High frequency current states in small size superconductors

V. P. Galaĭko
Physico-technical Institute of Low Temperatures, Ukrainian Academy of Sciences
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A Boltzmann kinetic equation, which contains the self-consistent field of the superconducting order parameter, is derived for quasiparticles scattered by impurities from the kinetic equation for the generalized electron-hole density matrix in the case of "pure" superconductors. After completion of the rapid formation of the superconducting condensate, the equation describes nonlinear nonstationary processes in the system, with frequencies $\omega \ll T_c$. The equation is employed to analyze the problem of high-frequency spatially homogeneous current states in small superconducting samples. The nonlinear response—a high-frequency electric field corresponding to the given external current passed through the sample—is also found.

1. Nonlinear, nonstationary phenomena in superconductors were investigated on the basis of the BCS microscopic theory[1] by Gor'kov and Éliashberg[2-4]. The dynamical scheme used by them to describe the kinetic phenomena is based on an analytic continuation of the Gor'kov equations[5] for the electron Green's functions into the region of real frequencies. The authors were mainly interested in the temperature region close to the critical temperature T_c of the superconducting transition. In this region, the rate of relaxation of the superconducting order parameter is small, and this relaxation which is described by different generalizations of the so-called time-dependent Ginzburg-Landau equations[6,7], is the dominating process.

In earlier papers (see[8,9]) the author has proposed to describe kinetic phenomena in superconductors on the basis of the BCS theory by using for a generalized electron density matrix a kinetic equation that is not diagonal in the isotopic spin in the "electron-hole" space, and thus takes into account the Cooper pairing of the superconducting electrons. For the density matrix γ (in the coordinate representation $\gamma(\mathbf{r}_1, \mathbf{r}_2)$), this equation takes the form

$$i\frac{\partial\gamma}{\partial t} = [\hat{\varepsilon}(\gamma) + \hat{U}, \gamma] + iL^{(2)}(\gamma),$$
$$\hat{\varepsilon}(\mathbf{r}_1, \mathbf{r}_2) = \hat{\varepsilon}_{r_1}\delta(\mathbf{r}_1 - \mathbf{r}_2),$$
$$\hat{\varepsilon}_r = \sigma_z\left[\mathscr{E}(\hat{\mathbf{p}} + \sigma_z\mathbf{p}_s(\mathbf{r})) + e\varphi(\mathbf{r}) + \frac{1}{2}\frac{\partial\chi(\mathbf{r})}{\partial t}\right] + \hat{\xi}(\mathbf{r}) + \sigma_x\Delta(\mathbf{r}), \quad (1)$$
$$\hat{\xi}(\mathbf{r}) = \sigma_z\frac{gN(\mathbf{r})}{2} \quad \left(N_\uparrow = N_\downarrow = \frac{N}{2}\right), \quad \mathscr{E}(\hat{\mathbf{p}}) = \frac{\hat{\mathbf{p}}^2}{2m}, \quad \hat{\mathbf{p}} = -i\nabla,$$
$$\mathbf{p}_s = m\mathbf{v}_s = {}^1/_2(\nabla\chi - 2e\mathbf{A}) \qquad (\hbar = c = 1).$$

Here e and m are the electron charge and mass, N is the electron density, φ and A are the scalar and vector potentials of the electromagnetic field, σ_x and σ_z are Pauli matrices, $g < 0$ is the constant of effective attraction between the electrons, and χ and Δ are the phase and the modulus of the superconducting order parameter, the latter being defined by the formula

$$\Delta(\mathbf{r}) = {}^1/_2|g|\,\mathrm{Tr}\,(\sigma_x\gamma(\mathbf{r}, \mathbf{r})). \quad (2)$$

The electron density N and the electric-current density j are expressed in terms of the density matrix:

$$N(\mathbf{r}) = \mathrm{Tr}[{}^1/_2\delta(\mathbf{r} - \mathbf{r}') - \sigma_z\gamma(\mathbf{r}, \mathbf{r}')]|_{r'=r},$$
$$\mathbf{j}(\mathbf{r}) = N(\mathbf{r})e\mathbf{v}_s(\mathbf{r}) + \frac{e}{2m}(\hat{\mathbf{p}} - \mathbf{p}')\,\mathrm{Tr}\left[\frac{1}{2}\delta(\mathbf{r} - \mathbf{r}') - \gamma(\mathbf{r}, \mathbf{r}')\right]\Big|_{r'=r}. \quad (3)$$

The additional equation that determines the phase χ of the order parameter is the continuity equation

$$e\partial N/\partial t + \mathrm{div}\,\mathbf{j} = 0,$$

which in this case (after introducing the phase χ) is not an identical consequence of Eqs. (1)—(3). In Eq. (1) we have separated, for future use, the potential U of the scattering of electrons by the impurities. In the coordinate representation we have

$$\hat{U}(\mathbf{r}_1, \mathbf{r}_2) = \sigma_z\delta(\mathbf{r}_1 - \mathbf{r}_2)\sum_k U(\mathbf{r}_1 - \mathbf{r}_k), \quad (4)$$

where $U(\mathbf{r}_1 - \mathbf{r}_k)$ is the potential for scattering by one impurity located at the point $\mathbf{r}_k$.

The last term in the right-hand side of (1) describes the energy relaxation of the electrons. The concrete form of the "collision integral" $L^{(2)}(\gamma)$ depends on whether the electron-electron or the electron-phonon collisions predominate. The characteristic frequencies of these collisions are relatively small ($1/\tau_2 \sim 10^8\ \mathrm{sec}^{-1} \ll T_c \sim 10^{12}\ \mathrm{sec}^{-1}$), so that the last term in (1) describes in the general case the slowest stage of the relaxation of the system to the equilibrium state.

Jointly with Maxwell's equations, Eqs. (1)—(3) form a complete system of equations describing the behavior of the superconductor in the electromagnetic field. In the case of real metals, these equations admit of certain simplifications. As is well known, owing to the large electron density in the metals, the equation $\mathrm{div}\,\mathbf{E} = 4\pi e\delta N$, which determines the electric field E, reduces to the simpler equation $\delta N = 0$. For this reason, we can neglect the displacement current in the equations for the magnetic field H:

$$\mathrm{rot}\,\mathbf{H} = 4\pi\mathbf{j} + \partial\mathbf{E}/\partial t \approx 4\pi\mathbf{j}, \quad \mathrm{div}\,\mathbf{H} = 0.$$

The continuity equation expresses accordingly in this case the transversality of the current: $\mathrm{div}\,\mathbf{j} = 0$.

