Theory of Stark broadening of hydrogen lines in plasma

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A theoretical analysis is given of the hydrogen line profiles in plasma, resulting from the static effect of ions and collision effect of electrons. Parabolic quantization is used to derive general expressions for the collision half-widths of the Stark components of any hydrogen line. A systematic analysis is carried out of the transition from the case of overlap to the case of isolated Stark components. It is shown that this transition is accompanied by the cancellation of nondiagonal matrix elements of the electron collision broadening operator and a sharp differentiation of the electron collisions into elastic and inelastic. The result is that the interference term in the electron collision broadening operator is substantially reduced. The agreement between theory and experiment for lines with strong central components is improved when this phenomenon is taken into account. A rigorous mathematical procedure is used to derive new formulas for the intensity distribution in the wings of the hydrogen lines. These formulas are then used to calculate the coefficients R(N, T) for the L_α, L_γ, H_α, and H_γ lines at different plasma densities N and temperatures T. A detailed comparison is carried out with the experimental data for the L_α line and the general features of the new formulas for other lines are discussed.

1. INTRODUCTION

The Stark profiles of atomic lines in plasma are produced under the action of the low-frequency fields of ions and under the action of high-frequency fields of electrons. The effect of the low-frequency electric fields is usually treated as quasistatic, and parabolic quantization with a static ion field **F**, is introduced for the hydrogen atom. The contribution of electrons to line broadening, on the other hand, is looked upon as collision broadening, and the most important effect of distant nonadiabatic encounters is taken into account in terms of perturbation theory.[1] The hydrogen atom has so far been regarded as a typical object for the application of collision broadening theory in the case of overlapping levels, and it has been assumed that it is unimportant which particular basis is used to evaluate the matrix elements of the operator Φ for electron collision broadening. Moreover, in most published papers the evaluation of Φ is based on the use of spherical wave functions although the natural basis for such calculations is provided by the above parabolic wave functions. The overlapping of Stark components, on the other hand, will not occur if the splitting in the static ion field exceeds the collision half-width of an individual component. There is a certain critical value of the ion field ($F = F_C$) beyond which the Stark sublevels can be regarded as isolated lines, and one can then speak of elastic and inelastic electron-atom collisions in the case of transitions between these sublevels. The difference between the individual half-widths of the Stark components and the average values introduced in spherical quantization may then become important.

The aim of the present paper was to take the hydrogen atom as an example of the dynamics of the transition from the case of overlapping to the case of isolated lines as the ion field **F** is varied. As in our previous paper,[2] we shall make extensive use of the Runge-Lenz vector since this leads to a substantial simplification of the calculations. Rigorous allowance for the transition from overlapping to isolated Stark components throws considerable light on such fundamental theoretical problems as the coherence of electron contributions to line broadening, and the intensity distribution in the wings. This allows us to achieve much better agreement with experimental data than was possible without taking this transition into account (see[3,4]).

2. ELECTRON COLLISION BROADENING OPERATOR IN THE PRESENCE OF A STATIC ION FIELD

According to the generalized collision broadening theory,[3,4] the intensity distribution in a line which is emitted when an atom undergoes a transition from an upper state n to a lower state n′ is given by

$$I_{nn'}(\omega)=\frac{1}{\pi}\int dF W(F)\,\mathrm{Re}\sum_{\alpha\alpha'\beta\beta'}\langle\alpha|\mathbf{P}|\beta\rangle\langle\beta'|\mathbf{P}|\alpha'\rangle\langle\alpha|\langle\beta|[i(\omega-\hbar^{-1}\{H_n^0 - H_{n'}^0\})-\Phi_{nn'}]^{-1}|\alpha'\rangle|\beta'\rangle, \quad (1)$$

where ω is the observed frequency, H_n^0 or $H_{n'}^0$ is a Hamiltonian which includes the interaction of the atom with the electric field of the ions **F**, **P** is the dipole moment operator, $\Phi_{nn'}$ is the electron collision broadening operator for the $n \to n'$ transition, the indices $\alpha\alpha'$ and $\beta\beta'$ identify the Stark states of the upper and lower levels, and W(F) is the distribution function for the electric fields of the plasma ions.

The operator Φ is related to the mean evolution operator $T(t, t')$ for the atom over a time interval Δt by the formula

$$\Phi_{nn'}=\frac{1}{\Delta t}\exp\left[-\frac{i}{\hbar}(H_n^0-H_{n'}^0)t\right]\{T_n(t+\Delta t,t)\,T_{n'}^{*}(t+\Delta t,t)-1\}_{\mathrm{av}}\times\exp\left[\frac{i}{\hbar}(H_n^0-H_{n'}^0)t\right]. \quad (2)$$

It is assumed that Δt is much greater than the mean electron transit time $\tau \sim \rho/v$ but is less than the time between collisions (ρ is the impact parameter and v the electron velocity). This choice of Δt enables us to replace the evolution operator $T_n(t + \Delta t, t)$ for a finite interval of time by the scattering matrix $S_n(s) = T_n(+\infty, -\infty)$ corresponding to the time of closest approach, s. As a result, the operator $\Phi_{nn'}$ assumes the form

$$\langle\alpha|\langle\beta|\Phi_{nn'}|\alpha'\rangle|\beta'\rangle = \int_0^\infty Nvf(v)dv \int_0^\infty 2\pi\rho\, d\rho \int_0^\infty \mathcal{P}(s)\exp[i(\omega_{\alpha\alpha'} - \omega_{\beta\beta'})s]ds$$

$$\times \{\langle\alpha|\langle\beta|[S_n(0)S_{n'}^*(0) - 1]|\alpha'\rangle|\beta'\rangle\}_{av}. \quad (3)$$

In this expression the symbol $\{\ldots\}_{av}$ represents averaging over the angular variables, f(v) is the electron velocity distribution function, ρ is the impact parameter, N is the density of the perturbing particles, s is the time of closest approach, $\mathcal{P}(s)$ governs the probability that the collision will occur in a time interval $0 < s < \Delta t$, and the factor $\exp[i(\omega_{\alpha\alpha'} - \omega_{\beta\beta'})s]$ reflects the dependence of the scattering matrix for $t = s$ on the scattering matrix at $t = 0$:[5]

$$\langle\alpha|S_n(s)|\alpha'\rangle = \exp(i\omega_{\alpha\alpha'}s)\langle\alpha|S_n(0)|\alpha'\rangle.$$

Averaging over the closest-approach times s allows for the fact that contributions to the broadening are provided not only by collisions for which s lies in the interval Δt, but also for other values of s.[6-8] This is not usually carried out explicitly, but it is simply assumed that $\mathcal{P}(s) = 1/\Delta t$ and the integral with respect to s is evaluated only over the time Δt. It will be seen below, however, that averaging over s with an adequately defined distribution $\mathcal{P}(s)$ is quite important for the understanding of the transition from the case of overlapping to that of isolated lines.

