Two-level system with damping in a plasma

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(Submitted February 22, 1973)
Zh. Eksp. Teor. Fiz. **65**, 152–167 (July 1973)

A theory of inelastic transitions in a two-level system is developed which extends the usual theory in two directions: (1) only one of the levels is considered stationary whereas the second is assumed to possess a prescribed damping γ; (2) the system is perturbed by the resultant microfield of all surrounding particles. The calculations are performed for the case of dipole interaction with the microfield of charged particles moving along classical linear trajectories. The whole range of perturbing particle velocities can be described in a unified way by the theory. Within this range, five different physical regions can be distinguished: static, "Weisskopf," adiabatic, "exponential," and Purcell (Born) regions. Only in some of the regions can the lifetime of the stationary (usually metastable) level be expressed in terms of an ordinary cross section for inelastic transitions between stationary levels whereas in other regions it can be expressed in terms of a kind of "cross sections" of a more complex nature (which contain γ) or is not related to any type of cross section at all. The concepts of elasticity and inelasticity, of adiabaticity and nonadiabaticity become "mixed up" if allowance is made for $\gamma \neq 0$. The possibility of applying the theory to the problem of destruction of a metastable level in a plasma is discussed.

1. INTRODUCTION

The present article represents a step towards a generalization of the traditional formulation of the problem about transitions in a two-level system ("atom")[1-3] in the following two directions: a) Only one of the levels is assumed to be stationary while the second is assumed to have a given damping γ (let us assume radiative damping); b) the perturbation of the system is not simply due to isolated (pair) collisions, but is due to the joint microfield created by the entire system of perturbing particles. In this connection the fundamental simplifying assumptions are as follows: 1) the classical nature and the rectilinearity of the motion of the perturbing particles, and 2) the homogeneity of the perturbing microfield over spatial dimensions of the size of an atom (the dipole interaction).

The problem under consideration is obviously related to a whole series of effects, in the first place, to the effects associated with the decay of metastable atomic levels located near radiative levels. Here the classic example is the removal of the metastability of the hydrogen 2s level as a result of transitions to the radiative level $2p_{1/2}$, which is separated from it by the Lamb shift $\hbar\omega_L$.[4] Estimates of the corresponding lifetime τ have been made only in the two simplest cases: for decay in the presence of a static electric field **F** (the field of a capacitor, let us say)[5,6] and for the case of decay caused by collisions with the charged particles of a plasma.[7-10] Both of the indicated approaches are reviewed in[4], but without any attempt to establish their interrelation and limits of applicability. An analogous picture also holds for similar problems pertaining to the intensities of forbidden components of the lines,[11,12] where an approach involving one of the two indicated methods is used more or less arbitrarily without clarifying the region of its applicability.

The solution of the posed problem will be a synthesis of ideas and calculational methods from collision theory and from the theory of the broadening of spectral lines. (Such a synthesis was contemplated in[13] for the problem of the 2s − 2p transition in hydrogen.) In this connection in view of the presence of the additional parameters, namely, the damping γ and the density N of the perturbing particles, a significant number of regions of the characteristic dimensionless parameters arises in the problem. To clarify the physical meaning of these regions, it is appropriate to start the investigation with the construction of the corresponding approximate treatments (Secs. 2 through 5) and only then do we generalize them to a single universal theory (Sec. 6).[1)]

The initial system of Schrödinger equations for the amplitudes a_0 and a_1 of the stationary (or metastable) and radiative states has the form

$$i\dot{a}_0 = V_{01}(t)e^{i\omega t}a_1, \quad i\dot{a}_1 = -i\gamma a_1 + V_{10}(t)e^{-i\omega t}a_0. \tag{1.1}$$

Here $\omega \equiv |E_0 - E_1|/\hbar$ is the distance between the levels, $\hbar V_{01}(t)$ is the matrix element of the dipole interaction of the atom with the electric microfield **F** of the system of $\mathcal{N}$ charged particles:

$$\hbar V_{01} = -(\mathbf{dF})_{01}, \quad \mathbf{F}(t) = \sum_{i=1}^{\mathcal{N}} \mathbf{F}_i = \sum_{i=1}^{\mathcal{N}} \frac{e(\mathbf{r}_{0i} + \mathbf{v}_i t)}{|\mathbf{r}_{0i} + \mathbf{v}_i t|^3}, \tag{1.2}$$

where $\mathbf{r}_{0i}$ and $\mathbf{v}_i$ are the initial coordinates and velocities of the perturbing particles with charge e, and **d** is the dipole moment of the atom.

The problem under consideration, concerning the lifetime τ of the stationary level—that is, the time it takes for the value of $|a_0(t)|^2$ to decrease by a factor of e—breaks up into two stages. The first stage consists in the solution of the dynamical problem, that is, the solution of the system (1.1) with the appropriate initial conditions; the second stage consists in averaging the result with respect to the statistical ensemble of the random variables $\mathbf{r}_{0i}$ and $\mathbf{v}_i$.

The conventional approach to the calculation of τ, developed by Purcell[7] and improved in[8-10],—the calculation of the cross section for isolated fast collisions—corresponds to the solution of the system (1.1) for $\gamma \equiv 0$ according to perturbation theory. The position of this method (henceforth called the Purcell method) within the framework of the general theory will be clarified below. Incidentally, without such a clarification it is not even completely obvious that this approach is internally consistent since it combines neglect of γ (i.e., it assumes $\gamma = 0$) with non-allowance for the possibility of the inverse transition from a_1 to a_0 even before the emission of the photon (which is equivalent, on the other hand, to the assumption that $\gamma \to \infty$).

The most realistic case $\gamma \ll \omega$ is considered below (since the ratio $\gamma/\omega \approx 0.1$ for the hydrogen levels $2s - 2p_{1/2}$). In this connection it turns out that the finiteness of γ itself leads to such interesting physical consequences as an unusual "intermingling" of the concepts of elasticity and inelasticity of the scattering (Sec. 3), an "intermingling" of the concepts of adiabaticity and nonadiabaticity (Sec. 4), and the appearance of effective cross sections of a more complicated (in comparison with ordinary cross sections) nature containing γ (Secs. 3 and 4), and sometimes it even leads to the result that τ cannot be reduced to a cross section in general (Secs. 2 and 3).

2. THE STATIC LIMIT

We start with the case of the decay of a metastable atomic level due to the presence of the constant electric field **F** created by the surrounding charged particles in the plasma. In the absence of radiative damping ($\gamma = 0$) a constant perturbation would simply lead to periodic oscillations of the amplitudes of both atomic states with frequencies $\omega^{(1,2)}$, which are determined by the roots of the corresponding secular equation.[14] Taking the attenuation into account ($\gamma \neq 0$) leads to the appearance of imaginary parts in the frequencies $\omega^{(1,2)}$ and, thereby, leads to an attenuation of the amplitude of the metastable state. In the case $\gamma \ll \omega$ under consideration, the probability $|a_0(t)|^2$ that the system remains in the metastable state is given by the expression (compare with[6])[2)]

$$|a_0(t)|^2 \approx \exp\{-\gamma t[1-\omega/(\omega^2+4|V_{01}|^2)^{1/2}]\}. \tag{2.1}$$

This formula is obtained from the exact expression, which is much more cumbersome, by making certain simplifications which are permissible in the lifetime problem of interest to us, namely: the terms which are most slowly damped with time are kept, and the time-independent pre-exponential factor, which is close to unity, has been replaced by unity. The appropriateness of such a simplification is also confirmed by the results of Secs. 4 and 6.

