

# Cascade emission of $\gamma$ -quanta by highly excited nuclei

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The thermodynamics of the electromagnetic radiation from heated nuclei is developed on basis of the Landau theory of a Fermi liquid.<sup>1</sup> The case of nonspherical nuclei is considered, in which the quasiparticle energy spectrum is not distorted by the residual interactions that affect appreciably the thermodynamic behavior of the system. The number of quanta per cascade and mean-square fluctuation are calculated; the  $\gamma$ -quantum spectrum of the whole cascade is also obtained. The formulas can be used to determine the entropy and temperature of the initial nucleus by various methods. The effective nucleon (quasiparticle) mass in nuclear matter is determined by comparison with the experimental data. The region of validity of the theory and some possibilities of its extension on the basis of new experiments are discussed.

## 1. INTRODUCTION

Radiative transitions between the lowest levels of nuclei exhibit a great diversity in their intensities and multipolarities. In addition to the usually intense transitions between the "collective" (rotational and vibrational) levels, there are encountered also isomeric  $\gamma$  transitions, the high degree of hindrance of which may be due to the large change in the nuclear spin and also to other causes. Even a schematic tentative classification and a very cursory discussion of the particular interest that may attach to any particular modification of the radiative transitions between any two concrete energy levels of the nucleus would greatly exceed the scope of the present article. In nuclei that are not too light, however, the number of levels that lend themselves to transitions with  $\gamma$ -quantum emission increases sharply, with increasing excitation energy, and the individual features then become gradually equalized. A well known example may be the observed spectra of  $\gamma$  rays of radiative capture of thermal neutrons<sup>[1]</sup>. It is clear that for a sufficiently heavy nucleus, the overall picture of such a phenomenon characterizes not the individual levels, but more readily a certain region of the energy spectrum of the nucleus as a whole. It is natural to assume that rather general laws of phenomenomic character come to the forefront here. Unification of the dominant mechanism of the process becomes manifest, e.g., in the fact that these electromagnetic-radiation spectra<sup>[2]</sup> of many different nuclei exhibit great similarities among themselves.

Incidentally, at the  $\sim 8$  MeV excitation energy I referred to in this case, certain striking qualitative differences still remain. Since they are due to phenomena that are of very great importance in nuclear physics, it behooves us to dwell on the examples that illustrate this fact in somewhat greater detail. We consider the situations on both sides of osmium ( $Z = 76$ ), where the spectra of the capture  $\gamma$  quanta<sup>[2]</sup> have been investigated experimentally in sufficient detail. Many nuclei of the chemical elements preceding osmium have spectra of rather standard form, namely the energy distribution of the quanta has a maximum at  $\epsilon \lesssim 2$  MeV, after which it drops off rapidly towards the limiting value  $\epsilon = E$  (the excitation energy of the nucleus in the initial state). However, this "temperature" maximum becomes considerably smoothed out even for the first element following osmium, namely iridium ( $Z = 77$ ), and the transitions in the hard part of the  $\gamma$  spectrum are simultaneously smoothed out. In addition this characteristic deformation of the spectrum of the captured

quanta is further developed, in the case of  $^{78}\text{Pt}$ , namely, roughly speaking, the areas under both maxima become comparable. The emission spectra of  $^{79}\text{Au}$  reveals in practice only one maximum adjacent to the hard edge  $\epsilon = \epsilon_n$  ( $\epsilon_n$  is the neutron detachment energy). In the elements that follow,  $^{80}\text{Hg}$  and  $^{81}\text{Tl}$ , the relative area under the hard part of the spectrum continues to increase. It is curious to note that this feature becomes much more sharply pronounced in the case of the doubly magic nucleus  $^{82}\text{Pb}_{126}^{208}$ . When the preceding isotope  $\text{Pb}^{207}$  captures a thermal neutron, practically 100% of all the radiative transitions go directly to the ground state; in other words, the spectrum of the cascade degenerates into a single line  $\epsilon = \epsilon_n$ .

The picture outlined above is probably brought by about a unique phenomenon that becomes manifest not only in the spectra of the  $\gamma$ -quantum cascades; the equilibrium shape of the nucleus also changes in the immediate vicinity of osmium. The properties of the energy spectrum of spherical nuclei located in the region adjacent to the doubly-magic nucleus are strongly influenced by the residual interaction between the quasiparticles<sup>[1]</sup>. To the contrary, on the other side of the phase transition point<sup>[4]</sup>, such an influence apparently ceases to be decisive in any manner. The nonspherical shape of the nuclei in this region is quite natural, for when there is no interaction whatever between the particles the instability of the spherical configuration is proved by direct calculation.

