Instability of transverse electromagnetic waves in a drifting electron-hole plasma

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The dispersion law for low-frequency electromagnetic waves propagating in the direction of stationary drift of electrons and holes is investigated for crystals in the shape of plates and cylinders. The investigation is carried out with allowance for the magnetic self-field of the direct current. The instability of an axially symmetric wave generated in the absence of an external longitudinal magnetic field in crystals of cylindrical geometry is investigated in detail. It is shown that this instability develops only when the radius of the cylinder exceeds some critical value. Numerical estimates of the wave increment in the instability region are made for indium antimonide.

1. The question of instability and of the possibility of amplifying transverse waves in a drifting electron-hole plasma has been repeatedly raised in the scientific literature (see, for example,[1-9]). Theoretical investigations have been carried out both with[5-7] and without[1-4] allowance for the magnetic self-field of the direct current. It is now established that the neglect of the magnetic self-field (MSF) of the direct carrier current is inconsistent—the terms of the dispersion equation arising as a result of the allowance for the MSF have the same magnitude and structure as the Doppler terms that directly take into account the drift of the carriers. The magnitude and distribution in space of the MSF depend on the geometry of the crystal, and therefore solutions obtained without allowance for the shape of the crystal also need to be critically examined.

The types of the instabilities discovered were not investigated in[1,4], and this led to the erroneous conclusion that it is possible to amplify waves in those regions of k space where the waves are not transmitted (special investigation shows that the instability can be absolute). At the same time the wrong conclusion is drawn in[2] that it is impossible to amplify helicons, since the instability always turns out to be absolute; here the error in the definition of the type of instability arose as a result of an incorrect analysis of the dispersion equation. In the present paper we aspire (wherever it is within the reach of simple means) to investigate the type of instability. In doing this, we shall utilize the Briggs discrimination criterion[10]. According to this work, the determination of the type of instability allows us to answer without further investigation the question as to the possibility of amplifying oscillations in a medium which is infinite in the direction of propagation of the waves (though without any indication of the amplification region). The possibility of amplification arises only in the case when a region of convective instability exists. To absolute instability corresponds the nontransmission of the oscillations.

The entire analysis below pertains to the case of a wave propagating in the direction of the stationary drift of the carriers (along the z axis). The external magnetic field H_0 (if one is present) is directed along the same axis. We assume that the components of the wave fields depend on z and t according to the law $\exp(-i\omega t + ikz)$. We investigate two of the simplest geometries of crystals: an infinite plate ($-\infty < x < \infty$, $-R \le y \le R$, $-\infty < z < \infty$) and an infinite cylinder ($0 \le \rho \le R$, $0 \le \varphi \le 2\pi$, $-\infty < z < \infty$). In the case of the cylinder the investigation is restricted to the axially symmetric mode (the instability of the higher helicon modes has been considered in[5]) and, in the case of the plate, to the solutions that are independent of the x coordinate.

2. The basic system of hydrodynamic equations is of the form

$$m_n\tau_n^{-1}\mathbf{v}_n = -e\mathbf{E} - \frac{e}{c}[\mathbf{v}_n\times\mathbf{H}],\quad m_p\tau_p^{-1}\mathbf{v}_p = e\mathbf{E} + \frac{e}{c}[\mathbf{v}_p\times\mathbf{H}],$$
$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t},\qquad \operatorname{rot}\mathbf{H} = \frac{4\pi e}{c}(p\mathbf{v}_p - n\mathbf{v}_n), \tag{1}$$
$$\operatorname{div}\mathbf{H} = 0,\quad \operatorname{div}\mathbf{v}_n = 0,\quad \operatorname{div}\mathbf{v}_p = 0.$$

Here $\mathbf{v}_n$ and $\mathbf{v}_p$ are the electron and hole velocities, **E** is the resultant electric field, including the pulling, Hall, focusing, and wave fields; **H** is the resultant magnetic field, including the external field H_0, the field H_I generated by the direct current, and the wave field **h**; p and n are the electron and hole densities, and $m_{p,n}$ and $\tau_{n,p}$ are their effective masses and momentum relaxation times.

