

Spatial field and intensity correlation functions of laser radiation

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The factors determining the spatial statistics of the various types of laser radiation are discussed. The results of an investigation of the transverse correlation functions of the radiation of single-mode and multi-mode lasers operating under diverse conditions and of an investigation of stimulated Raman scattering (SRS) are reported. The use of a sensitive circuit with a high-resolution polarization interferometer (correlation radii of up to 70μ were measured) allowed a detailed investigation of not only the spatial field correlation functions (SFCF), but also of the spatial intensity correlation functions (SICF) of solid lasers with pulse durations $\tau_p \approx 10^{-4}$ sec and $\tau_p \approx 10^{-8}$ sec, as well as of stimulated scattering. An investigation of solid-laser statistics showed that the solid-laser field is an inhomogeneous random process. For a laser with a spherical resonator, at the beam center $r_c \sim N^{-1/2}$, where N is the number of excited transverse modes. The simultaneous measurement of the SFCF and SICF allowed us to follow the formation of transverse Gaussian statistics in a multi-mode laser as N increased (up to $N \approx 10^4$). Data are presented which characterize the formation of spatial coherence during superluminescence (as illustrated by SRS without a resonator). The possibility is discussed of using the methods developed for detecting laser action in the absence of feedback in investigations of the non-Gaussian spatial statistics of scattered light (in particular, in investigations of the transformation of light statistics in inhomogeneous media).

1. INTRODUCTION

The investigation of the statistics of light fields is a persistently pressing problem of optics. The last few years have brought a number of important results in the investigation of the temporal statistics of laser fields. At least three directions of investigation can be distinguished here. There are, first, the investigations of the temporal statistics in single-mode lasers (investigations of the natural fluctuations in the amplitude and phase of the single-mode radiation^[1-3]). Another important direction is the investigation of the temporal statistics of the multi-mode radiation (see, for example, the paper by Armstrong and Smith^[3] and the recent review by Duguay et al.^[4]). Interest in these problems was stimulated in the first place by investigations on ultrashort pulses and the generation of light in systems with nonresonant feedback^[5,6]; due, to a large extent, to these investigations, the methods of measuring temporal intensity correlation functions of diverse orders have become an integral part of laser technology. Finally, there are the investigations of the transformations of the temporal statistics during the propagations of radiation in linear (including inhomogeneous) and nonlinear (see, for example,^[6,7]) media.

Of no less interest are the analogous problems for the spatial statistics. However, comparatively little has at present been done here. Spatial field correlation functions (SFCF) for gas lasers have been measured in a number of experiments^[8-18]; these measurements were made either for the lowest mode^[8,9,11-15,17], or for a mixture of two or three lowest modes^[8,10,14,16,18]. The level of the experimental technology used in the cited investigations was, however, such that detection of the spontaneous-transition-related deviations from the ideal spatial coherence was out of the question.

The question of the possibility of measuring them (i.e., of measuring the "natural" angular-spectrum broadening due to spontaneous noise) deserves separate consideration. Apparently, these effects are extremely

difficult to separate out from the scattering background due to the inhomogeneities and molecular scattering. We only note here that the quite strong deviations from the ideal spatial coherence detected in a number of the above-cited experiments (see, for example,^[9,12-15]) should be ascribed to an uncontrolled excitation of spatial coherence usually are of qualitative nature. In ruby^[19-23] and semiconductor^[24,25] lasers, the field-correlation coefficient is, as a rule, measured for two fixed points of the beam. In some papers the conclusions about the spatial correlation were qualitatively drawn on the basis of the data on divergence; only the papers^[22] by Voropaev et al. and^[25] by Bogatov et al. contain data on the field correlation functions. For lasers with nonresonant feedback the correlation radius has been estimated in^[5].

We are not aware of any experiments in which the transformation of the spatial statistics during nonlinear transformations of radiation has been measured.

In the present paper the main attention is focused on the quantitative investigation of the spatial statistics of multi-mode systems. Of greatest interest here is the question of the dependence of the statistics on the number of excited modes, i.e., on the geometry and amplification of the laser system. This problem turns out to be one of the major problems for laser superluminescence. It is of particular importance for superluminescent lasers in the vacuum ultraviolet. Analogous problems arise in connection with the investigation of stimulated scattering—spatially coherent Stokes radiation is formed from a spatially δ -correlated seed.

In the present paper we describe a sensitive circuit with a high-resolution (correlation radii of up to 70μ have been measured) polarization interferometer which has allowed us to carry out detailed measurements of the spatial field correlation functions (SFCF) and the spatial intensity correlation functions (SICF) of laser radiation. The measurements were performed for continuous-beam lasers, pulsed lasers operating in the free-generation ($\tau_p \approx 10^{-4}$ sec), Q-switched ($\tau_p \approx 10^{-8}$ sec), and stimula-

ted-Raman-scattering modes. The results obtained are compared with the theoretical estimates.

