Recombination and the role of negative ions in a high-frequency gas discharge

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A theory of a high frequency discharge in gases is developed and takes into account both attachment and recombination of electrons diffusing from the discharge. It is found that even in gases with a very low probability for negative ion formation the ions play an important role, since electron-ion recombination by itself cannot ensure a sufficiently rapid decrease of the electron concentration outside the discharge. The analytical formulas obtained in the paper can be employed, in particular, to determine the kinetic coefficients on basis of the experimental gas discharge characteristics.

1. INTRODUCTION

One of us[1] has previously investigated diffusion in a high-frequency high-pressure gas discharge, when the principal role among the various recombination processes is played by the attachment of electrons to neutral atoms to form negative ions.

In the present paper we study diffusion in a stationary high-frequency discharge in gases in which the probability of production of negative ions is small (e.g., in helium), so that it is necessary to take into account direct electron-ion recombination processes.

The distribution of the electron density in the transition layer on the boundary of the discharge depends essentially on the probability of production of the negative ions. Indeed, at a low degree of gas ionization, the probability of production of negative ions is proportional to the first power of the electron density. On the other hand, the probability of direct electron-ion recombination is proportional both to the concentration of the electrons and to the concentration of the ions. When account is taken of quasineutrality, this probability is proportional to at least to the second power of the electron density. Outside the discharge, the electron-ion recombination decreases, therefore more rapidly with decreasing electron concentration than the probability of production of negative ions. If the probability of production of a negative ion is low, then the electron density decreases with increasing distance outside the discharge, and then the direct electron-ion recombination ultimately gives way to attachment of electrons to neutral atoms. The electron-ion recombination itself does not ensure a sufficiently rapid decrease of the density of the diffusing electrons outside the discharge, so that the heat release does not decrease outside the discharge. Therefore, in the absence of electron attachment to neutral atoms, it is difficult to realize a stationary discharge with a boundary detached from the walls of a cooled vessel (or resonator)[1)]. The attachment of the electrons to the neutral atoms leads, as shown in[1], to an exponential decrease of the electron density outside the discharge, ensuring by the same token a rapid decrease of the heat release and the possibility of a stationary state.

Thus, the process of production of negative ions, even if its probability is low, is very important and must be taken into account when a theory is developed for a stationary high-frequency discharge in a gas.

2. INITIAL EQUATIONS

We take into account the electron-ion recombination in a high-frequency gas discharge under the same assumptions as in[1]. In the particle-balance equation (Eq. (2.3) of[1]) we include, besides terms describing the production and decay of the negative ions, also terms that describe electron-ion recombination and the corresponding inverse ionization process. Among the different free combination processes we consider only those whose probability decreases most slowly with decreasing electron density N_e, in proportion to N_e^2. Recognizing that the deviation from local thermodynamic equilibrium is due only to diffusion, we can write the equation for the ionization-recombination balance for the stationary state in the form[2)]

$$\frac{d}{dx}D\frac{dN_e}{dx}=\beta_{\rm rec}(N_e^2-N_{e\,\rm eq}^2\ (T))+\beta_{\rm cap}(N_e-N_{e\ \rm eq}\ (T)). \qquad (2.1)$$

Here D is the coefficient of ambipolar diffusion of the electrons, β_{rec} is the coefficient of electron-ion recombination, β_{cap} is the coefficient of capture (attachment) of an electron and an atom with production of a negative ion, $N_{e\ eq}(T)$ is the equilibrium concentration of the electrons and is determined by the system of Saha equations ([14], Sec. 106). The coefficients in (2.1) are functions of the temperature T, which satisfies the equation[1,5]

$$\frac{d^3}{dx^3}\frac{1}{\sigma}\frac{d}{dx}\varkappa\frac{dT}{dx}-\frac{64\pi^2\omega^2\sigma}{c^4}\varkappa\frac{dT}{dx}=0 \qquad (2.2)$$

(κ is the thermal conductivity and ω is the frequency of the electromagnetic field). We confine ourselves to a temperature region in which the collisions of the electrons with neutral atoms play the most important role. The electric conductivity of the gas σ is then proportional to the electron density

$$\sigma=e^2N_e/m\nu_{\rm eff} \qquad (2.3)$$

(ν_{eff} is the effective number of collisions, and e and m are the charge and mass of the electron).