We consider below the properties of electromagnetic radiation of all the nuclei pertaining to this nonspherical "normal" phase (see<sup>[4]</sup>).

## 2. THERMODYNAMICS OF ELECTROMAGNETIC RADIATION OF STRONGLY EXCITED NUCLEI

According to Fermi-liquid theory<sup>[1,6]</sup>, the behavior of this liquid is determined by quasiparticles that obey the Pauli principle and are sufficiently close to the Fermi boundary. In the state of thermal equilibrium, the usual Fermi distribution holds<sup>[2]</sup>

$$n(\epsilon') = (\exp(\epsilon'/T) + 1)^{-1}, \quad (1)$$

where  $\epsilon'$  is the energy of the quasiparticle reckoned from the chemical potential, and  $T$  is the temperature. We now explain the predominant mechanism of the process. In accordance with the accepted concepts, the quasiparticles move freely inside the nuclear matter. However, when they strike the transition region on the nuclear surface, they are reflected from the latter,

i.e., they are accelerated. This makes the emission of electromagnetic quanta possible<sup>3)</sup>.

On the average, we ascribe radiation to an individual quasiparticle in accordance with the law  $f(\epsilon)d\epsilon$ , where  $\epsilon$  is the energy of the  $\gamma$  quantum (the form of the function  $f(\epsilon)$  will be established below). To go over to the true probability distribution  $w(\epsilon)d\epsilon$ , we must take into account the entire aggregate (1) of the quasiparticles, and also allow for the Pauli principle. Actually this reduces to additional multiplication by the product  $n(\epsilon')[1 - n(\epsilon' - \epsilon)]$ , integrated over the fermion energies. Simple integration yields

$$\int_{-\infty}^{\infty} n(\epsilon')[1 - n(\epsilon' - \epsilon)]d\epsilon' = \frac{\epsilon}{e^{\epsilon/T} - 1},$$

$$w(\epsilon)d\epsilon = \frac{f(\epsilon)\epsilon d\epsilon}{e^{\epsilon/T} - 1}. \quad (2)$$

We now consider the question from a somewhat different point of view. The radiation wavelength is long relative to the dimensions of the nucleus and is principally of the electric-dipole type. The average level density of the system changes significantly only over energy intervals of the order of the temperature. In other words, when the excitation energy changes by an amount  $\epsilon \ll T$ , the energy characteristics of the spectrum of the nucleus as a whole, averaged over many quantum states, remain practically constant. Therefore in the limit as  $\epsilon \rightarrow 0$  there remains only the cubic dependence  $w(\epsilon) \propto \epsilon^3$  of the probability of the process on the transition energy, a dependence characteristic of dipole emission. Taking (2) into account we therefore have<sup>4)</sup>

$$f(\epsilon) = \text{const} \cdot \epsilon^3. \quad (3)$$

For our purposes there is no need to calculate the absolute value of the probability  $\Gamma_\gamma/\hbar$  of the radiation per unit time ( $\Gamma_\gamma$  is the radiative width). We can deduce even from (2) and (3), however, how this quantity depends on the temperature of the nucleus (or on the excitation energy; see (15)). Since the constant factor in the right-hand side of (3) depends neither on  $\epsilon$  nor on  $T$ , the integration of the second formula of (2) yields

$$\Gamma_\gamma \propto T^5. \quad (4)$$

The  $\gamma$ -quantum energy distribution  $w(\epsilon)d\epsilon$  will now be renormalized to a unit total probability of its radiation

$$w(\epsilon)d\epsilon = \frac{1}{24\zeta(5)T^5} \frac{\epsilon^4 d\epsilon}{e^{\epsilon/T} - 1},$$

$$\int_0^\infty w(\epsilon)d\epsilon = 1. \quad (5)$$

Here

$$\zeta(z) = \frac{1}{\Gamma(z)} \int_0^\infty \frac{x^{z-1} dx}{e^x - 1} = \sum_{n=1}^{\infty} \frac{1}{n^z}$$

is the Riemann  $\zeta$  function. We need also the average energy  $\bar{\epsilon}(T)$  of the quantum emitted by the nucleus whose temperature is  $T$

$$\bar{\epsilon} = \int_0^\infty \epsilon w(\epsilon)d\epsilon = \frac{\pi^6}{189} \frac{T}{\zeta(5)} \approx 4.91T. \quad (6)$$

As the nucleus radiates, it becomes cooler; the running values of the excitation energy and of the temperature will be denoted by  $E'$  and  $T'$ , respectively. For the average number  $\nu$  of the quanta in the cascade we obtain

$$\nu = \int_0^\infty \frac{dE'}{\bar{\epsilon}(T')} = \frac{189\zeta(5)}{\pi^6} \int_0^\infty \frac{dE'}{T'} \approx \frac{S}{4.91}, \quad (7)$$

where  $E$  is the energy of the initial state of the nucleus and  $S$  is its entropy. Relation (7) makes it possible to determine the latter from experiment.