2. EXPERIMENT

2.1. The transverse correlation functions

The field of a laser beam can be represented in the form

$$E(\mathbf{r}, z, t) = A(\mathbf{r}, z, t) e^{i(\omega t - kz)},$$

where z is the longitudinal coordinate measured along the beam, \mathbf{r} is the transverse radius vector, and t is the time. (For axially symmetric beams, which are the most frequently used beams, the field is isotropic:

$E = E(\mathbf{r}, z, t)$.) If what we are interested in is spatial statistics, then for the formed laser beam, the transverse field and intensity correlation functions for fixed z are of greatest interest: the field correlation function is

$$\begin{aligned} \Gamma(\mathbf{r}, \mathbf{r} + \mathbf{s}, t, t + \tau; z) &= \langle E(\mathbf{r}, z, t) E^*(\mathbf{r} + \mathbf{s}, z, t + \tau) \rangle \\ &= \langle A(\mathbf{r}, z, t) A^*(\mathbf{r} + \mathbf{s}, z, t + \tau) \rangle e^{i\omega\tau} \end{aligned} \quad (1)$$

(the longitudinal coordinate enters as a parameter), and the correlation function of the intensity $I(\mathbf{r}, z, t) = A(\mathbf{r}, z, t) A^*(\mathbf{r}, z, t)$ is

$$G(\mathbf{r}, \mathbf{r} + \mathbf{s}, t, t + \tau; z) = \langle I(\mathbf{r}, z, t) I(\mathbf{r} + \mathbf{s}, z, t + \tau) \rangle. \quad (2)$$

The measurements of the transverse correlation functions can be carried out with a high degree of accuracy if the time lags τ that invariably arise in real systems are much less than the radiation-correlation time:

$$\tau \ll \tau_c. \quad (3)$$

If the condition (3) is fulfilled, then for a stationary, locally homogeneous, and isotropic field, we can write (we write down only the real variables)

$$\Gamma(s, \tau) = \langle A(r) A^*(r+s) \rangle e^{i\omega\tau} = \gamma(s) e^{i\omega\tau}, \quad (4)$$

where $\gamma(s)$ is the spatial amplitude correlation function (SACF)¹⁾

$$G(s) = \langle I(r) I(r+s) \rangle. \quad (5)$$

For a Gaussian field, we have

$$G(s) = \gamma^2(0) + |\gamma(s)|^2. \quad (6)$$

Below, besides the functions $\gamma(s)$ and $G(s)$, we shall also use the normalized functions

$$\bar{\gamma}(s) = \gamma(s) / \gamma(0), \quad \bar{G}(s) = 1 + |\gamma(s)|^2 / \gamma^2(0). \quad (7)$$

Notice, finally, that for a stationary, homogeneous, and isotropic Gaussian process, we have for any τ

$$\Gamma(s, \tau) = \gamma(s) B(\tau), \quad G(s, \tau) = G(s) G(\tau). \quad (8)$$

2.2. The experimental setup

To measure the transverse correlation functions $\Gamma(s)$ and $G(s)$, we must lead the beam from the points \mathbf{r} and $\mathbf{r} + \mathbf{s}$ of the field in question that are of interest to us to some point \mathbf{Q} of the measuring device. If it is SFCF that we are measuring, then we must determine the luminosity $V(\mathbf{Q})$ of the interference pattern produced. $V(\mathbf{Q})$ can, when the condition (3) is fulfilled, be expressed directly in terms of the value of the transverse correlation function γ for the points under consideration:

$$V(\mathbf{Q}) = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{2(I_1 I_2)^{1/2}}{I_1 + I_2} |\bar{\gamma}| \quad (9)$$

(where I_1 and I_2 are the intensities of the interfering

beams). If it is SICF that is being measured, then the beams must be fed to a nonlinear device that effects the operation (2). In the majority of the cited papers^[8,14,17-19,23,25], the classical Young interferometer—a screen with two slits—was used to measure the SFCF. However, this interferometer has a small aperture ratio and its resolving power is limited by the dimensions of the slits (≈ 0.2 mm), and this does not allow the investigation of radiation with a small coherence radius.

Janossy et al.^[15,16] have investigated SFCF with the aid of the Mach-Zehnder interferometer, which has a large aperture ratio. But because of the system's strict adjustment requirements and the high sensitivity to external influences, its practical use did not become widespread. Furthermore, the spatial displacement of one of the interfering beams relative to the other that can be realized in this interferometer by shifting the mirror leads inevitably to the same path difference for the beams. As a result, the temporal coherence affects the measurement of the spatial coherence, which makes the indicated interferometer suitable only for investigating radiation with a very narrow spectral line.