In the presence of electron-ion recombination and electron capture by neutral atoms, there are two characteristic diffusion lengths: the recombination diffusion length $d_{rec} = (D/\beta_{rec}N_{e\ eq}(T_m))^{1/2}$, over which the electrons diffusing from the discharge recombine, and the capture diffusion length $d_{cap} = (D/\beta_{cap})^{1/2}$, over which the electrons diffusing from the discharge are captured by the neutral atoms (T_m is the maximum temperature of the plasma in the discharge).

Assuming that the kinetic coefficients change little over the capture diffusion length d_{cap}, and changing over to the dimensionless variables ζ, Θ, and n:

$$\zeta=\frac{x}{\delta_m},\quad \Theta=\frac{I(T_m-T)}{2T_m^2},\quad n=\frac{N_e}{N_{e\,eq}(T_m)},\quad \frac{N_{e\,eq}(T)}{N_{e\,eq}(T_m)}=e^{-\Theta} \tag{2.4}$$

(here $\delta_m = c/(8\pi\omega\sigma(T_m))^{1/2}$ is the depth of penetration of the field into the plasma with temperature T_m), we reduce Eqs. (2.1) and (2.2) to the form

$$\frac{d^2n}{d\zeta^2}=\gamma_r^2(n^2-e^{-2\Theta})+\gamma_c^2(n-e^{-\Theta}), \tag{2.5}$$

$$\frac{d^3}{d\zeta^3}\frac{1}{n}\frac{d^2\Theta}{d\zeta^2}-n\frac{d\Theta}{d\zeta}=0. \tag{2.6}$$

Here

$$\gamma_r=\delta_m/d_{rec},\quad \gamma_c=\delta_m/d_{cap}. \tag{2.7}$$

Just as in[1], the condition for the applicability of Eqs. (2.5) and (2.6), which calls for smallness of the change of the kinetic coefficients over the capture diffusion length is

$$\gamma_c\gg T_m/I, \tag{2.8}$$

I is the gas ionization potential ($I \gg T_m$).

In the present paper we investigate the effect of electron-ion recombination, on the discharge, assuming the probability of electron capture by the atom to be small. In accordance with this, we assume that

$$1\gg\gamma_c\gg T_m/I, \tag{2.9}$$

and γ_r can be arbitrary. The case of small probability of electron-ion recombination $1 \sim \gamma_c \gg \gamma_r$ was considered in[1].

Outside the plasma, as $\zeta \to -\infty$, the electron concentration N_e tends to zero, and the heat flux $S = -\kappa dT/dx$ tends to a specified value S_0, equal to the flux of the electromagnetic energy entering the discharge. In terms of the dimensionless variables (2.4), these conditions become

$$n\to 0,\quad \zeta\to-\infty, \tag{2.10}$$

$$d\Theta/d\zeta\to -S_0I\delta_m/2\varkappa_mT_m^2,\quad \zeta\to-\infty. \tag{2.11}$$

Inside the plasma, as $\zeta \to -\infty$, the temperature approaches exponentially its maximal value, and the electron density tends to equilibrium. In terms of the dimensionless variables, this means that

$$\Theta\to e^{-(\zeta-\zeta_0)},\quad \zeta\to+\infty, \tag{2.12}$$

$$n\to 1,\quad \zeta\to+\infty. \tag{2.13}$$

Equations (2.5) and (2.6) with boundary conditions (2.10), (2.12), and (2.13) describe a stationary high-frequency discharge at high gas pressure, with allowance for the processes of fusion, recombination, and formation of negative ions. Since ζ does not enter explicitly in (2.5) and (2.6), the solutions of these equations, satisfying the boundary conditions (2.10), (2.12), and (2.13), are functions of $\zeta - \zeta_0$ (ζ_0 is an arbitrary constant) and depend on γ_c and γ_r as parameters. In particular, as $\zeta \to -\infty$ (outside the discharge), the dimensionless heat flux $d\Theta/d\zeta$ does not depend on ζ and is a function of two parameters:

$$d\Theta/d\zeta=-F(\gamma_c,\gamma_r),\quad \zeta\to-\infty. \tag{2.14}$$

Comparing formulas (2.11) and (2.14), we find the electromagnetic-energy flux density S_0 necessary to heat the plasma in the discharge to the temperature T_m:

$$S_0(T_m)=2F(\gamma_c,\gamma_r)\varkappa_mT_m^2/I\delta_m. \tag{2.15}$$

The form of the function $F(\gamma_c, \gamma_r)$ and also of $n(\zeta)$ and $\Theta(\zeta)$ can be obtained only by solving Eqs. (2.5) and (2.6). The latter are a system of complicated nonlinear equations, so that it is impossible to obtain their analytic solution in the entire region of variation of the parameters. However, analytic solutions can be obtained in the most important limiting cases of strong and weak diffusion.

We shall show first that if only electron-ion recombination is taken into account and the attachment of the electrons to the neutral atoms is neglected, this does not ensure the existence of a stationary state of a discharge with a finite region from which general heat is released. Neglecting the processes of ionization and formation of negative ions outside the discharge, we rewrite (2.5) in the form

$$d^2n/d\zeta^2=\gamma_r^2n^2,\quad \zeta\to-\infty. \tag{2.16}$$

Its solution, which tends to zero as $\zeta \to -\infty$, decreases in inverse proportion to the square of the distance

$$n=6/\gamma_r^2(\zeta-\tilde{\zeta}_0)^2, \tag{2.17}$$

$\tilde{\zeta}_0$ is a certain constant. Consequently, the conductivity of the gas (2.3) also decreases in inverse proportion to the square of the distance. On the other hand, under conditions of strong skin effect, at distances from the discharge much shorter than the wavelength, the field is a linear function of the coordinate if the conduction current is not taken into account[3)]. The heat release, on the other hand, is proportional to $\sigma|E|^2$, does not decrease outside the discharge. Thus, the stationary state of a discharge with a transition layer that is much thicker than the wavelength of the high-frequency field does not take place in the absence of formation of negative ions.

If, however, in the equation of the ionization-recombination balance (2.5), we retain in the region outside the discharge the term describing the attachment of the electrons to the neutral atoms:

$$d^2n/d\zeta^2=\gamma_r^2n^2+\gamma_c^2n,\quad \zeta\to-\infty, \tag{2.18}$$

then we obtain for the electron concentration the expression

$$n=\frac{3}{2}\left(\frac{\gamma_c}{\gamma_r}\right)^2\operatorname{sh}^{-2}\frac{\gamma_c(\zeta-\tilde{\zeta}_0)}{2}, \tag{2.19}$$

which coincides with (2.17) if $|\zeta - \tilde{\zeta}_0| \ll 1/\gamma_c$ (so long as the recombination exceeds the attachment), and which tends exponentially to zero as $\zeta \to -\infty$. The electric conductivity of the gas and the heat release also decrease exponentially in this case with increasing distance outside the discharge. The possibility of production of negative ions thus ensures the existence of a stationary state of a discharge whose boundary has dimensions on the order of the diffusion capture length $d_{cap} \ll \lambda_0$.

3. STRONG DIFFUSION

We consider the limiting case of strong diffusion, when both the capture diffusion length and the recombination diffusion length are large in comparison with the depth of penetration of the field into the plasma at the maximum temperature, i.e., we assume in addition to (2.9) that

$$\gamma_r \ll 1. \tag{3.1}$$

The ratio

$$\lambda = \gamma_c / \gamma_r = d_{rec} / d_{cap} \tag{3.2}$$

of the recombination diffusion length to the capture diffusion length can be arbitrary. The conditions (2.9) and (3.1) make it possible to assume that temperature Θ is in the region n a slowly varying function in comparison with the electron density n.

