On the phenomenology of gravitation

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It is shown that within the limits of validity of the equivalence principle weak gravitation is determined by extremely general principles, as a consequence of which nonrelativistic effects can yield only phenomenological information. It is shown that a theory of the Brans-Dicke type[2] can lead to misinformation. For this reason nonrelativistic data should be considered phenomenologically. Exact criteria have been obtained for a broad class of equations, including the Einstein equations. It is proved that the deflection of light-rays can yield precise information on the canonical character of the equations and on the interaction of gravitational fields. Two types of gravitational waves are pointed out, waves which may yield direct data on short-range interaction. It is shown phenomenologically that if the present data on light-deflection and perihelion advance are corroborated, then the most likely conclusion is that within the limits of the equivalence principle, the gravitational field is a curvature-tensor field, but obeys equations of order higher than two. A basic difference between gravitation and the other interactions is pointed out. It is shown that the Eötvös experiment would not have caught all the violations of the equivalence principle; an experiment is suggested for their verification and estimates are given on the basis of electromagnetic data. An upper limit has been determined on the coupling constants of a direct interaction between matter and long-range nonmetric fields.

1. INTRODUCTION

As we shall see, nonrelativistic effects exhibit not the details but only the phenomenology of gravitation. Phenomenology is important because of the peculiarity of the Einstein equations. In these equations gravitation is considered to be metric, i.e., it can be reduced to the tensor g_{ik} which defines the 4-interval, and the equations themselves are: generally covariant (A_1), derivable from a variational principle (A_2), and canonical (C), i.e., do not involve derivatives beyond the second order. The requirements $A_{1,2}$ are compulsory for all equations, and since in a metric field the equations corresponding to the requirements C and $A_{1,2}$ are unique (Weyl[1]), any violation of the Einstein equations is a matter of principle: either the metric ones are noncanonical or the canonical ones are nonmetric, i.e., they require the presence of gravitational fields which do not reduce to the curvature of 4-space. Therefore, a confirmation of the Einstein equations can only be obtained through an experimental guarantee of their absolute accuracy.

In the literature it is not taken into account that the Einstein equations severely limit the relative difference δ_0 between the first approximation to the radially symmetric Schwarzschild metric and experiment. Indeed, if the Einstein equations are valid, then in the field of the sun

$$|\delta_0| = \sigma \lesssim 10^{-5}, \tag{1}$$

where σ are all the corrections required by the exact Einstein equations. The only universal constants[1)] which can occur in σ are the speed of light c and the Newton constant G; then σ is exhausted by contributions $\lesssim Gm_{\odot}c^{-2}a_{\odot}^{-1} \sim 10^{-6}$ (here $m_{\odot}$ and $a_{\odot}$ are the mass and radius of the sun), produced by relativistic effects and the nonstatic character of solar matter, by contributions $\lesssim e$ due to the sun's oblateness, of the order $e \lesssim 10^{-5}$, and by contributions $\lesssim 10^{-7}$ from accelerations produced by extrasolar masses. The available data on the solar field (cf. Sec. 5) do not necessarily guarantee the validity of the condition (1). Therefore the problem on the nature of the fields and the order of the equations is still open. In view of this it is important to establish exactly the contents of the phenomenological information contained in the nonrelativistic data.

All known non-Einsteinian theories are non-phenomenological, as a consequence of which their equations, as we shall see, may agree with experiment even if the assumptions were erroneous. Therefore at present the experiments should be considered phenomenologically, i.e., under assumptions which are sufficiently concrete for an effective verification, and at the same time, are sufficiently general to exclude disinformation. We shall show (apparently for the first time) that such a phenomenology is possible and that in the nonrelativistic case for such a phenomenology it suffices to have $A_{1,2}$ and the following conditions: the existence of particles for the gravitational field (A_3), the absence of a macroscopic nonlocality (i.e., acausality) in the Lagrangian (A_4) and the possibility of spontaneous decay of the vacuum (A_5). We shall take these points into account in the most general form, but not at all as a generalized theory of gravitation, but only for completeness and uniqueness of the interpretation of experiments. The results of such an interpretation differ from those generally accepted.

2. THE PHENOMENOLOGY OF WEAK FIELDS

The empirical data on the equations of gravitation are phenomenological only within the limits of applicability of the equivalence principle, for which the relative error ϵ is sufficiently small (cf. Sec. 5). The equivalence principle (cf. (4)) allows matter to interact directly only with the metric, in view of which all observable effects which are of order $\gg \epsilon$, firstly, do not depend on the exact form of the law (30) (cf. infra) describing the interaction of matter and the gravitational fields, and secondly, are manifestly produced by the action on the measuring instruments of the metric field only, and consequently are completely determined by the equations for g_{ik}. We consider weak fields. We shall use coordinates such that $x^0 = ct$ and

$$g_{ik} = \gamma_{ik} - 2h_{ik}, \quad |h_{ik}| \ll 1, \quad \gamma_{00} = -1, \quad \gamma_{i\alpha} = \delta_{i\alpha} \quad (\alpha = 1, 2, 3) \tag{2}$$

and shall derive within the limits of validity of the equivalence principle the equations for g_{ik} in the linear approximation with respect to h_{ik}.

According to the condition A_3 the equations of any weak field must reduce to the form

$$(\Box - k_1^2)(\Box - k_2^2)\ldots f = Q, \quad k_i^2 = \text{const} \geqslant 0, \tag{3}$$

where $\Box$ is the D'Alembertian and Q is the source of the given field. The source cannot vanish for $f = 0$, since otherwise solutions of (3) with zero initial and boundary conditions, i.e., the fields of the source proper, would vanish. Therefore weak fields of the type A_3 are necessarily linear in the sources. But then the tensor $G_i^k = R_i^k - \frac{1}{2}R_n^n\delta_i^k$, where R_i^k is the Ricci tensor, is linear with respect to matter in the weak-field approximation and for the conditions A_{1-5} is related to matter in a quite general form.

