Nonstationary theory of decay instability in a weakly inhomogeneous plasma

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The time evolution of the amplitudes of two parametrically coupled waves in a weakly inhomogeneous plasma is considered for the case when the oscillations can be described by the geometric-optics approximation and the decay conditions are satisfied only in a single resonant plane of the layer. Conditions for existence of a stationary solution are found. They are determined by the imaginary part of the product of the wave interaction matrix elements.

Decay instability in a weakly inhomogeneous plasma, when parametrically-coupled waves are described by the geometrical-optics approximation, was considered in a number of recent papers [1-3]. A characteristic feature of this case is that the decay conditions for the x components of the wave vectors (x is the coordinate on which the plasma parameters depend) can be satisfied only at certain values of x, and resonant interaction of the waves is possible in a narrow vicinity of these points. The natural assumption was made that in this case the absolute instability gives way to a finite amplification of the waves incident on a resonant layer, and the corresponding "amplification coefficients" were calculated.

In the present paper we examined, under the same conditions, the time evolution of the amplitudes of coupled waves. The solution of this problem is of interest for several reasons. First, it makes it possible to determine the conditions under which the previously considered stationary state is indeed reached. Second, there are grounds for assuming [4] that the change in the amplitude after turning on the pump is nonmonotonic, so that the most dangerous level of the oscillations is reached in the time of the transient process. Finally, the process of establishment of the oscillations can be observed experimentally and can be used both for detailed comparison of theory with experiment and possibly for plasma diagnostics.

To be specific, in the present paper we solve the following problem. A wave of frequency ω_1 is present in a plasma layer. At the instant t = 0, pumping with frequency ω_0 and wave vector $\mathbf{k}_0(x)$ is turned on in the entire volume. In the vicinity of the point where the decay conditions are satisfied for the waves ω_0 and ω_1 as well as for some natural plasma mode ω_2, the amplitude of the wave ω_1 begins to grow and the wave ω_2 appears. After a sufficiently large time interval, the spatial distribution of the amplitudes of these waves takes the form of expanding pulses, one boundary of which is immobile and is located at the resonant point, while the other moves with the group velocity of the wave. Inside the pulses, far from the boundary, the amplitudes are constant and are determined by a stationary gain, and outside the pulses, they retain their initial value. Thus, a field evolution that leads to establishment of a stationary state at any layer point sufficiently remote from the resonant region is connected with the passage of the pulse boundary. The shape of the boundary does not vary in time and depends on whether the group velocities of the interacting waves are parallel or antiparallel. At an identical direction of the velocities, the amplitude outside the pulse drops off exponentially, and inside the pulse it has an oscillatory structure, the principal maximum being approximately double the stationary level. In the case of mutually opposite directions of the velocities, the boundary is smooth and the amplitude decreases monotonically outside the pulse.

1. We consider a plane plasma layer whose parameters depend on the coordinate x, in the vicinity of the point x = 0, at which the condition

$$k_{0x} = k_{1x} + k_{2x}$$

is satisfied for three quasiclassical waves $\mathbf{E}_{\mathbf{k}\omega}$, whose constant wave-vector components k_{yi} and k_{zi}, and frequencies ω_i are also connected by the decay condition. One of the waves $\mathbf{E}_{\mathbf{k}_0\omega_0}$ will be assumed specified.

The picture of the decay interaction corresponds to the solution of the wave equation in the form of a pair of coupled waves

$$\mathscr{E}_{\mathbf{k}_1\mathbf{k}_2} = a_1\mathbf{E}_{\mathbf{k}_1\omega_1} + a_2^*\mathbf{E}_{\mathbf{k}_2\omega_2} + \text{c.c.}$$

In the vicinity of the resonant point x = 0, in a region small compared with the plasma inhomogeneity scale, the amplitudes satisfy the equations

$$\begin{aligned}\frac{\partial a_1}{\partial\tau} + \frac{\partial a_2}{\partial\zeta} &= \nu_1 \exp\left\{\frac{i\zeta^2}{2}\right\} a_2,\\ \gamma\frac{\partial a_2}{\partial\tau} + \frac{\partial a_2}{\partial\zeta} &= \nu_2 \exp\left\{-\frac{i\zeta^2}{2}\right\} a_1,\end{aligned} \tag{1}$$

where

$$\zeta = \frac{x}{l},\quad \tau = \frac{u_1 t}{l},\quad \gamma = \frac{u_1}{u_2},\quad l = \left|\frac{\partial}{\partial x}(k_{0x} - k_{1x} - k_{2x})\right|_{x=0}^{-1/2},$$

and u_i are the x-th components of the group velocity. The nonlinear-coupling coefficients ν_i are proportional to the amplitude of the pump wave. We assume them to be arbitrary complex numbers.

