogeneity of the plasma density. First, just as in the case of linear transformation, the frequencies (1.3) and (1.5) of the waves interacting in accordance with (1.10) and (1.11) differ little from each other. Second, for the nonlinear transformation it is necessary that the polarization vector $\mathbf{e}$ of the transverse wave have a component lying in the plane of the wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$. The case when the polarization vector $\mathbf{e}$ is perpendicular to the ($\mathbf{k}_1$, $\mathbf{k}_2$) plane corresponds in terms of the linear theory to s-polarization of the transverse wave (the transformation corresponds to $\boldsymbol{\kappa}_1\cdot\mathbf{e} = 0$). A vector $\mathbf{e}$ parallel to the ($\mathbf{k}_1$, $\mathbf{k}_2$) plane corresponds to p-polarization. It can be stated qualitatively that the wave vector $\mathbf{k}_1$ of the electron Langmuir oscillation in a homogeneous isotropic plasma singles out a direction similar to the direction of the change of the density in an inhomogeneous plasma. If the transverse wave is naturally polarized, the angle factor $(\boldsymbol{\kappa}_1 \cdot \mathbf{e})^2$ in (1.10) or (1.11) takes the form $(1/2)[\boldsymbol{\kappa}_1 \times \boldsymbol{\kappa}_2]^2$ ($\boldsymbol{\kappa}_2 \equiv \mathbf{k}_2/k_2$), and the nonlinear transformation is always possible when the direction of propagation of the interacting waves are not collinear (the "incidence" of the transverse wave is not normal). In this connection, the stationary nonlinear transformation of the waves in space is described, in accordance with (1.10) and (1.11), by the essentially non-one-dimensional equations

$$\sin\theta \frac{\partial I_1}{\partial x} + \cos\theta \frac{\partial I_1}{\partial z} = -2\mu_1 I_1 + A I_1 I_2 \sin^2\theta,$$

$$\frac{\partial I_2}{\partial z} = -2\mu_2 I_2 - 3\frac{k_1}{k_2}\frac{v_{Te}^2}{c^2} A I_1 I_2 \sin^2\theta,$$

in which z is the coordinate along the wave vector $\mathbf{k}_2$ of the transverse wave, and the function $I_{1,2}(x)$ characterize the variation, in energy space, of the waves in a direction perpendicular to the direction of $\mathbf{k}_2$, along the vector $\mathbf{e}$ of the "p-polarized" transverse wave, so that $\cos\theta = (\boldsymbol{\kappa}_1 \boldsymbol{\kappa}_2)$, $\sin^2\theta = (\boldsymbol{\kappa}_1 \mathbf{e})^2$. The coefficients $\mu_{1,2}$ determine the linear absorption of the waves (in reciprocal centimeters):

$$\mu_1 = \frac{1}{3}\frac{\tilde{\gamma}(\mathbf{k}_1)}{(k_1 r_{De}) v_{Te}}, \quad \mu_2 = \frac{1}{2}\frac{\omega_{Le}\nu_{ei}}{c^2 k_2},$$

and the quantity A determines the characteristic length of the nonlinear transformation ($Q = 3v_{Te}k_1 r_{De} A$). The only difference between the nonlinear transformation of the transverse wave into an electron Langmuir oscillation from linear transformation of these waves in an inhomogeneous plasma is (besides the nonlinearity of the process), the dependence of the kernel Q in (1.10), (1.11), and (1.13) on the ionic quantities (the ion mass M, its charge e_i, and the ion temperature T_i) in terms of the partial dielectric constant $\delta\epsilon_i$ of the ions, whereas the linear transformation occurs in an inhomogeneous electron plasma. Such a dependence of Q on the ionic quantities becomes all the more manifest because the induced scattering is mainly by ions (the second, ionic term in the right-hand side of (1.8) is as a rule larger than the first).

If Eq. (1.10) is considered in the given-intensity approximation $I_2 = (\pi E_0)^2$ of a transverse electromagnetic wave, then the evolution of the intensity and of the spectral energy density of the electron Langmuir oscillations can be treated as parametric instability excited in the plasma by a pump wave with electric field intensity $\mathbf{E}_0 = \mathbf{e}E_0$. In fact, if $k_2 \ll k_1$, $T_e \gg T_i$, $(k_1 r_{De})^2 \ll 1$, and we take into account in the kernel Q the term describing the scattering by electrons, as well as the fact that the detuning $\Delta\omega_0 \equiv \omega_2 - \omega_1 = \omega_0 - \omega_{Le} - \frac{3}{2}k_1^2 r_{De} v_{Te} > 0$ ($\omega_2 \equiv \omega_0$) is positive, we can rewrite (1.1) and (1.10) in the linear form