Indeed, the Lagrangian density of the whole system is

$$L = \sqrt{-g}(\Gamma + M), \quad M = M_0 + M_1, \quad g = \det g_{ik}, \tag{4}$$

where Γ and M (the Lagrangians of the gravitational fields and of gravitating matter, respectively) can differ in the case A_1 from the invariants only by a gauge transformation $L \to L + \partial_n L^n$, which does not change the equations. Here $\partial_n = \partial/\partial x^n$, M_0 is the Lagrangian of nongravitating matter, expressed in coordinates with the gravitational metric g_{ik} and M_1 describes the violations of the equivalence principle, which is exhausted by the requirement $M \equiv M_0$. It is obvious that if $M_1 = 0$ all gravitational fields generate directly or indirectly the tensor

$$T_i^k = -2g^{kn}\delta(\sqrt{-g}M)/\sqrt{-g}\delta g^{in}. \tag{5}$$

such that $T_{i;k}^k = 0$. Therefore in the field (3) the tensor G_i^k, is linear and homogeneous with respect to T_n^m in the linear approximation with respect to h_{ik} (cf. footnote 1), and to the accuracy under consideration $\partial_k G_i^k = 0$ (the Bianchi identities), $\partial_k T_i^k = 0$, and the relation of G_i^k with T_n^m is Lorentz invariant, according to A_1, but the relation of G_i^k with $\partial_l T_n^m$ is nonlocal, in view of the variational character of the equations, i.e., according to A_2. But then we have in the most general case in the coordinates (2)

$$G_i^k + \Box_i^k F = \hat{X}_1 T_i^k, \quad \Box F = \hat{X}_2 T_n^n, \quad \hat{X}_n = \sum_{p\geqslant 0} Q_n(p)\hat{Y}_p, \tag{6}$$

where

$$\hat{Y}_p = (1 - p^2\Box^{-1})^{-1}, \quad \gamma_{kn}\Box_i^n = \gamma_{ik}\Box - \partial_k\partial_i,$$

Q and p are constants, with $p^2 \geq 0$, according to A_3, and A_4 requires retarded Y_p.

Thus, within the accuracy of ϵ, the requirements A_{1-4} determine completely the structure of the linearized equations for observable quantities. By means of adequate selection of Q and p one can obtain from (6) the linearized equations of all known theories of gravitation. But since, according to nonrelativistic effects which are of much larger order of magnitude than ϵ, one cannot practically obtain all constants, as we shall see, these effects can only yield a phenomenology of gravitation, and therefore one must consider them only phenomenologically.

For example, in (6) the field F may well be metric, since the conditions A_{1-4} do not necessarily require nonmetric fields, and since the constants Q cannot be determined in general from gravitational waves (cf. infra), experimentally a nonmetric wave could not be distinguished from a metric one. Under these conditions nonmetric theories of scalar waves (e.g., the Brans-Dicke theory[2]) would lead to disinformation, since their equations, being of the type (6), could agree with experiment even in the absence of nonmetric fields. Therefore data on waves cannot be discussed under assumptions which are more concrete than A_{1-4}. But then, to have complete information one must detect, in addition to the transverse and scalar waves, the two other waves T and Λ which have not been discussed until now. Indeed, to accuracy ϵ only the components $R_{0\alpha}^{0\beta}$ of the Riemann-Christoffel tensor can be detected in a nonrelativistic wave (Weber[7]) and these components reduce to the transverse waves $\theta_\xi^\eta = 2R_{0\xi}^{0\eta} - S\delta_\xi^\eta$ (the only ones admitted by the Einstein equations), to the scalar $S = R_{0\xi}^{0\xi}$, to the longitudinal-transverse waves $T_\xi = R_{0\xi}^{03}$ and to the scalar $\Lambda = 4R_{03}^{03}$, where x^3 is the axis along which the wave propagates, orthogonal to the directions $\xi, \eta = 1, 2$. In the wave zone of the fields (2), we have, according to the equations (6)

$$\begin{aligned} \theta_\xi^\eta &= \sum (\omega/c)^2(2t_\xi^\eta - t_{\xi'}^{\xi'}\delta_\xi^\eta)Q_1, \quad T_\xi = \sum p^2 Q_1 t_\xi^3, \\ S &= \sum [(2Q_2 - Q_1)p^2 t_3^3 + 2(\omega/c)^2 Q_2 t_\xi^\xi], \\ \Lambda &= \sum p^2[(2Q_2 - Q_1)t_\xi^\xi + (cp/\omega)^2(2Q_2 + Q_1)t_3^3], \end{aligned} \tag{7}$$

where the sums include all the parameters of (6) and frequencies $\omega \geq cp$, and t_α^β are proportional to the Fourier components of the tensor (5). It is obvious that as long as t_α^β will not be measured independently and with good accuracy (which is practically impossible), the waves will only record the presence or absence of some of the Q, but not their magnitude. But then the θ-wave (which is necessary in view of the manifest smallness of the possible violations of the Einstein equations) would only prove the nonsphericity of the source and would not yield any information on the equations of gravitation, in the same manner as the total intensity. The S-wave (which is not excluded, although it was not detected in Weber's experiments[4]), would overthrow Einstein's equations and the remaining information would be determined by the T- and Λ-waves.

According to Eq. (7) the T- and Λ-waves transport fields of finite range $l = p^{-1}$. If l_1 and l_2 are the effective radii of $\hat{X}_1$ and F, respectively, and $\delta_{T,\Lambda,S}$ are the amplitudes of the corresponding waves, averaged over the sources, and measured in units of the θ-amplitude, then according to (7)

$$l_1 \sim \delta_T^{-1/2} c\omega_0^{-1}, \quad l_2 \sim (\delta_S/\delta_\Lambda)^{1/2} c\omega_0^{-1}, \tag{8}$$

where ω_0 is an effective frequency

$$\delta_T < 1, \quad \delta_T \lessapprox \delta_\Lambda \lessapprox \delta_S, \quad Q_2 \sim \delta_S Q_1. \tag{9}$$

Consequently, measuring all the polarizations (e.g., a rod directed toward the source would receive only the Λ-waves), one can obtain direct information on short-range action and verify not only Einstein's equations, but also (by means of the first two relations (9)) the applicability of the principles A_{1-4} to gravitation.

Experiment shows that short-range interactions are inessential in (6) (according to Weber[4], $\delta_S < 0.3$, i.e., $\delta_{T,\Lambda} < 10\%$, and according to static data (cf. Sec. 5) there might not be any short-range interaction at all). Therefore we take into account the short-range interaction only in the correction (11) (cf. infra) to the equations (10), which were obtained from (6) by deleting all $p \neq 0$. Then, in order to achieve agreement with the Newtonian Poisson equation it suffices to set $Q_1(0) + Q_2(0) = 8\pi c^{-4}G$ and we have

$$G_i^k + \Box_i^k F = 8\pi c^{-4}(l - w)GT_i^k, \quad \Box F = 8\pi c^{-4} wGT_n^n, \tag{10}$$

where $w = Q_2(0)[Q_1(0) + Q_2(0)]^{-1}$, with $w \sim \delta_S$, according to (9). Thus, up to short-range interactions, all the unknowns in (6) have been reduced to a single constant w.

In the Einstein equations (and only in these equations) $w = 0$, and for $w \neq 0$ the equations (10) coincide exactly with the linearized Brans-Dicke equations[2]. But since agreement of the equations (10) with experiment does not necessarily imply that the nonmetric field postulated in[2] exist in nature, then at least in the linear approximation the Brans-Dicke theory and all other non-phenomenological non-Einsteinian theories are disinformative.