Other types of interferometers have been used to investigate the spatial coherence of laser radiation: the Linnik technique^[19,26], the Michelson interferometer with a corner-reflector in place of one mirror^[27], and the Jamin interferometer^[21]. A holographic method^[28] and a method proposed by Sotskii^[29] and based on the analysis of the diffraction pattern from one slit^[22] have also been used for the same purpose. All of them, however, have to some extent the above-mentioned disadvantages of the Young and Mach-Zehnder interferometers and have not found widespread application. We also note that none of the enumerated experiments measured SICF.

The polarization interferometer previously proposed by us^[30] is, to a considerable degree, free from the enumerated disadvantages. It has a large aperture ratio, is suitable for investigating SFCF and SICF, and allows the measurement of small correlation radii of radiation with any width of the frequency spectrum. The design of the interferometer is shown in Fig. 1.

The combination of the fields in question in the indicated interferometer is realized with the aid of a double-refracting plate that bifurcates the incident beam and effects the necessary relative displacement s of the resulting beams. After passing through the polarizer, the beam under investigation fell onto a system of two identical plane-parallel plates 3 and 4, which were cut out from a double-refracting crystal along the optical axis, and a half-wave plate 5 located between them. These crystals could be rotated about their optical axes, which

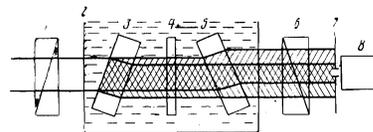


FIG. 1. Diagram of the polarization interferometer: 1—polarizer; 2—cuvette with immersion liquid (castor oil); 3—and 5—calcite plates of thickness $d = 25$ mm (or $d = 0.8$ cm); 4—half-wave plate; 6—analyzer; 7—diaphragm; 8—photorecording device. When measuring SICF the analyzer is replaced by a nonlinear crystal in which a doubling of the light frequency occurs. One thin plate is used when investigating narrow-band radiation with a small correlation radius.

were oriented parallel to each other, perpendicular to the incident beam, and at an angle of 45° to the polarization of the beam. On passing through the first plate the beam was split up into two beams with mutually perpendicular polarizations and a relative displacement of

$$s = \frac{1}{2}d \sin 2\alpha [(\xi_e^{-2} - \sin^2 \alpha)^{-1/2} - (\xi_o^{-2} - \sin^2 \alpha)^{-1/2}], \quad (10a)$$

$$\xi_e = n_i / n_e, \quad \xi_o = n_i / n_o,$$

where α is the angle of incidence of the beam on the plate, d is the plate thickness, and n_o , n_e , and n_i are the refractive indices for the normal and extraordinary waves in the crystal and surrounding medium respectively. They acquire in the process some path difference y , which, for small α , is equal to

$$y = d(n_o - n_e) (1 + \frac{1}{2}\xi_o\xi_e \sin^2 \alpha). \quad (10b)$$

Upon passage through the half-wave plate the beams have their polarizations interchanged and, therefore, upon passage through the second plate, inclined at the same angle but in the opposite direction, the path difference which arose in the plate 3 is fully compensated. The spatial displacements, on the other hand, add up.

Notice that the use of the plates 4 and 5 is not obligatory when investigating radiation with a narrow spectral line when $\tau = y/c \ll \tau_c$ (the train coherence length $L_{\text{coh}} \gg y$). Further, the radiation passed through the analyzer with a crossed polarizer (it brings the polarizations into one plane, and the beams interfere) and hit the photo-recording device (PM). Located in front of the PM was a diaphragm which defined the region of interaction of the beams. The dimension r of the region should be much less than the distance between the interference fringes:

$$r \ll R\lambda / s, \quad (11)$$

where R is the radius of curvature of the wave front and λ is the wavelength of the radiation.

The fine tuning to the maximum or minimum of the interference was effected by a small rotation of one of the crystals, which resulted in a change $\Delta y \sim \lambda$ in the path difference y . The change in the spatial displacement of the beams is insignificant in this case and can be neglected. In those cases when these changes turned out to be significant, an electrically controlled phase shifter was placed in front of the PM.

The selection of the beam cross section in which the spatial correlation function is measured is made with the aid of a lens located in front of the interferometer; this lens throws the image of the cross section in question onto the entrance of the measuring device. If the radiation intensities I_1 and I_2 at the two points under investigation are unequal, they can be equalized by rotating the analyzer (then, according to (9), $V(\mathbf{Q}) = \tilde{\gamma}(\mathbf{r}, \mathbf{r} + \mathbf{s})$).

In measuring the SICF, we replaced the analyzer in front of the PM by a nonlinear crystal in which the ordinary and extraordinary waves of the radiation under investigation synchronously excited a second harmonic (an OE-E type of interaction). In this case the intensity of the second harmonic is proportional to the SICF of the transformed radiation (see, for example, [7]). Since there is then no interference, the condition (11) can be met. If the synchronous doubling occurs not at an angle of 90° to the optical axis of the nonlinear crystal, the ordinary and extraordinary rays undergo a relative displacement of βl , where l is the crystal thickness and β is the angle of double refraction. The latter imposes on the length of the nonlinear crystal the requirement:

$$\beta l \ll r_c, \quad (12)$$

where r_c is the correlation radius of the radiation in question.