It is clear from Eq. (3) that the evaluation of $\Phi_{nn'}$ reduces to the determination of the average of $S_n(0)S_{n'}^*(0) - 1$. The simultaneous perturbation of the upper (n) and lower (n') states can be deduced relatively simply from the corresponding results for a single level. We shall therefore begin with the operator $S_n(0) - 1$ which describes the perturbation of only the upper level of the hydrogen atom in the external static ion field **F**. The field **F** removes the degeneracy of the hydrogen sublevels, and the operator $S_n(0) - 1$ can be written out in a form analogous to the collision broadening operator in the case of a nondegenerate system:[5,9]

$$\{\langle i|S_n(0) - 1|l\rangle\}_{av} = -\frac{3}{4}n^2\left(\frac{\hbar}{mv\rho}\right)^2\left(\frac{1}{\hbar^2}\sum_{\sigma,l'}\langle i|M_\sigma|l'\rangle\langle l'|M_\sigma|l\rangle \times [A(z_1, z_2) + iB(z_1, z_2)]\right), \quad (4)$$

$$A + iB = \frac{1}{2}\int_{-\infty}^{+\infty}dx_1\int_{-\infty}^{x_1}dx_2\exp[i(z_1x_1 - z_2x_2)]\frac{1 + x_1x_2}{(1 + x_1^2)^{3/2}(1 + x_2^2)^{3/2}}. \quad (5)$$

In these expressions, instead of the vector $\mathbf{P} = -e\mathbf{r}$ we use the Runge-Lenz vector:

$$\mathbf{M} = \left(\frac{m}{-2E_n}\right)^{1/2}\left\{\frac{[\mathbf{p}\times\mathbf{L}] - [\mathbf{L}\times\mathbf{p}]}{2m} - e^2\frac{\mathbf{r}}{r}\right\}$$

where m, e are the mass and charge of an electron, r, p, and L are, respectively, the coordinate, momentum, and angular momentum operators, E_n is the hydrogen level energy corresponding to the principal quantum number n, $z_1 = \rho\omega_{il'}/v$, $z_2 = \rho\omega_{il'}/v$, the indices i, l, and l' represent the parabolic states in the ion field **F**, and $\omega_{il'}$, $\omega_{ll'}$ are the frequencies corresponding to the Stark splitting of the levels il' and ll' and are given by

$$\omega_{ik} = \frac{3}{2}\frac{nea_0}{\hbar}F[(n_1^{(i)} - n_2^{(i)}) - (n_1^{(k)} - n_2^{(k)})].$$

The operator $\langle i|M_\sigma|l'\rangle$ has nonzero matrix elements only for transitions between neighboring Stark sublevels [see Eq. (12) below].[1)] Since the Stark sublevels are equidistant, $z_1 = z_2 = z$ for the diagonal matrix element ($i = l$) and $z_1 = -z_2 = z$ for the nondiagonal matrix elements ($i \neq l$). As a result, the functions A and B assume the form[9]

$$A(z, \pm z) \equiv A_\pm(z) = z^2[K_1^2(z) \pm K_0^2(z)], \quad (6)$$

$$B(z, \pm z) \equiv B_\pm(z) = \frac{2|z|}{\pi}\operatorname{sgn}\omega_{il'}\cdot\int_0^\infty\frac{A_\pm(z')}{z^2 - z'^2}dz', \quad (7)$$

where K_1 and K_0 are Macdonald functions and the signs $\pm$ correspond to the above cases of diagonal and nondiagonal matrix elements, while the crossed integral sign represents the principal value of the corresponding integral.

To obtain the final form of the matrix elements of the operator Φ_n we must integrate Eq. (4) with respect to ρ. Since only the functions A and B depend on ρ, we have

$$a_\pm(z_{min}; z_{max}) = \int_{|z_{min}|}^{|z_{max}|}\frac{A_\pm(z)}{z}dz, \quad (8)$$

$$b_\pm(z_{min}; z_{max}) = \int_{|z_{min}|}^{|z_{max}|}\frac{B_\pm(z)}{z}dz. \quad (9)$$

The parameter z_{min} corresponds to the cutting off of the integrals in Eqs. (8) and (9) at a certain impact parameter value ρ_{min} at which the validity of the perturbation theory is violated. It is clear that ρ_{min} is of the order of the Weisskopf radius $\rho_0 \sim n^2\hbar/mv$ and, therefore, the effective value is

$$(z_{min})_{eff} \sim \frac{\rho_{min}}{v}(|\omega_{il}|)_{eff}$$

$$\sim n^2\frac{\hbar}{mv^2}\frac{3}{2}\frac{ne^2a_0}{\hbar}N^{1/3} \ll 1.$$

As $z_{min} \to 0$, the integral in Eq. (8) diverges logarithmically at the lower limit. As regards the upper limit z_{max}, the introduction of this quantity is not connected with the divergence of the functions a(z) and b(z) for $z \to \infty$. On the other hand, the integrand in Eq. (8) decreases exponentially as $z \to \infty$, so that the region $z' \sim z_{min} \ll 1$ turns out to be the important one. Since in this region $A_\pm(z') \approx 1$, we can readily verify that

$$a_\pm(z_{min}; \infty) = \int_{|z_{min}|}^{\infty}\frac{A_\pm(z)}{z}dz \approx -\ln|z_{min}| = \ln\left(\frac{\rho_{max}}{\rho_{min}}\right), \quad (10)$$

where $\rho_{max} = 2v\hbar F/3nea_0$ and, consequently, $a_\pm(z_{min}; \infty)$ depends parametrically on the ion field **F**.

A similar analysis can be given for the function $b_\pm(z)$. In the integral in Eq. (9) we must then set $z_{min} \approx 0$ because as $z \to 0$ the integral remains finite:

$$b_+(0) = b_-(0) = {}^1/_2\pi\operatorname{sgn}\omega_{il'} \equiv b(z_{min}; \infty). \quad (11)$$

This result is obtained by the limiting transition $z_{min} \to 0$ described in[5]. The result of this is that in the expression given by Eq. (9) the important values of $A_\pm(z')$ are those at $z' = 0$ if we take Eq. (7) into account.[5] However, according to Eq. (6), $A_+(0) = A_-(0) = 1$ and this leads to Eq. (11). An important feature of

Eq. (11) is that $b_{\pm}(0)$ depends on the sign of $\omega_{il'}$. This must be taken into account when the sum in Eq. (4) is evaluated.[2)]

To evaluate the matrix of the operator Φ_n all that remains is to evaluate the sum of the matrix elements M_σ in Eq. (4) with the functions $a(z)$ and $b(z)$ given by Eqs. (10) and (11). The function $a(|z_{min}|)$ does not, in fact, depend on the summation indices because z_{min} is determined by the equidistant splitting of the Stark sublevels. The dependence on the summation indices in $b(z_{min})$ is determined only by sgn $\omega_{il'}$. As already noted, when these facts are taken into account, and the explicit form is used for the operator $\mathbf{M}\cdot\mathbf{M}$, it is readily verified that in the case of parabolic quantization[11] the diagonal matrix elements

$$\langle n_1 n_2 m|\Phi_n|n_1 n_2 m\rangle = -\langle n_1 n_2 m|w_n|n_1 n_2 m\rangle + i\langle n_1 n_2 m|d_n|n_1 n_2 m\rangle)$$

may be written in the form

$$\langle n_1 n_2 m|w_n|n_1 n_2 m\rangle = \frac{3}{2}\pi\left(\frac{\hbar}{m}\right)^2 N_e\left(\frac{2m}{\pi kT_e}\right)^{1/2}$$

$$\times n^2[n^2+(n_1-n_2)^2-|m|^2-1]\left\{\ln\left(\frac{\rho_{max}}{\rho_{min}(\langle v\rangle)}\right)+0{,}215\right\}, \quad (12)$$

$$\langle n_1 n_2 m|d_n|n_1 n_2 m\rangle = \frac{3}{2}\pi^2\left(\frac{\hbar}{m}\right)^2 N_e\left(\frac{2m}{\pi kT_e}\right)^{1/2} n(n_1-n_2), \quad (13)$$

where T_e and N_e are the electron temperature and density, $(\langle v\rangle)^2 = 2kT_e/m$, and the second term in the curly brackets in Eq. (12) corresponds to strong collisions.[3)]