For sufficiently strong fields ($|V_{01}|_{\text{eff}} \gg \omega$) the states a_0 and a_1 are strongly "intermixed" and the lifetime τ of both states turns out to be of the order of $1/\gamma$. Therefore, from the point of view of visualizing the dependence of τ on the plasma parameters, the most interesting case is $|V_{01}|_{\text{eff}} \ll \omega$, when (compare with Eq. (1.2))

$$\operatorname{Im}\omega^{(1,2)} = -\frac{\gamma}{\omega^2}|V_{01}|^2 = -\frac{\gamma}{\omega^2}\frac{|d_{01}|^2F^2}{\hbar^2} \equiv \frac{C_4}{e^2}F^2. \tag{2.2}$$

It follows from Eq. (2.2) that in this case we are dealing with a distinctive quadratic Stark effect and the value of the constant C_4 depends on the value of γ.

To determine τ it is necessary to average (2.1) with respect to the distribution of the plasma microfield, which we assume to be a Holtsmark distribution.[15] Then the equation for the determination of τ takes the form

$$\int_0^\infty \exp\left\{-\gamma\tau\left(1-\frac{1}{[1+(2\lambda\mu z)]^{1/2}}\right)\right\}\mathscr{H}(z)\,dz = e^{-1}. \tag{2.3}$$

Here $\mathscr{H}(z)$ is the Holtsmark function, $\lambda \approx 2.603$, $\mu \equiv e|\mathbf{d}_{01}|N^{2/3}/\hbar\omega$ (N denotes the density of the perturbing particles) is the characteristic dimensionless parameter which determines the order of magnitude of the ratio of the value of the splitting in the field for an average interparticle distance ($\sim eN^{2/3}$) to the distance between the levels.

For $\mu \gg 1$ the lifetime τ determined from Eq. (2.3) turns out to be of the order of $1/\gamma$, in agreement with what was said earlier. For $\mu \ll 1$ we obtain

$$\tau \approx \frac{1}{\gamma}\frac{\Delta}{(\lambda\mu)^2} \equiv \tau^{\text{st}}, \quad \Delta \approx 0.13. \tag{2.4}$$

It is interesting to determine what value F* of the constant field F corresponds to τ from Eq. (2.4). Using the expression for the lifetime in a constant field (obtained from (2.1) or (2.3) for $\mu \ll 1$) and equating it to τ as given by (2.4), we have: $F^* \approx 5.11\, eN^{2/3}$.

3. PERTURBATION THEORY. THE WEISSKOPF MECHANISM FOR INELASTIC TRANSITIONS

Let us consider the solution of the system (1.1) in the approximation of perturbation theory with regard to the magnitude of the interaction V.

We seek the solution for a_0 in the form $a_0 = e^{-i\varphi(t)}$, where in second-order perturbation theory we write down the following expression for the phase $\varphi(t)$ (assuming $a_0(0) = 1$):[2]

$$\varphi(t) = -i\int_0^t dt'\,V_{01}(t')\exp\{(i\omega-\gamma)t'\}\int_0^{t'} dt''\,V_{10}(t'')\exp\{(-i\omega+\gamma)t''\}. \tag{3.1}$$

Since the phase shifts are small in the case under consideration, we assume that the resulting shift $\varphi(t)$ is the summation of the shifts arising from individual collisions:

$$\varphi(t) = \sum_{k=1}^{\mathscr{N}} \varphi_k(t).$$

This enables us to reduce the averaging over the phase space of $\mathscr{N}$ particles (denoted below by $\langle\ldots\rangle_{\mathscr{N}}$) to an averaging over the phase space of a single particle.[3)] Introducing the normalization volume V containing $\mathscr{N}$ particles and going to the limit $\mathscr{N} \to \infty$, $V \to \infty$, $\mathscr{N}/V = N$ = const in the usual manner,[16] we obtain

$$\langle |a_0(t)|^2\rangle_{\mathscr{N}} = \left\langle \prod_{k=1}^{\mathscr{N}} \exp\{2\operatorname{Im}\varphi_k(t)\}\right\rangle = \exp\{-N\mathscr{V}(t)\} \tag{3.2}$$

where the "collision volume" $\mathscr{V}(t)$ is given by

$$\mathscr{V}(t) \equiv \int d\mathbf{r}[1-\exp\{2\operatorname{Im}\varphi(t)\}]. \tag{3.3}$$

Expressions (3.2) and (3.3) resemble the expressions for the correlation function in the adiabatic theory of line broadening[2] with, however, the difference that the phase $\varphi(t)$ is complex in the general case.

To begin with let us consider the static limit, which is connected in the present case with the perturbation of the atom by individual particles. Calculating (3.3) with (3.1) and (2.2) (where $F = e/r_0^2$) taken into consideration, we find

$$\mathscr{V}(t) = 4\pi\int_0^\infty r_0^2 dr_0\left[1-\exp\left\{-2\frac{C_4}{r_0^2}t\right\}\right] = \frac{4\pi}{3}\Gamma\left(\frac{1}{4}\right)(2C_4t)^{3/4} \tag{3.4}$$

According to Eqs. (3.2) and (3.4) we obtain the following result for the lifetime

$$\tau \approx \frac{1}{\gamma\mu^2}\frac{1}{2}\left(\frac{3}{4\pi\Gamma(1/4)}\right)^{4/3} \approx \frac{0.013}{\gamma\mu^2} \equiv \tilde{\tau}^{\text{st}}. \tag{3.5}$$

This "binary" $\tilde{\tau}^{\text{st}}$ differs from the exact expression (2.4) for τ^{st} by a numerical factor ≈ 0.7 which obviously characterizes the accuracy of the binary scheme (the additivity of the phases φ_k) in the static limit. The noted difference is (besides, for example, the Holtsmark broadening of the lines) one of a few examples of the nonbinary nature of the effect of the microfield on the atom. As one goes away from the static limit, the role of the nonbinary effects decreases more and more, and in the opposite, impact limit any effects due to the action of the microfield (line broadening, Coulomb collisions, and others) are completely converted into binary effects.[17,18]

Now let us take the time dependence of the perturbation V_{01} into account. We shall consider $V_{01}(t)$ in the so-called rotating coordinate system, where at any instant of time the axis of quantization z is directed toward the perturbing particle;[2] this permits us to greatly simplify the investigation while preserving all the characteristic dynamical features of the problem.[4)]

Then for the dipole interaction under consideration we have

$$V_{01}(t)=\frac{e(d_z)_{01}}{\hbar R^2(t)}\equiv\frac{\alpha}{R^2(t)},\quad R^2(t)=\rho^2+v^2t^2 \tag{3.6}$$

(ρ is the impact parameter and v is the velocity of the perturbing particle).