We use the low-frequency approximation: $|\omega|$, $|v_{n,p}k| \ll \tau_{n,p}$, $c|k|$. The carrier densities are assumed to be constants—the strong longitudinal magnetic field inhibits the appearance of the pinch effect; in the absence of a longitudinal magnetic field we assume the transverse (to the direct current) dimension R of the crystal to be large compared to the ambipolar diffusion length L, in which case there also does not arise an appreciable pinch in the volume. The conditions $|k|L \ll 1$ and $R^{-1}L \ll 1$ (in the impact-ionization regime it is also required that $s\tau_{rec}R^{-1} \ll 1$, where τ_{rec} is the colume recombination time) evidently permit us to also neglect the boundary inhomogeneities in the density when matching the solutions of the exterior and interior problems.

In solving the system (1) in the case when n and p are constant and kinetic paramagnetism is neglected (see below), the magnetic field of the direct current is ultimately only manifest in the form dH_x^I/dy (plate) or $dH_\varphi^I/d\rho$ (cylinder); these quantities do not depend on the space coordinates, and the equations of the system (1), when linearized with respect to the wave corrections, admit of solutions in the form of hyperbolic (plate) and cylindrical (cylinder) functions. By matching the solutions obtained inside the crystal with the solutions of the exterior problem, we obtain the dispersion equation. In the general case this equation in its explicit form contains the half-thickness R of the crystal as a parameter, and is extremely unwieldy. The situation gets appreciably simplified if $k^2R^2 \gg b^2$, where $b = \pi/2$ for the plate and $b = a_1$ for the cylinder, the a_1 here being the first finite root of the Bessel function of the first order. When this condition is fulfilled one of the branches of the dispersion equation practically coincides with the dispersion equation in an infinite crystal if we neglect the MSF. Let us first consider this branch. Notice that in the general case, when the MSF is taken into account, the separation of the waves into purely longitudinal and purely transverse waves does not occur.

The dispersion equation has the following form:

$$\varkappa^\pm(k)\left[\frac{2k-ik_2\pm ir}{k(1+h_0^2)-ik_2}+g\frac{2sk+ik_2\mp ir}{k(s^2+h_0^2)+isk_2}\right]=-2i\frac{k^2}{k_1}$$
$$+kw\left[\frac{2k-ik_2\pm ir}{k(1+h_0^2)-ik_2}-gs^{-1}\frac{2sk+ik_2\mp ir}{k(s^2+h_0^2)+isk_2}\right]. \quad (2)$$

Here

$$\varkappa=\frac{\omega}{c},\quad k_1=\frac{4\pi e^2 n\tau_n}{m_n c}=\frac{\omega_n^2\tau_n}{c},\quad k_2=\eta k_1 w(1+gs^{-1}),$$
$$g=\frac{p}{n},\quad s=\frac{m_p\tau_n}{m_n\tau_p},\quad h_0=\frac{eH_z^0\tau_n}{m_n c},\quad r=(4h_0^2k^2+k_2^2)^{1/2},$$

$\eta = 1$ for the plate and $\eta = 1/2$ for the cylinder, $w = -u_n/c$, where $(-u_n)$ is the stationary-drift velocity of the electrons in the z direction ($u_n = e\tau_n E_z^0/m_n$, the E_z^0 here being the stationary pulling electric field). The appearance in (2) of the quantity k^2 is wholly connected with the allowance for the magnetic self-field of the current[1)]; at $k_2 = 0$ Eq. (2) goes over formally into the dispersion equation obtained in[1,4]. The two signs in (2) correspond to the two different polarizations of the waves.

Investigation of Eq. (2) for arbitrary values of the quantities h_0, g, and s for a crystal in the form of a plate ($\eta = 1$) shows that waves of both polarizations are always stable, i.e., $\operatorname{Im}\omega(k) < 0$ for any real k. We note here that when the MSF is neglected, the analysis indicates the possibility of an instability for $h_0 \neq 0$, $E_z^0 \neq 0$, and $g \neq 0, \infty$.

Let us proceed to the consideration of a crystal in the form of a cylinder ($\eta = 1/2$). For this geometry, the investigation which takes the MSF into account indicates that an instability is possible (if, of course, $g \neq 0, \infty$, i.e., current carriers of both signs are present). Under certain conditions an instability can develop even in the absence of an external longitudinal magnetic field—the case with $h_0 = 0$. The instability of $l \neq 0$ helicon modes has been detected in[5] for $h_0 = 0$; the possibility that the axially symmetric ($l = 0$) mode is unstable is negated without sufficient grounds for it[2)].