$$dW_l(\mathbf{k})\,/\,dt = 2\gamma(\mathbf{k})W_l(\mathbf{k}),$$

in which the increment γ of the parametric buildup is given by the expression

$$\gamma(\mathbf{k}) = \frac{\pi^2}{2}Q(\boldsymbol{\varkappa}_1\mathbf{E}_0)^2 - \tilde{\gamma}(\mathbf{k}_1) \approx \frac{1}{4}\frac{(kr_E)^2}{(kr_{De})^2}\frac{\omega_0\Delta\omega_0\gamma_s}{\omega_s^2} - \tilde{\gamma}, \quad \mathbf{k} = \mathbf{k}_1, \quad r_E = \frac{eE_0}{m\omega_0^2},$$

which coincides with the increment (3.13) of [24] in the limit of small detuning $\Delta\omega_0 < kv_{Ti} < k\omega_{Li}r_{De} \equiv \omega_s$ in comparison with the acoustic frequency ω_s and in the case of a relatively large decrement of the Cerenkov damping $\gamma_s > \tilde{\gamma}$ of the ion sound by the electrons ($v_{Ti} \equiv (\kappa T_i/M)^{1/2}$). This fact enables us to interpret the change of the intensity of the electron Langmuir oscillations and the nonstationary nonlinear absorption of the transverse electromagnetic wave, which are described by the pair of coupled equations (1.10) and (1.11), as the process of stabilization of the parametric instability as a result of "depletion" of the pumping (see [25]).

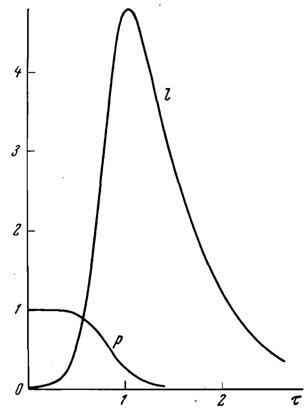

FIG. 1

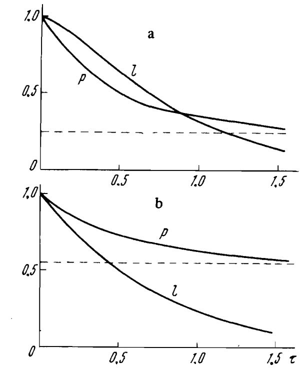

FIG. 2

FIG. 1. Dependence of the relative intensities of the transverse electromagnetic pump wave p and of the parametrically excited electron Langmuir oscillations l on the time τ at $q > 1$ ($q = 5$, $\beta = 10^{-3}$; see formulas (2.2) and (2.3)). The l curve corresponds to a decrease of the value $10^{-2}l$ by a factor of 100. The horizontal asymptote $p = p(\infty) \approx 6.7 \times 10^{-3}$, which determines the stationary level of the intensity of the transverse wave, is not shown in the figure.

FIG. 2. Relative intensities p and l of the transverse and Langmuir waves as functions of the time τ at: a–$1 > q > (1+\beta)^{-1}$ ($q = 0.8$, $\beta = 1$), b) at $q(1+\beta) < 1$ ($q = 0.4$, $\beta = 1$).

To illustrate the foregoing ideas, let us discuss the solution of the nonstationary equations (1.10) and (1.11) under conditions (1.12) and (1.13) of nonlinear absorption and spatially homogeneous intensities $I_1 = I_1(t)$ and $I_2 = I_2(t)$ of the interacting waves, neglecting completely the contribution of the dissipation due to the collisions ν_{ei} ($\tilde{\gamma}$ is determined in this case only by the second term in (1.4)):

$$\frac{\partial I_1}{\partial t} = -2\tilde{\gamma}I_1 + I_1 I_2 Q \sin^2\theta, \quad \frac{\partial I_2}{\partial t} = -I_1 I_2 Q \sin^2\theta \qquad (2.1)$$

Introducing the initial values $I_1(0)$ and $I_2(0)$ of the intensities of the interacting waves and the dimensionless variables

$$p \equiv \frac{I_2(t)}{I_2(0)}, \quad l \equiv \frac{I_1(t)}{I_1(0)}, \quad \beta \equiv \frac{I_1(0)}{I_2(0)}, \quad q \equiv \frac{QI_2(0)}{2\tilde{\gamma}}\sin^2\theta, \quad \tau \equiv \tilde{\gamma}t, \qquad (2.2)$$

we can represent the solutions of equations (2.1) in the form of two equations

$$\tau = \frac{1}{2}\int_p^1 \frac{dx}{x[q(1+\beta-x)+\ln x]}, \quad l = \frac{1}{\beta q}[q(1+\beta-p)+\ln p]. \qquad (2.3)$$