How are the probability distributions (5) of different quanta of the cascade interrelated? Since the function  $w(\epsilon, T)$  depends on  $T$  as a parameter, its form is determined by the prior history and by the total energy of the entire preceding radiation that caused the nucleus to cool down to the temperature in question. However, reasoning somewhat formally, such a relation between the  $\gamma$  quanta of the cascade is due to the fact that each of them is characterized by an energy  $\epsilon$ . We now change over to another variable  $s$ , which is the entropy carried away by the quantum from the nucleus. Obviously,  $s = \epsilon/T$ , and the distribution (5) can be rewritten in the form

$$w(s)ds = \frac{1}{24\zeta(5)} \frac{s^4 ds}{e^s - 1}. \quad (8)$$

Thus, in terms of  $s$  the  $\gamma$  quanta are statistically independent. This simplifies very greatly the calculation of the fluctuations of the number of quanta in the cascade. We write down the corresponding mean values

$$\bar{s} = \int_0^\infty s w(s) ds = \frac{\pi^6}{189\zeta(5)} \approx 4.91,$$

$$\overline{s^2} = \int_0^\infty s^2 w(s) ds = 30 \frac{\zeta(7)}{\zeta(5)} \approx 29.2, \quad (9)$$

$$\overline{(\Delta s)^2} = \overline{s^2} - \bar{s}^2 \approx 5.11$$

(the first of these formulas, in fact, is a restatement of (6) in terms of other units).

We consider next a portion of the cascade consisting of  $\nu'$  successively emitted quanta. For the rms fluctuation of the entropy  $S'$  pertaining to this section we have the expressions

$$\overline{(\Delta S')^2} = \nu' \overline{(\Delta s)^2} \approx 5.11\nu', \quad \left( \frac{\Delta S'}{\nu'} \right)^2 \approx \frac{5.11}{\nu'}. \quad (10)$$

The last of the formulas (10) determines the fluctuation of the entropy  $S'/\nu'$  per  $\gamma$  quantum. The same quantity admits also of another definition: the considered section can be determined by specifying the constant  $S'$ , and the number of quanta needed to produce this entropy drop can be regarded as fluctuating. Therefore

$$\left( \frac{\Delta S'}{\nu'} \right)^2 = \frac{S'^2}{\nu'^4} \overline{(\Delta \nu')^2}. \quad (11)$$

Combining (11) with (10) and taking (7) into account, we extend the final formula to include the entire cascade:

$$\overline{(\Delta \nu)^2} = \left\{ 30 \frac{\zeta(7)}{\zeta(5)} - \left[ \frac{\pi^6}{189\zeta(5)} \right]^2 \right\} \frac{\nu^3}{S^2} \approx 0.0433S. \quad (12)$$

Here we have one other method of measuring the entropy of the initial nucleus, but this time from fluctuations of the number of the  $\gamma$  quanta per cascade (cf. (7)). Eliminating  $S$  from (7) and (12), we obtain the relation

$$\overline{(\Delta \nu)^2}^{1/2} \approx 0.461\nu^{1/2}, \quad (13)$$

which may turn out to be useful to verify the mechanism of the process.

The energy spectrum  $W(\epsilon)d\epsilon$  for the  $\gamma$  quanta of the entire cascade is made up of distributions of the type (5) for each of them. Taking also (6) into account, we have

$$W(\epsilon)d\epsilon = d\epsilon \int_0^\infty w(\epsilon, T') \frac{dE'}{\bar{\epsilon}(T')} = d\epsilon \epsilon^4 \frac{63}{8\pi^6} \int_0^\infty \frac{C(T')dT'}{T'^6(e^{T'/T} - 1)} \quad (14)$$

where  $C(T) = dE/dT$  is the specific heat of the nucleus. Its dependence on the temperature is determined by the formulas

$$E = \frac{1}{2} a T^2, \quad C = S = a T = \sqrt{2aE}, \quad (15)$$

which follows from the Fermi-liquid theory<sup>[1,6]</sup> (see also<sup>[8]</sup>, where rather weighty arguments were first advanced favoring such an equation of state of the nucleus). Substituting (15) in (14) and introducing the integration variable  $x = \epsilon/T'$ , we obtain ultimately

$$W(\epsilon) d\epsilon = d\epsilon \frac{63}{8\pi^2} a \int_{\epsilon/T'}^{\infty} \frac{x^3 dx}{e^x - 1}, \quad \int_0^{\infty} W(\epsilon) d\epsilon = \nu \quad (16)$$

(in practice it frequently turns out to be convenient to express the coefficient  $a$  in terms of the thermodynamic quantities of the initial state of the nucleus in accordance with (15)). The integral in this formula

$$J(y) = \int_y^{\infty} \frac{x^3 dx}{e^x - 1} = \frac{\pi^4}{15} - \frac{y^3}{3} D(y) \quad (17)$$

can be easily investigated. It is expressed in terms of the Debye function  $D(y)$ , which determines the well-known interpolation for the specific heat of a solid (see, e.g.,<sup>[6]</sup>).