It is convenient to seek the solution of (2.5) separately in two regions: in the external region

$$-\infty < \zeta - \zeta_0 \ll 1/\gamma_c \tag{3.3}$$

and in the internal region

$$-1/\gamma_c \ll \zeta - \zeta_0 < \infty, \tag{3.4}$$

which overlap at $|\zeta - \zeta_0| \ll 1/\gamma_c$. In the region (3.3) outside the discharge, the terms $e^{-2\Theta}$ and $e^{-\Theta}$ in (2.5) can be neglected, after which its solution can be conveniently represented in the form

$$n(\zeta - \zeta_0) = \frac{3\lambda^2}{2} \operatorname{sh}^{-2} \frac{\gamma_3(\zeta - \zeta_0 - \zeta_1)}{2}, \quad -\infty < \zeta - \zeta_0 \ll 1/\gamma_c, \tag{3.5}$$

by separating from the arbitrary constant ζ_0, which is common to all ζ, a certain phase ζ_1 for the region (3.3). In the internal region (3.4), we can put $e^{-\Theta} = e^{-2\Theta} = 1$, and the solution of (2.5) takes the form

$$n(\zeta - \zeta_0) = 1 - \frac{3}{2}\left(1 + \frac{\lambda^2}{2}\right) \operatorname{ch}^{-2} \frac{(2\gamma_r^2 + \gamma_c^2)^{1/2}(\zeta - \zeta_0 + \zeta_2)}{2}, \quad -\frac{1}{\gamma_c} \ll \zeta - \zeta_0 < \infty. \tag{3.6}$$

Here ζ_2 is the phase in the region (3.4).

In the intermediate region $|\zeta - \zeta_0| \sim 1$, the right-hand side of (2.5) can in general be neglected, from which it follows that expressions (3.5) and (3.6) should coincide at $|\zeta - \zeta_0| \sim 1$ with accuracy up to terms of order γ^2. From (3.5) we have

$$n = \frac{3}{2}\lambda^2 \operatorname{sh}^{-2} \frac{\gamma_c \zeta_1}{2} + \frac{3}{2}\lambda^2 \gamma_c (\zeta - \zeta_0) \frac{\operatorname{ch}(\gamma_c \zeta_1/2)}{\operatorname{sh}^3(\gamma_c \zeta_1/2)}, \quad |\zeta - \zeta_0| \sim 1. \tag{3.7}$$

From (3.6) we have

$$n = 1 - 3\left(1 + \frac{\lambda^2}{2}\right) \operatorname{ch}^{-2} \frac{(2\gamma_r^2 + \gamma_c^2)^{1/2}\zeta_2}{2} + 3\gamma_c(\zeta - \zeta_0) \frac{(1 + 2/\lambda)^{3/2}\lambda^2}{2} \cdot \times \operatorname{sh} \frac{(2\gamma_r^2 + \gamma_c^2)^{1/2}\zeta_2}{2} \Big/ \operatorname{ch}^3 \frac{(2\gamma_r^2 + \gamma_c^2)^{1/2}\zeta_2}{2}, \quad |\zeta - \zeta| \sim 1. \tag{3.8}$$

Putting

$$x = \operatorname{cth} \frac{\gamma_3 \zeta_1}{2}, \quad y = \operatorname{th} \frac{(2\gamma_r^2 + \gamma_c^2)^{1/2}\zeta_2}{2} \tag{3.9}$$

and comparing (3.7) and (3.8), we obtain a system of equations that determine the still unknown phases ζ_1 and ζ_2:

$$\begin{aligned} {}^3/_2\lambda^2(x^2 - 1) &= 1 - 3(1 + \lambda^2/2)(1 - y^2), \\ x(x^2 - 1) &= (1 + 2/\lambda^2)^{3/2} y (1 - y^2). \end{aligned} \tag{3.10}$$

Solving the system (3.10) relative to x and y, we obtain

$$x = \frac{3\lambda^2 + 2}{3\lambda\sqrt{1 + \lambda^2}}, \quad y = \frac{3\lambda^2 + 4}{3\sqrt{(\lambda^2 + 1)(\lambda^2 + 2)}}. \tag{3.11}$$