Since the scalar and vector potentials of the electromagnetic field enter in Eq. (1) in gauge-invariant combinations $e\varphi + (1/2)\partial\chi/\partial t$ and $\nabla\chi - 2e\mathbf{A}$, it is expedient to choose as the independent quantities the momentum $\mathbf{p}_s = (\nabla\chi - 2e\mathbf{A})/2$ of the superconducting condensate and the local chemical potential μ, defined by the relation

$$-\mu = \frac{1}{2}\left(\frac{\partial\chi}{\partial t} + gN\right) + e\varphi + \frac{mv_s^2}{2}. \quad (5)$$

Then the operator $\hat{\varepsilon}_r$ (1) takes the form[1)]

$$\hat{\varepsilon}_r = \sigma_z \xi(\hat{p}) + \mathbf{v}_s \hat{\mathbf{p}} + \sigma_x \Delta, \quad \xi(\hat{p}) = \frac{\hat{\mathbf{p}}^2}{2m} - \mu. \tag{6}$$

The quantity μ is determined in this case from the electroneutrality condition $\delta N = 0$, and the condensate momentum p_s, by definition (see (1)), is determined from the equation curl $p_s = -eH$ and from the continuity equation div $j = 0$.

In view of the definition (5), the electric field is determined directly from the relation

$$\partial \mathbf{p}_s / \partial t = e\mathbf{E} - \nabla(\mu + m v_s^2 / 2).$$

It can be easily verified by taking the curls of both halves of the last equation that Maxwell's equation curl $E = -\partial H/\partial t$ is identically satisfied.

We shall illustrate the application of Eqs. (1)–(3) to the solution of concrete problems by using as an example the problem of high-frequency spatially-homogeneous current states in superconductors.

Owing to the Meissner effect, spatially-homogeneous flow of current is possible only in superconducting samples of sufficiently small dimensions (smaller than the depth of penetration of the field). In this case the magnetic field, at a given current, is small and the equations presented above, which determine the momentum of the condensate p_s, the chemical potential μ, and the electric field $\mathbf{E}$ corresponding to the given current, reduce to the following simple system

$$\mathbf{j} = \mathbf{j}_{ext}, \quad \delta N = 0, \quad \partial \mathbf{p}_s / \partial t = e\mathbf{E}, \tag{7}$$

where $\mathbf{j}$ and N are defined in (3) and $\mathbf{j}_{ext}$ is the given external current, which depends on the manner in which the superconducting element is connected in the external circuit.

Since the purpose of the present paper is to investigate nonlinear nonequilibrium processes (and not to calculate the linear response of the system to a small external perturbation), it is important to establish the main relations between the characteristic frequencies of the problem and to separate by the same token the fast and slow processes that occur in the system. At temperatures not to close to critical, the formation of a superconducting condensate, i.e., the relaxation of the order parameter Δ and the diagonalization of the density matrix in the representation of the quasiparticle energy operator $\hat{\epsilon}$ (6), occur (see[8]) at frequencies on the order of T_c (in volumes $\sim \xi_0^3$, $\xi_0 \sim v_F/T_c$). An important role is played by the relation between the quantity T_c and the electron-impurity collision frequency $1/\tau$. It is known (see, e.g.,[10]) that superconductors with $\tau T_c \gg 1$ and $\tau T_c \ll 1$ differ strongly in their magnetic properties. In the present paper we consider the simpler case of "pure" superconductors $\tau T_c \gg 1$, in which the electron mean free path $l = v_F \tau$ is large in comparison with the coherence length $\xi_0 \sim v_F/T_c$ ($l \gg \xi_0$) of the superconducting electrons. The inequality $\tau \ll \tau_2$ is likewise practically always satisfied in this case.

As to the frequency ω of the oscillations of the current and of other macroscopic quantities, in the case $\omega \gg T_c$ the superconductor behaves like a normal metal. It is therefore of interest to investigate the opposite case, $\omega \ll T_c$, and moreover, it is necessary to put $\omega \ll 1/\tau$, for otherwise the scattering by the impurities is negligible and the relaxation of the electrons is connected with the spatial dispersion, which is not taken into account in this problem. If at the same time the frequency ω is so small that $\omega \ll 1/\tau_2$, then an equilibrium state has time to be established in the system during the variation of the current, and is well described by the well known formula[11] for the dependence of the current $\mathbf{j}$ on the condensate velocity $\mathbf{v}_s$. Thus, the principal inequalities that will be used from now on take the following form:

$$T_c \gg 1/\tau \gg \omega \gg 1/\tau_2.$$

2. The first of the inequalities, $T_c \gg 1/\tau$, makes it possible, when considering collisions of electrons with impurities, to regard the fast process of formation of a superconducting condensate and of quasiparticles corresponding to the energy operator $\hat{\epsilon}$ (6) as completed, and consequently to describe these conditions with the aid of a Boltzmann kinetic equation. Using well known methods[12], it is easy to find the explicit form of the kinetic equation from Eq. (1) for the density matrix. According to the foregoing, in the absence of impurities the density matrix is synchronized within a time $\sim 1/T_c$ with the diagonal matrix $\gamma^{(0)}$:

$$\gamma \to \gamma^{(0)} = \sum_\lambda \varphi_\lambda E_\lambda(\varphi), \quad [\hat{\varepsilon}(\gamma^{(0)}), \gamma^{(0)}] = 0, \tag{8}$$

where φ_λ is the excitation distribution function, E_λ are the projection operators constituting the expansion of unity for the operator $\hat{\epsilon}$ (6) (λ is the total set of the quantum numbers, $\lambda = (p, \sigma)$, p is the momentum, $\sigma = \pm 1$):

$$E_\lambda(\mathbf{r}_1, \mathbf{r}_2) \equiv E_{\mathbf{p},\sigma}(\mathbf{r}_1, \mathbf{r}_2) = \frac{\exp[i\mathbf{p}(\mathbf{r}_1 - \mathbf{r}_2)]}{V} \frac{1}{2}\left(1 + \sigma \frac{\sigma_z \xi_p + \sigma_x \Delta}{\varepsilon_p}\right),$$
$$\hat{\varepsilon}(\gamma^{(0)}) = \sum_\lambda \varepsilon_\lambda(\varphi) E_\lambda(\varphi), \quad \varepsilon_\lambda \equiv \varepsilon_{p,\sigma} = \sigma \varepsilon_p + \mathbf{p}\mathbf{v}_s, \quad \varepsilon_p = \sqrt{\xi_p^2 + \Delta^2}, \tag{9}$$

V is a normalization volume.