3. THE CONSEQUENCES OF HIGHER APPROXIMATIONS

Let us estimate the relative difference λ between the results of Eq. (10) and the exact theoretical magnitude of some effect, in the first approximation which is linear in the gravitational fields. In the general case λ is the sum of a contribution $\lesssim Gmc^{-2}a^{-1}$ from the nonlinearities in h_{ik} (m and a are the mass and radius of the source), the contribution from M_1, of order ϵ, the correction k due to short-range interaction and the contribution ζ from non-Einsteinian nonlinear long-range interaction. Qualitatively, the ζ-correction reduces to replacing F by $F + \tau F^{n+1} = (1 + \zeta)F$, where τ is a dimensionless constant $n > 0$, and then, according to (10)

$$\lambda \leqslant Gmc^{-2}a^{-1} + k + \varepsilon + \zeta(r), \quad \zeta(r) \sim \tau(wGmc^{-2}r^{-1})^n, \tag{11}$$

where r is the distance from the center of the source. Consequently, in the field of the sun the theoretical relative difference between the radial static solution of (10) and experiment is

$$\delta(r) = \sigma + \lambda \approx \sigma + k + \varepsilon + \zeta(r), \tag{12}$$

where σ is the parameter (1) including also the first term of (11).

The quantity $\delta(r)$ can determine the applicability of the theory. Indeed, if the condition

$$|\delta(r)| \leqslant \mu = |\sigma + k + \varepsilon|_{max} \tag{13}$$

is violated for empirical values of μ, the theory is valid only for

$$\tau \geqslant (w/\delta)^{-1}[c^2 a_\odot G^{-1} m_\odot^{-1}]^n \sim 10^{6n}, \tag{14}$$

since then experimentally $\delta \sim w$ according to (25), below. Thus, for arbitrary integer n the nonvalidity of (13) would exclude in general scale invariance of gravitation (and together with it the Brans-Dicke theory[2]), according to which $\tau \sim 1$, and would lead in (6) to doubtful nonlinearities of the type $F + 10^6 F^2$. A theory which is possible only for

$$|\delta(r)| = \text{const} > \mu, \tag{15}$$

is incorrect, since according to (12) $\delta(r) = \zeta(r)$ for $|\delta| > \mu$.

The deflection of light in the field of the sun (cf. Sec. 5) yields directly an empirical value for $w + \delta(r)$. Therefore a phenomenology which restricts w also limits $\delta(r)$, which is sufficient for its verification according to the conditions (13)—(15). In case the conditions A_{1-5} are satisfied, this allows one to obtain exact data on nonmetric fields from the light deflection, since, as will be shown below, for a stable vacuum, such fields are not compatible with the equivalence principle for all values of w.

4. NONMETRIC RESTRICTIONS

Within the limits of validity of the equivalence principle obviously matter can not generate nonmetric fields, in view of which fact such fields would manifest themselves in effects of order of magnitude much larger than ϵ only if they would be generated by the curvature tensor. Under the assumptions A_{1-5} the presence of such a process (we shall call its R-process; a first example of such kind was indicated in[2]) severely limits the values of the constant w. According to (6) and (4) this constant is determined by the Lagrangian Γ, which on the basis of assumption A_4 can always be regauged in macroscopic fields into the local invariant

$$\Gamma = \text{const} \cdot (U_{lm}{}^{ik} R_{ik}{}^{lm} + U), \tag{16}$$

where U is the Lagrangian of the free nonmetric fields, the tensor U^{ik}_{lm} has the symmetry of the curvature tensor R^{ik}_{lm} and in the absence of gravitation it equals $\delta^i_{[l}\delta^k_{m]}$; therefore

$$U_{lm}{}^{ik} = (l - \overline{\varphi})\delta_{[l}{}^i\delta_{m]}{}^k - \bar{\eta}_{[l}{}^{[i}\delta_{m]}{}^{k]} - \bar{\psi}_{lm}{}^{ik}, \quad \bar{\eta}_n{}^n = \bar{\psi}_{in}{}^{kn} \equiv 0, \tag{17}$$

where $[ik] = ik - ki$, and $\overline{\varphi}_1\overline{\eta}$ and $\overline{\psi}$ are fields which are missing from the Einstein equations. It is obvious that w is determined only by the long-range part of the first approximation to U and the fields (17).

In the case (3) the Lagrangian U in the first approximation is quadratic in the fields, and in the canonical case can be gauge-transformed into a sum of expressions of the form $f(\Box - k^2)f$, where k^{-1} is the range of the interaction. Noncanonical corrections to this Lagrangian have the form $f\Box^n f$ with $n \geq 2$, i.e., they change $f(\Box - k^2)f$ into $f(\Box - k_1^2)\ldots(\Box - k_n^2)f$ and reduce completely to short-range action, in view of which a contribution to w comes only from the first canonical approximation to U with $k = 0$. According to the equations of gravitation this term is quadratic in matter, and has an order of magnitude equal to the contribution to Γ of the nonmetric parts of the fields (17). In first approximation it follows from the equations of gravitation that all nonvanishing fields (17) are always linear in matter and have a Newtonian asymptotic behavior. But if the asymptotic behavior of the metric is of the same nature (which seems to be required by experiment), the static curvature tensor decreases not slower than r^{-3} and then its components are manifestly nonlinear with respect to the metric part of those fields (17) with which it is contracted in Γ. But then the metric part of Γ which contributes to w is, in first approximation, biquadratic in the matter, i.e., is a relativistic correction $\lesssim Gmc^{-2}a^{-1}$ to the nonmetric part (if it exists), which in first approximation is quadratic in matter. Therefore the requirement A_5 which in the 00-component of the nonrelativistic energy-momentum pseudotensor limits only the part which is quadratic in matter, will affect w only in the presence of R-processes.

According to what was said above, in R-processes w is determined by the first canonical approximation to U, and for the same reason, by the first canonical approximation to the fields (17). It is easy to show that in the first canonical approximation the latter are nonmetric:

$$\overline{\varphi} = 2\sum a_1\varphi, \quad \overline{\eta}_i{}^k = \sum a_2\eta_i{}^k, \quad \overline{\psi}_{ik}{}^{ln} = 4\sum a_3\psi_{ik}{}^{lm}, \tag{18}$$

where everything is dimensionless and all sorts of φ, η, and ψ have been taken into account; they are those nonmetric fields which under the conditions A_{1-4} can appear in nonrelativistic R-processes. Within the limits of the equivalence principle nonmetric fields would, obviously, act on instruments only through the metric, and therefore for no tensorial rank would they influence the anisotropy of the observable 4-space. This invalidates the arguments of Peebles and Dicke[6,7] against tonsorial nonmetricity and proves the necessity of taking all fields (18) into account.