From (3.9) and (3.11) we obtain the phases ζ_1 and ζ_2 which enter in (3.5) and (3.6):

$$\zeta_1 = \frac{2}{\gamma_c} \operatorname{Arcth} \frac{3\lambda^2 + 2}{3\lambda\sqrt{1 + \lambda^2}}, \quad \zeta_2 = \frac{2\lambda}{\gamma_c\sqrt{\lambda^2 + 2}} \operatorname{Arth} \frac{3\lambda^2 + 4}{3\sqrt{(1 + \lambda^2)(2 + \lambda^2)}}. \tag{3.12}$$

The solution of Eq. (2.6) with slowly varying electron concentration n was obtained in[1] in the form

$$\Theta(\zeta - \zeta_0) = A \int_{\zeta - \zeta_0}^{\infty} d\zeta' \exp\left\{-\int_{-\infty}^{\zeta'} (n(\zeta''))^{1/2} d\zeta''\right\}, \quad n \gg \gamma_c^2. \tag{3.13}$$

To calculate the constants A, we integrate Eq. (2.5) from $-\infty$ to $+\infty$. Noting that $n'(-\infty) = n'(+\infty) = 0$, we obtain the relation

$$\int_{-\infty}^{+\infty} \{(n^2 - e^{-2\Theta}) + \lambda^2(n - e^{-\Theta})\} d\zeta = 0. \tag{3.14}$$

Using the explicit forms (3.5) and (3.6) of the function $n(\zeta - \zeta_0)$, we can transform (3.14) into

$$\int_{-\infty}^{\zeta_0} (e^{-2\Theta} + \lambda^2 e^{-\Theta}) d\zeta = \int_{\zeta_0}^{\infty} \{1 - e^{-2\Theta} + \lambda^2(1 - e^{-\Theta})\} d\zeta. \tag{3.15}$$

The main contribution to the integrals in (3.15) is made by the region $|\zeta - \zeta_0| \sim 1 \ll 1/\gamma_c$. Substituting (3.5) in (3.13) we obtain in this region the following expression for the dimensionless temperature Θ:

$$\Theta = A \sqrt{\frac{2}{3}} \frac{\operatorname{sh}(\gamma_c \zeta_1/2)}{\lambda} \left(\operatorname{th} \frac{\gamma_3 \zeta_1}{4}\right)^{\sqrt{6}/\gamma_p} \exp\left\{-\sqrt{\frac{3}{2}} \frac{\lambda(\zeta - \zeta_0)}{\operatorname{sh}(\gamma_c \zeta_1/2)}\right\}, \quad |\zeta - \zeta_0| \sim 1. \tag{3.16}$$

Substituting (3.16) in (3.15), using relation (3.12), and performing a number of transformations analogous to those described in[1], we obtain

$$A = \left(\frac{3\lambda^2 + 4}{6(1 + \lambda^2)}\right)^{1/2} 2^{-1/(1+\lambda^2)} \exp\left\{-C + \frac{\sqrt{6}}{\gamma_r} \operatorname{Arch} \frac{3\lambda^2 + 2}{3\lambda\sqrt{1 + \lambda^2}}\right\}. \tag{3.17}$$

Here C = 0.577... is Euler's constant.

To find the function $F(\gamma_c, \gamma_r)$ (2.14) it is necessary to find the solution of (2.6) in the region $n \lesssim \gamma_c^2$, in which the quasiclassical approximation employed above is not valid. A detailed analysis shows that the exact solution differs from the quasiclassical solution only by a pre-exponential factor. The quasiclassical approximation no longer holds in the region

$$\gamma_c |\zeta - \zeta_0 - \zeta_1| / 2 \sim \ln(1/\gamma_r^2) \gg 1,$$

in which the electron concentration (3.5) differs little from exponential. The solution of the corresponding equation does not differ from the solution of Eq. (3.19) in[1]. As the result we obtain

$$F(\gamma_c, \gamma_r) = 0.725 \cdot 2^{-1/(1+\lambda^2)} \left(\frac{3\lambda^2 + 4}{6(1 + \lambda^2)}\right)^{1/2} \cdot \times \exp\left\{-C + \frac{\sqrt{6}}{\gamma_r} \operatorname{Arch} \frac{3\lambda^2 + 2}{3\lambda(1 + \lambda^2)^{1/2}}\right\}, \quad \gamma_c \ll 1, \quad \gamma_r \ll 1, \quad \lambda \sim 1. \tag{3.18}$$