For the nondiagonal matrix elements in the case of complete overlap of the components $|\omega_{\alpha\alpha'}|\Delta t \ll 1$ we have

$$\langle n_1-1, n_2+1, m|\Phi_n|n_1, n_2, m\rangle = -3\pi\left(\frac{\hbar}{m}\right)^2 N_e\left(\frac{2m}{\pi kT_e}\right)^{1/2}$$

$$\times n^2[n_1(n-n_1)(n_2+1)(n-n_2-1)]^{1/2}\left\{\ln\left(\frac{\rho_{max}}{\rho_{min}(\langle v\rangle)}\right)+0.215\right\}, \quad (14)$$

$$\langle n_1+1, n_2-1, m|\Phi_n|n_1, n_2, m\rangle = -3\pi\left(\frac{\hbar}{m}\right)^2 N_e\left(\frac{2m}{\pi kT_e}\right)^{1/2}$$

$$\times n^2[n_2(n-n_2)(n_1+1)(n-n_1-1)]^{1/2}\left\{\ln\left(\frac{\rho_{max}}{\rho_{min}(\langle v\rangle)}\right)+0.215\right\}. \quad (15)$$

As the ion field F increases, the quantities $\omega_{\alpha\alpha'}$ increase also, so that the condition for the validity of the theory of isolated lines becomes valid, i.e.,

$$|\omega_{\alpha\alpha'}|\Delta t \sim |\omega_{\alpha\alpha'}|/w_n \gg 1. \quad (16)$$

It is well known[7,8] that the nondiagonal matrix elements should, in this case, be of a higher order of small quantities than the diagonal terms, and can therefore be neglected. It is readily verified that when distributions of the form $\mathcal{P}(s) = e^{-s/\Delta t}/\Delta t$ are used, averaging over the times of closest approach leads to the appearance in Eq. (3) of the characteristic dispersion factor

$$\mathrm{Re}\int_0^\infty \mathcal{P}(s)\exp\{i(\omega_{\alpha\alpha'}-\omega_{\beta\beta'})s\}ds = [(\omega_{\alpha\alpha'}-\omega_{\beta\beta'})^2\Delta t^2+1]^{-1}, \quad (17)$$

which automatically takes into account only those states which lie on the isoenergetic surface: $|\omega_{\alpha\alpha'}-\omega_{\beta\beta'}|\Delta t \sim |\omega_{\alpha\alpha'}-\omega_{\beta\beta'}|/w \ll 1$. As regards the diagonal ($\alpha=\alpha'$, $\beta=\beta'$) matrix elements of Φ_n, the averaging over s clearly has no effect on their magnitude because $\mathcal{P}(s)$ is normalized. Therefore, in the region in which the Stark components are isolated [Eq. (16)] we can use the broadening theory of isolated lines for the hydrogen sublevels and neglect the nondiagonal elements of Φ_n in comparison with the diagonal elements, since the former are small quantities of order $(\omega_n/\omega_{\alpha\alpha'})^2$.

Equations (12) and (13) determine the width and shift of the Stark components of the Lyman lines if all the electron encounters can be treated as binary and we assume in Eqs. (8) and (9) that $z_{max} = \infty$. This treatment is valid when the effective value of ρ_{max} in the resulting values of $a(z)$ and $b(z)$ is of the order of the mean distance between the particles $\rho_0 = N^{-1/3}$ and is at least smaller than the Debye length $\rho_D = v_{Te}/\omega_{pe}$. It is readily verified, however, that for the important region of values of the ion field $F \sim F_0$ the effective value is

$$\rho_{max} \sim \frac{2}{3}\frac{v_{Te}\hbar}{nea_0}N^{-1/3} \gg \rho_D.$$

In other words, when the integral with respect to ρ is extended to $\rho = \infty$, distant encounters for which correlation effects cannot be neglected must be taken into account. The integration with respect to ρ must therefore be cut off at $\rho = \rho_0 \sim N^{-1/3}$ or $\rho = \rho_D$, and more distant encounters must be looked upon as the effects of plasma oscillations. For equilibrium plasma this last effect is small[12] and, therefore, the half-widths of the Stark components are adequately described by Eq. (12), in which the argument of the logarithm is the ratio ρ_{max}/ρ_{min} which is independent of the ion field (ρ_{max} is determined exclusively by the mean distance between the particles or by the Debye length).

It is readily verified that in the case of the shift of the Stark component, on the other hand, the introduction into Eq. (9) of the upper limit

$$z_{max} \approx \frac{\rho_D}{v_{Te}}\frac{3}{2}\frac{nea_0}{\hbar}F \ll 1$$

reduces d_n by a factor of about $(n^2a_0N^{1/3})^{1/2}$ so that in the dipole approximation we have, in fact, $d_n \approx 0$. It is important to note, however, that this reduction in the shift is not produced by the symmetry properties of $B(z, \pm z)$, as indicated in[9], but is wholly determined by the Debye screening of the charges.

The transition from the case of overlap to the case of isolated Stark components enables us to introduce a definite differentiation of the electron collisions into elastic and inelastic with respect to the transitions between these levels: in the parabolic quantization all collisions which change only the electric-field component in the direction of the ion field F can be regarded as elastic. This is particularly important for the simultaneous broadening of both the upper and lower levels which are responsible for the line because the contributions of inelastic collisions due to the upper and lower states combine incoherently in the final broadening, and the coherent law of addition is valid only for elastic collisions.[3,6] The electron collision broadening operator Φ_n, evaluated with this fact taken into account, allows for a continuous limiting transition to the general quantum-mechanical formulas (in contrast to that used in[1]), and can be written in the form

$$\langle\alpha|\langle\beta|\Phi_{nn'}|\alpha'\rangle|\beta'\rangle = -\delta_{\beta\beta'}\langle\alpha|w_n|\alpha'\rangle - \delta_{\alpha\alpha'}\langle\beta'|w_{n'}|\beta\rangle$$
$$+ 6\pi\left(\frac{\hbar}{m}\right)^2 N_e\left(\frac{2m}{\pi kT_e}\right)^{1/2} nn'(\hbar^{-2}\langle\alpha|M_z|\alpha'\rangle\langle\beta'|M_z|\beta\rangle)$$
$$\times\left\{\ln\left(\frac{\rho_{max}}{\rho_{min}(\langle v\rangle)}\right) + 0{,}215\right\}, \qquad (18)$$

$$\langle\alpha|\langle\beta|w_{nn'}|\alpha\rangle|\beta\rangle = \langle\alpha|w_n|\alpha\rangle + \langle\beta|w_{n'}|\beta\rangle$$
$$- 6\pi\left(\frac{\hbar}{m}\right)^2 N_e\left(\frac{2m}{\pi kT_e}\right)^{1/2} nn'(\hbar^{-2}\langle\alpha|M_z|\alpha\rangle\langle\beta|M_z|\beta\rangle)\cdot$$
$$\times\left\{\ln\left(\frac{\rho_{max}}{\rho_{min}(\langle v\rangle)}\right) + 0{,}215\right\}, \qquad (19)$$

$$\hbar^{-2}\langle\alpha|M_z|\alpha\rangle\langle\beta|M_z|\beta\rangle = [(n_2^{(\alpha)} - n_1^{(\alpha)})(n_2'^{(\beta)} - n_1'^{(\beta)})].$$

The quantities w_n in these expressions are given by Eq. (12), and the third term in Eq. (19) describes the interference effect associated with elastic scattering.