A rigorous calculation of the time dependence of V_{01} would require utilization of the quantity $R^2(t)=\rho^2+v^2(t-t_0)^2$ in expression (3.6), where t_0 denotes the time of closest approach. The customary simplification $t_0\equiv 0$ is obviously equivalent to taking account of only one of the completed trajectories, which is completely analogous to the impact approximation in the theory of broadening. By using the standard arguments corresponding to this approximation,[2] we obtain the following result from Eqs. (3.1), (3.3), and (3.6):

$$N\mathscr{V}(t)=\Gamma t, \tag{3.7}$$

$$\Gamma=Nv\int_0^\infty 2\pi\rho\, d\rho[1-\exp\{2\,\mathrm{Im}\,\varphi_1(\infty)\}]\equiv Nv\sigma(v), \tag{3.8}$$

$$\mathrm{Im}\,\varphi_1(\infty)=-\mathrm{Re}\int_0^\infty e^{(i\omega-\gamma)\tau}d\tau\int_{-\infty}^{\infty}V_{01}(t)V_{10}(t-\tau)\,dt. \tag{3.9}$$

As is clear from Eqs. (3.2), (3.7), and (3.8), in the approximation under consideration the quantity τ is expressed in terms of a certain cross section $\sigma(v)$, which corresponds to the binary nature of the collisions in this approximation. Since the connection between the quantities τ and $\sigma(v)$ has a well-known form in this case, we shall often write out only one of these.

Let us evaluate expression (3.9) by using (3.6) and the condition $\gamma\ll\omega$. Direct integration gives

$$\mathrm{Im}\,\varphi_1(\infty)=-\frac{\pi}{2}\left(\frac{\alpha}{\rho v}\right)^2\left[\pi e^{-2\rho\omega/v}+\frac{\gamma}{\omega}F\left(2\frac{\rho\omega}{v}\right)\right], \tag{3.10}$$

where the function F(x) has the form

$$F(x)=x[e^{-x}\overline{\mathrm{Ei}}(x)+e^x\mathrm{Ei}(-x)]\approx\begin{cases}2/x, & x\gg 1\\ 2x(\ln x+C), & x\ll 1\end{cases} \tag{3.11}$$

(Ei denotes the exponential integral and C = 0.577 is Euler's constant).

It follows from Eqs. (3.10) and (3.11) that the first term in (3.10) gives the major contribution to the cross section (3.8) for large velocities, and the second term gives the major contribution for small velocities. According to Eq. (3.1) expression (3.10) is the inelastic scattering phase, where the first term corresponds to the usual Born approximation of the type used by Purcell.[7] In regard to its structure the second term resembles the elastic scattering phase in the theory of broadening associated with the presence of a perturbing level,[2] which is related to the usual adiabatic (non-Weisskopf) mechanism for line broadening. The appearance of such a phase shift in expression (3.10), characterizing the inelastic scattering, means that the presence of an attenuation γ leads to an "intermingling" of the elastic and inelastic scattering amplitudes. Thus, here we encounter a new, unique inelastic transition mechanism, resulting from the presence (to "sufficient degree") of the elastic phase in the ratio γ/ω. In analogy with the theory of line broadening, in what follows we shall call this mechanism the Weisskopf mechanism.

Now let us trace the transition between the Purcell and the Weisskopf mechanisms for inelastic scattering. Changing to dimensionless variables $x=2\rho\omega/v$ and $\beta=\omega\alpha/v^2$ in Eqs. (3.8) and (3.10), we obtain

$$\Gamma=\pi^{5/3}\Gamma\left(\frac{1}{3}\right)N\frac{v^{1/3}\alpha^{4/3}}{\omega^{2/3}}I_{\gamma/\omega}(\beta), \tag{3.12}$$

where

$$I_{\gamma/\omega}(\beta)=\frac{\beta^{-4/3}}{2\pi^{2/3}\Gamma(1/3)}\int_0^\infty dx\,x[1-\exp\{-\beta^2\chi_{\gamma/\omega}(x)\}], \tag{3.13}$$

$$\chi_{\gamma/\omega}(x)=4\pi\left(\frac{\pi e^{-x}}{x^2}+\frac{\gamma}{\omega}\frac{F(x)}{x^2}\right)\approx 4\pi\begin{cases}2\gamma/\omega x^3, & x\gg 1\\ \pi/x^2, & x\ll 1\end{cases} \tag{3.14}$$

As $\beta\to\infty$, $I_{\gamma/\omega}(\beta)$ approaches $(\gamma/\omega)^{2/3}$, and we arrive at the Weisskopf limit where, according to Eq. (3.12), the lifetime and the cross section turn out to be given by

$$\begin{aligned}\tau&\approx\frac{1}{\pi^{5/3}\Gamma(1/3)}\frac{\omega^{4/3}}{\gamma N v^{1/3}\alpha^{4/3}}\equiv\tau^{\mathrm{W}},\\ \sigma&\approx\pi^{5/3}\Gamma\left(\frac{1}{3}\right)\frac{\gamma^{2/3}\alpha^{4/3}}{v^{2/3}\omega^{4/3}}\equiv\sigma^{\mathrm{W}}.\end{aligned} \tag{3.15}$$

For $\beta\to 0$ we have

$$I_{\gamma/\omega}(\beta)=\frac{\pi^{4/3}\beta^{2/3}}{\Gamma(1/3)}\ln\frac{1}{\beta},$$

which gives the Purcell limit (to within logarithmic accuracy) after substitution into expression (3.13)

$$\tau\approx\frac{v}{\pi^3N\alpha^2\ln 1/\beta}\equiv\tau^{\mathrm{P}},\quad\sigma\approx\pi^3\left(\frac{\alpha}{v}\right)^2\ln\frac{1}{\beta}\equiv\sigma^{\mathrm{P}}. \tag{3.16}$$

Expression (3.16) differs from Purcell's result[7] by the numerical factor $\pi^2/4$, which is typical for the difference between results obtained in rotating and fixed coordinate systems—see[2] and also Sec. 6 below).

In addition to the limiting cases $\beta\to\infty$ and $\beta\to 0$, formula (3.31) also generally admits the existence of a certain intermediate range of values of β, where the dimensions of this region are determined by the value of γ/ω. However, such a region cannot be correctly described within the framework of perturbation theory and requires a more rigorous investigation (see Sec. 4).

The transition from the "collisional" case (3.7) under consideration (more precisely, from its Weisskopf limit) to the static case (3.4) is given by the general formula (3.3). Such a transition is quite analogous to the transition from the impact limit to the static limit in the theory of broadening associated with the quadratic Stark effect. It is not possible to evaluate the integral (3.3) analytically. We are primarily interested in the boundary between the regions of applicability of (3.4) and (3.7). It is well known from the theory of broadening that the static and impact results are valid for times $t\ll\rho_0/v$ and

$t \gg \rho_0/v$, respectively, where ρ_0 is the characteristic radius of the collision. In our case it is related to the "Weisskopf" cross section: $\rho_0 \sim \sqrt{\sigma W}$ (see (3.15)). We note that the quantity ρ_0 is, in a sense, the analog of the Weisskopf radius in the theory of broadening;[2] here it has a more complicated nature (which appears in the explicit dependence of ρ_0 on γ). The "transition" value $t \sim \rho_0/v$ can also be obtained by directly setting the "collisional" (3.7) (in its Weisskopf limit) and the static (3.4) expressions for $\mathscr{V}(t)$ equal to each other.