The resolving power of the interferometer is, in principle, determined only by the thickness d of the double-refracting plate. The minimum width R_0 of the instrumental function of the above-described interferometer can be obtained in the investigation of radiation with a narrow spectral line; it is obviously determined by the relation

$$kR_0^2 \approx d.$$

A realistic value for the visible region ($\lambda \approx 0.5 \mu$) is $R_0 \geq (4-5) \times 10^{-4}$ cm. Notice that at such resolutions a definite role may be played by the nonparallelism of the faces of the plate: an inclination $d\varphi$ between the faces gives rise to an additional spatial displacement S' of the beams given by

$$S' = d \frac{n_o - n_e}{n_i} d\varphi. \quad (13)$$

In the experiments described below, we investigated radiation with correlation radii $r_c \approx 10^{-2}$ cm; therefore, it was quite sufficient to have $R_0 \approx 4 \times 10^{-3}$ cm. The minimum values of the correlation radius measured in the experiment were $\sim 70 \mu$ (see, for example, Fig. 6 below); the maximum angular aperture of the instrument attained 8° .

The degree of exactness of the compensation of the temporal path difference limits the spectral width of the radiation that can be investigated. The uncompensated path difference by arising as a result of the inexact equality of the angles of rotation of the crystals is equal to

$$dy = \frac{1}{2}d(n_o - n_e)\xi_o\xi_e \sin 2\alpha d\alpha. \quad (14)$$

For a comparatively thick calcite plate with $d = 2$ cm and $d\alpha = 10^{-2}$, $dy \approx 10^{-3}$. Consequently, we can investigate radiation with a train coherence length $L_{\text{coh}} \geq 10^2 - 10^1$ cm.

2.3 Experimental results

Our main attention was focused on the investigation of pulsed YAG: Nd³⁺ lasers, the large amplification of whose active elements allows a wide variation of the number of excitable transverse modes. (Besides this, to calibrate and check the resolution, we performed standard measurements with continuous-beam gas lasers.) In all the experiments we used a pulsed YAG: Nd³⁺ laser (maximum pulse repetition rate of up to 50 Hz); the laser rod was 50 mm long and 5 mm in diameter.

The measurement time T_{meas} was usually several seconds, so that the number of realizations of the random field during a measurement was more than $10^2 - 10^3$, and it is obvious that $T_{\text{meas}} \gg \tau_c$, the correlation time of the radiation. Figures 2 and 3 show the normalized spatial amplitude correlation functions (SACF) of single-mode (one longitudinal and one transverse mode) lasers operating respectively in the Q-switched (pulse duration $\tau_p \approx 10^{-8}$ sec) and free-generation ($\tau_p \approx 3 \times 10^{-4}$ sec) modes. It can be seen that spatial coherence is extremely high in the single-mode operation. The foreground pertains especially to the free-generation mode of operation (Fig. 3). Here the quantity $\tilde{\gamma} = 1 \pm 0.005$ even for $s = 1.2$ mm for which the intensity drops by a factor of more than 20.

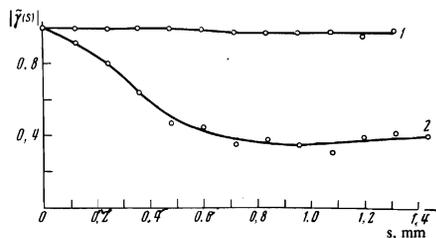


FIG. 2. Normalized SACF of the radiation of a YAG:Nd³⁺ laser operating in the Q-switched mode (switching with the aid of an electro-optical shutter). The resonator is plane-spherical. The curve 1 corresponds to a TEM₀₀ mode and the curve 2 corresponds to a multi-mode operation.

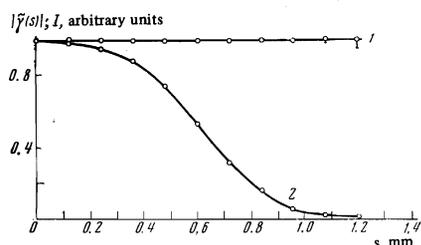


FIG. 3. Normalized SACF (curve 1) of the radiation of YAG:Nd³⁺ laser operating in the free-generation mode for the TEM₀₀ mode. The resonator is plane-spherical; the mode selection was accomplished with the aid of a diaphragm ($\phi \approx 1$ mm). The curve 2 is a plot of the distribution of intensity *I* over the cross section of the beam.

The curve 1 in Fig. 3 was obtained when the threshold pumping was exceeded by a factor of five; similar data were obtained when the threshold was exceeded by a factor of 1.19. The error in the measurement of the normalized SACF over the entire cross section of the beam was ~ 0.005 ; the error was higher only at the very edge of the beam, where it was 0.05. The data obtained thus indicate the possibility of obtaining extremely high spatial coherence in the single-mode solid-state laser³.