The nondiagonal matrix elements of $\Phi_{nn'}$ are important only for transitions between the sublevels of the upper (n) and lower (n') states and, as before, are determined by Eqs. (14) and (15) in the overlap region. It follows from Eq. (19) that the operator splits into matrix blocks which are diagonal in the quantum numbers m and m'. The diagonalization of $\Phi_{nn'}$ within each such block presents little difficulty in the case of the weakly excited lines (L_α and L_β). As n increases, however, the order of the matrix increases and the problem becomes more complicated. When parabolic quantization is used in the operator Φ_n, we have the formal differentiation of the electron collisions into elastic and inelastic even in the region of overlap of the component, where, in general, this differentiation is physically somewhat dubious. The overlap region, however, provides a relatively unimportant contribution to the line profile, and there is little point in introducing more exact values for the half-widths of the individual components in this region than are provided by Eqs. (12) and (19).

When the additional smallness of the nondiagonal matrix elements is taken into account, this produces an important modification of the expressions used in the next averaging over the microfields due to the plasma ions. Thus, for example, in the case of the L_α line and a fixed field $F > 2w\hbar/3nea_0$ the intensity can be written as the sum of the central intensity I_0 and the intensities I_1 and I_2 of two side components. The profile $I_0(\omega)$ has the Lorentz shape, while without this additional smallness of the nondiagonal matrix elements the profiles I_1 and I_2 never lead to the Lorentz shape and are given by[13,14]

$$I_1(\omega) + I_2(\omega) = \frac{1}{\pi}\left[\frac{w + \beta(\omega - \omega_0 + \Omega)/\Omega}{(\omega - \omega_0 + \Omega)^2 + w^2} + \frac{w - \beta(\omega - \omega_0 - \Omega)/\Omega}{(\omega - \omega_0 - \Omega)^2 + w^2}\right]$$
$$\beta = -(\Phi_2)_{21}, \quad w = -(\Phi_2)_{22} = -(\Phi_2)_{11},$$
$$\Omega = (\Delta^2/4 - \beta^2)^{1/2}, \quad \Delta = 6ea_0F/\hbar. \qquad (20)$$

On the other hand, when the factor $[(\omega_{\alpha\alpha'}\Delta t)^2 + 1]^{-1}$ is taken into account it is possible to neglect β in comparison with w when $\Delta/2 \gg \omega$, so that the intensity distribution of the side components takes the form of the sum of two Lorentz profiles. The situation in the case of the other lines is quite similar.

Let us now consider how the isolated nature of the Stark components affects the half-width of lines with strong central components (H_α, H_γ, etc.). The half-widths of such lines are largely governed by collision broadening of the corresponding central components, whilst the side components form a broad quasistatic "base" of the line well away from the center. Moreover, in the usual spherical quantization one uses not the individual half-widths of the Stark components but only a certain average collision half-width given by[15]

$$w = \frac{4\pi}{3}\left(\frac{\hbar}{m}\right)^3 N\left(\frac{2m}{\pi kT_e}\right)^{1/2} I(n,n')\ln\left(\frac{\rho_{max}}{\rho_{min}}\right); \qquad (21)$$

$$I(n,n') = a_0^{-2}G\sum_{aa',bb'}\mathbf{P}_{a'b'}\mathbf{P}_{ba}\left(\delta_{bb'}\sum_{a''}\mathbf{r}_{aa''}\mathbf{r}_{a''a'}\right.$$
$$\left. + \delta_{aa'}\sum_{b''}\mathbf{r}_{b'b''}\mathbf{r}_{b''b} - 2\mathbf{r}_{aa'}\mathbf{r}_{b'b}\right); \qquad (22)$$

$$G = \left(\sum_{ab}|\mathbf{P}_{ab}|^2\right)^{-1}. \qquad (23)$$

The indices aa' and bb' identify spherical quantum numbers of the levels n and n'. The expressions given by Eqs. (21)–(23) do not take into account the effect of the ion field and refer, generally speaking, only to the region in which the components merge ($3nea_0F/2\hbar \lesssim w$).

When the ion field is taken into account, the Stark side components are "expelled" from the overlap region to distances $\Delta/2 \gg w$ for fields $F \sim F_0 \gg 2\hbar w/3nea_0$ which are important in the integrals. In such fields the Stark components behave as if they were isolated lines, so that the side components have practically no effect on the half-width of the line as a whole, which is now determined by the collision broadening of only the central components. The half-widths w_c of the central components are given by

$$w_c = \sum_{\alpha\beta}|\mathbf{P}_{\alpha\beta}|^2 w_{\alpha\alpha,\beta\beta}\Big/\sum_{\alpha\beta}|\mathbf{P}_{\alpha\beta}|^2, \qquad (24)$$

where the summation over the parabolic quantum numbers α, β is taken over the transitions responsible for the formation of the central components, and $w_{\alpha\alpha,\beta\beta}$ is determined from Eq. (19).

Let us now compare the half-widths of H_α calculated from Eq. (21), taken from[15], and calculated from Eq. (24). When the perturbation of the lower level is neglected we have $w/w_c = 56/47$. This narrowing of the lines is connected with the "individualization" of the half-width of the central component in the case of parabolic quantization in the ion field. When the perturbation of both levels is taken into account, we have the opposite situation: $w/w_c = 27/56$. The broadening of the line in this case is explained by the absence of interference terms in the half-widths of the central components.[4)] For the $H\gamma$ line the estimated role of the interference terms shows that the true line half-width w_c should be greater than that calculated from the usual theory[4] by about 30–40%. It is precisely this effect that explains the discrepancy between the measured $H\gamma$ profiles[16,19] and the calculated values.

3. ASYMPTOTIC INTENSITY DISTRIBUTION IN THE HYDROGEN LINE WINGS

When the contributions of inelastic collisions are combined incoherently the collision half-widths become

substantially greater than was usually assumed in calculations,[4] and the substitution of these half-widths into the asymptotic Griem formulas[20] should lead to a still greater discrepancy between the calculated intensity distribution in the line wings and the experimental values than was previously established.[21-24] If at the same time we recall that the electron broadening probably does not go over from the collision limit to the quasistatic value,[25] the discrepancy between theory and experiment becomes greater still. Hence, the problem arises as to whether the mathematical procedure used by many workers[4,20,24] to obtain the asymptotic intensity distribution is, in fact, valid.

In actual fact, the overlap region in which the individual states cannot be distinguished within the line profile was not taken into account in[4,40,24]. Mathematically this means that the integration cannot be extended to $F = 0$. Moreover, the order of integration was changed in the improper integrals in[4,20,24] and these are not uniformly convergent. We shall now consider these points with the necessary degree of mathematical rigor, and derive a new formula for the asymptotic intensity distribution in hydrogen line wings.

We shall be interested in the intensity distribution well away from the line center

$$|\omega-\omega_0| \gg |\langle\alpha|\langle\beta|\Phi_{nn'}|\alpha\rangle|\beta\rangle| \equiv w_{nn'}.$$

The integrand in Eq. (1) behaves in a different way, depending on the ion field $\mathbf{F}$. When

$$|\mathbf{F}| \leqslant \frac{2}{3}\frac{\hbar}{nea_0}|\Phi_{nn'}| = \frac{2}{3}\frac{\hbar w_{nn'}}{nea_0}$$

the Stark components merge into a single unshifted line with an effective half-width $w_{eff}(F)$. On the other hand, when $2\hbar w/3nea_0 < F < \infty$, the Stark components behave as if they were isolated lines with their own individual collision half-widths $w_{\alpha\beta}$ and practically zero collision shifts. The nondiagonal matrix elements of the operator $\Phi_{nn'}$ in this region behave as quantities of a higher order of smallness in the parameter $(w_{\alpha\beta}/\omega_F) \ll 1$ and can therefore be neglected, $\omega_F \equiv 3nea_0F/2\hbar$.