The weak damping of the waves in (1) can be taken into account by assuming the wave vectors k_{1x} and k_{2x} to be complex. The resonant point then lies in the complex plane, so that real x corresponds to complex ζ. Although the general solution obtained below for the system (1) is also suitable in this case, we shall assume for simplicity that there is no dissipation.

2. We consider first the time-independent solutions of the system (1). Eliminating a_2 in this case, we obtain for a_1 the equation

$$a_1'' - i\zeta a_1' - z a_1 = 0,\quad z = \nu_1\nu_2,$$

the solution of which we write in the form

$$a_1 = C_1\Psi_{iz}^{(+)}(\zeta) + C_2\Psi_{iz}^{(+)}(-\zeta). \tag{2}$$

Here C_1 and C_2 are arbitrary constants and the functions $\Psi_\lambda^{(\pm)}$ are expressed in terms of the parabolic-cylinder functions D_λ as follows:

$$\Psi_\lambda^{(\pm)}(\zeta)=D_\lambda\left(\exp\left\{\pm\frac{i\pi}{4}\right\}\zeta\right)\exp\left\{\mp\frac{i}{4}(\pi\lambda-\zeta^2)\right\}. \quad (3)$$

(The functions Ψ_λ^- will be needed later on).

For a_2 we obtain from the second equation of the system (1)

$$a_2=i\nu_2[C_1\Psi_{iz-1}^{(+)}(\zeta)-C_2\Psi_{iz-1}^{(+)}(-\zeta)] \quad (4)$$

(we have used here the relation $d\Psi_\lambda^{(\pm)}/d\zeta=\lambda\Psi_{\lambda-1}^{(\pm)}$, which follows from the recurrence formulas for D_λ [5]). Taking into account the asymptotic behavior of the function $\Psi_\lambda^{(\pm)}(\zeta)$:

$$\Psi_\lambda^{(\pm)}\to\zeta^\lambda,\quad \zeta\to\infty; \quad (5)$$

$$\Psi_\lambda^{(\pm)}\to e^{\mp i\pi\lambda}(-\zeta)^\lambda+\frac{(\mp2\pi i)^{1/2}}{\Gamma(-\lambda)}\exp\left\{\mp\frac{i}{2}(\pi\lambda-\zeta^2)\right\}(-\zeta)^{-\lambda-1},\quad \zeta\to-\infty,$$

we see that the behavior of the amplitudes at $|\zeta|\to\infty$ depends strongly on the value of the imaginary part z'' of the parameter $z=z'+iz''$. Namely, at $-1/2<z''<1/2$, the principal terms in (5) are slowly varying terms, so that

$$a_{1,2}\to\text{const}\cdot|\zeta|^{\pm iz}. \quad (6)$$

On the other hand, if $|z''|>1/2$, the principal term in the asymptotic form of one of the amplitudes is the one proportional to $\exp(\pm i\zeta^2/2)$.

Physically, the picture of resonant interaction in a narrow region $|\zeta|\sim1$ corresponds to the fullest degree to the case of real z, since in this case the wave energy far from the resonant layer becomes constant. Assuming that in this case the constants of (6), which correspond to incident waves, are specified, and determining the amplitudes of the outgoing waves from the solution, we can find the "gain" matrix. (The direction of wave incidence is determined by the sign of the group velocity; the constant in (6), which corresponds to the incident wave of type i, will be designated by A_i, and the constant corresponding to the outgoing wave will be designated B_i.) By virtue of the linearity of the problem (relative to waves 1 and 2), it suffices to consider the solutions $\mathscr{E}^{(i)}$ describing the incidence of a wave of type i.

We consider first, by way of example, the solution $\mathscr{E}^{(2)}$ at $u_1>0$ and $u_2<0$. In this case the boundary condition for the amplitudes takes the form

$$A_1^{(2)}=0,\quad A_2^{(2)}=1. \quad (7)$$

Determining the constants C_1 and C_2 in (2) and (4) with the aid of (5) and (7), and taking into account the linear relations between the functions with different indices [5], we obtain

$$a_2^{(2)}=\Psi_{-iz}^{(-)}(-\zeta),\quad a_1^{(2)}=i\nu_1\exp\{i\zeta^2/2\}\Psi_{-iz-1}^{(-)}(-\zeta). \quad (8)$$

With the aid of (5) we get from this

$$B_2^{(2)}=e^{\pi z},\quad B_1^{(2)}=\nu_1(2\pi i)^{1/2}e^{\pi z/2}/\Gamma(1+iz). \quad (9)$$