The analysis which has been carried out permits us to relate the transition between the static and Weisskopf mechanisms of inelasticity to a variation of the parameter $g \equiv N\rho_0^3$, which obviously characterizes the degree of the non-binary nature of the microfield's effect ($g \ll 1$ corresponds to the binary nature of the collisions, and $g \gg 1$ corresponds to the multiple nature of the collisions). In fact, as one can easily verify, the value of g determines into which of the regions of time evolution of the collision volume $\mathscr{V}(t)$—namely, (3.4) or (3.1)—the lifetime τ determined by the relation $N\mathscr{V}(\tau) = 1$ will fall. We thus obtain $\tau \gg \rho_0/v$ for $g \ll 1$, that is, the lifetime of the atom is determined by the Weisskopf mechanism of inelasticity, and we have $\tau \ll \rho/v$ for $g \gg 1$, which corresponds to the static mechanism. We recall that a parameter of the type of g plays a similar role in connection with the comparison of the roles of the impact and static mechanisms in the generation of the half-widths of spectral lines.[2]

Strictly speaking, the binary scheme used above is only valid for $g \ll 1$; however, even for $g \gg 1$ it leads to results which do not differ markedly from the exact results, as has already been indicated in connection with the difference between expressions (3.5) and (2.4).

4. THE ADIABATIC APPROXIMATION

Now let us consider another approximation which is valid for perturbations which vary sufficiently slowly—the adiabatic approximation. Such an approximation is applicable in the region $\beta \gg 1$, where $\beta \equiv \alpha\omega/v^2$; see Eq. (3.12).[5)] In this case the solution is usually based on the static result (2.1), where the perturbation V_{01} is assumed to parametrically depend on the time.[1] On the other hand, one can also approach the solution of this problem in the spirit of the work by Smirnov and Chibisov.[19] Starting from Eq. (2.1), let us determine the steady-state value of the transition probability per unit time, $W = -d \ln |a_0|^2/dt$. If the perturbation varies adiabatically, then one can assume that the change of the total (at the moment t) probability P(t) for a transition from the state a_0 to a_1 is equal at each moment of time to the product of the probability $1 - P$ of observing the system in the state a_0 by the probability W, which depends parametrically on the time:

$$dP/dt = -W(t)(1-P). \tag{4.1}$$

Integrating (4.1) with the initial condition $P(-\infty) = 0$, we obtain the following result with (2.1) taken into account:

$$P(\infty) = 1 - \exp\left\{-\gamma \int_{-\infty}^{\infty} \left(1 - \frac{\omega}{[\omega^2 + 4|V_{01}(t)|^2]^{1/2}}\right) dt\right\}. \tag{4.2}$$

Defining on the basis of Eq. (4.2) the "adiabatic" cross section σ^{ad} in the usual manner (compare with Eq. (3.8)) and introducing the dimensionless parameter $\xi = \gamma\sqrt{2\beta/\omega}$, we obtain:

$$\sigma^{ad}(\xi) = 4\pi \frac{\alpha}{\omega} \Lambda(\xi), \tag{4.3}$$

where

$$\Lambda(\xi) = \int_0^\infty \frac{dz}{z^3}[1 - e^{-\xi p(z)}], \tag{4.4}$$

$$p(z) = \frac{1}{z}\int_{-\infty}^{\infty} dx \left[1 - \left(1 + \frac{z^4}{(1+x^2)^2}\right)^{-1/2}\right] \approx \begin{cases} \pi z^3/4, & z \ll 1 \\ B(3/4, 3/4) \approx 1.70, & z \gg 1 \end{cases} \tag{4.5}$$

(B is the beta function).

For $\xi \gg 1$ we have $z_{eff} \ll 1$ and $\Lambda(\xi) \approx 2^{-7/3}\pi^{2/3}\Gamma(1/3)\xi^{2/3}$, which now gives for (4.3) the well-known Weisskopf cross section (3.15).

For $\xi \ll 1$ we have $z_{eff} \gg 1$ and $\Lambda(\xi) \approx (4/3)B(1/4, 5/4)\xi$, which gives

$$\sigma^{ad} \approx \frac{8\pi}{3} B\left(\frac{1}{4}, \frac{5}{4}\right) \frac{\gamma\alpha^{3/2}}{v\omega^{3/2}} \approx 31.1 \frac{\gamma\alpha^{3/2}}{v\omega^{3/2}}. \tag{4.6}$$

The lifetime corresponding to this cross section is given by

$$\tau \approx \frac{3}{8\pi B(1/4, 5/4)} \frac{\omega^{3/2}}{N\gamma\alpha^{3/2}} \approx 0.032 \frac{\omega^{3/2}}{N\gamma\alpha^{3/2}} \equiv \tau^{ad}. \tag{4.7}$$

We note that this "adiabatic" lifetime does not depend on the velocity, just like the static τ^{st} given by expression (2.4). It is interesting to compare these two lifetimes:

$$\tau^{ad}/\tau^{st} \sim (\alpha N^{2/3}/\omega)^{1/2} \sim \sqrt{\mu} \ll 1. \tag{4.8}$$

We note that the value $\xi \sim 1$ corresponds to the value $\beta \sim (\omega/\gamma)^2 \gg 1$.

The investigation which has been carried out essentially exhausts the adiabatic region for the case $\gamma/\omega \neq 0$ and furthermore for not too small values of γ/ω. The latter limitation is associated with the fact that the investigation is primarily based on the static result (2.1), whose structure is valid only for $\gamma \neq 0$, as is clear from what was said in Sec. 2. The result is that the cross section σ^{ad} given by Eq. (4.6) vanishes in the limit $\gamma/\omega = 0$ (for a fixed value of v), whereas in actual fact the cross section does not vanish for $\gamma = 0$ and $\beta \gg 1$ (which simply corresponds to the transition between two stationary levels) although it is exponential small.[1-3] We note that our result for the adiabatic cross section (4.3) when $\gamma/\omega \neq 0$ has a power-law character. This qualitative difference indicates the important role in which the non-stationary nature of the level plays in scattering, which essentially leads to an "intermingling" of the concepts of adiabaticity and non-adiabaticity (in analogy to the way this occurred for the concepts of elasticity and inelasticity in Sec. 3).

To establish the value of β at which the transition from the cross section (4.6) to the usual exponential cross section σ^e occurs, it is sufficient to match these cross sections in the intermediate region. Confining our attention to the most important dependence, we have[1,2]

$$\sigma^e(\beta) \approx \frac{\alpha}{\omega}\beta^{c_1}\exp\{-c_2\sqrt{\beta}\}, \qquad \tau^e \approx \frac{\omega}{Nv\alpha\beta^{c_1}}\exp\{c_2\sqrt{\beta}\}, \tag{4.9}$$

where c_1 and c_2 are numbers of the order of unity. By equating (4.6) and (4.9) we find

$$\beta \sim [\ln(\omega/\gamma) + \ln\ln(\omega/\gamma) + \ldots]^2.$$

Upon a further decrease in the value of β there occurs (for $\beta \sim 1$) a transition from the "exponential" region (4.9) to the Purcell (Born) region $\beta \ll 1$ (compare with Sec. 3), which is no longer adiabatic. This transition fits into the framework of the standard problem about inelastic scattering on stationary levels ($\gamma \equiv 0$) and may be traced, for example, on the basis of the interpolation formulas presented in[1,2]. A more complete analysis of the transition between the various types of cross sections (for arbitrary $\gamma/\omega \ll 1$) will be given in Sec. 6.