Bearing all this in mind, we can divide the integral with respect to F in Eq. (1) into three regions

$$1)\ -\infty < F < -F_c;\quad 2)\ -F_c \leqslant F \leqslant F_c;\quad 3)\ F_c < F < \infty$$

and represent the profile of the line of Eq. (1) in the form

$$I_{nn'}(\omega) = \frac{1}{\pi}\Big[\int_{-F_c}^{F_c} W(F)\,dF\,\mathrm{Re}\sum_{\alpha\alpha'\beta\beta'}\langle\alpha|\mathbf{P}|\beta\rangle\langle\beta'|\mathbf{P}|\alpha'\rangle \times\langle\alpha|\langle\beta|\Big[i\Big(\omega-\frac{1}{\hbar}\{H_n^0-H_{n'}^0\}\Big)-\Phi_{nn'}\Big]^{-1}|\alpha'\rangle|\beta'\rangle + \Big\{\int_{-\infty}^{-F_c}+\int_{F_c}^{\infty}\Big\}W(F)\,dF\sum_{\alpha\beta}|\langle\alpha|\mathbf{P}|\beta\rangle|^2 \times\Big(\frac{w_{\alpha\beta}}{(\omega-\omega_0-C_{\alpha\beta}F)^2+w_{\alpha\beta}^2}+\frac{w_{\alpha\beta}}{(\omega-\omega_0+C_{\alpha\beta}F)^2+w_{\alpha\beta}^2}\Big) + \sum_{\alpha_0\beta_0}|\langle\alpha_0|\mathbf{P}|\beta_0\rangle|^2\frac{w_{\alpha_0\beta_0}}{(\omega-\omega_0)^2+w_{\alpha_0\beta_0}^2}\Big]. \qquad (25)$$

In these expressions $\omega_0 \equiv \hbar^{-1}\{H_n^0(0) - H_{n'}^0(0)\}$ is the unshifted frequency of the $n \to n'$ transition, the indices $\alpha\alpha'$, $\beta\beta'$ identify the side Stark components and α_0, β_0 the central Stark components, $w_{\alpha\beta} \equiv \mathrm{Re}(\Phi_{nn'})_{\alpha\alpha'\beta\beta}$,

$$C_{\alpha\beta} \equiv \frac{3}{2}\frac{ea_0}{\hbar}[n(n_1^{(\alpha)}-n_2^{(\alpha)})-n'(n_1^{'(\beta)}-n_2^{'(\beta)})];$$

and n_1, n_2 are the parabolic quantum numbers.

The first term on the right-hand side of Eq. (25) describes the contribution of the overlap region to the line profile of the side components. Since we are interested in the case where $|\omega - \omega_0| \gg w_{\alpha\beta}$, this contribution can be written in the form $(\Delta\omega = \omega - \omega_0)$

$$\int_{-F_c}^{F_c} W(F)\,dF\Big[\mathrm{Re}\sum_{\alpha\alpha',\beta\beta'}\langle\alpha|\mathbf{P}|\beta\rangle\langle\beta'|\mathbf{P}|\alpha'\rangle \times\langle\alpha|\langle\beta|\Big[i\Big(\omega-\frac{1}{\hbar}\{H_n^0-H_{n'}^0\}\Big)-\Phi_{nn'}\Big]^{-1} \times|\alpha'\rangle|\beta'\rangle\Big] \leqslant \mathrm{Re}\sum_{\alpha\alpha'\beta\beta'}\frac{\langle\alpha|\mathbf{P}|\beta\rangle\langle\beta'|\mathbf{P}|\alpha'\rangle}{\Delta\omega^2}(-\Phi_{nn'})_{\alpha\alpha',\beta\beta'} \times\int_{-F_c}^{F_c}W(F)\,dF < \frac{w_{\mathrm{eff}}(0)}{\Delta\omega^2}\sum_{\alpha\beta}|\langle\alpha|\mathbf{P}|\beta\rangle|^2\int_{-F_c}^{F_c}W(F)\,dF. \qquad (26)$$

In deriving this equation we used the fact that, when $\Delta\omega \gg w$, all the terms which depend on the field cancel out.[14]

Moreover, it is important to note that the exact diagonalization of the resolvent

$$[i(\omega-\hbar^{-1}\{H_n^0-H_{n'}^0\})-\Phi_{nn'}]^{-1}$$

leads to the narrowing of the line and to a reduction in w_{eff}, since the inequality sign in Eq. (26) is fully justified. Since the critical value of the ion field F_c is always much less than the "normal" field $F_0 = 2.6eN^{2/3}$, it follows that if we use the Holtzmark distribution[26] for $W(F)$ we can readily verify that the contribution of the side component from the overlap region is negligible because, when $F_c \ll F_0$,

$$\int_{-F_c}^{F_c} W(F)\,dF \approx \frac{4}{9\pi}\Big(\frac{F_c}{F_0}\Big)^3 \ll 1.$$

The second term in Eq. (25) describes the contribution of the side Stark components outside the overlap region. In the sum over α, β we have isolated the pairs of symmetric Stark side components which represent isolated lines with the dispersion profile. Finally, the third term in Eq. (25) represents the contribution of the central components to the total line profile. Here the integration with respect to F is readily carried out because the central components are not shifted in the ion field.

To obtain the asymptotic distribution in the wing, let us consider in greater detail the second term in Eq. (25). Substituting $\beta = F/F_0$ and writing out a typical term in the sum over α, β, we obtain

$$i(\Delta\omega) = \Big\{\int_{-\infty}^{-\beta_c}+\int_{\beta_c}^{\infty}\Big\}W(\beta)\,d\beta\Big[\frac{w}{(\Delta\omega-C\beta)^2+w^2}+\frac{w}{(\Delta\omega+C\beta)^2+w^2}\Big]; \qquad \beta_c \equiv F_c/F_0,\quad C \equiv C_{\alpha\beta}F_0,\quad w \equiv w_{\alpha\beta}. \qquad (27)$$

We shall take the distribution $W(\beta)$ of the electric microfields in the form of the Holtzmark function[26]

$$W(\beta) = \frac{\beta}{\pi}\int_{-\infty}^{+\infty}\sin\beta\eta\,\exp(-\eta^{3/2})\,\eta\,d\eta. \qquad (28)$$

The integral $i(\Delta\omega)$ can be written as a series in the reciprocal powers of $\Delta\omega$ if the function $W(\beta)$ is expanded in powers of the reciprocal of β and the convergent improper integrals which appear as a result of this procedure are evaluated. Let us therefore substitute the following expression into the right-hand side of Eq. (28):

$$\exp(-\eta^{3/2}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \eta^{3n/2}.$$

The integrals which appear as a result of this are defined merely as

$$\lim_{a^+ \to 0} \int_0^{\infty} e^{-a\eta} \eta^{(3n+2)/2} \sin \beta\eta \, d\eta;$$

They are essentially the Fourier transforms of the generalized function $\eta^{(3n+2)/2} \operatorname{sgn} \eta$:[2π]

$$\int_0^{\infty} \eta^{(3n+2)/2} \sin \beta\eta \, d\eta = \Gamma\left(\frac{3n+4}{2}\right) \cos\left(\frac{3n+2}{4}\pi\right) |\beta|^{-(3n+4)/2} \operatorname{sgn} \beta. \quad (29)$$