To describe the slow process of relaxation of the excitations on the impurities, it is necessary to represent the density matrix γ in the form

$$\gamma = \gamma^{(0)}(\varphi) + \gamma^{(1)}(\varphi) = \sum_\lambda \varphi_\lambda E_\lambda(\varphi) + \gamma^{(1)}(\varphi), \tag{10}$$

where $\gamma^{(1)}$ is a small nondiagonal correction to the matrix $\gamma^{(0)}$:

$$\mathrm{Sp}\,(E_\lambda \gamma^{(1)}) = 0, \tag{11}$$

and the slow variation of the distribution function φ_λ should be described by a kinetic equation in the form

$$\partial \varphi_\lambda / \partial t = I_\lambda(\varphi). \tag{12}$$

Substituting expression (10) in (1) and omitting small terms of order $(\gamma^{(1)})^2$, we obtain, in view of relations (8) and (12),

$$[\hat{\varepsilon} + \hat{U}, \gamma^{(1)}] = A,$$
$$A = i\sum_\lambda I_\lambda \Big(E_\lambda + \sum_{\lambda'} \varphi_{\lambda'} \partial E_{\lambda'} / \partial \varphi_\lambda\Big) - [\delta\hat{\varepsilon} + \hat{U}, \gamma^{(0)}] - iL^{(2)}(\gamma^{(0)}), \tag{13}$$
$$\delta\hat{\varepsilon} = \hat{\varepsilon}(\gamma) - \hat{\varepsilon}(\gamma^{(0)}) \sim \gamma^{(1)}.$$

A boundary condition that fixes the solution of the homogeneous equation follows for Eq. (13) from the synchronization requirement (8) and, in accordance with Eq. (1) (at $\hat{U} = 0$ and $L^{(2)} = 0$), can be written down for the off-diagonal matrix $\gamma^{(1)}$ in the form

$$t \to \infty, \quad e^{-i\hat{\varepsilon}t} \gamma^{(1)} e^{i\hat{\varepsilon}t} \to 0.$$

Taking this condition into account, Eq. (13) takes the form of the integral equation

$$\gamma^{(1)} = i\int_0^\infty dt\, e^{-2\eta t} e^{-i\hat{\varepsilon}t}(A - [\hat{U}, \gamma^{(1)}])\, e^{i\hat{\varepsilon}t}, \quad \eta \to +0.$$

It follows therefore that Eq. (13), together with the boundary condition, becomes

$$[\hat{\varepsilon} + \hat{U}, \gamma^{(1)}] - 2i\eta\gamma^{(1)} = A, \quad \eta \to +0.$$

The formal solution of this equation (see [13]) is

$$\gamma^{(1)} = \frac{i}{2\pi}\int_{-\infty}^{+\infty} d\omega\, R^+(\omega) A R^-(\omega), \quad \eta \to +0, \tag{14}$$

where R(z) is the resolvent operator:

$$R(z) = [z - (\hat{\varepsilon} + \hat{U})]^{-1}, \quad R^\pm(\omega) = R(\omega \pm i\eta),$$
$$R(z) = \frac{1}{d(z)} + \frac{1}{d(z)} T(z) \frac{1}{d(z)}, \quad d(z) = z - \hat{\varepsilon}, \tag{15}$$

T(z) is the operator for scattering by impurities.

Substituting the obtained solution (14) (A is defined in (13)) in the condition (11) and taking into account the easily-proved identities

$$\mathrm{Sp}\,(E_\lambda \partial E_{\lambda'}) = 0 \quad \mathrm{Sp}\,(E_\lambda [\delta\hat{\varepsilon}, \gamma^{(0)}]) = 0,$$

we obtain, after discarding small terms of higher order, the following expressions for the collision integral (12):

$$I_\lambda = I_\lambda^{(i)} + I_\lambda^{(2)}, \quad I_\lambda^{(2)} = \mathrm{Sp}(E_\lambda L^{(2)}(\gamma^{(0)})),$$
$$I_\lambda^{(i)} = \frac{\eta}{i\pi}\int_{-\infty}^{\infty} d\omega\, \mathrm{Sp}(E_\lambda R^+(\omega)[\hat{U}, \gamma^{(0)}] R^-(\omega)), \quad \eta \to +0. \tag{16}$$

Here $I_\lambda^{(i)}$ is the quasiparticle-impurity collision integral and $I_\lambda^{(2)}$ is the collision integral describing the quasiparticle energy relaxation. The explicit form of $I_\lambda^{(2)}$ is given in [8] for the case of electron-electron collisions.

Further formal transformations in expression (16) for the collision integral with impurities $I_\lambda^{(i)}$ are based on the properties of the resolvent operator (15) and coincide with those given in [13]. When account is taken of formulas (4) and (19) in the approximation linear in the impurity concentration and in the Born approximation for the electron-impurity scattering amplitude, these calculations lead to the following kinetic equation (in the limit as $V \to \infty$):

$$\partial\varphi_{p,\sigma}/\partial t = I_{p,\sigma}^{(i)}(\varphi) + I_{p,\sigma}^{(2)}(\varphi),$$
$$I_{p,\sigma}^{(i)} = n\int \frac{d^3p'}{(2\pi)^2} |U(\mathbf{p} - \mathbf{p}')|^2 \sum_{\sigma'=\pm 1} \frac{1}{2}\left(1 + \sigma\sigma' \frac{\xi_p \xi_{p'} - \Delta^2}{\varepsilon_p \varepsilon_{p'}}\right)$$
$$\times (\varphi_{p',\sigma'} - \varphi_{p,\sigma})\,\delta(\varepsilon_{p',\sigma'} - \varepsilon_{p,\sigma}), \tag{17}$$
$$\varepsilon_{p,\sigma} = \sigma\varepsilon_p + \mathbf{p}\mathbf{v}_s, \quad \varepsilon_p = \sqrt{\xi_p^2 + \Delta^2},$$

where n is the impurity concentration and U(p) is the Fourier transform of the electron-impurity interaction potential.

Substituting the expression (3) for the density matrix γ and formulas (9) in Eq. (2) and in expressions (3), we obtain in the principal approximation

$$\Delta = \frac{|g|}{2}\int \frac{d^3\mathbf{p}}{(2\pi)^3} \sum_\sigma \frac{\Delta}{\varepsilon_p} \sigma\varphi_{p,\sigma}, \tag{18}$$

$$N = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\left(1 - \frac{\xi_p}{\varepsilon_p}\sum_\sigma \sigma\varphi_{p,\sigma}\right), \tag{19}$$

$$\mathbf{j} = Ne\mathbf{v}_s + \frac{e}{m}\int \frac{d^3\mathbf{p}}{(2\pi)^3}\mathbf{p}\left(1 - \sum_\sigma \varphi_{p,\sigma}\right). \tag{20}$$

The kinetic equation (17) is outwardly similar to that used earlier (see [14,15]) to calculate of the linear responses in the microscopic theory of superconductivity. It is necessary, however, to emphasize that here this equation has been obtained on the basis of the inequality $1/\tau \ll T_c$ as a result of consolation in the description of the system with the aid of density matrix γ and Eq. (1) (and not as a result of the probability of the transition per unit time for a weakly excited system of quasiparticles). Therefore Eq. (17) together with the self-consistency equation (18) constitutes a complicated nonlinear system of integro-differential equations, in which the external "fields" p_s, Δ and μ depend on the time.