The part U_1 of the Lagrangian U which determines w, has the following form in the general case:

$$U_1 = -2\sum \left(\varphi_{;n}\varphi^{\,n} + \eta^{\,l}_{m;n}\eta_l^{\,m;n} - b_1\eta_n{}^m{}_{;m}\eta^{nl}{}_{;l} + \psi_{ik}{}^{lm}{}_{;n}\psi_{lm}{}^{ik;n} - 2b_2\psi_{lm;k}{}^{nk}\psi_{ns}{}^{lm;s}\right), \tag{19}$$

where $b_{1,2}$ are constants; the conditions $\eta^n_n = \psi^{kn}_{in} = 0$ which follow from (17) have been taken into account and only $\eta^l_{m;n}\eta_l^{n;m}$ and $\psi^{lm;n}_{ik}\psi^{ik}_{nm;l}$ have been excluded, since they add only gauge terms to (4), as well as the commutators of covariant derivatives which do not enter into (10). The fields (18) are determined up to the required accuracy by equations which follow from (4), (16) and (18) for $U = U_1$ and $M_1 = 0$; substituting their solutions into the corresponding equations for g_{ik} and linearizing with respect to all fields, including (2), we obtain the equations (10) with

$$w = (d_1 + d_2)(3 + 4d_1 + d_2)^{-1}, \tag{20}$$

where

$$d_1 = \sum [6a_1{}^2 + {}^8/_9(1 - {}^3/_4 b_1)^{-1}a_2{}^2], \quad d_2 = 4\sum [a_2{}^2 + (2 - b_2)^{-1}a_3{}^2], \tag{21}$$

which determines w in the presence of R-processes.

In the case (17)—(19) the Lagrangian (4) reduces to the expression L_0, which does not contain derivatives of order higher than one; it corresponds to a Hamiltonian with density $\dot{f}\partial L_0/\partial\dot{f} - L_0$, where $\dot{f} = \partial f/\partial t$, f are the field functions and the requirement A_5 boils down to the condition $H > 0$ which has been discussed by W. Thirring[8]. It is easy to show that it imposes the conditions

$$b_{1,2} \leqslant 1, \quad 4\sum [a_2{}^2 + (1 - b_2)^{-1}a_3{}^2] \leqslant 1, \tag{22}$$

for which, according to (20), (21), w is situated within the limits

$$0 \leqslant w \leqslant {}^1/_4, \tag{23}$$

which do not depend on the tensor rank of the nonmetric fields occurring in (18). Therefore within the accuracy imposed by (11), w is unobservable.

Thus, R-processes of the type A_{1-5} are possible only under the condition (23).

5. DATA DERIVED FROM THE FIELD OF THE SUN

For the phenomenological interpretation of the data on the field of the sun it is necessary to estimate ϵ and k which occur in (13). Judging by the direct measurements of Shamir and Fox[9] for photons in the field of the sun $\epsilon \lesssim 10^{-3}$, which is an upper limit for ϵ; it is likely that it is strongly exaggerated, since, according to Braginskiĭ and Panov[10], for matter $\delta\epsilon \lesssim 10^{-12}$, where $\delta\epsilon$ is the spread among probes of the value of the nongeodesicity ϵ of the gravitational acceleration (this is the quantity occurring in (13)), and then $\epsilon \lesssim 10^{-9}$, since it is easy to show that in matter $\delta\epsilon/\epsilon$ is smaller than the deviation of the average from $(v/c)^2$ for nucleons, the latter being $\sim 10^{-3}$. The quantity k is probably equal to zero. This follows from a phenomenological estimate $l > k_L^{-1/2} r_L$ of the mean range of short-range interactions, where k_L is the upper limit of the relative short-range correction to the acceleration of the Moon and r_L is the radius of its orbit. From observational data the nonnewtonian accelerations of the Moon are not larger than the contribution $\sim 10^{-6}$ introduced into its fundamental acceleration by the oblateness of the Earth, i.e., $k_L < 10^{-6}$ and $l > 10^{13}$ cm which is at least by six orders larger than the noncanonicity radius indicated by Ginsburg, Kirzhnits, and Lyubushin[11]. But then the average mass of the graviton is smaller than 10^{-24} times the electron mass and in view of the doubtful character of such elementary masses it is natural to assume the gravitational short-range action can have only microscopic ranges and does not manifest itself at macroscopic distances. Independently one can show phenomenologically that $k < 0.1(r_M/r_L)^2 k_L < 10^{-3}$, where r_M is the radius of Mercury's orbit, in view of which in (13)

$$\mu \leqslant 10^{-3}, \tag{24}$$

but it is most likely that $\mu = \sigma \lesssim 10^{-5}$.

Let us look at the gravitational deflection of a light ray. Let $\gamma(r) = \beta(r)\beta_0(r)^{-1} - 1$, where β is the observed deflection angle of the beam passing at a distance $r \geq a_\odot$ from the center of the Sun, $a_\odot$ is the radius of the Sun and $\beta_0 = 4Gm_\odot c^{-2}r^{-1} = 1''.75 a_\odot r^{-1}$. Within the accuracy (11) the ray is conformal and the angle β is linear in the Weyl tensor W, where according to the equations (10) $W(w) = (1 - w)W(0)$, and $W(0)$ corresponds to the angle β_0. Therefore, in the case A_{1-4} we have without approximations

$$\gamma(r) + w + \delta(r) = 0, \tag{25}$$

where w is a constant, (10), and $\delta(r)$ is the quantity (12). It can be directly estimated from the red shift, since in the case A_{1-4} and for arbitrary w it equals the δ_0 defined in (1) for the red shift. The data on the red-shift so far guarantee only the too rough estimate $|\delta_0| \lesssim 10^{-1}$.

According to (1) and (25) the Einstein equations ($|\delta| = \sigma$, $w = 0$) are valid only for $|\gamma(r)| \lesssim 10^{-5}$, so that for an estimate of the accuracy it is important to know the relative error in the quantity $\gamma(r)$, rather than in the angle $\beta(r)$. According to Blamont[12] the optical measurements yield on the average $\gamma = \gamma(a_\odot) = +0.23 \pm 0.13$, but with additional corrections. Therefore reliable values of the optical angles $\beta(a_\odot) = 1.17''$ (Ginzburg[13]) and $1.79'' \pm 0.06''$ (MacVittie[14]), i.e., $\gamma = +0.02 \pm 0.10$ and 0.02 ± 0.03, respectively. These numbers are confirmed in a much more reliable way according to the estimates of the authors of[15,16], the microwave values $\gamma = +0.01 \pm 0.11$ (Seielstad et al.[15]) and $+0.04^{+0.15}_{-0.10}$ (Muhleman et al.[16]), which are methodically independent of the optical data and of one another. The radio data have been obtained by direct interferometry of the quasars 3C273 and 3C279 and are therefore more reliable than the indirect radar values $\gamma = -0.01 \pm 0.2$ (Shapiro[12]) and 0.0 ± 0.1 (Blamont[12]). The average with the least dispersion $\gamma = \Sigma p_i \gamma_i$ of all the indicated data, except the first, is

$$\gamma = +0.016 \pm 0.026, \tag{26}$$

where $p_i = \text{const} \cdot \Delta_i^{-2}$, where Δ_i are the standard deviations, $\Sigma p_i = 1$.