5. THE TOTAL PICTURE OF THE PHYSICAL REGIONS AND THE NATURE OF THE VARIATION OF THE LIFETIME

Before proceeding to the construction of the general solution, it is useful to summarize qualitatively the results of the conducted investigation. Figure 1 shows the density N as a function of v with two curves drawn: one of these curves is given by the equation $\tau(N, v) \sim 1/\gamma$, which determines (in order of magnitude) when lifetime takes on its minimal value $1/\gamma$; all of the regions lying above this curve obviously correspond to the value $\tau \approx 1/\gamma$ and therefore their mutual demarcation is not of interest. The corresponding value of τ taken from formulas (2.4), (3.15), (3.16), (4.7), and (4.9) was used as the lifetime τ in each of the regions below the curve.

The second curve shown on Fig. 1 corresponds to the boundary of the region of applicability of the binary approximation, which is determined by the value of the parameter $g_{eff} \equiv N\rho^3_{eff} \sim 1$. In each of the physical regions of Fig. 1, the quantity ρ_{eff} has the meaning of the effective impact parameter: in order of magnitude ρ_{eff} is determined by those values of ρ which introduce the major contribution to the corresponding cross section—integrals over ρ of the form (3.8) and so forth. Thus, for the Weisskopf region we obtain: $\rho_{eff} \sim \rho_0 \sim \sqrt{\sigma^W}$; for the adiabatic and exponential regions we find: $\rho_{eff} \sim \sqrt{\alpha/\omega}$; and, finally, for the Purcell region we have: $\rho_{eff} \sim \sqrt{\sigma^P} \sim \alpha/v$. (We note that the value of ρ_{eff} does not always coincide with the square root of the corresponding cross section.) Thus, the criterion of the binary nature itself, $g_{eff} \ll 1$, changes during a transition from one region to another, and therefore the parameter g_{eff} shown on Fig. 1 is supplied with the superscript of the corresponding region.

As is evident from Fig. 1, the violation of the binary nature primarily manifests itself in the static region, since in the remaining regions the curve $g_{eff} = 1$ either coincides with the curve $\tau \sim 1/\gamma$ or else lies above it.

The dependence of the lifetime on the velocity v for a fixed value of the density (i.e., for μ = const) is schematically shown in Fig. 2. As is clear from the figure, τ is maximal in the static and in the Purcell limits. This has a simple meaning: In both of these cases the characteristic frequencies of the perturbing field are far from the natural frequency ω of the two-level system.

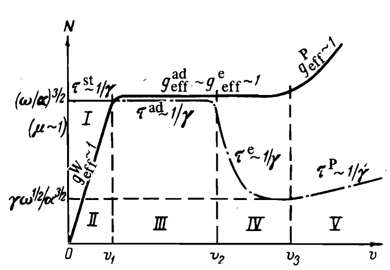

FIG. 1. Demarcation of the regions in the (N, v) plane: I—the static region, II—the Weisskopf region, III—the adiabatic region, IV—the exponential region, and V—the Purcell region. The dotted curve corresponds to the emergence of the lifetime at the limiting value $1/\gamma$; the solid curve corresponds to the boundary of the binary region.

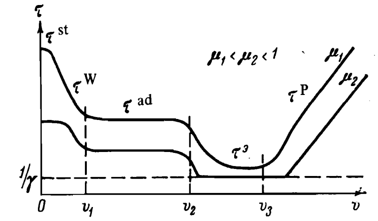

FIG. 2. Velocity dependence of the lifetime (for two values of the density);

The minimum lifetime corresponds to the case of a "resonance": The frequency v/ρ_{eff} of the perturbing field is comparable with ω. For $\gamma = 0$ an exponential growth of τ would be observed in the region of small v. For $\gamma \neq 0$ τ assumes the finite (static) limiting value, successively passing through the adiabatic and Weisskopf values. The physical reason for this is that the attenuation of one of the levels even for $v \to 0$ is equivalent to a certain nonstationary behavior of the perturbation and thereby is equivalent to the preservation of the resonant Fourier harmonics of this perturbation.

According to Eq. (4.8) the difference between the static and adiabatic lifetimes decreases with increasing density (i.e., with increasing μ; see the lower curve on Fig. 2). When a certain segment of the $\tau(v)$ curve reaches the line $\tau \sim 1/\gamma$, there is naturally no further decrease of τ on this segment.

6. THE GENERAL SOLUTION

The results obtained above for separate physical regions, can be generalized on the basis of a sufficiently general formal solution. It is clear that obtaining the exact solution of the system (1.1) for $V_{01}(t)$ of arbitrary form and moreover for $\gamma \neq 0$ is apparently a problem which is insoluble in analytic form. Additional difficulties arise at the stage of statistical averaging owing to the multiple nature of the microfield. In view of these difficulties we shall, in the first place, confine our attention to the set of regions in which τ is expressed in terms of the cross section (see Secs. 3 and 4) and, in the second place, we shall resort to an approximation for the general formal solution of the system (1.1) in the spirit of[20].[6)]

The indicated general solution will be constructed by starting from the formalism of a factorized (or multiplicative) integral.[21,22]

Let us write down the initial system (1.1) in matrix form. To do this we introduce the state vector $\hat{a} = (a_0, a_1)$ and the evolution operator $\hat{S}$ according to the relation $\hat{a}(t) = \hat{a}(t_0)\hat{S}(t_0, t)$. Then the system (1.1) takes the form ($\hat{S}(t_0, t_0) = 1$)

$$i\dot{\hat{S}}(t_0, t) = \hat{S}(t_0, t)\hat{V}(t),$$
$$\hat{V}(t) = -\tfrac{1}{2}i\gamma(1-\sigma_z) + V_{01}\sigma_x \exp\{i\sigma_z\omega t\}, \tag{6.1}$$

where σ_x and σ_z are the Pauli matrices. The amplitudes a_0 and a_1 are obtained from Eq. (6.1) by the action of the projection operators P_0 and P_1:

$$|a_{0,1}(t)|^2 = \mathrm{Sp}\{\hat{P}_0\hat{S}(t_0, t)\hat{P}_{0,1}\hat{S}^+(t_0, t)\},$$
$$P_{0,1} = (1 \pm \sigma_z)/2. \tag{6.2}$$

One can write the general formal solution of the system (6.1) in the form of the so-called multiplicative integral e $\int$ (the other notation is T exp), compare with[22]:

$$\hat{S}(t_0, t) = \mathrm{e}\int_{t_0}^{t} [-i\hat{V}(\tau)]d\tau \equiv T\exp\Big[-i\int_{t_0}^{t}\hat{V}(\tau)d\tau\Big]. \tag{6.3}$$

One can rewrite Eq. (6.1) in the following integral form:

$$\hat{S}(t_0, t) = 1 - i\int_{t_0}^{t} dt_1 \hat{S}(t_0, t_1)\hat{V}(t_1). \qquad (6.4)$$

Setting $\hat{S}(t_0, t_1) = 1$ on the right-hand side, we obtain first-order perturbation theory. By applying this procedure to the right-hand side several times, one can obtain any arbitrary number of terms in the perturbation-theory series. The obtained approximations obviously do not describe strong perturbations. Therefore, let us construct on the basis of Eq. (6.4) a certain other approximation, permitting us to describe the case of weak perturbations as well as the case of strong but adiabatically varying perturbations. To do this, we substitute for the exact S-matrix the adiabatic S-matrix $\hat{S}^{ad}(t_0, t)$ in the right-hand side of Eq. (6.4), and stipulate that $\hat{S}^{ad}(t_0, t)$ tend to unity as $V \to 0$ and the results of perturbation theory be thereby preserved. In this connection, in the case $V \approx$ const such a substitution of $\hat{S}^{ad}(t_0, t)$ guarantees that we obtain the exact solution.