If the recombination is less effective than the capture of the electrons by the neutral atoms, i.e., $\gamma_c \gg \gamma_r$, then γ_r drops out, and (3.18) goes over into formula (3.23) of[1]:

$$F(\gamma_c, \gamma_r) = f(\gamma_c) = 0.725 \frac{e^{-C}}{\sqrt{2}} e^{\sqrt{2}/\gamma_c}, \quad \lambda \gg 1, \quad \gamma_c \ll 1. \tag{3.19}$$

In the opposite limiting case of strong recombination $\lambda \ll 1$, we obtain from (3.18)

$$F(\gamma_c, \gamma_r) = 0.725 \frac{e^{-C}}{\sqrt{6}} \left(\frac{4\gamma_r}{3\gamma_c}\right)^{\sqrt{6}/\gamma_r}, \quad \lambda \ll 1, \quad \gamma_c \ll 1. \tag{3.20}$$

Expression (3.20) depends significantly, as before, on γ_c even if $\gamma_c \ll \gamma_r$. This circumstance is brought about by the fact that the dimension of the heat-release region is of the order of the diffusion capture length.

4. STRONG RECOMBINATION

If the probability of formation of negative ions in the gas is low, and the gas pressure is sufficiently high, a situation arises wherein the recombination diffusion length d_{rec} is much smaller than the depth of penetration δ_m of the field into the plasma, which in turn is much smaller than the capture diffusion length d_{cap}:

$$d_{\rm rec} \ll \delta_m \ll d_{\rm cap}, \qquad \gamma_c \ll 1, \qquad \gamma_r \gg 1. \tag{4.1}$$

We assume that the last of the conditions (4.1) is so strong, that even $\ln \gamma_r$ can be regarded as large:

$$\ln \gamma_r \gg 1. \tag{4.2}$$

Under conditions (4.2), it follows from (2.5) that the electron density n differs little from the equilibrium value $e^{-\Theta}$ in the region $|\zeta - \zeta_0| \sim 1$. However, with increasing distance outside the discharge, when $|\zeta - \zeta_0|$ becomes of the order of $\ln \gamma_c$, the electron density n deviates noticeably from the equilibrium value, owing to the diffusion. The slow decrease of the concentration of electrons that diffuse over large distances leads (in the case of strong recombination and weak attachment) to the existence, besides the heat-release maximum in the transition layer $|\zeta - \zeta_0| \sim 1$, of one more gently-sloping maximum of the heat release at large distances $|\zeta - \zeta_0| \sim 1/\gamma_c$, on the order of the diffusion capture length.

We integrate Eq. (2.6) from $+\infty$ to values of $\zeta - \zeta_0$ such that

$$\ln \gamma_r \gg \zeta_0 - \zeta \gg 1, \tag{4.3}$$

and obtain

$$\frac{d^2}{d\zeta^2}\frac{1}{n}\frac{d^2\Theta}{d\zeta^2} = \int_{+\infty}^{\zeta} n\frac{d\Theta}{d\zeta}d\zeta \approx \int_{-\infty}^{+\infty} e^{-\Theta}\frac{d\Theta}{d\zeta}d\zeta = \int_0^{\infty} e^{-\Theta}d\Theta = 1, \tag{4.4}$$

since the difference between the electron density and the equilibrium value is negligible in the region (4.3). Integrating (4.4) from $-\infty$ to a value of ζ that satisfies (4.3), we get

$$\frac{1}{n}\frac{d^2\Theta}{d\zeta^2} = \frac{1}{2}\zeta^2 + c_1\zeta + c_2, \tag{4.5}$$