In view of the large error, the result (26) may well exclude not only the Einsteinian values $|\gamma(r)| \lesssim 10^{-5}$, but also the R-processes, since to the condition (23) corresponds in the case (26), according to (25), the whole interval $-0.30 \le \delta(a_S) \le +0.01$, which on the basis of (24) contains also the deviations from (13). Therefore it is extremely important to establish experimentally the radial dependence of $\gamma(r)$, since for

$$\gamma(r) = \text{const} \tag{27}$$

the violation of the condition (13) would, according to (25), be of the type (15), i.e., would guarantee the violation of the Einstein equations and the absence of R-processes of the type A_{1-5}. The law (27) is compatible with observations, but in order to test it it is necessary (and probably also sufficient) to increase the accuracy of the data on γ by one order of magnitude. Such an accuracy of the measurements of the parameter γ and reliable data on the radial dependence $\gamma(r)$ could, consequently, solve the question of the Einstein equations and the generation of nonmetric fields by the curvature tensor.

We now consider the perihelion advance. Let $\xi = 1 - \vartheta\vartheta_0^{-1}$, where ϑ is the exact relativistic perihelion shift, ϑ_0 is the first Schwarzschild approximation. With the accuracy (11) the shift is phenomenologically determined by the first approximation to a geodesic, where for $g_{\alpha\beta}$ it suffices to use the equations (10), and for g_{00} it is necessary to consider all quadratic long-range corrections. It is easy to show that if R-processes are possible, then under the assumptions A_{1-5} we have in the first approximation[2)]

$$3\xi = 4w - w_1, \quad w_1 = \sum a_3^2 (1 - b_2)(2 - b_2)^{-2} \geqslant 0$$

with the constants (18), (19), w is the constant (10), the sign of w_1 is guaranteed by the conditions (22) and in the coefficient (both here and below in (28)) corrections of the order $w \ll 1$ have been omitted. Thus, if R-processes are possible, then under the assumptions A_{1-5} we have without neglecting anything

$$w = {}^1/_4[w_1 + 3\xi(r)] + \delta_1(r) \geqslant {}^3/_4\xi(r) + \delta_1(r), \tag{28}$$

where $\delta_1(r)$ is a quantity of type (12), including all necessary corrections and r is the radius of the orbit.

A comparison of (28) with experiment is made difficult due to the absence of direct values of $\xi(r)$. Experimentally $\xi = \psi_0\vartheta_0^{-1} - \eta$ where $\eta = \psi\vartheta_0^{-1} - 1$, with $\vartheta = \psi - \psi_0$, where ψ is the perihelion shift calculated from astronomical data, which does not reduce to kinematics and to planetary interactions and ψ_0 is the shift produced by all long-range nonrelativistic causes not taken into account in ψ and which are compatible with the equivalence principle (ϵ and k are taken into account in δ_1). In ψ_0 one may neglect the proper rotation and the effects of the extrasolar gas (for real densities its contribution to ξ would be $\lesssim 10^{-10}$). Then ψ_0 is, apparently, determined only by the possible planet Zoe between the sun and Mercury[17] and the quadrupole potential of the Sun

$$Gm_\odot q a_\odot^2 (\cos^2\alpha - {}^1/_3) r^{-3},$$

where $q \le 0$ is a dimensionless quadrupole moment, α is the angle between the radius-vector of the observation point and the axis of rotation of the sun, where within the limits of accuracy of the astronomical data it suffices to use the first approximation with $\alpha = \pi/2$. Then

$$\xi(r) = x r_M r^{-1} - \eta(r), \quad x = 1.9 \cdot 10^3 q + 3.2(m_Z r_Z^2 / m_M r_M^2), \tag{29}$$

where r_M, r_Z, m_M and m_Z are the orbit radii and masses of Mercury and Zoe, respectively.

According to (1) and (28) the Einstein equations are valid only for $|\xi(r)| \lesssim 10^{-5}$, i.e., according to (29), either for $x \lesssim 10^{-5}$, or for $\eta(r) = x r_M r^{-1} \gg 10^{-5}$. According to the astronomical data (Blamont[12]) for Mercury we have $\eta = +0.002 \pm 0.010$, for Venus $\eta = +0.10 \pm 0.58$, and for Earth $\eta = +0.32 \pm 0.32$, which guarantees neither the law $|\eta| \lesssim 10^{-5}$, nor the law r^{-1}, and is too inaccurate for establishing the genuine properties of $\eta(r)$. We also note that not only $x \lesssim 10^{-5}$, but all $x \lesssim 10^{-2}$ (which are necessary for $\eta(r) = x r_M r^{-1}$, since $\eta(r_M)$ has a better likelihood than the values for Venus and Earth) can be lower than the actual values. Indeed, if $x \lesssim 10^{-2}$, then according to (29) $q < 0.3e$, since the surface of the sun is at equilibrium for $q = e - b$, where $b = 1.0 \times 10^{-5}$ is the centrifugal potential in units of the gravitational potential, and e is the relative difference of the semiaxes. It is easy to show that $q/e = 0.6(c_1/c_0)^2$, where c_0 and c_1 are the distances between the foci of the solar surface and its interior layers, respectively, in view of which the Einstein equations are possible only for $c_1 < 0.7\, c_0$, i.e., if the oblateness of the sun decreases toward the center. But this may contradict the differential rotation of the Sun, for which the equatorial angular velocity is larger than the polar one, hence the condition $|\xi(r)| < 10^{-5}$ may turn out to be impossible to realize. Thus, direct data on the perihelia, as well as the data on light deflection, so far do not guarantee the validity of the Einstein equations and may even contradict these equations.