Owing to the shift of the quasiparticle energy $\sigma\epsilon_p$ by an amount $\mathbf{p}\cdot\mathbf{v}_s$ ($\epsilon_{p,s} = \sigma\epsilon_p + \mathbf{p}\cdot\mathbf{v}_s$), which is contained in the impurity collision integral (17), the energy conservation law is written out in a reference frame connected with the impurity-containing crystal lattice. The equilibrium distribution function is therefore

$$\varphi_{p,\sigma}^{eq} = 1 - \left[\exp\left(\frac{\sigma\varepsilon_p + \mathbf{p}\mathbf{v}_s}{T}\right) + 1\right]^{-1} \tag{21}$$

and corresponds, as it should, to a complete stoppage of the normal component.

Using a mixed Wigner representation for the density matrix γ, we can show that in the spatially-inhomogeneous case (with a characteristic inhomogeneity radius $r \gg \xi_0 \sim v_F/T_c$), Eq. (17) should be supplemented in the left-hand side by convective terms that represent classical Poisson brackets[2)]:

$$(\varepsilon_{p,\sigma}, \varphi_{p,\sigma}) \equiv \frac{\partial\varepsilon_{p,\sigma}}{\partial\mathbf{p}}\frac{\partial\varphi_{p,\sigma}}{\partial\mathbf{r}} - \frac{\partial\varepsilon_{p,\sigma}}{\partial\mathbf{r}}\frac{\partial\varphi_{p,\sigma}}{\partial\mathbf{p}}. \tag{22}$$

3. In view of the inequality $\omega \ll 1/\tau$, the solution of (17) can be obtained by expanding in the frequency ω, Since the density change δN (19), unlike the parameter Δ (18) and the current density j (20), is determined by a function $\varphi_{p,\sigma}$ that is odd in the electron energy ξ_p reckoned from the Fermi boundary, it suffices to confine oneself henceforth to a distribution function $\varphi_{p,\sigma}$ that is even in ξ_p. Then the electroneutrality condition $\delta N = 0$ is identically satisfied and we can assume μ = const. To separate in explicit form the small terms in (17), which are proportional to the rate of change of the macroscopic quantities, it is expedient to change variables in phase space. Changing over in the collision integral (17), in the usual manner, to integration with respect to the angles and the energy ξ in the vicinity of the Fermi boundary, and recognizing that the distribution function is even in ξ, we choose as an independent variable in addition to the unit vector $\mathbf{n} = \mathbf{p}/p$, the quasiparticle energy:

$$E = \sigma\varepsilon + \mathbf{n}\mathbf{w}, \quad \mathbf{w} = p_F\mathbf{v}_s, \quad \varepsilon = \sqrt{\xi^2 + \Delta^2},$$
$$\varphi_1(\varepsilon, \mathbf{n}) = \varphi(\varepsilon, \mathbf{n}), \quad \varphi_{-1}(\varepsilon, \mathbf{n}) = \varphi(-\varepsilon, \mathbf{n}). \tag{23}$$

In terms of these variables, Eq. (17) takes the form

$$\left[\frac{\partial}{\partial t} + (u(E, \mathbf{n})\dot{\Delta} + \mathbf{n}\dot{\mathbf{w}})\frac{\partial}{\partial E}\right]\varphi(E, \mathbf{n}) = I(\varphi) + I^{(2)}(\varphi),$$
$$I(\varphi) = \frac{1}{\tau}\int \frac{dO'}{4\pi}\nu(E, \mathbf{n}')\,\theta((E - \mathbf{n}'\mathbf{w})^2 - \Delta^2)\,(1 - u(E, \mathbf{n})u(E, \mathbf{n}')) \tag{24}$$
$$\times (\varphi(E, \mathbf{n}') - \varphi(E, \mathbf{n})).$$

The dot denotes here differentiation with respect to time and, in addition,

$$u(E, \mathbf{n}) = \frac{\Delta}{E - \mathbf{n}\mathbf{w}}, \quad \nu(E, \mathbf{n}) = \frac{|E - \mathbf{n}\mathbf{w}|}{\sqrt{(E - \mathbf{n}\mathbf{w})^2 - \Delta^2}}, \quad \theta(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}, \tag{25}$$

$1/\tau = nmp_F U_0^2/\pi$ is the frequency of the collisions with impurities, the scattering by which is assumed for simplicity to be isotropic ($U(p - p') = U_0$ const on the Fermi surface). In the zeroth approximation in the parameter $\omega\tau \ll 1$, Eq. (24) takes the form

$$I(\varphi) \equiv a\tau^{-1}[\langle\varphi\rangle - \varphi - u(\langle u\varphi\rangle - \langle u\rangle\varphi)] = 0, \qquad (26)$$

while the angle brackets denote averaging over the angles in accordance with the formula

$$\langle f(E, \mathbf{n})\rangle = \frac{1}{a}\int \frac{dO}{4\pi}\nu(E, \mathbf{n})\theta((E - \mathbf{nw})^2 - \Delta^2) f(E, \mathbf{n}), \qquad (27)$$

$$a = \int \frac{dO}{4\pi}\nu(E, \mathbf{n})\theta((E - \mathbf{nw})^2 - \Delta^2). \qquad (28)$$

A unique solution of (26) is an arbitrary isotropic function of the energy $\varphi = \varphi^{(0)}(E)$. In view of the inequality $\omega \gg 1/\tau_2$, Eq. (24) takes in the next-higher approximation the form

$$\frac{a}{\tau}[\langle\varphi^{(1)}\rangle - \varphi^{(1)} - u(\langle u\varphi^{(1)}\rangle - \langle u\rangle\varphi^{(1)})] = \left[\frac{\partial}{\partial t} + (u\dot{\Delta} + \mathbf{n}\dot{\mathbf{w}})\frac{\partial}{\partial E}\right]\varphi^{(0)}. \qquad (29)$$

From the condition that this equation have a solution, namely

$$\frac{\partial\varphi^{(0)}}{\partial t} + (\langle u\rangle\dot{\Delta} + \langle\mathbf{n}\dot{\mathbf{w}}\rangle)\frac{\partial\varphi^{(0)}}{\partial E} = 0 \qquad (30)$$

we get an expression for the function $\varphi^{(0)}(t, E)$:

$$a(t, E)\frac{\partial\varphi^{(0)}(t, E)}{\partial t} + b(t, E)\frac{\partial\varphi^{(0)}(t, E)}{\partial E} = 0, \qquad (31)$$

where

$$b = a(\langle u\rangle\dot{\Delta} + \langle\mathbf{n}\dot{\mathbf{w}}\rangle) = \int\frac{dO}{4\pi}\nu(E, \mathbf{n})\theta((E - \mathbf{nw})^2 - \Delta^2)(u(E, \mathbf{n})\dot{\Delta} + \mathbf{n}\dot{\mathbf{w}}). \qquad (32)$$