In order to obtain $\hat{S}^{ad}(t_0, t)$ we apply the method used in the work by Kolkunov and Rostokin.[22] First let us consider the case $\gamma = 0$. By using the easily checked relationship

$$\hat{S}(t_0, t) = \hat{R}^{-1}(t_0)\,\mathrm{e}\int_{t_0}^{t}\{-i\hat{R}(\tau)\hat{V}(\tau)\hat{R}^{-1}(\tau) - \hat{R}(\tau)\hat{R}^{-1}(\tau)\}d\tau\hat{R}(t), \qquad (6.5)$$

where $\hat{R}(t)$ is an arbitrary matrix having an inverse, we transform Eq. (6.3) to the form

$$\hat{S}(t_0, t) = \exp\left[-i\sigma_z\frac{\omega t_0}{2}\right]\mathrm{e}\int_{t_0}^{t}\left\{-iV_{01}(\tau)\sigma_x - i\frac{\omega}{2}\sigma_z\right\}d\tau\exp\left[i\sigma_z\frac{\omega t}{2}\right]. \qquad (6.6)$$

Let us write the matrix inside the curly brackets in (6.6) in the form

$$\begin{gathered} -iV_{01}(\tau)\sigma_x - i\frac{\omega}{2}\sigma_z = -i\sigma_z w(\tau)e^{i\sigma_y\theta}, \\ w(\tau) = \left(V_{01}^2(\tau) + \frac{\omega^2}{4}\right)^{1/2}, \quad \mathrm{tg}\,\theta = \frac{2V_{01}}{\omega}. \end{gathered} \qquad (6.7)$$

By separating the oscillating factor $\exp(-i\sigma_y\theta)$ from (6.6) with the aid of the transformation (6.5), we have

$$\begin{aligned} \hat{S}(t_0, t) &= \exp\left[-i\sigma_z\frac{\omega t_0}{2}\right]\exp\left[-i\sigma_y\frac{\theta(t_0)}{2}\right] \\ &\times \mathrm{e}\int_{t_0}^{t}\left\{-i\sigma_z w(\tau) - i\sigma_y\frac{\dot{\theta}(\tau)}{2}\right\}d\tau\exp\left[i\sigma_y\frac{\theta(t)}{2}\right]\exp\left[i\sigma_z\frac{\omega t}{2}\right]. \end{aligned} \qquad (6.8)$$

According to Eq. (6.7) the derivative $\dot{\theta}$ in (6.8) is proportional to the derivative $\dot{V}$ and, therefore, is small in the limit of an adiabatic perturbation. The desired matrix $\hat{S}^{ad}(t_0, t)$ is obtained by expanding (6.8) in a series up to terms of first order in $\dot{\theta}$. Then substituting $\hat{S}^{ad}$ into (6.4) and integrating over frequencies,[7] we obtain the following result after making certain transformations and taking (6.2) into account:

$$|a_1^0(\infty)|^2 = \left|\int_{-\infty}^{\infty} dt\, V_{01}(t)\exp\left[-2i\int_0^t w(\tau)d\tau\right]\right|^2, \qquad (6.9)$$

where t_0 is set equal to $-\infty$ $(V(-\infty) = V(\infty) = 0)$.

Formula (6.9) agrees with the result of Vaĭnshteĭn, Presnyakov, and Sobel'man;[20] this agreement is natural since similar assumptions were used in the derivation of both formulas.

Now let us take the damping of the state $a_1(t)$ into account ($\gamma \neq 0$). It is obvious that for arbitrarily small $\gamma \neq 0$ and $t \to \infty$ the amplitude $a_1(t)$ tends to zero, and by the same token the probability of finding the system in the state a_1 also tends to zero. However, the probability to remain in the state $a_0(t)$ for $t \to \infty$ and $\gamma \neq 0$ is finite. Thus, $|a_1(\infty)|^2$ is a discontinuous function of γ ($|a_1(\infty)|^2 = 0$ for any arbitrary $\gamma \neq 0$, but it is given by expression (6.9) for $\gamma = 0$). However, the function $|a_0(\infty)|^2$ is continuous as $\gamma \to 0$.

We further note that with the aid of the transformation (6.5) it is possible to represent the matrix $\hat{S}_\gamma(t_0, t)$ given by expression (6.3) in the form

$$\hat{S}_\gamma(t_0, t) = \hat{S}_{1\gamma}(t_0, t)\hat{S}_{\gamma\to 0}(t_0, t), \qquad (6.10)$$

where

$$\hat{S}_{1\gamma}(t_0, t) = \lim_{\gamma'\to 0}\mathrm{e}\int_{t_0}^{t}\{-(\gamma - \gamma')\hat{S}_{\gamma'}(t_0, \tau)\hat{P}_1\hat{S}_{\gamma'}^{-1}(t_0, \tau)\}d\tau. \qquad (6.11)$$

Since the values of $\hat{S}_{\gamma'}(t_0, \tau)$ enter into (6.11) for finite times τ, one can simply set $\gamma' = 0$ inside the integral sign.

Taking what has been said in connection with $\hat{S}_{\gamma\to 0}(-\infty, \infty)$ into account, with the aid of Eqs. (6.1), (6.2), (6.10), and (6.11) we obtain the following result for the probability to remain in the state a_0:

$$|a_0(\infty)|^2 = [1 - |a_1^0(\infty)|^2]\exp\left\{-2\gamma\int_{-\infty}^{\infty}|a_1^0(t)|^2 dt\right\}. \qquad (6.12)$$

The physical meaning of this result is clearly evident from its structure: For $\gamma \neq 0$ the probability to remain in the state a_0 is equal to the product of the probability to remain in any of the two levels (the exponential factor) times the probability to be in the state a_0 for $\gamma = 0$. The probability of remaining in either of the two levels is determined in turn by the product of the probability of being in the state a_1 at the moment t and the probability to "immediately" leave the system: $2\gamma dt$.

We emphasize that the result (6.12) is not connected with the use of the approximation (6.9).

Passing on to a concrete calculation, let us substitute the approximate value of $|a_1^0|^2$ given by (6.9) into expression (6.12). In this connection, since we are interested in the case $\gamma/\omega \ll 1$ (when, as was indicated in Secs. 3 and 4, the effects associated with $\gamma \neq 0$ only give a contribution in the region of slow adiabatic collisions), one can limit oneself to the even simpler purely adiabatic approximation for $|a_1^0(t)|^2$ in the argument of the exponential function in (6.12). The latter approximation is obtained with the aid of $\hat{S}^{ad}$ (for $\dot{V} = 0$) and relations (6.1) and (6.2):

$$|a_1^{ad}(t)|^2 = \tfrac{1}{2}[1 - \omega/2w(t)]. \qquad (6.13)$$

We note that, just as should happen, $a_1(\infty) = 0$ in the adiabatic limit, that is, the adiabatic perturbation by itself doesn't cause any transitions between the levels. However, even an adiabatic perturbation leads to a nonvanishing transition probability when $\gamma \neq 0$. As one can easily see, this probability corresponds to the probability (4.2), which was derived above from different considerations.