In addition to the number of quanta  $W(\epsilon)$  per unit change of the variable  $\epsilon$ , we introduce also the distribution of the energy  $\mathcal{E}(\epsilon) d\epsilon = \epsilon W(\epsilon) d\epsilon$  and the spectrum of the  $\gamma$  quanta of the cascade:

$$\mathcal{E}(\epsilon) d\epsilon = d\epsilon \frac{63}{8\pi^2} a \epsilon J\left(\frac{\epsilon}{T}\right), \quad \int_0^{\infty} \mathcal{E}(\epsilon) d\epsilon = E. \quad (18)$$

Taking (16), (7), and (15) into account, we obviously have

$$\bar{\epsilon} = \frac{\int_0^{\infty} \mathcal{E}(\epsilon) d\epsilon}{\int_0^{\infty} W(\epsilon) d\epsilon} = \frac{\pi^4}{189\zeta(5)} \frac{E}{S} = \frac{\pi^4}{189\zeta(5)} \frac{T}{2} \approx 2.45T \quad (19)$$

for the quantum energy  $\bar{\epsilon}$  averaged over the spectrum (cf. (6)). The function  $\mathcal{E}(\epsilon)$  has a maximum<sup>5)</sup>. To determine its position, we equate the derivative  $d\mathcal{E}/d\epsilon$  to zero. The transcendental equation

$$\int_y^{\infty} \frac{x^3 dx}{e^x - 1} = \frac{y^4}{e^y - 1} \quad (20)$$

is easy to solve:  $y \approx 2.89$ . Thus,

$$\epsilon_{\max} \approx 2.89T. \quad (21)$$

This is possibly one of the most convenient methods of determining the temperature of a nucleus.

In view of the importance of the question, we present also a convenient formula that expresses the initial spectrum  $W(\epsilon)$  directly in terms of  $\epsilon_{\max}$ :

$$W(\epsilon) \approx 0.137 \frac{E}{(\epsilon_{\max})^2} J\left(\frac{\epsilon}{T}\right). \quad (22)$$

The limits of applicability of the theory are determined by the requirement

$$\epsilon(T') \ll E'. \quad (23)$$

In this sense, the nature of the violations near the hard edge  $\epsilon = E$  of the spectrum is quite clear. The energy conservation law forbids the emission of quanta with  $\epsilon > E$ , and the theoretical expressions (16) and (18) lead us to nonzero intensities, although they do prove to be exponentially small. However, in the softest regions of the spectrum, the theory also ceases to be

valid. Indeed, according to (6) the characteristic energy  $\bar{\epsilon}(T')$  for the  $\gamma$  quantum is proportional to the temperature of the radiating nucleus, since its excitation energy  $E'$  depends on the temperature quadratically (see (15)), i.e., it decreases more rapidly.

To make the criterion more precise, it is simplest to take into account the fact that, according to (6) and (7), we have the proportion

$$\bar{\epsilon}(T') / T' = S / \nu. \quad (24)$$

Replacing the numerator of the left-hand side by the running excitation energy  $E'$ , we transform the resultant relation in accordance with (15):

$$\frac{E'}{T'} = \frac{1}{2} \sqrt{2aE'} = \frac{1}{2} \sqrt{\frac{S^2}{E'} \sqrt{E'}} = \frac{S}{2} \sqrt{\frac{E'}{E}}.$$

We now equate the result to the right-hand side of (24):

$$\frac{S}{2} \sqrt{\frac{E'}{E}} = \frac{S}{\nu}, \quad E' = 4 \frac{E}{\nu^2}.$$

We have obtained here the excitation energy at which  $\epsilon \sim E'$ ; it determines the lower limit (with respect to the energy of the emitted quantum) of the applicability of the theory. Therefore the sought criterion takes the form

$$4E / \nu^2 \ll \epsilon \ll E. \quad (25)$$

Consequently, the region of thermal emission of the nucleus exists under the condition