Let $k_2 > 0$ and $h_0 = 0$. Then the $\kappa^-(k)$ wave is unstable. Let us consider the $\kappa^+(k)$ wave for $g = 1$ (the electron and hole densities are equal). We obtain

$$\operatorname{Re}\varkappa^+(k)=\frac{s-1}{2s}wk,\qquad \operatorname{Im}\varkappa^+(k)=\frac{k_2^2-sk^2}{k_1(1+s)} \quad (3)$$

or in dimensional notation:

$$\operatorname{Re}\omega^+(k)=\frac{1}{2}u_n k\left(\frac{m_n\tau_p}{m_p\tau_n}-1\right),$$
$$\operatorname{Im}\omega^+(k)=\frac{\omega_n^{-2}\tau_n^{-1}}{m_n\tau_p+m_p\tau_n}\left(\frac{\omega_n^4u_n^2}{4c^2}\tau_n^2\tau_p m_n-m_p\tau_n c^2k^2\right).$$

The phase velocity $k^{-1}\operatorname{Re}\omega^+(k)$ of the waves under consideration does not depend on k. The instability is wholly connected with the allowed for magnetic self-field of the direct current and it encompasses the region

$$0\leqslant|k|<k_0=\frac{|u_n|}{2c^2}\omega_n^2\tau_n\sqrt{\frac{m_n\tau_p}{m_p\tau_n}}.$$

The increment is a maximum at $k = 0$, i.e., in the model under consideration a homogeneous distribution along the z axis is the most unstable (see, however, below the solution obtained with allowance for the finiteness of the thickness of the crystal). Let us estimate $\operatorname{Im}\omega^+(0)$ and k_0 for the semiconductor InSb at $T = 77^\circ$K in a pulling electric field inducing interband breakdown[8]. Suppose $E_z^0 = 250$ V/cm, $\tau_n = \tau_p = 10^{-12}$ sec, $m_n = 1.5\times10^{-29}$ g, $m_p/m_n = 40$, $n = p = 10^{17}$ cm^{-3}. We obtain $k_0 = 50$ cm^{-1} and $\operatorname{Im}\omega^+(0) \approx 10^8$. The quantity $\operatorname{Im}\omega^+(k = 0)$ is different from zero only when $g = 1$. If $g \neq 1$, then $\operatorname{Im}\omega^+(k) \to 0$ as $k \to 0$.

Investigation of the nature of the instability showed that the instability is absolute when $n = p$ if the condition

$$(3+\sqrt{8})^{-1}<s<3+\sqrt{8} \quad (4)$$

is fulfilled; when this condition is violated the instability is a convective instability. For arbitrary values of g and s the $\kappa^+(k)$ wave is unstable if the inequality

$$g(1+s)^2-s(1-g)^2>0. \quad (5)$$

is satisfied. The instability encompasses the region

$$0<k^2<\frac{k_2^2}{(s+g)^2}[gs^{-1}(1+s)^2-(1-g)^2].$$

It is impossible to ascertain the nature of the instability at $g \neq 1$ without numerical computations.

Let us now consider the case when the external longitudinal magnetic field is different from zero ($h_0 \neq 0$). Equation (2) admits of a simple analysis for arbitrary g and h_0 if $s = 1$. In this case the instability exists when

$$(3+\sqrt{8})^{-1}<g<3+\sqrt{8}.$$

Notice that this condition coincides exactly with the condition for absolute instability of the solution obtained when the MSF is neglected.

The instability encompasses the region

$$0<k^2<k_2^2\frac{(6g-1-g^2)[(1+g)^2+8gh_0^2]}{(1+g)^4(1+h_0^2)^2}.$$

It is easy to verify that the maximum wave number decreases with increasing h_0^2. A plot of $\operatorname{Im}\omega(k)$ for different values of h_0^2 for the case $s = g = 1$ is shown in Fig. 1. It was experimentally detected in[9] that an increase in the external longitudinal magnetic field impedes the development of the instability.

In the particular case $s = g = h_0^2 = 1$, the relation (2) is greatly simplified. We obtain

$$\varkappa^+(k)=\frac{i}{2k_1}\left[-2k^2+\frac{1}{2}k_2^2+|k_2|\sqrt{k^2+\frac{1}{4}k_2^2}\right].$$

Here all the harmonics with $k^2 < \frac{3}{4}k_2^2$ are unstable, the instability being absolute.