Figures 1 and 2 show the dependence of the intensities of the transverse and Langmuir waves p and l in the time τ, which is determined by formulas (2.3). It is seen from the figures that the intensity of the powerful transverse electromagnetic waves decreases monotonically with time τ to a nonzero stationary value $p(\infty)$, determined by the condition

$$q[1+\beta-p(\infty)] + \ln p(\infty) = 0.$$

Obviously, the nonlinear absorption of the transverse wave is sufficiently effective only under the conditions $p(\infty) \approx \exp(-q - q\beta) \ll 1$, when the nonlinear transformation and the initial intensity of the transverse wave are sufficiently high: $q(1+\beta) \gg 1$ (see (2.2)). During the initial stage $\tau \to 0$ the intensity of the transverse wave decreases linearly with time, $p = 1 - 2\beta q\tau$, and the stationary value $p(\infty)$ is reached in exponential fashion: $p(\tau) \approx p(\infty)[1 - e^{-2\tau}]^{-1}$.

Of the three possible variants of the evolution of p and l on Figs. 1 and 2, greatest interest attaches from the point of view of nonlinear dissipation to the case $q > 1$ (Fig. 1). The intensity of the electron Langmuir oscillations in this case first increases (the energy coming from the transverse wave "has no time" to become dissipated because of Landau damping), reaches a maximum value

$$l_{max} = \frac{1}{\beta q}[q(1+\beta) - 1 - \ln q]$$

at the instant of time

$$\tau_{max} = \frac{1}{2}\int_{1/q}^{1} \frac{dx}{x[q(1+\beta-x)+\ln x]}$$

and then decreases to zero (neglecting the contribution of the spontaneous radiation of the Langmuir oscillations). If the initial intensity level of the Langmuir oscillations is small in comparison with the initial intensity of the transverse wave absorbed by the plasma, $\beta \ll 1$, and the nonlinear transformation is sufficiently high, $q \gg 1$, then the maximum value $I_1(t)$ is equal to $I_2(0)$. During the initial stage of absorption ($p \approx 1$), the intensity of the Langmuir oscillations changes linearly with the intensity of the transverse waves $l \approx 1 + (q-1)(1-p) \times (q\beta)^{-1}$, while the stationary value is assumed in accordance with the law

$$l \approx \frac{1}{\beta q}\frac{p - p(\infty)}{p(\infty)}[1 - qp(\infty)] = \frac{1 - qp(\infty)}{q\beta}e^{-2\tau}.$$

The intersection of the curves l and p, when the relative intensity of the transverse waves into Langmuir oscillation becomes equalized, occurs at the instant of time τ_0, which is determined by the equality

$$q(1+\beta)(1-p) + \ln p = 0$$

and by the first of the relations (2.3).

The rate of nonlinear nonstationary absorption of the transverse electromagnetic wave is determined by the effective frequency ν_{eff}:

$$I_2(t) = I_2(0)\exp(-\nu_{eff}t),$$

which in this case is a transcendental function of the intensity of the transverse wave:

$$\nu \equiv \frac{\nu_{eff}}{2\tilde{\gamma}} = -\left\{\int_p^1 \frac{dx}{x[q(1+\beta-x)+\ln x]}\right\}^{-1} \ln p. \qquad (2.4)$$

In the case of weak nonlinear transformation ($q < 1$), the effective frequency $\nu/\beta q$, according to (2.4), decreases monotonically from unity to zero with decreasing intensity of the transverse wave from 1 to $p(\infty)$. Under conditions of nonlinear dissipation ($q > 1$), the effective frequency (2.4) reaches a maximum value $\nu_{max} = \beta_q l$ at an instant of time determined by the equation

$$l(\tau) = \frac{1}{\tau}\int_0^\tau d\tau' l(\tau').$$

At the initial instants of time, the effective frequency of the nonlinear dissipation is almost constant:

$$\nu \approx \beta q + {}^1/_2(q-1)(1-p), \quad p \approx 1,$$

and in the concluding stage of the nonlinear absorption it takes the form

$$\nu \approx -q(1+\beta)\ln^{-1}[p - p(\infty)].$$

A plot of the effective frequency of the nonlinear dissipation on the intensity of the absorbed transverse wave p is shown in Fig. 3 (at $q > 1$).