where c_1 and c_2 are constants on the order of unity and do not depend on γ_c. In the region $\zeta_0 - \zeta \ll 1$, the exponentials in (2.5) can be neglected and we can represent the electron concentration in the form (3.5). Leaving only the higher-order terms in the parameter $1/\gamma_c$, we get from (4.5)

$$\Theta'(\zeta - \zeta_0) = c_0 + \int_{\zeta^*}^{\zeta} \left(\frac{1}{2}\zeta^2 + c_1\zeta + c_2\right) n(\zeta - \zeta_0)d\zeta. \tag{4.6}$$

Choosing ζ^* in such a way that $\ln \gamma_r \gg \zeta_0 - \zeta^* \gg 1$ and noting that the solution does not differ from the equilibrium value in this region, we obtain $c_0 = -1.57$. (We have used here the fact that $\Theta'(-\infty) = -1.57$, see formula (4.16) of[5]). Substituting (3.5) in (4.6), we obtain

$$|\Theta'(-\infty)| = 1.57 + \frac{6}{\gamma_r^2\gamma_c}\int_{-\infty}^{0}\frac{x^2\,dx}{\mathrm{sh}^2 x} = 1.57 + \frac{\pi^2}{\gamma_r^2\gamma_c}$$

and thus

$$F(\gamma_c, \gamma_r) = 1.57 + \pi^2/\gamma_r^2\gamma_c, \qquad \gamma_r \gg 1, \qquad \gamma_c \ll 1. \tag{4.7}$$

Thus, at a low probability of electron attachment to neutral atoms, the limiting transition to the formulas for the equilibrium plasma occurs not simply in the case of strong recombination $\gamma_r \gg 1$, and not even at $\ln \gamma_r \gg 1$, but generally speaking under the stronger condition $\gamma_r\gamma_c^2 \gg 1$, which means that the contribution of the second maximum to the total heat release is negligible.

5. WEAK ATTACHMENT

In the case of low probability of production of negative ions $\gamma_c \ll 1$, the dependence of the discharge parameters on γ_c for any γ_r can be obtained analytically. At $\gamma_c \ll 1$ and $\gamma_r \sim 1$, the maximum of the heat release occurs at the larger distance, on the order of the diffusion capture length. In the region $|\zeta - \tilde{\zeta}_0| \ll 1/\gamma_c$, the processes connected with the negative ions are negligible and γ_c should not enter in the formulas. For electron concentration in the region $1 \ll |\zeta - \tilde{\zeta}_0| \ll 1/\gamma_c$, expression (2.7) is valid, where $\tilde{\zeta}_0$ is a suitable constant. The solution of (2.6) in this region is

$$d\Theta/d\zeta = A_0(\gamma_r)\,|\zeta - \tilde{\zeta}_0|^{\alpha(\gamma_c)}, \tag{5.1}$$

where

$$\alpha(\gamma_r) = [{}^1\!/_2 + ({}^1\!/_4 + 36/\gamma_r^4)^{1/2}]^{1/2} \tag{5.2}$$

and $A_0(\gamma_r)$ is a function of γ_r only. On the other hand, in the region of the maximum heat release $1 \ll |\zeta - \zeta_0| \sim 1/\gamma_c$, the electron density takes the form (2.19). Substituting (2.19) in (2.6) and putting $x \equiv \gamma_c(\zeta - \tilde{\zeta}_0)/2$ and $\varphi(x) \equiv d\Theta(x)/d\zeta$, we get

$$\frac{d^3}{dx^3}\mathrm{sh}^2 x\frac{d\varphi}{dx} - \frac{36}{\gamma_r^4}\mathrm{sh}^{-2}x\varphi = 0, \qquad |x| \gg \gamma_c. \tag{5.3}$$