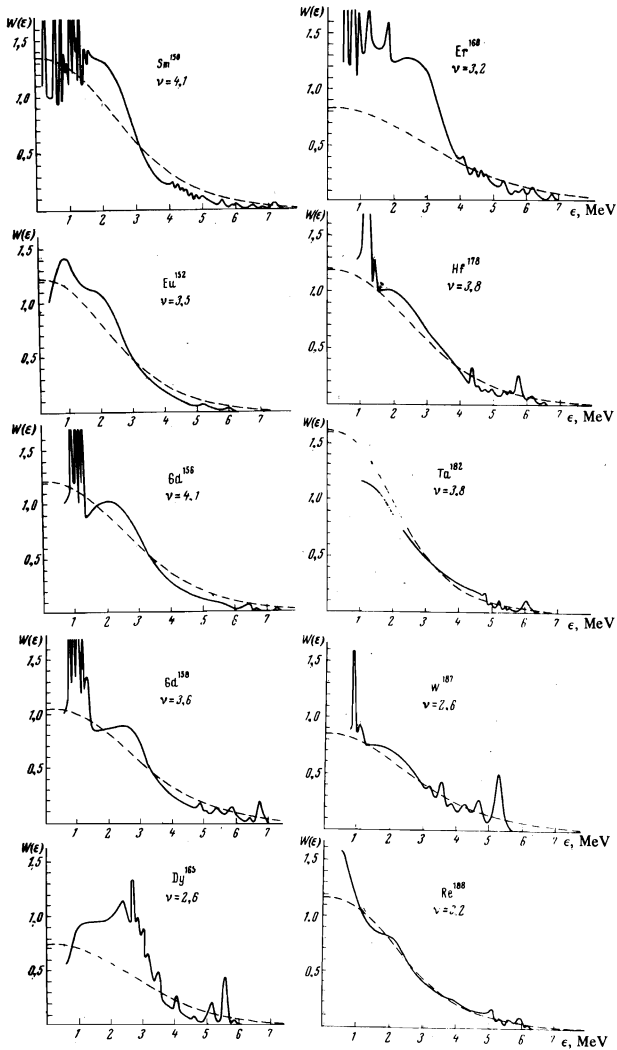
$$\nu^2 \gg 4. \quad (26)$$

### 3. COMPARISON WITH EXPERIMENT. EFFECTIVE MASS OF THE NUCLEON (QUASIPARTICLE)

The region of heavy nonspherical nuclei, in which sufficiently systematic experimental investigations of the  $\gamma$ -cascade spectra were made, extends from samarium-gadolinium to osmium<sup>6)</sup>. A comparison of the data of<sup>[2]</sup> with the theory developed in the preceding section was made for the spectra of radiative capture of thermal neutrons by ten different nuclei. According to formula (21) (see also the texts pertaining thereto) we determined the temperature of the initial compound nucleus. Since its excitation energy  $E = \epsilon_n$  is also known, relations (15) enable us to calculate the entropy and the specific heat. All these results are listed in the table.

The condition (26) may not be well satisfied in the case of capture of thermal neutrons. From this point of view, an advantageous method of monitoring the obtained temperatures is a comparison of the theoretical spectra of the  $\gamma$  quanta with the experimental ones. The figure shows the measured spectra<sup>[2]</sup> (in units of quanta/megavolt) with those calculated by formula (22). There is apparently a definite correlation with the total number  $\nu$ , calculated in accordance with (7), of the "evaporation" quanta in the cascade (i.e., due to the considered thermal mechanism). To the extent that the area under the theoretical spectrum  $W(\epsilon)$  approaches 2, the agreement, generally speaking, becomes much worse. To the contrary, even at  $\nu \approx 4$  satisfactory agreement is observed in a certain spectral region that does not contradict the criterion (25)<sup>7)</sup>. On the whole, qualitative considerations suggest that, owing to the excessively small number of the quanta in the cascade, the method considered here may overestimate somewhat the nuclear temperature.

Compound nucleus	E, MeV	T, MeV	s - c	$\Delta T$ , MeV	$\frac{m^*}{m_n}$	Compound nucleus	E, MeV	T, MeV	s - c	$\Delta T$ , MeV	$\frac{m^*}{m_n}$
$^{150}_{82}\text{Sm}$	7.98	0.80	20.0	0.18	1.04	$^{168}_{68}\text{Er}$	7.77	1.00	15.6	0.25	0.56
$^{152}_{83}\text{Eu}$	6.29	0.74	17.0	0.18	0.91	$^{178}_{72}\text{Hf}$	7.62	0.83	18.4	0.19	0.75
$^{156}_{64}\text{Gd}$	8.53	0.87	19.6	0.20	0.88	$^{182}_{73}\text{Ta}$	6.06	0.66	18.4	0.15	0.92
$^{158}_{64}\text{Gd}$	7.93	0.90	17.6	0.21	0.75	$^{187}_{74}\text{W}$	5.46	0.83	13.2	0.23	0.51
$^{165}_{69}\text{Dy}$	5.64	0.90	12.5	0.25	0.51	$^{188}_{75}\text{Re}$	5.73	0.73	15.7	0.18	0.68



The experimental spectra of the  $\gamma$ -quantum cascades are shown by solid lines, and the theoretical ones by dashed ones. The peaks in the soft parts of the spectra are not of thermal origin; some of them could not be drawn at all in the chosen scale.