By itself the perihelion shift can hardly give exact information on the R-processes. For this it is necessary, according to (28), that either $\bar{\xi} > +{}^1/_2$ (which in the case (13) excludes the possibility (23)) since according to (28) and (15) $\xi(r) = \text{const}$; according to (29) for observable $\eta(r)$ in the first case hardly possible $q > 10^{-4}$ are necessary, and in the second case negative x are necessary, which contradict (29). But if the disputed measurements of Dicke and Goldenberg[18] are true and $e = (5.0 \pm 0.7) \times 10^{-5}$, then together with the data on light deflection, the data on the perihelia severely limit the R-processes. Indeed, if $q = e - b = (4.0 \pm 0.7) \times 10^{-5}$, then for Mercury, according to (29) $\xi \ge (+0.08 \pm 0.02) \times 10^{-2}$, and then, according to (28) and (24) we have $w \ge +0.06 \pm 0.02$ if the condition (13) is satisfied. According to (10) this is compatible with the result $\delta_S < 0.3$, of Weber's measurement of scalar waves[4], but then according to (26) we will have $|\delta(a_S)| \ge 0.08 \pm 0.03$ in (25) and on the basis of (24) the possibility (13) is excluded. Consequently, if the experimental data of Dicke and Goldenberg[18] and (26) are true, the Einstein equations are not valid and R-processes are incompatible with (13). Under these conditions the scale invariance of gravitation, and together with it all variants of the Brans-Dicke theory[2] are rigorously excluded, and the R-processes require the doubtful nonlinearity (14). Consequently, if the available (not too reliable) data are confirmed, it will be natural to assume that the R-processes are absent, and then, within the limits of validity of the equivalence principle, the real gravitational field will correspond to the basic principle of Einstein's general theory of relativity and will be of purely metric nature, but the equations of gravitation will involve derivatives of the curvature tensor which are absent from the Einstein equations.

So far, this exhausts the conclusions drawn from the data about the solar field.

6. INTERACTIONS WITH MATTER

The phenomenological information on short-range interactions, on the order of the equations describing the gravitational field and on the R-processes which can be obtained from the processes considered above, depend, as we have seen, on the law of interaction between matter and the gravitational fields. We now consider the phenomenology of this interaction.

The source of gravitation is the mass, i.e., the total energy which is not necessarily conserved in the absence of a flow of matter. Therefore gravitation cannot have a vector character (vector charges are conserved), which causes the nonconservation of inertial mass of particles in their decays. The latter are determined by nongravitational interactions for which the nonconservation of constants never produces particles which are not required by the given interactions. This is the main distinction from gravitation, since no decay which is compatible with the equivalence principle, can generally speaking, be accompanied by the emission of gravitons. Indeed, for $M = M_0$ (cf. (4)) gravitation is included not only in the Lagrangians of matter proper, but also in the Lagrangians of all nongravitational interactions, which are consequently independent sources of gravitation. This is naturally to be considered a specific property of gravitation even when the equivalence principle is violated, and then gravitation is separated into a completely special class of interactions, characterized by the Lagrangian

$$L = L_0 + L_1(L_0), \tag{30}$$

where L_0 is the total Lagrangian in the absence of gravitation and L_1 depends, generally speaking, on all parts L_0, contains the metric field, but may also contain nonmetric gravitational fields. In nongravitational interactions L_1 does not depend on L_0.

The presence of nonmetric fields in (30) would not influence the R-processes and would not violate the equivalence principle, but, as we shall see, it would not be detectable in the Eötvös experiment. More complete information on the relation (30) is included in the electromagnetic data, as we shall show.

7. THE INSUFFICIENCY OF THE EÖTVÖS EXPERIMENT

In the presence of gravitation and of a nongravitational interaction V a particle is phenomenologically described by the Lagrangian

$$L = [-m(1+X) + V]ds/dt, \quad ds = (-g_{ik}dx^i dx^k)^{1/2}. \tag{31}$$

The quantity X vanishes in the absence of gravitation, m is the field-independent part of the inertial mass, i.e. $X \neq$ const for $X \neq 0$. Experimentally the part of X that does not depend on the particle is automatically included in g_{ik}, i.e., $X \neq 0$ only if the equivalence principle is violated. It is sufficient to consider that X does not contain derivatives of $u^i = dx^i/ds$ and (in view of the absence of vector gravitation) the first power of u^i:

$$X = Y + Y_{ik}u^i u^k + \ldots, \tag{32}$$

where the quantities Y depend on gravitation, and according to (30) may depend on V; in macroscopic bodies the latter effect is inessential (of the order of the ratio of the external field to that inside the atom). The universal part of the scalar Y vanishes (cf. supra). According to (31) we have $\partial L/\partial \dot{x}^i = m_{ik}u^k$, where for $V = 0$

$$m_{ik} = m(1+X)g_{ik} + mu_k\partial X/\partial u^i - mu_i\partial X/\partial u^k. \tag{33}$$

The tensor (33) is the tensor of inertial mass: for $X \neq 0$ it replaces the inertial mass in the conservation laws and enters into the equations of motion only in the form of derivatives. The effects determined by the fields (32) themselves, and not by their derivatives will be called nongradient effects.

In the tensor (33) the nonstatic fields of external bodies would on the average cancel each other, and the static ones could (in the absence of antigravitation) have a constant sign. Then a mass M at distance r would contribute to the metric part of the tensor (33) (determined by the curvature tensor) a contribution of the order Mr^{-k} with $k > 2$. But already for $k > 1$ the Olbers paradox (the importance of the fields produced by remote masses) is excluded, so tha latter is possible only in the nonmetric part of (33). Fields with the Olbers paradox are determined by the whole Universe (we shall call such fields galactic). Therefore experimentally such fields are constant and would manifest themselves only in nongradient X-effects, the difference of which from the gradient effects would consequently be a direct experimental proof of the presence of nonmetric fields in (30).

According to (31), in the presence of gravitation an electric charge moves in the field of an electromagnetic 4-potential A_i according to the law

$$\frac{du^k}{ds} + \Gamma^k_{nm}u^n u^m = (\delta_i^k + Z_i^k)(L^i + G^i), \tag{34}$$

where Γ^k_{nm} are the Christoffel symbols,

$$L^i = (e/mc^2)g^{in}(\partial_n A_l - \partial_l A_n)u^l, \quad G^i = u^{in}(X_{n;\,m}u^m - X_{;n}),$$
$$Z_i^k = [(X_n u^n - X)\delta_i^l + X_{in}u^{nl}](\delta_l^k + Z_l^k), \tag{35}$$

and $u^{ik} = g^{ik} + u^i u^k$, $X_i = \partial X/\partial u^i$, $X_{ik} = \partial^2 X/\partial u^i \partial u^k$ and $B_{;n}$ denotes the replacement of B in all fields (32) by their covariant derivatives with respect to x^n. The nongradient X-effects in this case are exhausted by the tensor Z_i^k. In general form this tensor has not been discussed.

In laboratory fields in (34) linearity with respect to gravitation is sufficient. Then the tensor Z_i^k would manifest itself only in electromagnetic accelerations, so that galactic (i.e., manifestly nonmetric) fields cannot be detected by the Eötvös experiment. This experiment would detect only the relative deviation $\delta_{Eö}$ of the gravitational (gradient) acceleration of probes, acceleration which according to (34) is determined by the vector G^i. In laboratory fields $G^i \sim \nabla X \sim a_T^{-1} X_T$ (in view of the absence of the Olbers paradox) and according to (34)

$$|\delta X_T| \sim c^{-2} g_0 a_T \delta_E \leqslant 10^{-21}, \tag{36}$$

where δX_T is the dispersion of the contributions X_T of the terrestial field in the invariants (32) of the probes, a_T is the radius of the Earth, g_0 is the acceleration of gravity and $\delta_{E\ddot{o}} < 10^{-12}$[10]. Below it will be shown that the result (36) refers, most likely, only to the scalar Y entering (32).