Let us write down the expression for the cross section σ_0, corresponding to the probability of leaving the state a_0. With the aid of Eqs. (6.9), (6.12), and (6.13) we obtain

$$\sigma_0(v) = 2\pi\int_0^{\infty}\rho\, d\rho\left\{1 - [1 - |a_1^0(\infty)|^2]\exp\left[-\gamma\int_{-\infty}^{\infty}\left(1 - \frac{\omega}{2w(t)}\right)dt\right]\right\}. \qquad (6.14)$$

Introducing, just as in Secs. 3 and 4, the dimensionless parameters $\beta = \omega\alpha/v^2$, $\xi = \gamma\sqrt{2\beta}/\omega$ and defining the dimensionless cross section $\sigma(\xi, \beta)$ with the aid of the relation $\sigma_0(v) = 2\pi(\alpha/v)^2\sigma(\xi, \beta)$, we find

$$\sigma(\xi,\beta)=\int_0^\infty x\,dx\left(1-[1-G_\beta(x)]\exp\left\{-\xi p\left(\frac{\sqrt{2}}{x}\right)\right\}\right), \quad (6.15)$$

where

$$G_\beta(x)=\frac{4}{x^2}\left\{\int_0^\infty\frac{dy}{1+y^2}\cos[\sqrt{\beta}\,\eta(x,y)]\right\}^2,$$
$$\eta(x,y)=x\int_0^y dz\left[1+\frac{4}{x^4(1+z^2)^2}\right]^{1/2}. \quad (6.16)$$

The function p(x) is defined by formula (4.5).

It is clear from Eq. (6.15) that the total cross section $\sigma(\xi, \beta)$ can be represented in the form of the sum of the already known "adiabatic" cross section $\sigma^{ad}(\xi)$ given by expression (4.3) and a certain other cross section $\tilde{\sigma}(\xi, \beta)$, which is defined as the product of the function $G_\beta(x)$ and the function $\exp\{-\xi p(\sqrt{2}/x)\}$. For $\xi = 0$ the cross section $\tilde{\sigma}(\xi, \beta)$ goes over into the cross section corresponding to the usual treatment of the scattering problem ($\gamma = 0$).[2,20] As we have seen in Sec. 4, the transition of the total cross section $\sigma(\xi, \beta)$ from $\sigma^{ad}(\xi)$ to $\sigma^{e}(\beta)$ given by Eq. (4.9) (which is one of the limits $\tilde{\sigma}(0, \beta)$) occurs for $\xi \ll 1$. Therefore, in the expression for $\sigma(\xi, \beta)$ one can everywhere approximately set $\xi = 0$.

Let us trace the behavior of the cross section $\tilde{\sigma}(0, \beta) \equiv \tilde{\sigma}(\beta)$ in the limiting cases of large and small β.

For $\beta \gg 1$ the values y_{eff} which give the major contribution to the integral (6.16) are small ($y_{eff} \ll 1$). By expanding the integral in the argument of the cosine in (6.16) in powers of y, we evaluate the integral over y (by the method of residues). The following integral with respect to x is evaluated by the saddle-point method on the real axis, which gives ($x_0 = \sqrt{2}$ is the saddle point)

$$\tilde{\sigma}(\beta)\propto\exp\{-c_2\sqrt{\beta}\},\quad c_2=2x_0(1+4/x_0^4)^{1/2}=4. \quad (6.17)$$

Values $x_{eff} \gg 1$ are important for $\beta \ll 1$, which allows us to neglect (in the argument of the cosine in (6.16)) the difference of the radical appearing there from unity. Evaluating the simple integrals with respect to y and x which arise, we arrive at the result (3.16).

The calculations of $\tilde{\sigma}(\beta)$ in the region $\beta \sim 1$ encounter considerable difficulties (even with the aid of an electronic computer). In order to simplify the calculations one can, for example, follow[20] and replace the square root in the argument of the cosine by a sum of the roots of both terms. One can show[23] that such a simplification permits us to express the function $G_\beta(x)$ in terms of a Whittaker function, and then the calculation is performed on an electronic computer.

Thus, all of the limiting cases investigated in Secs. 3 and 4 follow from the general formulas (6.15) and (6.16), which were derived on the basis of a formal solution of the initial system (1.1).

The approximation of a rotating coordinate system (3.6) was utilized in the cited investigation. As mentioned in Sec. 3, this only leads to an inaccuracy in the value of the numerical coefficient appearing in the Purcell cross section; in the remaining regions, corresponding to slow collisions, it is obviously not very essential to take the effects of rotation into account.

It is not difficult to generalize the result (6.9) in such a way that the effects of rotation are taken into account in it. Since these effects are most important in the region where perturbation theory is applicable, it is sufficient to replace expression (3.6) for $V_{01}(t)$ by its exact value $-(\mathbf{d}\cdot\mathbf{F}(t))_{01}$, $\mathbf{F} = e(\boldsymbol{\rho} + \mathbf{v}t)/|\boldsymbol{\rho} + \mathbf{v}t|^3$, where this replacement is only made in the term which is responsible for the result of perturbation theory (in other words, the values of $V_{01}(t)$ appearing under the integral sign in (6.9) are not subject to replacement). Averaging over the angles of the vectors $\boldsymbol{\rho}$ and $\mathbf{v}$ after making this replacement, we obtain a somewhat more complicated function $G_\beta^1(x)$ instead of the function $G_\beta(x)$ given by (6.16):

$$G_\beta'(x)=\frac{4\beta}{3x^2}\left\{\left[\int_0^\infty\frac{dy}{(1+y^2)^{3/2}}\cos(\sqrt{\beta}\,\eta(x,y))\right]^2+\left[\int_0^\infty dy\frac{y}{(1+y^2)^{3/2}}\cos(\sqrt{\beta}\,\eta(x,y))\right]^2\right\}. \quad (6.18)$$

For $\beta \gg 1$ expression (6.18) guarantees as usual the derivation of the exponential cross section (6.17). For $\beta \ll 1$ formula (6.18) leads to Purcell's result with the correct numerical coefficient in front of the logarithm:

$$\tilde{\sigma}(\beta)=2\beta\ln\frac{b}{\beta}\equiv\tilde{\sigma}^{\Pi}(\beta), \quad (6.19)$$

where b is a number of the order of unity. The value of b depends in general on the specific nature of the problem. For example, for the problem concerning the 2s − 2p transition in hydrogen, $b \approx 0.60$.[9,10] 8)

7. DISCUSSION

The investigation which has been carried out indicates first of all the important role of the damping γ in slow collisions, leading to the decay of metastable states. The question of experimental observation of the effects associated with the damping γ depends on the specific transition. For example, for the $2s - 2p_{1/2}$ transition in hydrogen, the lifetime τ of the 2s state is determined by the Purcell cross section for collisions with plasma ions up to velocities $v_i \sim 10^6$ cm/sec, i.e., for the majority of real cases. In the Purcell limit the contribution of the $2p_{3/2}$ level (which is approximately ten times farther from the 2s level than the $2p_{1/2}$ level is) to τ is comparable with the contribution of the $2p_{1/2}$ level.[7] We note, however, that for $v_i \sim 10^6$ cm/sec the contribution of the $2p_{3/2}$ level may no longer be determined (in view of the large value of β in comparison with the $2p_{1/2}$ level) by the Purcell cross section, but it may be determined, for example, by the corresponding minimum of the curve on Fig. 2. In this case the decay of the metastability is largely determined by the transitions to precisely this (more remote!) level.