Assuming relation (30) to be satisfied and substituting the value of the derivative $\partial\varphi^{(0)}/\partial t$ from formula (30) in (29), we obtain the solution of this equation under the additional condition $\langle\varphi^{(1)}\rangle = 0$:

$$\varphi^{(1)} = (1 - \langle u\rangle u)^{-1}\left[f - u\left\langle\frac{f}{1 - \langle u\rangle u}\right\rangle\left\langle\frac{u}{1 - \langle u\rangle u}\right\rangle^{-1}\right], \quad \langle\varphi^{(1)}\rangle = 0,$$
$$f = \frac{1}{a}\frac{\partial\varphi^{(0)}}{\partial E}\tau[\dot{\Delta}(\langle u\rangle - u) + \langle\mathbf{n}\dot{\mathbf{w}}\rangle - n\dot{\mathbf{w}}]. \qquad (33)$$

In the usual situation in a normal metal, the change of the quasiequilibrium distribution function $\varphi^{(0)}(t, E)$ is determined by the Joule heat, i.e., by the terms quadratic in the field or, equivalently, in the frequency ($E^2 \sim \dot{p}_s^2 \sim \omega^2$). In the given nonlinear case, owing to the appreciable influence of the field on the quasiparticle spectrum, the distribution function varies in accordance with Eq. (30) at the same rate as the field. Therefore the dynamics of the distribution of the superconducting electrons turns out to be more complicated and the energy E of the quasiparticles is not a "good" variable. However, by direct differentiation of the functions $a(t, E)$ (28) and $b(t, E)$ (32), with allowance for the definitions (25), it can be shown that Eq. (31) for the function $\varphi^{(0)}(t, E)$ has the following properties: in the corresponding characteristic equation $adE - bdt = 0$, the coefficients a and b satisfy the identity $\partial a/\partial t \neq -\partial b/\partial E$ for all Δ and p_s, and consequently the left-hand of this equation is a complete differential of a function that can be obtained as a result of elementary integration:

$$d\Xi = adE - bdt, \quad \Xi = \frac{\Delta^2}{2w}\int_{x_-}^{x_+} dx\,\theta(x^2 - 1)\,\mathrm{sign}\,x\sqrt{x^2 - 1}, \quad x_\pm = \frac{E \pm w}{\Delta}. \qquad (34)$$

From the definition of (28) we see that the derivative $\partial\Xi/\partial E = a > 0$ and the function $\Xi(E)$ is monotonic in its entire range of definition. This region, generally speak-

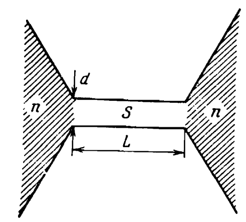

FIG. 1.

ing, has a gap on the energy axis $E(-\Delta + w, \Delta - w)$ at $w < \Delta$, and on the boundaries of the gap (and in the discontinuity inside the gap) we have $\Xi = 0$. Accordingly, the inverse function $E(\Xi)$ has in this case a discontinuity at the point $\Xi = 0$. From (34) we obtain the limiting relations

$$\Xi \approx E, \quad \Delta \to 0; \quad \Xi \approx \sqrt{E^2 - \Delta^2}\,\mathrm{sign}\,E\,\theta(E^2 - \Delta^2), \quad w \to 0.$$

The quasiequilibrium distribution function $\varphi^{(0)}(t, E)$ satisfying Eq. (31) is an arbitrary function of the "integral of motion" Ξ (34): $\varphi^{(0)}(t, E) = \varphi^{(0)}(\Xi)$, which is thus independent of the prior history of the motion of the system and is determined by the instantaneous values of the parameters Δ and v_s ($w = p_F v_s$). The problem consists now of finding the explicit form of the function $\varphi^0(\Xi)$. To this end it would be necessary to take into account in the kinematic equation (24), in second order in the parameter $\omega\tau$, the term $I^{(2)}(\varphi^{(0)})$, which describes the electron energy relaxation. Such a formulation of the problem, however, is not practically reasonable. Realistically, good heat-conduction conditions that ensure the feasibility of stationary states are the result of the contact (thermal and electric) between a small superconducting element and bulky metallic samples in such a way, that the principal energy exchange between the electrons occurs in the interior of these samples, which are in the equilibrium state (Fig. 1). In order for all the electron excitations to be able to penetrate into the interior of these samples and not to be partially trapped inside the element by the Andreev reflection from the boundaries[16], these samples must be in the normal state. In such a formulation, the problem is spatially-inhomogeneous. Using in addition to the expansion in $\omega\tau \ll 1$ also an expansion in the small spatial gradients (in which case it is necessary to take into account the part of the distribution function which is odd in ξ), we derive from the kinetic equation (17) together with the terms (22) the following generalization of (31):

$$a\frac{\partial\varphi^{(0)}}{\partial t} + b\frac{\partial\varphi^{(0)}}{\partial E} = \frac{\partial}{\partial x_i}\left(D_{ik}\frac{\partial\varphi^{(0)}}{\partial x_k}\right), \quad \varphi_\infty^{(0)} = \frac{1}{2}\left(1 + \mathrm{th}\frac{E}{2T}\right). \qquad (35)$$

The physical meaning of this equation is clear. After relaxation on the impurities, the subsequent slow evolution of the electron distribution in space and in time has a diffusion character. The diffusion coefficients D_{ik} are of the order of $v_F^2\tau$ and are complicated functions of the E (and of the parameters Δ and w), the explicit form of which is not necessary here.

The dimensions of the superconducting channel L and d indicated in Fig. 1 (L is the length and d is the diameter of the channel, $L \gg d$) should satisfy definite conditions. The role of the characteristic length in (35) is played by the diffusion length $\sqrt{D/\omega}$. If $L \ll \sqrt{D/\omega}$, then the distribution function coincides in first-order approximation with the boundary equilibrium function (35), which "penetrates" in the interior of the sample.

Interest attaches therefore to the opposite case: $L \gg \sqrt{D/\omega}$. However, the channel should not be too long: $L \ll \sqrt{D\tau_2}$, so as to be able to neglect the collision integral $I^{(2)}$ in (17). The condition for the applicability of (35) is smallness of the spatial gradient: $1/d \ll 1/l$, $l = v_F\tau \sim \sqrt{D\tau}$. We assume also the inequality $d \ll \sqrt{D/\omega}$, which enables us to continue the boundary condition (35) all the way to the end of the channel. It is convenient to combine these equalities into the following chain[3)]

$$1/\tau_2 \ll D/L^2 \ll \omega \ll D/d^2 \ll 1/\tau.$$

Putting $E \to E(X, t)$ in (35) with the aid of (34), we obtain in the interior of the channel

$$a\frac{\partial\varphi^{(0)}(\Xi)}{\partial t} = D\frac{\partial^2\varphi^{(0)}(\Xi)}{\partial x^2}, \quad \varphi^{(0)}_{x=0} = \varphi^{(0)}_{x=L} = \frac{1}{2}\left(1 + \operatorname{th}\frac{E(\Xi, t)}{2T}\right), \tag{36}$$

where x is the coordinate reckoned from the channel axis.