In the presence of a longitudinal magnetic field we must allow for the possibility of kinetic paramagnetism manifesting itself when the longitudinal magnetic field becomes inhomogeneous, increasing toward the center of the crystal (see[11,7]). The inhomogeneity is slight if the condition ($\mu_n = e\langle\tau_n\rangle/m_n$)

$$2\pi\frac{\mu_n^2}{c^2}(1+s^{-1})\frac{eE_z^0p}{k_2}\ln(1+s^{-1}k_2^2R^2)\ll 1.$$

is fulfilled. This inequality is violated in InSb for the above-chosen values of the parameters. It can however be satisfied in semimetals with a low mobility and a high density of the pairs (the latter is necessary for the fulfilment of the condition $|k_2|R \gg 1$, in which case the instability encompasses a region of reasonable wavelengths).

3. We have thus far virtually assumed in the analysis that the transverse dimension R is infinitely large. Let us investigate in greater detail the case $h_0 = 0$ in a system of finite thickness. The case $h_0 = 0$ is in some sense special—the structure of the solutions to the wave equation for this case is markedly different from the structure for $h_0 \neq 0$. In the absence of an external longitudinal magnetic field, the waves in a crystal in the form of a cylinder split up into two independent waves $(0, h_\varphi, 0)$ and $(h_\rho, 0, h_z)$ with different dispersion laws. The wave $(h_\rho, 0, h_z)$ is always stable for all real k. The wave $(0, h_\varphi, 0)$ can be unstable. Solving the wave equation in a crystal of finite radius R and matching this solution with the solution of the exterior problem, we obtain the following dispersion equation:

$$\begin{aligned}(a_m/R)^4 + (a_m/R)^2[2k^2 + ikk_2(s^{-1}-1) - ik_1(\varkappa - kw)\\ - ik_1gs^{-1}(\varkappa + ks^{-1}w)] + k^2(k - ik_2)(k + is^{-1}k_2)\\ - ik_1k(\varkappa - kw)(k + is^{-1}k_2)\\ - ik_1gs^{-1}k(k - ik_2)(\varkappa + ks^{-1}w) = 0.\end{aligned} \qquad (6)$$

Here the a_m are the zeroes of the Bessel function of the first order (with the exception of the root $a_0 = 0$). Allowing in (6) the quantity R to tend to infinity, we obtain the dispersion law (2) for the $\kappa^+(k)$ wave for $h_0 = 0$.

Investigation of Eq. (6) shows that the instability arises when the condition

$$\psi(s,g) \equiv \frac{k_2^2s^{-1}}{(1+gs^{-1})^2}[g(1+s^{-1})^2 - s^{-1}(1-g)^2] > \left(\frac{2a_m}{R}\right)^2, \qquad (7)$$

is fulfilled, or since $k_2 \sim E_z^0$, when $|E_z^0| > E_1$, i.e., there exists a certain threshold pulling electric field that depends on the crystal radius.

It can be seen from (7) that the threshold field E_1 decreases in inverse proportion to the radius R of the system. It can also be seen that the instability is realized only in a definite region of variation of the parameters $g = p/n$ and $s = m_p\tau_n/m_n\tau_p$, namely in the region located between the branches of the hyperbola

$$(\tfrac{1}{2}\sqrt{g+g^{-1}})^2 - (\tfrac{1}{2}\sqrt{s+s^{-1}})^2 = 1.$$

The dependence of the threshold value of the pulling electric field on the ratio of the band electron and hole densities is shown for a particular case in Fig. 2.

The field E_1 has a minimum at $g = (3+s)/(3+s^{-1})$. For such g and for $s \gg 1$, we obtain

$$|k_2(E_1)| \approx 4\sqrt{2}a_mR^{-1}.$$

Substituting into (7) values typical of the gaseous plasma, we verify that, in virtue of the large values of s and the small values of k_2^2, the condition for instability can practically never be fulfilled here for systems of reasonable thickness. In semiconductors and semimetals the condition (7) can be fulfilled; it is most easily satisfied for the root $a_1 \approx 3.83$ (we shall henceforth consider only the case $a_m = a_1$). The harmonics for which $k_-^2 < k^2 < k_+^2$, where

$$k_\pm^2 = -\left(\frac{a_1}{R}\right)^2 + \frac{\psi}{2} \pm \sqrt{\frac{\psi^2}{4} - \psi\left(\frac{a_1}{R}\right)^2}.$$

are unstable. As we can see, allowance for the finiteness of the crystal thickness eliminates the instability in the immediate neighborhood of the point $k = 0$. For $g = 1$ (i.e., $n \simeq p$) the increment attains a maximum at the point $k = \tilde{k}$, where

$$\tilde{k}^2 = -\left(\frac{a_1}{R}\right)^2 + \frac{a_1}{R}\sqrt{k_2^2s^{-1}}.$$