The meaning and accuracy of the "radiative" temperature, i.e., the one calculated from the position of the maximum in the spectrum  $\mathcal{E}(\epsilon)$ ; see formula (21) and the table), will be easier to analyze if account is taken of certain features of the thermodynamics of such a cooled body as a concrete nucleus. Owing to the absence of fluctuations of its total energy  $E$ , the equilibrium temperature of such a system becomes to a certain degree an approximate concept. The scale of the related temperature uncertainty is given by the well known thermodynamic formula (see [6])

$$\Delta T = T / \sqrt{\gamma C} \quad (27)$$

for its rms fluctuation. The calculated values of  $\Delta T$  are given in next to the last column of the table.

The statements made above pertain to a definite nucleus with fixed composition. One cannot exclude, e.g.,

the possibility that such limitations may become less stringent when an attempt is made to ascribe a common temperature to an entire aggregate of relatively close nuclei. It is therefore of interest to verify whether an appreciable averaging of the radiative temperature takes place over the entire atomic-weight interval  $150 \leq A \leq 188$  where the comparison was made. Using (15), we reduce all the temperatures to a single excitation energy, say  $E = 8$  MeV. We can then see that the swing of the fluctuations is in fair agreement with the thermodynamic estimates of the variance  $\Delta T$ . Thus, the characteristic period of the fluctuations along the  $A$  axis is apparently small in comparison with the section under consideration, so that the averaging referred to above indeed has time to occur<sup>8)</sup>. We are nevertheless left with the question of the systematic overestimate of the temperature of the nucleus when the values of  $\nu$  are too small (see above).

To highlight the distinction between the random (fluctuation) and systematic errors more lucid, it is desirable to determine from experiment a quantity that characterizes directly, if possible, the nuclear Fermi liquid as such. Satisfying these requirements is the effective mass  $m^*$ ; its value is also of definite interest in itself. The effective mass determines the specific heat and the entropy of the Fermi liquid<sup>[1,6]</sup>. The combinatorial expression for the entropy reduces to an integral that can be calculated without difficulty (the well-known problem of the specific heat of a degenerate Fermi gas reduces to a similar procedure; see, e.g., [6]). Summing the contributions from the neutron and proton quasiparticles, we have

$$S = \frac{4\pi}{9} \frac{R^2}{\hbar^2} m^* (\rho_i^N + \rho_i^Z) T. \quad (28)$$

Just as in many other problems of nuclear physics, an important role is played by the dimensionless product

$$\rho_i R \gg 1, \quad (29)$$

where  $k_f$  is the limiting momentum of the quasiparticles of the corresponding type and  $R$  is the radius of the nucleus (more accurately, of equivalent volume). When (28) is compared with (15), it is convenient to express the coefficient  $a$  in the temperature dependence of the specific heat in terms of the energy and the temperature. We then obtain for the effective mass

$$\frac{m^*}{m_n} = \frac{9/\pi}{\rho_i^N + \rho_i^Z} \frac{\hbar^2}{2m_n R^2} \frac{E}{T^2}, \quad (30)$$

where  $m_n$  is the mass of the free nucleon. The question of the connection between the "limiting momentum"  $\rho_f$  (see (29)) and the number of true particles in the nucleus was considered earlier<sup>[9]</sup>. The corresponding formula is

$$N, Z = \frac{4}{9\pi} \rho_i^3 - s\rho_i^2 + q\rho_i. \quad (31)$$

Since the relative accuracy of such an approximate expansion becomes worse with increasing number of nucleons, it is most natural to use data concerning the magic numbers 82 and 126. This yields  $s = 1.1$  and

$q = 6.8$  for the values of the parameters that enter in (31)<sup>9)</sup>.