Equation (5.3) is linear, homogeneous, and does not contain the parameter γ_c. Therefore in the entire region $|x| \gg \gamma_c$ the parameter γ_c enters in the expression for $d\Theta/d\zeta$ only in the constant factor, just as when $|x| \ll 1$. At $|x| \ll 1$ we get from (5.1)

$$d\Theta/d\zeta = A^*(\gamma_c, \gamma_r)\,|x|^{\alpha(\gamma_r)}, \qquad |x| \ll 1. \tag{5.4}$$

By virtue of the linearity and homogeneity of Eq. (5.3), its solution, which is given by (5.4) at $|x| \ll 1$, is

$$d\Theta/d\zeta = A^*(\gamma_c, \gamma_r)\,\Phi(\gamma_r, x), \qquad |x| \sim 1. \tag{5.5}$$

In particular, $d\Theta/d\zeta$ should not depend on ζ at $|x| \gg 1$, and we obtain

$$d\Theta/d\zeta = A^*(\gamma_c, \gamma_r)\,\Phi(\gamma_r), \tag{5.6}$$

where $\Phi(\gamma_r) = \Phi(\gamma_r, -\infty)$. Comparing (5.4) and (5.1), we have

$$A^*(\gamma_c, \gamma_r) = A_0(\gamma_r)\,(2/\gamma_c)^{\alpha(\gamma_r)}.$$

Thus

$$\frac{d\Theta}{d\zeta} = B(\gamma_r)\,\gamma_c^{-\alpha(\gamma_r)}, \quad \gamma_c \ll 1, \ \gamma_r \sim 1, \ \gamma_r^2\gamma_c \ll 1. \tag{5.7}$$

The functions $\Phi(\gamma_r, x)$ and $B(\gamma_r)$ can be determined only by numerical integration. In the limiting cases we obtain from (3.20) and (4.7)

$$B(\gamma_r) = \begin{cases} 0{,}725\cdot 6^{-1/4}e^{-C}({}^4\!/_C\gamma_r)^{\sqrt{6}/\gamma_r}, & \gamma_r \ll 1, \\ \pi^2/\gamma_r^2, & \gamma_r \gg 1. \end{cases}$$

The function $\alpha(\gamma_r)$ is given by formula (5.2). Ultimately we have

$$F(\gamma_c, \gamma_r) \sim \gamma_c^{-2(\gamma_r)}, \quad \gamma_c \ll 1, \ \gamma_r \sim 1. \tag{5.8}$$

We emphasize once more that γ_c does not drop out of the expressions for the discharge parameters, no matter how small it may be (of course, within the framework of the condition (2.8)). Namely, attachment of electrons to atoms with formation of negative ions determines the dimension of the heat-release region.

When the general formulas obtained above are used to calculate the parameters of a particular discharge, it is necessary to know the temperature dependence of the recombination coefficient. From among the gases that lend themselves best to experiment, the lowest probability of formation of negative ions is possessed by helium. We, however, found no reliable published data on the recombination coefficient from helium at temperatures on the order of one electron volt. The published theoretical calculations, which refine the work by Belyaev and Budker [6] or Pitaevskiĭ and Gurevich [7] have a range of applicability that is strictly limited in temperature. For this reason, the formulas obtained by us are best used at present stage to solve the inverse problem, namely determine the recombination coefficient and the attachment coefficients from the experimentally measured parameters of the gas discharge.

We note that since the probability of electron attachment to helium atoms is very low, the role of impurities of other gases, which are prone to form negative ions, becomes obviously important. In particular, the introduction of a small hydrogen impurity into helium decreases the diffusion capture length and the dimensions of the heat-release region, thus favoring the establishment of a stationary state of a discharge detached from the cold walls at a lower power. It is possible that the pinching of the discharge in helium following addition of a small hydrogen impurity [8] is also due to processes of negative-ion formation.

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1) The enhanced contraction of arc discharges following introduction of electronegative impurities was pointed out in [2,3].

2) Just as in [1], we neglect the difference between the temperature of the electrons and the temperatures of the ions and neutral atoms, as is generally the case at sufficiently high pressure.

3) In fact, when the conduction current is taken into account, the electric field outside the discharge increases even more rapidly, owing to the slow decrease of the electron density (see below).

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