A characteristic feature of the solutions of the diffusion equation (36) is that the high-frequency harmonics penetrate into the channel to a distance on the order of the diffusion length $\sqrt{D/\omega}$, and only the zeroth harmonic penetrates into the interior of the channel. Therefore the stationary solution $\varphi^{(0)}(\Xi)$ should take in the superconducting channel the form

$$\varphi^{(0)}(\Xi) = \frac{1}{2}\left(1 + \overline{\operatorname{th}\frac{E(\Xi)}{2T}}\right), \tag{37}$$

where the superior bar denotes averaging with respect to time. It must be emphasized, however, that actually formula (37) is not an exact solution of the problem, for owing to the self-consistency equations (18), (7), and (20) the quantities Δ and p_s, and with them also the coefficients a (28), b (32), and D_{ik} in (35) are themselves functions of the coordinates. At the same time, expression (37) is a reasonable approximation of the exact solution, which gives the correct transition to the limit as $\omega \to 0$ and takes into account the general fact that nonzero harmonics attenuate in the solution of the diffusion equation. In view of this, the results that follow are mainly approximate.

4. Formulas (37) and (33), together with the definitions (34), (23), (25), (27), and (28), solve our problem formally. This solution is quite complicated and even its numerical analysis is difficult. For a qualitative interpretation we can confine ourselves to sufficiently high temperatures, when we can use an asymptotic expansion in terms of $\Delta/T_c < 1$.[4)] It is convenient to subtract and add the equilibrium distribution function (21) in the integrands of (18) and (20). The terms containing the equilibrium function are expanded in the known manner (see, e.g.,[10]) in terms of the parameter Δ/T_c. Changing over in the remaining terms to integration with respect to the angles and the energy near the Fermi boundary, and replacing the integration with respect to E by integration with respect to (34), we obtain, taking formulas (37), (23), (35), (27), and (28) into account

$$\begin{gathered}\frac{7\zeta(3)}{8(\pi T_c)^2}\left[\Delta_0^2 - \left(\Delta^2 + \frac{2}{3}w^2\right)\right] + \frac{1}{\Delta}\int_{-\infty}^{\infty} d\Xi\left[\langle u\rangle\frac{1}{2}\left(\overline{\operatorname{th}\frac{E(\Xi)}{2T}} - \operatorname{th}\frac{E(\Xi)}{2T}\right) + \langle u\varphi^{(1)}\rangle\right] = 0,\\ \mathbf{j} = Ne\mathbf{v}_s\frac{7\zeta(3)}{4}\left(\frac{\Delta}{\pi T_c}\right)^2 - \frac{3Ne}{p_F}\int_{-\infty}^{\infty} d\Xi\left[\langle\mathbf{n}\rangle\frac{1}{2}\left(\overline{\operatorname{th}\frac{E(\Xi)}{2T}} - \operatorname{th}\frac{E(\Xi)}{2T}\right) + \langle\mathbf{n}\varphi^{(1)}\rangle\right],\end{gathered} \tag{38}$$

where Δ_0 is the BCS gap[1] in the superconductor spectrum, and the function $\varphi^{(1)}$ is defined in accordance with (33) by the following formulas:

$$\begin{gathered}\varphi^{(1)} = (1 - \langle u\rangle u)^{-1}\left[f - u\left\langle\frac{f}{1 - \langle u\rangle u}\right\rangle\left\langle\frac{u}{1 - \langle u\rangle u}\right\rangle^{-1}\right],\\ f = f_\Delta + f_w \quad (\varphi^{(1)} = \varphi^{(1)}_\Delta + \varphi^{(1)}_w),\\ f_\Delta = \tau\dot\Delta(\langle u\rangle - u)\frac{\partial\varphi^{(0)}}{\partial\Xi}, \quad f_w = \tau(\langle\mathbf{n}\dot{\mathbf{w}}\rangle - \mathbf{n}\dot{\mathbf{w}})\frac{\partial\varphi^{(0)}}{\partial\Xi}.\end{gathered} \tag{39}$$

In the obtained expressions (38), all the integrals with respect to Ξ converge at values $\Xi \sim \Delta \ll T_c$, with the exception of the integral containing the function $\varphi^{(1)}_w$ (39) in the expression for the current j (38). In the last integral, the characteristic values are $\Xi \sim T_c$, and we can put $\Delta \approx 0$, which reduces this term to the usual expression $Ne^2\tau E/m = Ne\tau v_s$ for the conductivity of the normal metal. In the remaining terms, using the inequality $E(\Xi) \sim \Delta \sim T_c$, we expand $\tanh(E/2T)$ in terms of E/T. We thus obtain from (38)

$$\begin{gathered}-\frac{7\zeta(3)}{8(\pi T_c)^2}\left[\Delta_0^2 - \left(\Delta^2 + \frac{2}{3}w^2\right)\right] + \frac{1}{2\Delta T_c}\int_0^\infty d\Xi\langle u\rangle(\overline{E(\Xi)} - E(\Xi)) + \frac{1}{\Delta}\int_{-\infty}^{\infty} d\Xi\,\langle u\varphi^{(1)}\rangle = 0,\\ \frac{p_F}{Ne}\mathbf{j} = \frac{7\zeta(3)}{4}\left(\frac{\Delta}{\pi T_c}\right)^2\mathbf{w} + \tau\dot{\mathbf{w}} - \frac{3}{2T_c}\int_0^\infty d\Xi\,\langle\mathbf{n}\rangle(\overline{E(\Xi)} - E(\Xi)) - 3\int d\Xi\,\langle\mathbf{n}\varphi^{(1)}_\Delta\rangle.\end{gathered} \tag{40}$$