Figures 4–8 pertain to multi-mode systems. As the objects of investigation, we chose the stimulated Raman scattering (SRS) and the YAG:Nd³⁺ laser with a quasi-concentric cavity resonator. Involved in both cases is radiation with an extremely narrow spectral line ($\Delta\nu$ is less than 1 cm^{-1}). The line of the first Stokes component of the SRS is narrower than the spontaneous-scattering line; as has previously been shown^[31], in the YAG:Nd³⁺ laser with a quasi-concentric resonator, in spite of the excitation of a large number of transverse modes, the line narrows down considerably owing to the fact that the modes strongly overlap.

Figure 4 shows the SACF for the first Stokes component of the stimulated Raman scattering, stimulated in liquid hydrogen by the second harmonic of the single-mode YAG:Nd³⁺ laser. One of the main problems which must be solved in performing such measurements is the reduction to a minimum of the distortions of the spatial correlation functions (SCF) caused by self-focusing. In this sense, liquid nitrogen, which has a relatively small nonresonance cubic nonlinearity and a high amplification factor, is one of the most preferable media (at the same time correct measurements in the standard organic liquids are difficult; this was, apparently, not given due consideration in the work by Dreiden et al.^[21]).

The SACF measurements were performed in a focused pumping beam ($F = 20$ cm). The parameter of the curves

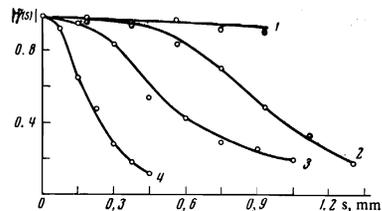


FIG. 4. Normalized SACF of the first Stokes component of the SRS, stimulated in liquid nitrogen by the second harmonic of the Q-switched YAG:Nd³¹ laser (the laser operated at the TEM₀₀ mode). The curve 1 is a plot of the SACF of the pump, while the curves 2, 3, and 4 are plots of the SACF of the Stokes component for pump power exceeding the threshold by factors of 1.5, 1.9, and 2.7 respectively.

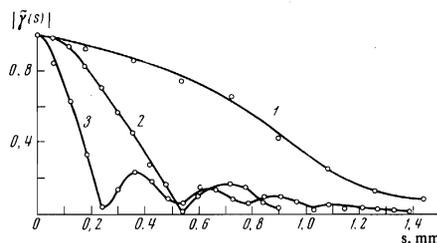


FIG. 5. Normalized SACF of the radiation of the YAG:Nd³⁺ laser with a quasi-concentric resonator operating in the free-generation mode. The curves 1, 2, and 3 correspond to different numbers of excitable transverse modes (the curve 1 was measured with a diaphragm inside the resonator, the diameter of the crystal being 5 mm; the curves 2 and 3 were measured without a diaphragm, and the diameters of the crystals were in these cases respectively equal to 5 and 3 mm).

is the excess P_p of the pump power over the SRS threshold (P_{thr}); the power is measured at the entrance of the lens. It can be seen that the correlation radius of the Stokes wave is less than the pump correlation radius, and that the experimental dependence of the correlation radius of the Stokes wave on the pump intensity is satisfactorily described by the relation

$$r_c(z) \sim a(z) (P_p - P_{thr})^{1/2}.$$

Here a is the fitting parameter and the dependence $a(z)$ is connected with the variation of the SCF as it propagates.

Figures 5–7 depict the properties of the SCF of multi-mode YAG:Nd³⁺ lasers operating in the free-generation mode. In all these experiments we used a laser with a quasi-concentric resonator (length of the crystal $l = 50$ mm, diameter 5 mm, length of resonator $L \approx 100$ cm). In the absence of selection, up to 10^4 transverse modes are effectively excited in such a laser (see^[31]); in a sense (see^[6]) this laser can be regarded as one of the variants of the generator with nonresonant feedback. At the same time, by placing diaphragms inside the resonator, or by changing the distance between the mirrors, we can strongly vary the number of excitable modes.

Figure 5 depicts the dependence of the SACF measured at the beam center on the number of the excitable transverse modes. The smallest field-correlation radius corresponds to a resonator without a diaphragm—in this case the number of excitable transverse modes is maximal. At the cross section of the beam where its diameter $D = 1$ cm, the correlation radius $r_c = 1.5 \times 10^{-2}$ cm.

The correlation radius, like the beam width, is naturally a function of the longitudinal coordinate z ; the measurements showed that for the multi-mode operation $D(z)/r_c(z) = \text{const}$ as we move away from the exit mirror.