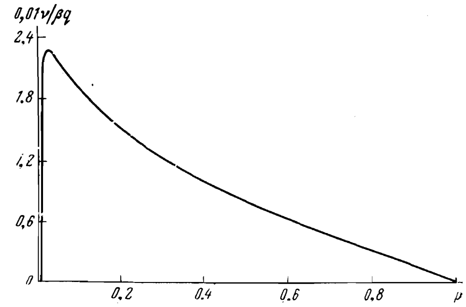

FIG. 3. Effective frequency ν of nonlinear dissipation (2.4) of a transverse electromagnetic pump wave as a function of its relative intensity p at $q > 1$ ($q = 5, \beta = 10^{-3}$, see (2.2)).

The final value of the stationary level $p(\infty)$ of the intensity of the nonlinearly absorbed transverse wave is due to the finite nature of the dissipation $\tilde{\gamma}$ of the electron Langmuir oscillations. If we neglect the damping decrement $\tilde{\gamma}$ in (2.1), then the absorption of the transverse wave is determined only by the nonlinear transformation

$$I_1(t) = I_1(0)[I_1(0) + I_2(0)]\{I_1(0) + I_2(0)\exp[-(I_1(0) + I_2(0))Qt\sin^2\theta]\}^{-1},$$
$$I_2(t) = I_2(0)[I_1(0) + I_2(0)]\{I_1(0) + I_2(0)\exp[-(I_1(0) + I_2(0))Qt\sin^2\theta]\}^{-1}\exp\{-[I_1(0) + I_2(0)]Qt\sin^2\theta\}, \qquad (2.5)$$

as a result of which the intensity of the transverse electromagnetic wave decreases to zero, and the intensity of the electron Langmuir oscillation grows to a value $[I_1(0) + I_2(0)]$. The incompleteness of the resultant physical picture of nonlinear dissipation necessitates the foregoing inclusion of the Landau damping $\tilde{\gamma}$.

The nonlinear absorption is large in comparison with the absorption of the transverse electromagnetic wave as a result of the Coulomb collisions of the plasma particles, if the effective frequency $\nu_{eff} = 2\nu\tilde{\gamma}$ greatly exceeds the collision frequency ν_{ei}:

$$\tilde{\gamma}\left\{\int_p^1 \frac{dx}{x}[q(1+\beta-x)+\ln x]^{-1}\right\}^{-1}\ln\frac{1}{p} \gg \nu_{ei}, \quad p = \frac{I_2(t)}{I_2(0)}. \qquad (2.6)$$

The nonstationary solution for the intensities of the electron Langmuir oscillation l, of the transverse electromagnetic wave p, and of the effective absorption frequency ν shows that this inequality (2.6) cannot be satisfied at any instant of time. For example, at the concluding stage of the nonlinear dissipation, when the intensity of the transverse wave assumes the stationary value $p(\infty)$, the effective frequency ν_{eff} decreases and cannot compete with ν_{ei}. In essence, during this stage there is no nonlinear dissipation since the intensity of the transverse wave becomes very small. However, during a very large time interval (see Fig. 3), the effective frequency is large.

A particularly simple form is assumed by ν_{eff} during the stage of "pure" transformation, when the Landau damping is still unable to interfere with the growth of the intensity of the Langmuir oscillation (the section $dl/d\tau) \gtrsim 0$ in Fig. 1 in the time interval $\tau' \lesssim \tau_{max}$). Then, in accordance with the formulas (2.5), we can assume that $\nu_{eff} \approx Q[I_2(0) + I_1(0)]\sin^2\theta$. Under the conditions $\sin^2\theta \approx 1$ and $I_2(0) \gg I_1(0)$ with allowance for the nonlinear transformation due to scattering by the ions, when the beat frequency $\omega_2 - \omega_1$ of the transverse and Langmuir waves (in terms of the theory of parametric resonance—the detuning $\Delta\omega_0$) is of the order of the frequency of the Doppler shift ($k_1 v_{Ti}$) due to the thermal motion of the ions, the effective frequency of the nonlinear dissipation does not depend on the Landau damping decrement and takes the form (E_0 is the intensity of the electric field of the transverse wave at the initial instant of time)

$$\nu_{\text{eff}} = \frac{\sqrt{2\pi}}{4}\omega_{Le}\frac{E_0^2}{8\pi N_e \varkappa T_i}\frac{e_i}{|e|}\left(1+\frac{e_i}{|e|}\frac{T_e}{T_i}\right)^{-2}. \tag{2.7}$$