8. ELECTROMAGNETIC DATA

If the equivalence principle is violated the geomagnetic deflection of any particle would be masked, as the energy increases, by the gravitational deflection. Therefore the tensor fields (32) are limited, so it seems, not by the Eötvös experiment, but by the relativistic geomagnetic effect.

Indeed, in a gravitational field, according to (34), a charge will be deflected by an electromagnetic field only for $s = G^i/L^i \lesssim 1$. The vector G^i is nonglactic. Therefore for a relativistic particle in a magnetic field H near the surface of the Earth we will have according to (35)

$$s \sim (mc^2/ea_T H) \sum_{n\neq 1} Y_n{}^T u^{n+1}, \tag{37}$$

where $u = E/mc^2$ and Y_n^T is the contribution of the terrestial field to the tensor (32) of rank n (including the scalar). Let s_0 be the order of magnitude of s for cosmic ray protons with $u \sim 10^{12}$ in the geomagnetic field. Then

$$Y_n{}^T \sim 10^{-(10+12n)} s_0, \tag{38}$$

where the estimate is more likely valid both for neutrons and for electrons, since at their masses they hardly interact more strongly than the proton with the gravitational field. In cosmic rays s is manifestly small up to $u \sim 10^5$ and it is impossible that $s_0 < 1$. But also for $s_0 \sim 10^3$ the tensor part of δX_T in (36) is, according to (38), by ten orders of magnitude smaller than the upper limit of the right-hand side, and cannot be detected by the Eötvös experiment.

The result (38) strongly limits the constants G_n of the galactic interaction, i.e., the direct interaction of matter with the nonmetric gravitational field which decreases like r^{-1}. Such fields, created by the earth, would give in Y_n^T a contribution $G_n m_T c^{-2} a_T^{-1}$, where m_T is the mass of the earth, and from (38) it follows that

$$|G_n| \leqslant 10^{-(1+12n)} s_0 G, \tag{39}$$

where G is the Newtonian constant. The galactic part of the scalar Y is unobservable (since it is a constant it is automatically included in m). Therefore the galacticity would manifest itself only for $n \geq 2$. But for all admissible s_0 only G_n with $n \geq 2$ are so much smaller than G, that this could also indicate the rigorous absence of the corresponding nonmetric fields.

We consider the direct data on galacticity. Its total contribution y_n to the Y-tensor of rank n must be of order $G_n M c^{-2} R^{-1} \sim 0.1\, G_n G^{-1}$, where M and R are the mass and radius of the Universe. Consequently,

$$y_n \leqslant 10^{-(2+12n)} s_0 \leqslant 10^{-26} s_0, \tag{40}$$

and then for a relativistic particle of energy umc^2

$$\theta \leqslant 10^{-(2+12n_1)} s_0 u^{n_1}, \quad n_1 \geqslant 2, \tag{41}$$

where θ is the galactic part of the invariant (32), n_1 is the rank of the effective Y-tensor; if the series (32) is a polynomial in u^i the estimate (41) is valid for all. For $s_0 \lessapprox 1$ all y_n are much below the sensitivity $\sim 10^{-19}$ of the measurements of the effect (42) and of the sensitivity $\sim 10^{-23}$ of the measurements by Drever[19] of the galactic mass anisotropy, and θ is negligible compared to the tolerances $\gtrsim 10^{-6}$ of fields in accelerators. Therefore for $s_0 \lesssim 1$ laboratory measurements of galacticity are senseless. But for a positive result (which is unlikely in view of (39)) they would be important, since they would prove the presence of nonmetric fields in (30) and the existence at ultrahigh energies of geomagnetic anomalies which are not necessarily galactic and have $s \gg 1$, for which so far there are no direct data. The mass anisotropy is hardly observable, since it is easy to show that it is determined by those space components of the Z-tensor (35), for which the order is lower than (40) by a quantity $\sim Gmc^{-2}a^{-1}$ for the source. And since for accessible energies the accelerators would yield only very rough estimates of the quantity (41), it is meaningful to consider the effect (42) below, which has not yet been discussed in the literature.

According to Eq. (34), gravitation can imitate an electric charge. Indeed, according to (34) in a gravitational field a homogeneous electric field will produce on a charge e an acceleration $m^{-1} e_\beta^\alpha E_\alpha^\beta$, where e_β^α is a three-dimensional tensor which is not equal to $e\delta_\alpha^\beta$. For an elementary particle $e_\beta^\alpha = (\delta_\beta^\alpha + \chi_\beta^\alpha)e$ where χ_β^α is the effect of gravitation, produced by the tensorial character of the mass (33). Generally it depends on the properties and the speed of the particle, so that the total charge tensor of a system is not proportional to its total charge, $e_0 = \Sigma e$, as well as the tensor

$$\Delta_\beta{}^\alpha = e_\beta{}^\alpha - e_0 \delta_\beta{}^\alpha, \tag{42}$$

which, consequently would manifest itself as a tensor charge supplementary to e_0, producing an additional deflection of the particle in an electric field. The Δ-effect of ions and elementary particles is undetectable, since for these objects it is $\lesssim c^{-2}\varphi e_0$, where φ is the Newtonian potential. But for nonionized atoms and molecules the Δ-effect can be experimentally estimated from the difference between the charge e_0 (measured by means of the method of Piccard and Kessler[20]) and the tensor e_β^α determined directly from the electrostatic deflection of an atomic or molecular beam. Zorn, Chamberlain and Hughes[21] have tried to measure e_0 by means of the beam method, but their results refer, obviously, not to e_0, but to e_β^α. So far, for nonionized particles neither e_0 nor e_β^α have been detected so far. From data on SF_6 (Shull, Brillman and Wedgwood[22])

$$|e_0| \leqslant 10^{-22} A |e_e|, \tag{43}$$

where A is the total number of nucleons, e_e is the electron charge; from the data of[21] for cesium, one gets for nonionized particles

$$|e_\beta{}^\alpha| \leqslant 10^{-19} A |e_e|. \tag{44}$$

consequently

$$|\Delta_\beta{}^\alpha| \leqslant 10^{-19} A |e_e|, \tag{45}$$

which limits at present the Δ-effect.