In perfect analogy, we have for the solution $\mathscr{E}^{(1)}$

$$B_1^{(1)}=e^{\pi z},\quad B_2^{(1)}=\nu_2(-2\pi i)^{1/2}e^{\pi z/2}/\Gamma(1-iz). \quad (10)$$

Let now $u_1>0$ and $u_2<0$. In this case the boundary condition for wave 2 is specified at $\zeta\to+\infty$, and we have from (5)—(7)

$$a_2^{(2)}=e^{-\pi z}\Psi_{iz}^{(-)}(-\zeta),\quad a_1^{(2)}=i\nu_1\exp\{-\pi z+i\zeta^2/2\}\Psi_{-iz-1}^{(-)}(-\zeta). \quad (11)$$

For the matrix $B_k^{(i)}$ we get

$$B_1^{(1)}=B_2^{(2)}=e^{-\pi z},\quad B_2^{(1)}=\frac{\nu_2(-2\pi i)^{1/2}e^{-\pi z/2}}{\Gamma(1-iz)},\quad B_1^{(2)}=\frac{\nu_1(2\pi i)^{1/2}e^{-\pi z/2}}{\Gamma(1+iz)}. \quad (12)$$

Thus, amplification takes place at $z>0$ when $\gamma>0$ and at $z<0$ when $\gamma<0$. The parameter z can be expressed in the form

$$z=\nu^2l^2/u_1u_2,$$

where ν is the increment of the decay instability of the considered waves in a homogeneous plasma with parameters corresponding to the resonant point. The amplification condition then takes the obvious form $\nu^2>0$.

The diagonal elements of the matrix $B_k^{(i)}$ are in fact the amplification coefficients of the incident wave; they were given in [1,2] for the case $\gamma>0$. The off-diagonal elements determine the amplitude of the produced beat-frequency wave. In the most interesting case $|z|\gg1$, we can obtain a clear-cut representation of the structure of the field in the resonant region with the aid of asymptotic expressions for the functions $\Psi_\lambda^{(\pm)}(\zeta)$, which can easily be found by standard methods, using the integral representations of the parabolic-cylinder functions.

By way of example, we consider the amplitude $a_1^{(2)}$. At $\gamma>0$, $z>0$, and $-1<\kappa<1$, where $\kappa=\zeta/2z^{1/2}$, we have

$$|a_1^{(2)}|^2=\frac{\nu_1^2}{2z\varkappa(1-\varkappa^2)^{1/2}}\exp\{2z[\pi-\arccos\varkappa+2\varkappa(1-\varkappa^2)^{1/2}]\};$$

at $\kappa\sim1$

$$|a_1^{(2)}|^2=2\nu_1^2z^{-1/3}v^2[2z^{2/3}(1-\varkappa)]e^{2\pi z},$$

where v is the Airy function; at $\kappa\gg1$ we have

$$|a_1^{(2)}|^2=\frac{\nu_1^2\varkappa}{z(\varkappa^2-1)^{1/2}}e^{2\pi z}\left(1+\frac{\sin\Phi(\varkappa)}{\varkappa}\right),$$

$$\Phi(\varkappa)=4z\left[\varkappa(\varkappa^2-1)^{1/2}-\ln\frac{\varkappa+(\varkappa^2-1)^{1/2}}{\varkappa-(\varkappa^2-1)^{1/2}}\right].$$

Thus, the modulus of the amplitude increases exponentially in the resonant region $|\zeta|>2z^{1/2}$ with an increment $\sim z^{1/2}$ that is linear in the field E_0, reaches a maximum value that exceeds the stationary level given by formula (9) by a factor $1.9z^{1/6}$, and then oscillates with decreasing amplitude and period. It should be noted that the asymptotic value is reached only at very large ζ, namely $\zeta\gg2z^{1/2}$.

At $\gamma<0$ and $(-z)\gg1$, the following relation is satisfied for all ζ:

$$|a_1^{(2)}|^2=\frac{\nu_1^2\tau_0^2}{-z(\tau_0^2-z)}e^{-2\pi z},\qquad \tau_0=\frac{(\zeta^2-4z)^{1/2}+\zeta}{2}.$$

The structure of the field turns out in this case to be entirely different: the modulus of the amplitude varies smoothly and there is no clearly pronounced resonance region.

Formulas (8)—(12) remain in force also in the case of complex z, if $|z''|<1/2$, but expressions (9)—(12) now no longer describe the total change of the wave energy in the layer, since the modulus of the amplitudes does not tend to a constant limit at $|\zeta|\to\infty$. The spatial increment at large ζ is equal to z''/x, i.e., it is quadratic in the pump field. It has the same order of magnitude as the contribution from the terms not taken into account in (1). For this reason, calculation of the total change of the wave energy in the layer turns out to be impossible within the framework of the considered approximation. At a still higher value of the imaginary part of z, the asymptotic form of the amplitude at $|\zeta| \gg 1$ is altered; the physical meaning of the solutions and the choice of the boundary condition becomes indeterminate. The question of the existence of stationary solutions can be resolved in this case only by considering the time-dependent problem.