Using this relation we have

$$i(\Delta\omega) = \frac{1}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \Gamma\left(\frac{3n+4}{2}\right) \cos\left(\frac{3n+2}{4}\pi\right) \left\{ \int_{-\infty}^{-\beta_c} + \int_{\beta_c}^{\infty} \right\} d\beta$$
$$\times |\beta|^{-(3n+2)/2} \left[\frac{w}{(\Delta\omega - C\beta)^2 + w^2} + \frac{w}{(\Delta\omega + C\beta)^2 + w^2} \right]. \quad (30)$$

Let us now consider separately the case of integral odd, integral even, and half-integral exponents of $|\beta|$ in the integrand in Eq. (30). The integral odd exponents correspond to $n = 4m$ ($m = 0, 1, 2, \ldots$) for which

$$\frac{3n+2}{2} = 6m+1 \text{ and } \cos\left(\frac{3m+1}{2}\pi\right) = 0.$$

Therefore, the contribution of integral odd exponents to $i(\Delta\omega)$ is zero. The even exponent of $|\beta|$ corresponds to $n = 4m + 2$ for which $(3n + 2)/2 = 6m + 4$. For even exponents we can remove the modulus sign in Eq. (7) and consider the integral of the following complex quantity over a closed contour:

$$f_1(z) = z^{-6m+1} \left[\left(z - \frac{\Delta\omega + iw}{C} \right)^{-1} - \left(z + \frac{\Delta\omega + iw}{C} \right)^{-1} \right]. \quad (31)$$

We shall take this contour in the form of a large semicircle C_R in the upper half plane, the segments $[-R, -\beta_c]$ and $[\beta_c, R]$ along the real axis, and the semicircle C_{β_c} of radius β_c drawn around the origin (Fig. 1). It is readily verified that the integral of $f_1(z)$ over C_{β_c} is zero. As $R \to \infty$ the integral over C_R will also vanish so that the integral of $f_1(z)$ in the interval $(-\infty, -\beta_c)$, (β_c, ∞) along the real axis is expressed in terms of the sum of the residues of this function (to be specific, we shall assume that $\Delta\omega > 0$). This enables us to write the contribution $i_r(\Delta\omega)$ of the even terms to $i(\Delta\omega)$ in the following form:

$$i_r(\Delta\omega) = \frac{1}{\pi} \sum_{m=0}^{\infty} \frac{\Gamma(6m+5)}{(4m+2)!} \cos[(3m+2)\pi] \operatorname{Im} \frac{1}{C} \left\{ \int_{-\infty}^{-\beta_c} + \int_{\beta_c}^{\infty} \right\} d\beta$$
$$\times \beta^{-(6m+4)} \left[\left(\beta - \frac{\Delta\omega + iw}{C} \right)^{-1} - \left(\beta + \frac{\Delta\omega + iw}{C} \right)^{-1} \right]$$
$$= \frac{2}{C} \operatorname{Re} \sum_{m=0}^{\infty} \frac{\Gamma(6m+5)}{(4m+2)!} \cos[(3m+2)\pi] \left(\frac{C}{\Delta\omega + iw} \right)^{6m+4}. \quad (32)$$

The half-integral powers of $|\beta|$ correspond to $n = 2m + 1$ for which $3n + 2 = 6m + 5$. For such terms we must take into account the branching of the integrand at the origin. We shall therefore cut the complex plane of β as follows. To evaluate the integral for the negative values of $\beta(-\infty, -\beta_c)$ we shall take the cut along the real axis between $\beta = -\infty$ and $\beta = 0$. The integration contour C^- is shown in Fig. 2a. It consists of two concentric circles C_R^- and $C_{\beta_c}^-$ with centers at the origin and two segments $[-R, -\beta_c]$ joining these circles which lie on either side of the cut. Consider the integral over this contour of the complex function

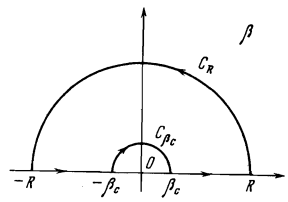

FIG. 1.

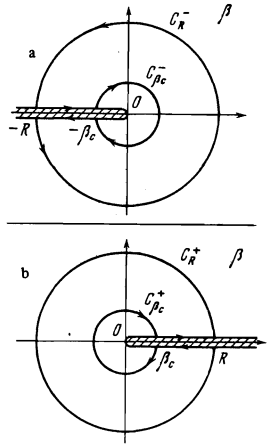

FIG. 2.

FIG. 1. Integration contour C for the evaluation of the contribution of even powers of $|\beta|$ to $i(\Delta\omega)$.

FIG. 2. Integration contours C^-(a) and C^+ (b) for the evaluation of the contribution of half-integral powers of $|\beta|$ to $i(\Delta\omega)$

$$f^-(z) = \exp\left\{ -\frac{i\pi(6m+5)}{2} \right\} z^{-(6m+5)/2}$$
$$\times \left[\left(z - \frac{\Delta\omega + iw}{C} \right)^{-1} - \left(z + \frac{\Delta\omega + iw}{C} \right)^{-1} \right]. \quad (33)$$

Along the segment $[-R, -\beta_c]$ the imaginary part of this function is identical with the integrand in $i(\Delta\omega)$ for the half-integral powers. Because of branching at $z = 0$ the integrals of $f^-(z)$ over the upper and lower edges of the cut are equal, so that as $R \to \infty$ the doubled value of the required integral for half-integral powers of $|\beta|$ in the interval $(-\infty, -\beta_c)$ is expressed in terms of the integral of $f^-(z)$ over the circles C_R^- and $C_{\beta_c}^-$ and the sum of the residues.

To evaluate the integrals for positive values of β, let us cut the complex plane along the real axis between $\beta = 0$ and $\beta = \infty$, and take the contour of integration C^+ in the form of the mirror reflection of C^- (Fig. 2b). Consider the integral of the following complex function over this contour:

$$f^+(z) = z^{-(6m+5)/2} \left[\left(z - \frac{\Delta\omega + iw}{C} \right)^{-1} - \left(z + \frac{\Delta\omega + iw}{C} \right)^{-1} \right]. \quad (34)$$

As in the previous case, the integrals of $f^+(z)$ over the upper and lower edges of the cut yield twice the value of the required integral for half-integral powers of $|\beta|$ in the interval (β_c, ∞), which are expressed in terms of the integral of $f^+(z)$ over the circles C_R^+, $C_{\beta_c}^+$, and the sum of the residues. Since $\Delta\omega \gg C\beta_c$, the residues of $f^{\pm}(z)$ always lie inside the contours $C^{\pm}$.

Therefore, the integral of the half-integral powers of $|\beta|$ is expressed throughout the region $-\infty < \beta < \beta_c$, $\beta_c < \beta < \infty$ in terms of the half-sum of the integrals of $f^-(z)$ over C_R^-, $C_{\beta_c}^-$, and of $f^+(z)$ over C_R^+, $C_{\beta_c}^+$, and the half sum of the residues of $f^{\pm}(z)$ inside the con-

tours $C_{\beta}^{\pm}$. Simple calculations show that the integral of $f^{+}(z)$ over $C_{\beta c}^{+}$ is exactly cancelled by the integral of $f^{-}(z)$ over $C_{\beta c}^{-}$. Moreover, as $R \to \infty$ the integrals of $f^{\pm}(z)$ over $C_{R}^{\pm}$ will vanish so that the resulting expression for the sum of the terms with half-integral powers of $|\beta|$ are determined only by the residues of $f^{\pm}(z)$ and can be written as

$$i_{cc} = \frac{2}{C}\,\mathrm{Re}\sum_{m=0}^{\infty}\frac{(-1)^{2m+1}}{(2m+1)!}\Gamma\left(\frac{6m+7}{2}\right)\cos\left(\frac{6m+5}{4}\pi\right)\left(\frac{C}{\Delta\omega+iw}\right)^{(6m+5)/2}. \tag{35}$$