These formulas can be simplified further because as will be shown later, one can assume $w \ll \Delta$ for all reasonable values of the current. Denoting by $\delta\Delta$ the small alternating increment to the order parameter ($\Delta \to \Delta + \delta\Delta = 0$), we obtain the first nonvanishing terms in the expansion of formulas (40) in the quantities $\delta\Delta$ and w/Δ. To this end, we consider the difference

$$\overline{E(\Xi)} - E(\Xi) \approx \frac{\Delta(\overline{\delta\Delta} - \delta\Delta)}{\sqrt{\Xi^2 + \Delta^2}} + (\overline{E(\Xi)} - E(\Xi))_w, \tag{41}$$

where the second term denotes the first nonvanishing term in the expansion in w/Δ. When (41) is substituted in Eq. (40) for Δ, the integral with respect to Ξ, which contains the first term of (41), converges at $\Xi \sim \Delta$. For these Ξ we have $\langle u\rangle \approx \Delta/\sqrt{X^2 + \Delta^2}$ (see formula (25)) and we thus obtain

$$\int_0^\infty d\Xi\,\langle u\rangle\frac{\Delta(\overline{\delta\Delta} - \delta\Delta)}{\sqrt{\Xi^2 + \Delta^2}} \approx -\frac{\pi}{2}\Delta\,\delta\Delta. \tag{42}$$

In the second interval, which contains the second term of (41), the characteristic values of Ξ are small: $\Xi \sim \sqrt{w\Delta} \ll \Delta$. For these Ξ we can obtain from (34) the following asymptotic expansion:

$$\begin{gathered}\Xi \sim \sqrt{w\Delta} \ll \Delta, \quad E \approx \Delta + w\psi(3\Xi/\sqrt{2w\Delta});\\ 0 < x < 2^{2/3}, \quad -1 < \psi < 1, \quad x = (\psi + 1)^{3/2};\\ 2^{3/2} < x, \quad 1 < \psi, \quad x = (\psi + 1)^{3/2} - (\psi - 1)^{3/2};\\ x \to \infty, \quad \psi \sim x^2/9 + 3/4x^2.\end{gathered} \tag{43}$$

The function $x(\psi)$ is monotonic and continuous together with the derivative $dx/d\psi$. At the same values of Ξ, in accordance with formula (25), we have $\langle u\rangle \approx 1$. Hence, taking the expansion (43) into account, we find the integral of the second term of (41) in Eq. (40) for Δ is equal to

$$\int_0^\infty d\Xi\,\langle u\rangle\,\overline{(E(\Xi) - E(\Xi))}_w \approx \frac{\alpha\Delta^2}{6}\left[\overline{\left(\frac{2w}{\Delta}\right)^{3/2}} - \left(\frac{2w}{\Delta}\right)^{3/2}\right],$$

$$\alpha = \int_0^\infty dx\,(\psi(x) - x^2/9). \tag{44}$$

Straightforward but cumbersome calculations show that in equation (40) for Δ the last term containing the correction $\varphi^{(1)}$ (39) to the distribution function makes a small contribution to this equation, and can be left out. Taking into account formulas (40) and (41), (42) and (44) and discarding terms of higher order of smallness, we obtain the following equation for Δ:

$$\frac{7\zeta(3)}{8(\pi T_c)^2}(\Delta_0^2 - \Delta^2) - \frac{\pi\delta\Delta}{4T_c} + \alpha\frac{\Delta}{12T_c}\left[\overline{\left(\frac{2w}{\Delta}\right)^{3/2}} - \left(\frac{2w}{\Delta}\right)^{3/2}\right] = 0.$$

From this, by virtue of the condition $\overline{\delta\Delta} = 0$, we get

$$\Delta \approx \Delta_0, \quad \delta\Delta \approx \alpha\frac{\Delta}{3\pi}\left[\overline{\left(\frac{2w}{\Delta}\right)^{3/2}} - \left(\frac{2w}{\Delta}\right)^{3/2}\right]. \tag{45}$$

The current j (40) is calculated analogously. The small alternating increment $\delta\Delta$ (45) makes no contribution to the current and the last term in formula (40) for the current, which contains the increment $\varphi_\Delta^{(1)}$ (39) to the distribution function, is small, just as in the case of the equation for Δ. The calculations yield the following:

$$\frac{p_F}{Ne}\mathrm{j} \approx \frac{7\zeta(3)}{4}\left(\frac{\Delta}{\pi T_c}\right)^2 \mathrm{w} + \tau\dot{\mathrm{w}} + \frac{\Delta}{2T_c}\mathrm{w}\sqrt{\frac{2w}{\Delta}}\int_0^\infty dx\,\eta(x) \times \left[\psi(x) - \left\langle \frac{w(t')}{w}\psi\left(\sqrt{\frac{w}{w(t')}}x\right)\right\rangle_{t'}\right], \tag{46}$$
$$-1 < \psi < 1, \quad \eta = {}^1/_3(2\psi - 1), \quad 1 < \psi, \quad \eta = {}^1/_3(\psi - \sqrt{\psi^2 - 1}),$$

where the function $\psi(x)$ is defined in (43), and the symbol $\langle\cdot\rangle_{t'}$ stands for averaging over t'.

Attention is called to the circumstance that in the resultant expression (46), unlike the first term, which is calculated with the aid of the equilibrium function (41) by expanding in powers of Δ/T_c in formula (20), and which is proportional to $(\Delta/T_c)^2$, the third term is proportional only to the first power of this parameter ($\sim \Delta/T_c$). Accordingly, even at relative small currents, for which $w/\Delta \gg (\Delta/T_c)^2$, the last term becomes the principal one. Physically this term is connected with the oscillations of the boundaries ($\Delta - w$ and $-\Delta + w$) of the gap in the quasiparticle spectrum, and it is these oscillations that make the essentially nonlinear non-equilibrium contribution to the current j (46). Simple estimates show that the inequality $w/\Delta \ll 1$, used in the derivation of formulas (45) and (46), remains valid up to current amplitudes on the order of the equilibrium critical value:

$$j \sim j_c \sim \frac{Ne}{p_F}\left(\frac{\Delta}{T_c}\right)^2 \Delta \to \frac{w_c}{\Delta} \sim \left(\frac{\Delta}{T_c}\right)^{2/3} \ll 1.$$

Confining ourselves to investigation of currents that are not too small ($w/\Delta \gg (\Delta/T_c)^2$), we omit the first term in (46). In view of the approximate character of formula (37) and of the entire subsequent analysis, it is advantageous to approximate expression (46) by a simpler analytic expression. An analysis of the last term in the current j (46) leads to the conclusion that this approximation can be taken in the form

$$\mathrm{j} \approx \frac{Ne}{p_F}\left[\tau\dot{\mathrm{w}} + C\frac{\Delta}{T_c}\mathrm{w}\sqrt{\frac{\overline{w}}{\Delta}}\left(1 - \frac{\overline{w^q}}{w^q}\right)\right] \quad (q = {}^3/_4), \tag{47}$$

where C is a constant on the order of unity and q is a certain exponent: $0 < q < 1$. For convenience we can put $q = 3/4$.