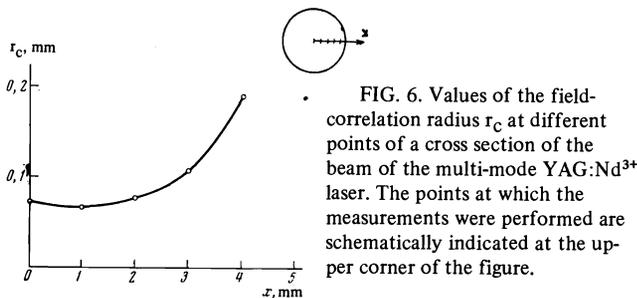


FIG. 6. Values of the field-correlation radius r_c at different points of a cross section of the beam of the multi-mode YAG:Nd³⁺ laser. The points at which the measurements were performed are schematically indicated at the upper corner of the figure.

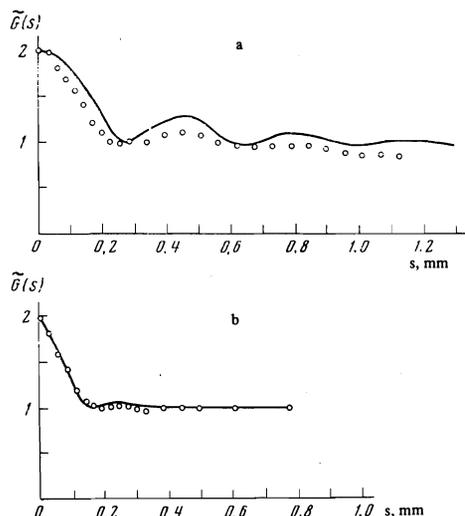


FIG. 7. Normalized spatial intensity correlation functions of the YAG:Nd³⁺ laser. a—Experimental data for laser with a quasi-concentric resonator with a transverse-mode selector (diaphragm $\phi = 4$ mm). The points represent the experimental values of the normalized SICF $\tilde{G}(s)$; the continuous curve was constructed from the formula $F(s) = 1 + |\tilde{\gamma}_{\text{exp}}(s)|^2$ on the basis of the simultaneously measured SACF for the same laser. b—Analogous data for laser with a quasi-concentric resonator in which the transverse modes are not specially selected.

The SACF were also measured at other cross sections of the beam; it was established that the laser field is an inhomogeneous field. The variation of the correlation radius over the cross section of the beam can be considerable: one of the typical plots obtained for the laser with a quasi-concentric resonator is shown in Fig. 6.

Finally, Fig. 7 shows SICF measured for the laser with a transverse-mode selector (Fig. 7a) and without the selector (Fig. 7b). Simultaneously, the SFCF were measured at the same cross section of the beam for the investigated modes of operation. In Figs. 7a and 7b the continuous curves represent the experimental values of the function $F(s) = 1 + |\tilde{\gamma}_{\text{exp}}(s)|^2$. It can be seen that if, for the laser with a mode selector, the difference between $F(s)$ and $\tilde{G}(s)$ is appreciable, then for the multi-mode operation $F(s)$ is very close to the experimentally measured $\tilde{G}(s)$, i.e., $\tilde{G}(s) \approx F(s)$.

3. DISCUSSION

3.1. Single-mode lasers

Experiment (see Fig. 3) indicates the possibility of obtaining extremely high spatial coherence in a laser with a carefully selected transverse mode in the free-generation operating mode ($\tau_p \approx 10^{-4}$ sec). The normalized SACF is, to within 10^{-2} , equal to unity for practically the whole beam (including the "wings," where the inten-

sity is 10% of the intensity at the peak), even for pumping very close to the threshold.

In fact, for the major part of the beam, we were not able to detect deviations from the ideal spatial coherence that fell outside the limits of the experimental errors. At the same time, the deviations from the ideal spatial coherence are already appreciable in the Q-switched operating mode. In our opinion, these deviations should be attributed to the fact that in this case the time τ_{tr} for the establishment of the steady-state transverse field distribution is comparable with the pulse duration, i.e., $\tau_{\text{tr}} \sim \tau_p^4$ (cf. [32,33]).

3.2 Stimulated raman scattering

In SRS, the boundedness of the pumping beam is the primary cause of the generation from the initially spatially δ -correlated seed (the spontaneous transitions from the levels under investigation) of the Stokes radiation whose transverse correlation radius is of the order of the beam radius (see [34,35]).

In a linear, passive medium, the correlation radius of initially δ -correlated radiation which has passed through an aperture of diameter a increases with distance according to the law (the Van Zittert-Zernicke theorem)

$$r_c^{(0)} \approx \lambda z / a. \quad (15)$$

In an active medium the formula (15) is inapplicable; the transformation of the transverse statistics with distance depends on the pump power—the pump does not constitute a diaphragm, but an active wave guide. In the geometrical-optics approximation, for the Stokes component of the SRS

$$r_c(z) \approx r_c^{(0)} G^h,$$

where G is the amplification factor; this formula has the same structure as the formula describing the growth of the temporal correlation of the Stokes wave in the process of amplification. As has already been pointed out, the experimental data agree satisfactorily well with this formula—at any rate, in the vicinity of the SRS threshold⁵⁾.