In a laser plasma with electron density $N_e \approx 10^{21}$ cm^{-3} and temperature $\kappa T_e \approx \kappa T_i \approx 1$ keV, which is produced by heating with a neodymium laser, the effective frequency (2.7) can be represented by the expression $\nu_{\text{eff}} \approx 3\times 10^{-2} S$ sec^{-1}, in which S is the light flux in W/cm^2 (we assume here the $e_i = |e|$). Under these conditions, the frequency of the Coulomb collisions of the electrons with the ions is $\nu_{ei} \approx 10^{12}$ sec^{-1}. It is obvious that even at the presently attainable light fluxes $S \approx 3\times 10^{14}$ W/cm^2 the effective frequency of the nonlinear dissipation ν_{eff} is higher by one order of magnitude than the frequency of the Coulomb collisions. The electric field intensity is $E_0 \approx 3.4\times 10^8$ V/cm, and the energy density ($E_0^2/4\pi$) in the light waves is lower by two orders of magnitude than the plasma pressure ($N_e\kappa T_e + N_i\kappa T_i$), i.e., the figures presented here are fully compatible with the developed theory. With increasing light flux and with increasing plasma temperature, the inequality $\nu_{\text{eff}} \gg \nu_{ei}$ becomes easier to satisfy. We emphasize that the light fluxes indicated here, which are needed for effective nonlinear absorption, agree with those previously used[16] to estimate the anomalous absorption of light in a weakly-turbulent plasma, where the mechanism of saturation of the ion-acoustic oscillations that grow during the course of the parametric buildup was taken to be induced scattering by ions. It can be assumed that light fluxes on the order of 10^{14} W/cm^2 are perfectly sufficient for the development of nonlinear dissipation in a laser plasma.

In light in the analogy noted above between the here-investigated nonlinear transformation of an electromagnetic wave in a homogeneous plasma and the linear transformation in inhomogeneous plasma, it is of interest to compare the absorption efficiencies of the transverse waves in both cases. The characteristic length $a\ln^{-1}(a/r_{De})$ of the linear absorption of p-polarized light in a laser plasma with inhomogeneity dimension a, obtained in[22], exceeds the characteristic length $\sqrt{3}k_1 r_{De}c/\nu_{\text{eff}}$ of the nonlinear absorption discussed here:

$$\nu_{\text{eff}} > \sqrt{3}\,k_1 r_{De}\frac{c}{a}\ln\frac{a}{r_{De}} \tag{2.8}$$

under the conditions of the foregoing estimate ($\kappa T_e \approx \kappa T_i \approx 1$ keV, $N_e \approx 10^{21}$ cm^{-3}, $S \approx 3\times 10^{14}$ W/cm^2) and at an inhomogeneity dimension $a \approx 5\times 10^{-3}$ cm (at $\sqrt{3}k_1 r_{De} \approx 0.1$). Obviously, the growth of the light flux contributes to predominance of nonlinear dissipation, and the decrease of the dimension of the plasma inhomogeneity increases the efficiency of the linear transformation. In this sense, the inequality (2.8) enables us to examine the conditions for the applicability of the homogeneous-plasma approximation used in the present paper to study nonlinear nonstationary dissipation of a high-power transverse wave. It is also clear that the effects of linear[22] and nonlinear transformation do not cancel each other, but add up and, in accordance with the foregoing, ensure a rather intensive absorption (in comparison with the Coulomb absorption) of the high-power electromagnetic waves in a plasma with arbitrary characteristic dimension of the inhomogeneity.

CONCLUSION

Let us dwell briefly on certain limitations of the developed theory and on the prospects of further study of nonlinear absorption. It must be noted first that the growth of the intensity of the electron Langmuir oscillation during the course of the nonlinear transformation (see Fig. 1) can lead to an additional decrease of $I_1(t)$ via emission of the Langmuir-oscillation energy at double the frequency of the radiation absorbed by the plasma (when two Langmuir oscillations coalesce into a transverse wave). In addition, as already noted in Sec. 1, neglect of the spectral redistribution within the line of the Langmuir oscillations (i.e., neglect of the first nonlinear term in (1.1)) imposes limitations (we assume that the kernels of (1.6) and (1.7) are of the same order) on the width of their spectral density $W_l(k)$ with respect to the wave numbers (Δk) and the angles ($\Delta\theta$):

$$I_2 > 3I_1\frac{v_{Te}}{v_{Ti}}\frac{\Delta k}{\Delta\theta}r_{De}.$$

Allowance for the spectral redistribution of the energy of the electronic Langmuir oscillations, which decreases their frequency, contributes to fulfillment of the nonlinear-transformation conditions (see (1.12)), i.e., to an increase in the effective nonlinear dissipation. As applied to the problem of the laser thermonuclear reaction, in which the target volume is illuminated with several light beams[26], it is apparently also of interest to study the limit inverse to (1.9), when the spectral density $W_l(\mathbf{k})$ is isotropic and depends only on the wave number k.

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