To obtain the asymptotic distribution in the line wing it is sufficient to retain in the expansion in terms of the powers of the reciprocal of $\Delta\omega$ only the leading terms corresponding to $m = 0$ in Eq. (35). As a result the wing of the hydrogen line turns out to be simply the sum of the dispersion contribution of the central component and the quasistatic contribution of the side components:

$$I_{nn'}(\Delta\omega) \approx \Delta\omega^{-2}\sum_{\alpha_0\beta_0}|\langle\alpha_0|\mathbf{P}|\beta_0\rangle|^2 w_{\alpha_0\beta_0} + 2\pi N\Delta\omega^{-5/2}\sum_{\alpha\beta}(eC_{\alpha\beta})^{3/2}|\langle\alpha|\mathbf{P}|\beta\rangle|^2. \tag{36}$$

If the line is even ($n + n' = 2k$) and consequently has no central component, then the dispersion contribution to the wing intensity can arise only from the overlap region $|\mathbf{F}| < F_c$. It is readily verified, however, that the integrated contribution by this region is so small that the dispersion shape has practically no effect on the behavior of the wing right up to distances from the center corresponding to overlapping with neighboring terms of the series. Therefore, in accordance with Eq. (36), the density distribution in the wings of even terms of the series is wholly determined by the quasistatic broadening of the side components in the ion field.

The dispersion contribution of the odd terms in the series ($n + n' = 2k + 1$) to the wings is also different from that predicted by Griem's formula.[4,20] It is wholly determined by the contribution of the central components and can therefore be correctly evaluated only by the use of the parabolic quantization and exact individual collision half-widths. Generally speaking, the dispersion contribution turns out to be much smaller than predicted by Griem.[4,20] even when the incoherent nature of the composition of amplitudes for inelastic electron scattering is taken into account.

4. NUMERICAL RESULTS AND COMPARISON WITH EXPERIMENT

In an experimental verification of the above asymptotic formulas it is convenient to transform from the frequency scale to the wavelength scale, and write the line profile given by Eq. (35) in the form

$$S_{nn'}(\Delta\lambda) = \frac{1}{c}\left(\frac{\lambda_0}{2\pi c}\right)^{1/2}\left(\sum_{\alpha\beta}(eC_{\alpha\beta})^{3/2}|\langle\alpha|\mathbf{P}|\beta\rangle|^2\right) \times N\left|\frac{\lambda_0}{\Delta\lambda}\right|^{5/2}[1+|\Delta\lambda|^{1/2}R(N,T)], \tag{37}$$

$$R(N,T) = \frac{(2\pi c)^{1/2}}{\lambda_0}\sum_{\alpha_0\beta_0}|\langle\alpha_0|\mathbf{P}|\beta_0\rangle|^2 w_{\alpha_0\beta_0}\Big/ 2\pi N\sum_{\alpha\beta}(eC_{\alpha\beta})^{3/2}|\langle\alpha|\mathbf{P}|\beta\rangle|^2,$$

$$\lambda_0 = 2\pi c/\omega_0.$$

According to[1,4,20], the function R(N, T) determines the deviation from the quasistatic behavior in the wing due to electron collision broadening. However, in contrast to[1,4,20], the quantity R(N, T) in Eq. (37) is wholly determined by the contribution of central Stark components and for even terms in the series ($n + n' = 2k$) we have $R(N, T) = 0$. To determine R(N, T) for the most important odd terms in the series we must use the individual collision half-widths of the central Stark components determined by Eq. (19) and the corresponding oscillator strengths.[28]

TABLE. R(N, T) in $Å^{-1/2}$

T, °K	N, cm⁻³					T, °K	N, cm⁻³				
	10^{14}	10^{15}	10^{16}	10^{17}	10^{18}		10^{14}	10^{15}	10^{16}	10^{17}	10^{18}
	Ly-α						*H_α*				
$0.5\cdot10^4$	1.03	0.845	0.65	0.46	—	$0.5\cdot10^4$	0.252	0.197	0.141	—	—
10^4	0.81	0.675	0.54	0.405	0.27	10^4	0.196	0.159	0.120	0.082	0.044
$2\cdot10^4$	0.63	0.535	0.44	0.344	0.249	$2\cdot10^4$	0.155	0.128	0.101	0.075	0.047
$4\cdot10^4$	0.48	0.413	0.346	0.278	0.21	$4\cdot10^4$	0.121	0.102	0.083	0.064	0.045
	Ly-γ						*H_γ*				
$0.5\cdot10^4$	0.481	0.366	0.251	—	—	$0.5\cdot10^4$	0.046	0.034	0.023	—	—
10^4	0.388	0.308	0.226	0.146	0.065	10^4	0.037	0.029	0.021	0.012	—
$2\cdot10^4$	0.312	0.254	0.196	0.139	0.081	$2\cdot10^4$	0.031	0.025	0.018	0.012	—
$4\cdot10^4$	0.21	0.177	0.14	0.105	0.070	$4\cdot10^4$	0.023	0.020	0.016	0.011	—

The table gives the values of R(N, T) obtained from Eq. (37) for the L_α, L_γ, H_α and H_γ lines for different values of the plasma density and temperature. Comparison with calculations based on the old formulas for the wings[4,20,24] does, in fact, show that the new formulas predict a much smaller contribution of the dispersion terms in spite of the large value of the individual Stark half-widths. It is interesting that, when $n \gg n'$, the value of R(N, T) does not increase in proportion to n or n^2, as predicted by previous theory,[4,20,24] but tends to a limit which depends on n only in a logarithmic fashion:

$$\lim_{n\to\infty}[(\lambda_0)^{1/2}R(N,T)] = 3{,}6\cdot10^2(n')^{-1}T^{-1/2}\ln\left(\frac{\rho_{max}}{\rho_{min}}\right).$$

This is due to the reduction in the relative oscillator strengths of the central Stark components which, according to[28], behave as $1/n$ for $n \gg n'$ and, consequently, the numerator in the expression for R(N, T) is proportional to n^3. The denominator in this formula also behaves as n^3 and this ensures that the prelogarithmic factor in R(N, T) is a constant.

The main conclusion of the present work is that the even-line wings ($n + n' = 2k$) exhibit purely quasistatic behavior and this is in good qualitative agreement with the corresponding experimental data.[21,23] With regard to quantitative agreement, this will require more careful analysis of experimental data with the necessary control of the charged-particle distribution in the high-frequency discharge.