The threshold, which was determined in our experiments to within ~ 20 – 30% , corresponds to the pump intensity at which the spontaneous Raman scattering is replaced by the stimulated Raman scattering.

3.3. The multi-mode laser

The behavior of the SACF and SICF of the multimode-laser radiation as the diaphragm placed inside the resonator is varied (variation of the number of excitable transverse modes) is in good accord with the ideas based on the description of the transverse laser-field distribution as the result of a superposition of statistically independent transverse modes.

The foregoing pertains not only to the general qualitative conclusions (as N increases, the transverse statistics of the field becomes more and more Gaussian: see Fig. 7b), but to certain quantitative data as well. Let us begin with estimate the field-correlation radius of the multi-mode radiation. The transverse field distribution of a mode of a spherical resonator of length L , having identical mirrors of spherical shape (radius of curvature R), is given by the expression [36]

$$E_{n,m}(r, \varphi, z) = t^{m/2} L_n^m(t) e^{-t/2} \cos m\varphi. \quad (16)$$

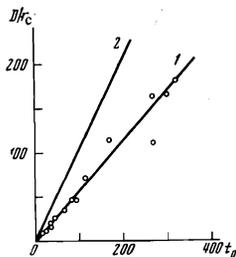


FIG. 8. Dependence of the ratio of the diameter D of the beam to the field correlation radius r_c on the parameter $t_0 = 1/2kD(0) \tan \phi$, where $D(0) = 2w(0)$ is the smallest diameter of the beam and ϕ is the angular divergence of the beam in the long-range field. The points represent the experimental values (1), and the straight line (2) is a theoretical curve computed from the formula (19); $t_0 \approx (12N)^{1/2}$.

Here $L_n^m(t)$ are the Laguerre polynomials, n and m being the radial and angular indices,

$$t = t_0 r^2 / w^2(z), \quad w^2(z) = w^2(0) (1 + 4z^2 / b^2),$$

$w(z)$ is the radius of the mode at a distance z from its constriction, $b = (2RL - L^2)^{1/2}$ is the confocal parameter of the modes of the resonator, $w(0) = (bt_0/2k)^{1/2}$ is the radius of the mode at the constriction, where the index

$$t_0 = (m + 2n + 1) + [(2n + 1)(2m + 2n + 1)]^{1/2},$$

and k is the wave number. The total divergence $2\phi_0$ of the mode can be expressed in terms of the index t_0 of the mode.

The distribution (16) has for large values of n the asymptotic form:

$$E_{n,m} \approx (\pi^2 t)^{-1/2} n^{1/2} m^{-1/2} \cos[2(nt)^{1/2} - 1/4\pi(m+1)] \cos m\phi. \quad (17)$$

It follows from (17) that at the level 0.5 the scale of the radial oscillations (the dimension of the spots) is equal to

$$r' = 1/3\pi w(z) (nt)^{-1/2}. \quad (18)$$

If the number N of the transverse modes excited in a laser is large ($N \gg 1$), then the mean dimension (averaged over the radial indices from 0 to N) of the spots can be used to estimate the correlation radius. Taking into account at the same time the structure connected with the azimuthal index, we obtain for the correlation radius for $N \gg 1^{(6)}$

$$r_c(z) \approx w(z) / N^{1/2}. \quad (19)$$

According to^[31], in the laser with the quasi-concentric diaphragmless resonator investigated by us, $N \approx 10^4$. In accordance with (19), we should have for such a laser $w(z)/r_c \approx 10^2$, which is in good agreement with the data presented in Fig. 5 (see also Fig. 8, where the relation

$$D/r_c = 2w/r_c$$

has been incorporated into the function of the parameter t_0). Also in agreement with experiment is the conclusion following from (19) that

$$r_c/D = \text{const}, \quad D = 2w.$$

The dependence of the field-correlation radius on the coordinates of the investigated region, shown in Fig. 6, can also be interpreted on the basis of the structure of the field of the modes in the spherical resonator. As can be seen from (16), the azimuthal scale $r_\phi \approx \pi r/m$ of the field inhomogeneity grows as we approach the beam boundary (cf. Fig. 6). For an exact computation of the form of the transverse correlation functions, we must have available information on the amplitudes of the modes excited in the laser; in the multi-mode operation such measurements are practically impossible. It should be noted at the same time that for $N \approx 10^4$ (see the curve 3 in Fig. 5) the field correlation function is close in form

to the correlation function of the homogeneous δ -correlated noise which filters through the circular diaphragm.

As is well known, at a distance z from a diaphragm of radius a the latter has the form:

$$|\bar{v}(s)| = 2J_1(v) / v, \quad (20)$$

where $J_1(v)$ is the Bessel function of order one and $v = kas/z$. Comparing (16) with Fig. 5, we find for the effective value of a (we assume that the diaphragm is located at the constriction of the beam): $a_{\text{eff}} \approx 1$ mm (in this case the radius of the laser rod is 1.5 mm). The differences between the shape of the SACF shown in Fig. 5 and the plot of the Bessel function are explained by the inhomogeneity in the intensity distribution over the cross section of the beam.