Here

$$\operatorname{Im}\varkappa(\tilde{k}) = \frac{k_2^2s^{-1} - 2a_1R^{-1}\sqrt{k_2^2s^{-1}}}{k_1(1+s^{-1})}.$$

It is not possible to investigate the type of this instability by simple means.

Let us make estimates for InSb for the previously indicated values of the parameters. For $g = 1$, Eq. (7) assumes the form $R > 0.153$ cm. Let $R = 0.77$ cm. Then $\tilde{k} \approx 15\ \text{cm}^{-1}$, $k_+ \approx 49.5\ \text{cm}^{-1}$, $k_- \approx 0.5\ \text{cm}^{-1}$, and $\operatorname{Im}\omega(\tilde{k}) \approx 10^8\ \text{sec}^{-1}$.

Figure 3 shows the dependence of $\operatorname{Im}\omega$ on the wave vector k for different values of the crystal radius.

The estimates made show that in real semiconductors (when account is taken of the fact that the maximum current density should not exceed a few hundred thousand amperes per cm^2), instability of the considered type is practically attainable only in the longwave region of the spectrum. Note that microwave radiation in the absence of a longitudinal magnetic field has been experimentally detected in the region of strong currents in[8].

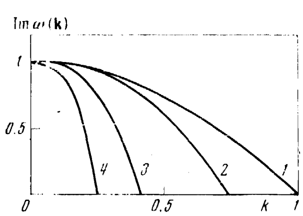

FIG. 1. Dependence of the increment of Imω on the wave vector k (in arbitrary units) for different values of the external longitudinal magnetic field h_0 in the case $s = g = 1$, $R \to \infty$, 1) $h_0^2 = 0$, 2) $h_0^2 = 2$, 3) $h_0^2 = 10$, 4) $h_0^2 = 30$.

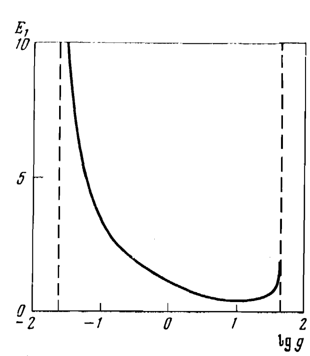

FIG. 2

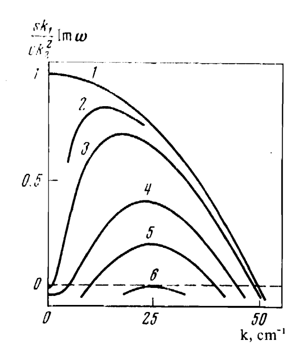

FIG. 3

FIG. 2. Dependence of the threshold field E_1 for the instability (in arbitrary units) on the ratio $g = p/n$ of the densities of the band carriers for $s = 40$.

FIG. 3. Dependence of the increment of Imω on the wave vector k in cylindrical crystals of different thicknesses for $g = 1$, $s = 40$, and $h_0 = 0$: 1) $a_1/R = 0$, $R = \infty$; 2) $a_1/R = \sqrt{15}\ \text{cm}^{-1}$, $R \approx 0.990$ cm; 3) $a_1/R = \sqrt{50}\ \text{cm}^{-1}$, $R \approx 0.543$ cm; 4) $a_1/R = 15\ \text{cm}^{-1}$, $R \approx 0.255$ cm; 5) $a_1/R = 20\ \text{cm}^{-1}$, $R \approx 0.192$ cm; 6) $a_1/R = 25\ \text{cm}^{-1}$, $R \approx 0.153$ cm.

[1]In a crystal in the form of a cylinder the field H_l is connected with k_2 by the relation $|e\tau_n H_l(\rho)/m_n c| = |k_2|\rho$.

[2]Analysis of the basic equations of [5] shows that the axially symmetric mode can, in principle, be unstable.

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