Such a type of increase in the value of β may also lead to that region of Fig. 2 where the lifetime τ still significantly depends on γ. This may occur, for example, for hydrogen-like ions with charge $Z > 1$. The point is that the Lamb shift ω_L increases rapidly with increasing values of Z ($\propto Z^4$).9) For example, the value of ω_L for the He^+ ion with n = 2 exceeds the value of ω_L in hydrogen by 14 times.[4] This leads to a shift of the boundary of the Purcell region toward larger velocities, $v \propto Z^2$, and thereby leads to an increase in the role of the new effects considered above. A similar situation may also be realized for non-hydrogen-like systems in the case when the metastable level is sufficiently far away from the radiating level (for example, for the case of the metastable 2^1S_0 and 2^3S_0 levels in helium).

As far as the effects of the non-binary nature in the action of the plasma microfield are concerned, they mainly appear in very slow (almost static) collisions. Experiments on the decay of metastability caused by static (external) fields are in good agreement with the theory.[24] The possibility of observing similar effects in a plasma is hindered in the first place by the masking of the static effects of the ions by the collisional effects of the electrons.

In conclusion we note that the question of the relative role of the ions and electrons of the plasma in the decay of metastability must be resolved in each specific case on the basis of Fig. 2 (with the difference in characteristic velocities arising from the mass difference taken into consideration).

The authors express their sincere gratitude to V. I. Rostokin for valuable discussions in the initial stages of this work.

[1]) To avoid misunderstandings, we note that the superscripts ("static", "adiabatic", and so forth) associated with the lifetimes corresponding to these treatments (see Eqs. (2.4) and (4.7)) sometimes may not pertain to the entire range of the approximation but only to one of its limits. We do this in order to simplify the notation in those cases when the other limit is trivial (Sec. 2) or when an overlapping of the approximations occurs; the latter situation refers to Sec. 4 where the cases of slow perturbations of arbitrary force and weak perturbations of arbitrary velocity (Sec. 3) overlap.

[2]) We have corrected certain errors in the formulas given in [6].

[3]) Here and below the velocity v of the perturbing particles is assumed for simplicity to be fixed and equal to a certain characteristic velocity of the Maxwell distribution, so that the averaging over the phase space of a single particle simply reduces to an integration with respect to its coordinate $\mathbf{r}_k$.

[4]) The effects associated with rotation (leading to a more accurate determination of the numerical coefficients in the Purcell limit) are analyzed in Sec. 6.

[5]) The quantity β is the analog of the well-known Massey parameter, [1] which characterizes the ratio of the natural frequency of the system to the frequency of the external perturbation.

[6]) As will be clear below, the approximation we are using touches essentially only on the traditional aspects of the problem, corresponding to the case $\gamma = 0$, which is not of major importance for our investigation.

[7]) Here it is clear that the expansion in powers of $\hat{\theta}$ contains, in particular, first-order perturbation theory, so that one can also obtain Eq. (6.9) by the direct substitution of $\hat{S}^{ad}$ into (6.2).

[8]) The calculation methods used in [9,10] are equivalent. The difference between the values of b obtained in these articles is simply due to the fact that in [9] Euler's constant was not taken into account upon expanding the Macdonald function $K_0(\rho\omega/v)$ for small values of its argument.

[9]) The order of magnitude of the ratio γ/ω_L does not change in this case.

[1] N. F. Mott and H. S. W. Massey, The Theory of Atomic Collisions, Oxford, 3rd ed., 1965 (Russ. Transl., Mir, 1969).

[2] I. I. Sobel;man, Vvedenie v teoriyu atomnykh spektrov (Introduction to the Theory of Atomic Spectra), Fizmatgiz, 1963 (English Transl., Pergamon Press, 1972).

[3] B. M. Smirnov, Atomnye stolknoveniya i élementarnye protsessy v plazme (Atomic Collisions and Elementary Processes in a Plasma), Atomizdat, 1968.

[4] H. A. Bethe and E. E. Salpeter, Quantum Mechanics of One- and Two-Electron Atoms, Springer-Verlag, 1957 (Russ. Transl., Fizmatgiz, 1960).

[5] W. E. Lamb, Jr., and R. C. Retherford, Phys. Rev. **79**, 549 (1950).

[6] A. B. Migdal and V. P. Kraĭnov, Priblizhennye metody kvantovoĭ mekhaniki (Approximate Methods of Quantum Mechanics), Nauka, 1966.

[7] E. M. Purcell, Astrophys. J. **116**, 457 (1952).

[8] M. J. Seaton, Proc. Phys. Soc. London **A68**, 457 (1955).

[9] M. I. Chibisov, Opt. Spektrosk. **27**, 9 (1969) [Opt. Spectrosc. **27**, 4 (1969)].

[10] A. I. Gurevich, V. M. Dubovik, and L. M. Satarov, v sb. Voprosy teorii atomnykh stolknoveniĭ (in the collection: Problems in the Theory of Atomic Collisions), Atomizdat, 1970.

[11] H. Sadjian, H. K. Wimmel, and H. Margenau, J. Quant. Spectrosc. Radiat. Transfer **1**, 46 (1961).

[12] L. I. Grechikhin, Opt. Spektrosk. **22**, 548 (1967) [Opt. Spectrosc. **22**, 301 (1967)].

[13] V. I. Kogan and A. D. Selidovkin, in Proceedings of the IXth International Conference on Phenomena in Ionized Gases, Bucharest, 1969, p. 601.

[14] L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics), Fizmatgiz, 1963 (English Transl., Transl., Pergamon Press, 1965).

[15] S. Chandrasekhar, Stochastic Problems in Physics and Astronomy, Revs. Mod. Phys. **15**, 1 (1943) (Russ. Transl., IIL, 1947).

[16] P. W. Anderson, Phys. Rev. **76**, 647 (1949).

[17] V. I. Kogan and V. S. Lisitsa, J. Quant. Spectros. Radiat. Transfer **12**, 881 (1972).

[18] V. I. Kogan, Dokl. Akad. Nauk SSSR **135**, 1374 (1960) [Sov. Phys.-Doklady **5**, 1316 (1961)].

[19] B. M. Smirnov and M. I. Chibisov, Zh. Eksp. Teor. Fiz. **49**, 841 (1965) [Sov. Phys.-JETP **22**, 585 (1966)].

[20] L. A. Vaĭnshteĭn, L. V. Presnyakov, and I. I. Sobel; Sobel'man, Zh. Eksp. Teor. Fiz. **43**, 518 (1962) [Sov. Phys.-JETP **16**, 370 (1963)].

[21] F. R. Gantmakher, Teoriya matrits (Theory of Matrices), Nauka, 1966.

[22] V. A. Kolkunov and V. I. Rostokin, v sb. Fizika gazorazryadnoĭ plazmy (in the collection: The Physics of Gas-Discharge Plasmas), Atomizdat, 1969.

[23] V. S. Lisitsa, Dokl. Akad. Nauk SSSR **201**, 823 (1971) [Sov. Phys.-Doklady **16**, 1059 (1972)].

[24] Yu. L. Sokolov, Zh. Eksp. Teor. Fiz. **63**, 461 (1972) [Sov. Phys.-JETP **36**, 243 (1973)].

Translated by H. H. Nickle
17