Comparison of the measured field and intensity correlation functions, $\gamma(s)$ and $G(s)$ (see Fig. 7), shows that in a laser with a quasi-concentric diaphragmless resonator, they satisfy to a high degree of accuracy the relation (6), which is valid for Gaussian processes (see Fig. 7b). We must at the same time bear in mind that the condition (6) does not, strictly speaking, guarantee that the process will be Gaussian.

The foregoing allows us to draw the conclusion that in spite of the strong bunching of the modes in a laser with a quasi-concentric resonator, their phases are statistically independent. At the same time, in a laser with a diaphragm inside the resonator (a smaller number of modes), the deviation from the relation (6) becomes appreciable (see Fig. 7a).

4. CONCLUSION

In the present paper we have presented the results of a systematic investigation of the spatial statistics of laser radiation. The high resolution of the polarization interferometer used in the experimental part of the work allowed us to carry out quantitative investigations of the multi-mode radiation having small (up to 50μ) transverse-correlation radii. The large aperture ratio of the interferometer enabled us to carry out sufficiently detailed measurements of the spatial intensity correlation functions of laser radiation.

It has been shown that in a multi-mode laser with a spherical resonator the correlation radius at some cross section z of the beam is

$$r_c(z) \approx w(z) / N^{1/2},$$

where w is the beam radius at the same cross section. For a quasi-concentric resonator $N = 10^4$ and $r_c/w \approx 10^{-2}$.

It is remarkable that for large N the "transverse" statistics turns out to be Gaussian for a laser with overlapping transverse modes.

The results obtained in the investigation of the multi-mode lasers indicate that the laser field is inhomogeneous and isotropic. The laser-beam divergence, which is determined by the angular spectrum of the highest mode N_{max} (see^[39]), is, generally speaking, not directly connected with the spatial coherence, determined by the total number N of excitable modes. Naturally, if $N \gg 1$, the difference between N_{max} and N becomes not very significant, and the divergence is determined by the value of the correlation radius. This conclusion is confirmed by the data of our experiments in which we simultane-

ously measured the SACF and the divergence of the laser beam.

The formation of the transverse statistics of superluminescence that has been discussed in this paper in application to SRS is of special interest for superluminescent lasers, especially in the ultraviolet region^[40]. A wide range of problems is connected with the transformation of spatial statistics in nonlinear media. The effect computed by us, whereby the correlation radius increases upon the doubling of the frequency in a double-refracting crystal^[41], was observed with the aid of the above-described technique^[35].

We note, in conclusion, that the results presented in the present paper pertain in the majority of cases to fields whose space-time correlation functions factor into a product of only time and only spatial correlation functions⁷⁾. There are, however, important examples of optical fields for which such a partition is, strictly speaking, impossible. Besides the already noted giant-pulse generation operating mode, this pertains, to a certain extent, to stimulated scattering; in an experiment, the pump power depends on the time (and is different at different points of the beam profile), so that the spatial correlation functions of the Stokes radiation measured by us are, generally speaking, the result of the time averaging of the instantaneous space-time functions. To a still larger extent, the foregoing pertains to parametric processes. The detailed investigation of the instantaneous space-time correlations for such fields is a separate, extremely interesting problem.

¹We shall here and below be concerned with beams whose temporal behavior is practically the same at all considered cross-section points, i.e., $E(r, t) = R(r)T(t)$. If the time characteristics for different r differ significantly from each other (the field does not break up into a product of coordinate and time functions), then we can speak of only space-time correlation functions. We encounter such a situation in certain Q-switched lasers and in the investigation of parametric processes.

²There was no correct estimate for the limiting resolution in the first version of the paper, a fact which was pointed out to us by A. M. Leontovich and B. Ya. Zel'dovich.

³Notice that in the Q-switched operating mode, the deviations of $\tilde{\gamma}(s)$ from unity are more noticeable (see Fig. 2). One of the possible causes of this may be the distinctive features of the development of a giant pulse across the beam: $E(r, t) \neq R(r)T(t)$.

⁴In the free-generation operating mode, $\tau_{tr} \leq \tau_p$ and the nonstationarity of the spatial correlations is practically not manifested.

⁵Marked deviations from the formula (15) should also be observed in the spontaneous scattering of a collimated beam, especially at 0°- or 180°-scattering angle.

⁶We must bear in mind, however, that because of the inhomogeneities of the laser rod, spherical-cavity modes are often excited in a resonator with plane mirrors for which $r_c \sim N^{-1}$ (see [37,38]).

⁷For such fields the data obtained allow us to improve the known [38,42] estimates of the effect of spatial coherence on the accuracy of the measurement of the temporal correlations (concerning the inverse effect, see Sec. 2 of the present paper).

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