The theoretical order of magnitude of the Δ-effect can be determined phenomenologically from the classical equation (34), and the quantum properties of the system may be taken into account only in the averaging of the final expressions. Up to the manifestly unobservable quantities $\lesssim c^{-2}\varphi e_0$ and the nonlinearities in the gravitation, one obtains for an atom or a molecule

$$\Delta \sim \Delta_0 + Ze\epsilon(\delta X_T + \nu + \delta' + (v_0/c)^2 y), \tag{46}$$

where Δ_0 is the order of magnitude of the part of the tensor (42) which is compatible with the equivalence principle, and the remainder is the contribution of the tensor Z_{ik}; Z is the number of electrons, v_0 is the maximal intraatomic velocity, δX_T is defined in (36), $\nu < 5 \times 10^{-23}$ [19] is the galactic mass anisotropy, δ' is the difference of the galactic magnitudes of the proton and electron (cf. above), which may be absent, in view of its galacticity, and y is the sum of the uncompensated galactic quantities of the type (40). The effect Δ_0 is produced by the impossibility of excluding rigorously gravitation from the laboratory metric of the atom along its whole trajectory, and depends on the choice of the coordinates. But since experimentally the coordinates of the atomic beam are in fact measured by means of the Wigner method[23], at the origin of the beam the conformal metric must be flat and should not involve first derivatives, but then at a distance from the origin, in any coordinates $\Delta_0 \sim (v_0/c)^2 W l^2 Ze$, where $W \sim c^{-2} g_0 a_T^{-1}$ is the Weyl tensor, i.e.,

$$\Delta_0 \sim 10^{-23}(v_0/c)^2 l^2 Ze\epsilon, \tag{47}$$

where l is measured in meters. We note that the harmonic coordinates of de Donder and Fock do not satisfy the indicated initial condition and lead to the estimate

$$\Delta_0 \sim (v_0/c)^2 W a_T^2 Ze_\epsilon \sim 10^{-11} Ze_\epsilon,$$

which contradicts the experimental result (45).

According to (36), (47) and (46) the upper limit (45) refers to $y(v_0/c)^2 Ze$; consequently

$$y_n \leqslant y \leqslant 10^{-17}, \quad |G_n| \leqslant 10^{-16} G, \tag{48}$$

in the notations (39) and (40). In distinction from (39) and from the data on the anisotropy of mass this estimate no longer contains any unknowns. According to (41) the upper limits (48) and (45) seem to be exaggerated at least by one order of magnitude, so that, judging by the x-ray spectra of nebulae, there are no anomalies in the electromagnetic acceleration of electrons up to energies of $\sim 10^9 mc^2$, and then $\theta \lesssim 1$, at least up to $u \sim 10^9$. Thus, from all estimates of the constants G_n they are so small that in reality they can hardly be distinguished from exact zero. At any rate, one can assert rigorously that if there is a direct interaction of matter with nonmetric fields of the type r^{-1} (only such an interaction would lead, according to (34) to an anomalous long-range interaction decreasing according to Newton's law), then at least for energies $\lesssim (10^8 - 10^9) mc^2$, it is $\lesssim 10^{-16}$ times the Newtonian. Consequently, within these limits, a long-range antigravitation is strictly excluded, and if the Lagrangian L_1 in (30) does not contain explicitly the curvature tensor for other gradients, which are important at short distances, then at least up to the indicated energies, gravitational collapse will be subject to the equivalence principle.

Since the accuracy of measurements of the electromagnetic deflection of atomic beams can hardly be improved[22] by more than three orders of magnitude, a more precise value for the Δ-effect will either insignificantly lower the upper limits (which are exaggerated by at least one order of magnitude, so it seems) (48) and (45), or, and this is rather doubtful, will prove the existence of an effect $\Delta \gtrsim 10^{-22} Ze$. According to (47) such an effect would establish a violation of the equivalence principle, and according to (46) and (36) an effect $\Delta \gtrsim 10^{-21} Ze$ would prove the presence in (30) of galactic fields with $G_n \geq 2 > 10^{-18}$ G. This would mean, on the basis of (39), that $s_0 > 10^7$, and in this case, according to (37) the geomagnetic deflection of protons with energy $> 10^5 mc^2$ would strongly differ from the usual one. For a positive result, exact measurements of the Δ-effect would be extremely important.

9. CONCLUSION

So far there are no clearcut experimental arguments in favor of the Einstein equations or any other concrete theory of gravitation, since in the field of the Sun the empirical limits of δ_0 are so far much above the limit (1) and could correspond to non-Einsteinian equations; in order to select among the latter more accurate data will be necessary.

It is desirable to have detectors for the three gravitational waves which are excluded by the Einstein equations.

The data on the deflection of light-rays in the gravitational field of the sun (and apparently only these data) could solve the problem on the canonicity of the gravitational equations and on the generation of nonmetric fields by the curvature tensor field. According to (15) and (23)—(25), for any $\gamma(r) = \text{const} > +10^{-3}$, under the conditions $A_{1\text{-}5}$, gravitation must be subject to partial differential equations of order higher than second, and the curvature tensor cannot generate nonmetric fields. This would also be guaranteed by all $\gamma(r) = \text{const} < -0.25$, but these (in distinction from the former) are excluded by the available data. For definitive conclusions one must establish the radial dependence of the light deflection in the field of the sun and measure angles to an accuracy not lower than several 0.001″, which would guarantee the $|\gamma| \sim 10^{-2}$, expected according to (26). This should be considered as one of the most important experimental problems.

Within the limits of their low reliability, the available data on light deflection and the oblateness of the sun indicate that the curvature tensor does not engender nonmetric fields and that the equations of gravitation are noncanonical. One can show that the quantum gravitational polarization of the vacuum introduces into the matter tensor corrections involving derivatives of the curvature, containing derivatives of g_{ik} of order not lower than fourth. Therefore the genuine equations of gravitation are manifestly noncanonical (in spite of the Einstein equations), but only future exact measurements will show whether the equations of gravitation contain noncanonicities other than the one due to vacuum polarization.

Gravitation is a peculiar interaction, for which the sources are not only particles but also any decays of particles.

The Eötvös experiment cannot clarify the nature of gravitational fields which interact directly with matter, and only limits scalar violations of the equivalence principle. The tensors (32) are limited by geomagnetic effects and by the charge of nonionized atoms; according to these matter either does not interact at all with nonmetric fields which decrease like r^{-1}, or that interaction is $\lesssim 10^{-16}$ times the Newtonian one, for energies $\lesssim 10^8 mc^2$, at least. Therefore, gradient violations of the equivalence principle would be important, violations which would manifest themselves in the Eötvös experiment, in the anomalies of the relativistic geomagnetic effect or in ultrarelativistic stages of gravitational collapse, for which so far there are no data at all.

At present the most acceptable assumption is that gravitation corresponds to the principles of Einstein's general theory of relativity, but may be subject to noncanonical equations.

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[1)]Cosmological and quantum corrections to the effects under discussion are not observable.
[2)]The eccentricity has been omitted (its contribution to the shift is $\lesssim 1\%$).

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