For L_α lines there are reliable experimental data[29] and one can carry out detailed comparison with the above theory. This is illustrated in Fig. 3 where, in addition to the experimental points, we show the result of line-shape calculations for the wings for different ratios ρ_{max}/ρ_{min}. Although this ratio enters R(N, T) only in its argument, the theoretical indeterminacy in its definition may lead to a spread of about 30%, as can be seen from the figure. The best agreement with experiment is observed for ρ max equal to the mean interparticle distance ρ_0 and $\rho_{min} = 6\lambda_{Te}$.[5)]

5. CONCLUSION

Comparison of the method for obtaining asymptotic expressions for line wings developed in the present paper with the method developed in[4,20,24] shows that

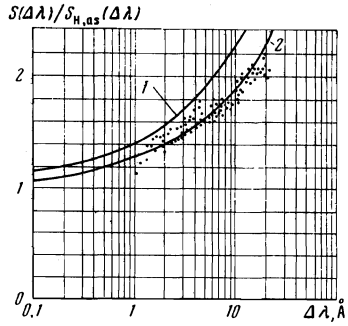

FIG. 3. Comparison of the experimental intensity distribution in the L_α wing[29] with calculations based on Eq. (37): points – experiment, curve 1 – usual value of $\rho_{max}/\rho_{min} = \rho_D/n^2\lambda_{Te}$, curve 2 – using $\rho_{max}/\rho_{min} = \rho_0/6\lambda_{Te}$.

the difference between the final expressions is due to the change of the order of integration in the improper integrals which are not uniformly convergent, as used in[4,20,24]. Moreover, in the present paper we have introduced a clear subdivision of the ion field F into a small internal overlap region $(0, F_c)$ and the region of isolation (F_c, ∞), which is important for the integrated effects, and have shown that the dispersion contribution to the line wing is determined by the electron collision broadening of the central Stark components. The critical field F_c is determined from the condition for the overlap of the Stark components of a given line, which are connected with one another through the nondiagonal matrix elements of $\Phi_{nn'}$, and varies from one set to another even for a given line. However, to estimate the maximum effect of the overlap region on the line profile in the wing, one can use an effective value $(\beta_c)_{eff}$ for the line as a whole:

$$\left(\frac{F_c}{F_0}\right)_{eff} = (\beta_c)_{eff} \sim 1.23 \cdot 10^{-6} n^2 \frac{N^{1/3}}{T^{1/2}} \ln\left(\frac{\rho_{max}}{\rho_{min}}\right). \qquad (38)$$

The maximum value of $(\beta_c)_{eff}$ as a function of n, T, and N for $T \gtrsim 5 \times 10^3$°K and density $n \lesssim 10^{18}$ cm^{-3} is readily shown to be less than unity ($\lesssim 0.3$).

All that remains is to consider in somewhat greater detail the accuracy of the above wing formulas. It was assumed in the derivation that there exists a sharp boundary at $\beta = \beta_c$ of the region in which the Stark components are isolated. In reality, there is no sharp boundary but merely a bounding region of width $\sim\beta_c$ within which one must use the eigenfunctions and eigenvalues of the operator $H_0 + i\Phi$. Therefore, if we take this transition region into account, the contribution of the side components to R(N, T) can be estimated from the formula

$$\Delta_s R(N,T) \approx \int_{-2F_c}^{2F_c} W(F)\,dF\,[R_{Tp}(N,T) - R(N,T)], \qquad (39)$$

where $R_{Gr}(N, T)$ are the coefficients tabulated by Griem,[4] and $\int_{-2F_c}^{2F_c} W(F)dF$ is the total statistical weight of the overlap region and the transition region. For values of R(N, T) tabulated in the present paper with the Holtzmark distribution function for the ion microfields, we have $\int_{-2F_c}^{2F_c} W(F)dF \approx 0.03$–$0.04$ and, therefore, the contribution of the side components to the dispersion term in the asymptotic intensity distribution can be neglected.

As the plasma becomes increasing nonideal, $\rho_D/\rho_0 \to 1$, the microfield distribution function for small F becomes appreciably different from the Holtzmark function,[3] and one would expect an increase in the dispersion contribution to the wing intensity due to the increase in the integral $\int_{-2F_c}^{2F_c} W(F)dF$. Estimates show, however, that even in the most critical region of the parameters, $T \sim 5 \times 10^3$°K and $N \sim 10^{18}$ cm^{-3}, we find that for actual levels $\int_{-2F_c}^{2F_c} W(F)dF \lesssim 0.15$, i.e., the dispersion contribution of the side components is always smaller by almost an order of magnitude than that calculated from Griem's formulas.[4,20,24]

We note, finally, the qualitative difference between the above expressions and the modified Griem formulas for the wing,[4,20] which use the assumed transition of the electron broadening to the quasistatic limit. Firstly, in Eq. (37) the quasistatic contribution is due only to the ions and, therefore, the term proportional to $|\Delta\lambda|^{-5/2}$ includes the ion density and not the total density of the charged particles, as in the case of Griem.[20] Secondly, the contribution of the neutral components never exceeds the quasistatic limit and, therefore, for lines with strong central components the intensity in the sufficiently distant wing may exceed the asymptotic Holtzmark distribution $S_{H,As}(\Delta\lambda)$ by a factor of more than 2. Thirdly, the contribution of collision broadening to the wing of lines with central components does not increase with increasing n for all the higher members of the series but, on the contrary, it decreases logarithmically. [For the Balmer series, for example, $R(N, T) \lesssim 0.05$ for $n > 6$ and all values of N and T for which the Stark profiles of the corresponding lines can be observed.] Therefore, according to Eq. (37), the higher series members will have a quasistatic wing intensity distribution, since for such small values of R(N, T) the collision contribution is appreciable at distances $\Delta\lambda$ from the line center, which exceed the distance to the neighboring series terms.

Under the conditions usually realized in practice, $\beta_c \gg 1$ and, therefore, the intensity distribution in the wings of the even terms of the series $(n + n' = 2k)$ is, in fact, described by the quasistatic contribution of the side components, whereas in the case of the odd members $(n + n' = 2k + 1)$ it is described by the dispersion contribution of the central and quasistatic side components. These qualitative conclusions are probably best verified by experiment.

The results of Sections 3 and 4 were obtained by G. V. Sholin and A. V. Demura, whilst all the authors contributed to Sec. 2.

[1] The connection between the electric dipole moment and the Runge-Lenz vector $\mathbf{P} = -3ea_0n\mathbf{M}/2\hbar$ was noted in the theory of the Bohr atom given by Sommerfeld. [10] In quantum theory the use of this operator equation is allowed when one restricts one's attention to a set of eigenfunctions with a fixed quantum number n. It is precisely this situation which occurs in the problem of the broadening of hydrogen lines.

[2] Numerical tests were used in [9] to postulate the symmetry property $B(z_1, z_2) = B(z_2, z_1)$, from which it follows that $B(z, -z) = 0$ and, therefore, $b_-(0) = 0$. Our results show that this symmetry property is not valid and the fact that $B(z, -z)$ is numerically small is completely compensated by the integration between infinite limits in the course of transition to $b_-(z)$.

[3] We note, by the way, that in the monograph of Sobel'man [3] the formulas for collision broadening of the hydrogen sublevels [Eqs. (38.32) and

(38.33)] contain a numerical error: to describe the contribution of strong collision to $\ln(\rho_{max}/\rho_{min})$ it is necessary to add 0.09 and not 0.33.

[4)]Precision measurements on H_α have shown [16] that the experimental profile is, in fact, narrower than that calculated neglecting interference effects [17] but is broader than predicted in [18] with the interference terms taken in the form given by Eq. (22).

[5)]The use of the formula $\rho_{min} = 6\lambda_{Te}$ instead of the approximate result $\rho_{min} \approx n^2\lambda_{Te}$ is based on the definition of the Weisskopf radius through the comparison of the electric and "magnetic" interactions with the parabolic wave function as the basis. [2, 11] In the notation of [2] the Weisskopf radius ρ_W is given by

$$(L_x)_{\alpha_0\alpha}\frac{v}{\rho_B} = \frac{1}{\hbar}\frac{e}{\rho_B^2}[(P_z)_{\alpha\alpha} - (P_z)_{\alpha_0\alpha_0}],$$

$$\rho_B = 3\frac{n(n_1^{(\alpha)} - n_2^{(\alpha)})}{[n_1^{(\alpha)}(n - n_1^{(\alpha)})]^{1/2}}\lambda_{Te}.$$

Hence for L_α we have $\rho_W = 6\lambda_{Te}$.

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