We assume

$$R = 1.2 \cdot 10^{-12} A^{1/3} \text{ cm.} \quad (32)$$

The effective-mass values determined from (30) and (31) are given in the last column of the table. The arithmetic mean is  $m^*/m_n = 0.72$  (the point pertaining to the nucleus  $\text{Th}^{233}$  is also taken into consideration here; see footnote 7). This result, however, was influenced by a systematic error, which lowers the effective mass of the quasiparticle. Indeed, there is a striking correlation between the ratio  $m^*/m_n$  calculated by this method and the number of quanta  $\nu$  (see the table and the figure). Consequently, a certain overestimate of the temperatures at extremely small  $\nu$ , referred to above, indeed took place. In this case this source of error seems even somewhat exaggerated, since the right-hand side of (30) contains the square of the temperature. On the other hand, in the region  $\nu = 3.5-4.1$ , the criterion (26) already seems to be fulfilled satisfactorily, as is confirmed also by the good agreement over the spectra (see the figure). The best value is therefore probably the one calculated for the six pertinent nuclei

$$m^*/m_n = 0.87 \pm 0.04 \quad (33)$$

(we cite here the purely statistical mean-squared variance).

#### 4. CONCLUSIONS

1. The known treatment of the spectra of the evaporation neutrons by the detail-balancing principle (see, e.g., [10]) makes it possible to carry out relatively rough estimates of the nuclear temperature. The thermodynamics of electromagnetic radiation of nuclei, which was developed in the present paper, is apparently more quantitative in character. Therefore, given the corresponding experimental data, it will be possible to measure more systematically and quantitatively the temperature of other thermodynamic quantities at different excitation energies.

2. At the present time, however, such a program could be realized only in part for one of the regions of the nonspherical nuclei (see the preceding section). Another interesting region begins with radium. Since many of the actinides in this region are fissile, a study of the captured quanta should be carried out under conditions of anticoincidences with fission fragments.

An even more noticeable shortcoming of the experimental data is due to their limitation with respect to the excitation energy. At  $E \sim 8$  MeV (the energy of attachment of a thermal neutron), the conditions for the applicability of the theory are frequently not very favorable, owing to the excessively low number of  $\gamma$  quanta in the cascade (see the preceding section, and also (26) and footnote 5). Observation of electromagnetic radiation of much more strongly excited nuclei is hindered by the evaporation neutrons. Therefore, for all their complexity, the performance of corresponding experiments for anticoincidence with neutrons is extremely desirable. They make it possible to overcome finally those difficulties in the reduction of the experimental data, which we attempted to analyze in the preceding section<sup>10)</sup>. One can even hope that further development of the theory (see, in particular, the next item), such

experiments will cast light on the interesting question of the phase transition that takes place when a spherical nucleus is sufficiently excited.

3. Spherical nuclei owe their very existence to the residual interaction between the quasiparticles. In [3] there were established only the most general, macroscopic features of its structure. It may turn out, however, that the form of the thermodynamic relations at low temperatures follows from it in a sufficiently unique manner. After establishing the dependence of the specific heat on the temperature (it is apparently not described by formula (15) in the given case), one can attempt to develop also the theory of radiation of such nuclei.

Qualitative considerations give grounds for assuming that spherical nuclei are characteristics by relatively high temperatures, and accordingly, by low entropies. Therefore, in particular, the region most easily accessible to practice,  $E = 6-8$  MeV, calls for a critical review. If it turns out that at such excitation energies the fluctuations are still large, then the thermodynamic relations will probably be suitable here only for rough estimates.

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<sup>1)</sup>This residual interaction has a macroscopic structure that influences the behavior of the nucleus as a whole. It was analyzed in [3].

<sup>2)</sup>We call attention to the connection between the condition  $T \ll \epsilon_n$  ( $-\epsilon_n$  is the chemical potential) for the applicability of the theory, on the one hand, and the very possible existence of a compound nucleus, on the other. In the opposite case, the neutrons having a binding energy  $\epsilon_n$  would be emitted from the nucleus "instantaneously," bypassing the stage of establishment of thermal equilibrium.

<sup>3)</sup>We mention also another possibility: radiation could be produced also when quasiparticles collide with one another. However, the number of such collisions is proportional to the cube of the temperature, whereas the effect calculated by us is linear in the temperature (see formula (2) below). Therefore the mechanism of collisions of individual quasiparticles with the "wall" of the nucleus should be regarded as predominant; see also footnote 2.

<sup>4)</sup>It is well known that owing to recoil of the nucleus as a whole, which has a charge  $Ze$ , the neutron component is also capable of electric dipole emission (see, e.g., [7]). In fact therefore, both proton and neutron quasiparticles take part in the process in question. However, it is clear from the character of the presented results that the form of the required expressions is determined only by the temperature of the nucleus and explicit allowance for the two-component character of the nuclear matter would not influence them. We therefore start for the time being from a simplified picture of quasiparticles of one type, the statistical distribution of which is given by formula (1).