To find the electric field $E = \dot{\mathrm{w}}/ev_F$ corresponding to a specified external current j_{ext}, it remains to equate (47) to the quantity j_{ext} and to solve the resultant equation. Two cases are possible here. In the case of "high" frequencies $\omega\tau \gg (\Delta/T_c)\sqrt{w/\Delta}$ the second term in (47) plays the role of the small correction, and the sample behaves like a normal metal. The more interesting case is that of low frequencies: $\omega\tau \ll (\Delta/T_c)\sqrt{w/\Delta}$. In terms of the dimensionless variables

$$\mathrm{w} = \Delta\left(\frac{\Delta}{T_c}\right)^{2/3}\mathrm{v}, \quad \mathrm{j} = C\frac{Ne}{p_F}\left(\frac{\Delta}{T_c}\right)^2\Delta\mathrm{i}, \quad \omega t = \chi,$$

the equation that determines the field **E** is

$$\gamma\frac{dv}{d\chi} = i_0\sin\chi - i(v), \quad \gamma = \omega\tau/C\left(\frac{\Delta}{T_c}\right)^{4/3} \ll 1,$$
$$i(v) = \operatorname{sign} v\,(|v|^{3/2} - |v|^{3/4}|\overline{v^{3/4}}|). \tag{48}$$

We have taken into account here the one-dimensional character of the problem and have specified the law governing the variation of the external current, $j_{ext} = j_0\sin\omega t$. The function i(v) in (48) is plotted qualitatively in Fig. 2.

Since $\gamma \ll 1$, Eq. (48) describes a rapid relaxation of the system to the rest points $r(v) = i_0\sin\chi$, after the elapse of which the current i(v) follows adiabatically the external current $i_0\sin\chi$. According to (48), in the vicinity of the rest point v_0 the relaxation proceeds in accordance with the law

$$v - v_0 \approx \mathrm{const}\cdot\exp\left(-\frac{i'(v_0)}{\gamma}\chi\right), \quad i' \equiv \frac{dv}{d\chi}.$$

We see therefore that the points at which $i'(v) > 0$ are stable, and conversely, $i'(v) < 0$ are unstable. When the external current increases from negative values (see Fig. 2), i(v) follows adiabatically and continuously the external current up to the maximum point $i'(v) = 0$. This is followed by a rapid transition along the descending and ascending sections of the i(v) curve to the equivalent rest point $i(v) = i_{max}$, after which the adiabatic following of the external current by i(v) continues again. The process proceeds analogously in the opposite direction (Fig. 2). In the principal time scale, $\delta t \sim 1/\omega$, these fast transitions are replaced by jumps from one branch of the i(v) curve to the other, and on the whole the picture exhibits hysteresis.

Let us verify the existence of a solution of Eq. (48) as $\gamma \to 0$. Taking into account the choice indicated in Fig. 2 for the branches of the i(v) curve, we obtain from (48) at $\gamma = 0$

$$y = \frac{\bar{y}}{2} + \left[\left(\frac{\bar{y}}{2}\right)^2 + \operatorname{sign} v\cdot i_0\sin\chi\right]^{1/2}, \quad y = |v|^{3/4}, \quad i = i_0\sin\chi.$$
$$\operatorname{sign} v = \begin{cases}-1, & i < (\bar{y}/2)^2\\ 1, & i > (\bar{y}/2)^2\end{cases} \text{ and } i' > 0;$$
$$\operatorname{sign} v = \begin{cases}1, & i > -(\bar{y}/2)^2\\ -1, & i < -(\bar{y}/2)^2\end{cases} \text{ and } i' < 0. \tag{49}$$

This leads to the equation

$$\bar{y} = \frac{\bar{y}}{2} + \overline{\left[\left(\frac{\bar{y}}{2}\right)^2 + \operatorname{sign} v\cdot i_0\sin\chi\right]^{1/2}},$$

or in other words

$$1 = \overline{\left(1 + \operatorname{sign} v\frac{\sin\chi}{\sin\chi_0}\right)^{1/2}} = \frac{1}{2\pi}\int_0^{2\pi} d\chi\left(1 + \operatorname{sign} v\frac{\sin\chi}{\sin\chi_0}\right)^{1/2},$$
$$\sin\chi_0 = \frac{1}{i_0}\left(\frac{\bar{y}}{2}\right)^2 < 1. \tag{50}$$

After simple transformations, with formulas (49) taken into account, Eq. (50) can be reduced to the form

$$1 = \frac{1}{\pi}\int_0^\pi d\varphi\,(1 + \cos\varphi + \operatorname{ctg}\chi_0\sin\varphi)^{1/2}.$$

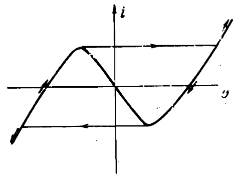

FIG. 2.

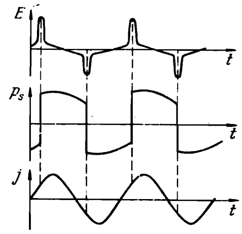

FIG. 3.

The last equation, as can be easily verified, does indeed have a root $0 < \chi_0 < \pi/2$.

The time dependence of j, p_S, and $\dot{p}_S = eE$ is shown qualitatively in Fig. 3. A characteristic feature of the described process is the presence of electric-field peaks corresponding to fast transitions from one branch to the other on the current curve i(v) (Fig. 2).

The results pertain to a relatively simple theoretical case, when the dynamic behavior of the superconducting electrons in the nonlinear region can be described in terms of the quasiparticles in the kinetic Boltzmann equation (17). It was noted above (see footnote 3) that such a situation is difficult to realize in practice. Regardless of this, however, the problem of current states in a superconductor is of fundamental interest and, as seen from the foregoing, even in this simplest case the superconducting condensate exhibits a nontrivial behavior.

Of greater practical interest in the case of "dirty" superconductors with $l \ll \xi_0$. The kinetic equation (17) certainly does not hold for such superconductors, since the characteristics distances ($\sim \xi_0$) over which the very concept of the quasiparticle can be introduced are larger in comparison with the mean free path. A kinetic description with these systems, which differs more radically from the description of normal methods, calls for a special investigation.

1) In view of the inequality $p_F = \gg 1/\delta$ (where δ is the depth of penetration of the field and $p_F = mv_F$ is the Fermi momentum), we can neglect the non-commutativity of the quantities $\hat{p}$ and v_s.

2) It should be noted that Eq. (17) together with the terms (22) have limited applicability to the description of the proper electromagnetic processes in a superconductor. Since $\delta \ll \xi_0$ in most pure superconductors (δ is the depth of penetration of the field), the weakly inhomogeneous states considered here are not realized in such superconductors. See footnote 3 below in this connection.

3) Since the Meissner effect can be neglected only in the case $d \leqslant \delta$ (δ is the depth of penetration of the field), the presented inequalities are quite stringent in practice and can probably be satisfied only in slightly contaminated samples of Nb.

4) The temperature must not be too close to critical, so that the formation of the condensate and of the quasiparticles remains the fastest process as before.

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