<sup>5)</sup>To the contrary, the theoretical expression for  $W(\epsilon)$ , determined by formula (16), is a monotonic function. When its maximum is observed in experiment (see the Introduction above), this is due the fact that the theory is not applicable to the softest part of the spectrum. With respect to the presently available experimental data, the main shortcoming of the proposed theory is the narrowness of the region of its applicability. The corresponding criterion will be obtained below, see formulas (25) and (26). The pertinent questions will be analyzed in greater detail in the concluding sections of the article.

<sup>6)</sup>It is difficult to identify more specifically the chemical element pertaining to its lower limit, since the phase state of the nucleus is more sensitive here to the number of neutrons  $N$ . To the contrary, for the spherical nuclei in the vicinity of lead, the position of the phase-transition point depends to a greater degree on the number of protons. Insofar as can be judged from the experimental data, osmium ( $Z = 76$ ) is located precisely at the point of transition of the nonspherical nuclei to spherical ones, or, at any rate, is very close to it; see also the Introduction.

<sup>7)</sup>We mention in this connection the compound actinide nucleus  $\text{Th}^{233}$ . The following results were obtained for it:  $T = 0.72$  MeV,  $E = 4.96$  MeV,  $\nu = 2.8$ ; the agreement over the spectrum turned out to be poor.

The case of the even-even nucleus  $\text{Sm}^{150}$  is curious in the following respect: it is well known that in accordance with the spectroscopic data it is spherical in the ground state, but the Curie point lies very close to its position in the periodic table. In this case, the characteristics of the  $\gamma$ -quantum cascade (see the table and the figure) do not deviate in any striking manner from the general picture, and consequently, at  $E = 8$  MeV we already have a nonspherical phase (see also the introduction). It is still difficult to indicate more accurately the excitation energy at which the phase transition takes place. To avoid misunderstandings, we note that there is apparently no phase transition whatever when nonspherical nuclei are excited; see [<sup>4,5</sup>].

<sup>8)</sup>This can also be confirmed qualitatively by means of theoretical estimates. The variance  $\Delta T$  of the temperature is closely related with the thermodynamic fluctuation  $\Delta S$  of the entropy of the closed system (see [<sup>6</sup>]). On the other hand, in the particular case of an energy spectrum of the Fermi-liquid type, the entropy, in order of magnitude, can be interpreted as the number of quasiparticles that fall in the zone of the temperature smearing of the Fermi distribution. By connecting the addition of not too large a number of nucleons to the nucleus, on the one hand, with the possible change of the entropy of the quasiparticle, on the other, we estimate the characteristic period of the fluctuations. It turns out that about two such periods are subtended by the investigated interval  $150 \leq A \leq 188$ . We do not present details of these estimates, which are more readily qualitative.

<sup>9)</sup>Notice should be taken of the following: the expansion (31), and incidentally the very concept of the radius  $R$  of the nucleus, is of macroscopic accuracy. Therefore the contributions of the lowest power of  $\rho_f$  to the number of particles (e.g., terms of order  $\rho_f^0 \sim 1$ ) are disregarded here. We obtained initially the magic values  $\rho_f$  (see [<sup>9</sup>], formula (19)), so that  $s$  and  $g$  were determined on the basis of data pertaining to spherical nuclei. It is easy to see, however, that relation (31), with the same values of the parameters, remains valid also for the case of nonspherical nuclei. Indeed, the first term in the right-hand side is the quasiclassical limit for the number of cells in phase space; it depends only on the volume of the system. The second (surface) term, on the other hand, can generally speaking be influenced by the deformation. For equilibrium deformations of nonspherical nuclei, however, we have  $\alpha \sim \rho_f^{-1}$ , and consequently the relative change in the area of the surface of the nucleus is only a quantity on the order of  $\alpha^2 \sim \rho_f^{-2}$ . This would yield, in final analysis, a correction of the order of unity to the number of nucleons in the nucleus. Thus, the actually realized equilibrium deformations exert no microscopic influence on the form of the function  $N(\rho_f)$ , and the possible corrections would lie beyond the limits of the accuracy of (31).

<sup>10)</sup>At large  $E$ , the region where the criterion (25) is satisfied becomes much wider. This makes it possible, in particular, to verify and refine the result (33) for the effective mass of the nucleon (quasiparticle). We call attention to a possible more profound shortcoming of the proposed method of measuring the effective mass, namely, the right-hand side of (30) is inversely proportional to the square of the nuclear radius  $R$ . It is clear to a transition to higher excitation energies does not change the situation in this respect. The value (32) assumed above seems to agree fairly well with the data on the scattering of electrons and on the internal structure of the nuclei. However, the isotropic Fermi liquid is an object that is relatively exotic and is not frequently encountered in nature, so that the value of the effective mass in a nuclear Fermi liquid can be of certain fundamental interest. From this point of view, the question of the choice of the best value of  $R$  can still not be regarded as completely solved.

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