3. We turn now to Eq. (1) and obtain its solutions that satisfy the specified initial conditions. By direct substitution we can easily verify that the general solution, which depends on two arbitrary functions F and Φ, can be represented in the form

$$a_1 = \exp\left\{\frac{i\eta s^2}{2}\right\} \int_{-\infty}^{\infty} \{F(x)u(x+\eta\tau) + \Phi(x)v(x+\eta\tau)\}e^{isx}\,dx,$$
$$a_2 = \frac{\eta}{\nu_1}\exp\left\{\frac{i}{2}(\eta s^2 - \zeta^2)\right\} \int_{-\infty}^{\infty} [Fu'(x+\eta\tau) + \Phi v'(x+\eta\tau)]e^{isx}\,dx, \tag{13}$$

where $\eta = 1/(1-\gamma)$, $s = \zeta - \tau$, and u and v are two linearly independent solutions of the equation

$$y'' - \alpha(ixy' - zy) = 0, \quad \alpha = (1-\gamma)^2/\gamma. \tag{14}$$

Representation of the general solution in the form (13) makes it possible to determine immediately the functions F and Φ in such a way that the solution satisfies the specified initial conditions at $\tau = 0$:

$$a_1(\zeta, 0) = a_{10}(\zeta), \quad a_2(\zeta, 0) = a_{20}(\zeta). \tag{15}$$

Indeed, at $\tau = 0$ the right-hand side of (13) has the form of Fourier integrals, so that

$$F(x)u(x) + \Phi(x)v(x) = \hat{a}_{10}(x),$$
$$F(x)u'(x) + \Phi(x)v'(x) = \hat{a}_{20}(x), \tag{16}$$

where $\hat{a}_{10}$ and $\hat{a}_{20}$ are the respective Fourier transforms of the functions $a_{10}\exp\{-i\eta\zeta^2/2\}$ and $a_{20}\exp\{-i(\eta-1)\times\zeta^2/2\}$. By virtue of (14), the determinant Δ of the system (16) takes the form $\Delta = \Delta_0\exp\{-i\alpha x^2/2\}$, where Δ_0 is a constant. We choose the functions u and v in the form

$$u(x) = \Psi_{iz}^{(\pm)}(|\alpha|^{1/2}x), \quad v(x) = \Psi_{iz}^{(\pm)}(-|\alpha|^{1/2}x) \tag{17}$$

(here and throughout, the upper superscript pertains to the case $\gamma > 0$ and the lower to $\gamma < 0$); then

$$\Delta_0 = (2\pi i\alpha)^{1/2}e^{\pm\pi z/2}/\Gamma(-iz). \tag{18}$$

Formulas (13) and (16)—(18) express, in terms of quadratures, a solution of (1) satisfying the arbitrary initial conditions (15).

4. By way of example, we consider the case of a_{i0} that do not depend on ζ. These boundary conditions mean that before the pump is turned on, the waves $\mathbf{E}_1$ and $\mathbf{E}_2$ filled the entire layer of plasma, which formally remains infinite. If the interaction of the waves at large distances is negligible, this automatically ensures a stationary flow of waves in the direction of the resonant layer at all times. Consequently, the condition a_{i0} = const should describe the process of establishment of a stationary state after a sudden application of the pump at t = 0. Just as in Sec. 2, it suffices to consider the fundamental solution $\mathscr{E}^{(2)}$ that satisfies the initial conditions

$$a_{10}^{(2)} = 0, \quad a_{20}^{(2)} = \text{const}. \tag{19}$$

The solution $\mathscr{E}^{(1)}$ can be obtained, if $\mathscr{E}^{(2)}$ is known, with the aid of the relation

$$a_i^{(1)}(\nu_1, \nu_2, u_1, u_2) = a_i^{(2)*}(\nu_2^*, \nu_1^*, u_2, u_1),$$

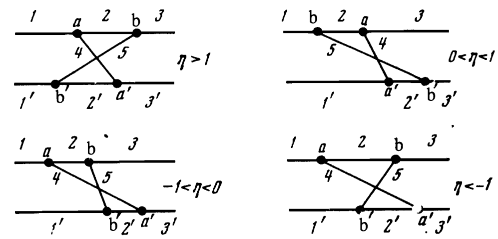
which follows from (1) and (19). Substituting (19) in (16) and (13), using the integral representation of the functions $\Psi_\lambda^{\pm}$, and varying the order of integration, we obtain after certain transformations

$$a_1^{(2)} = \nu_1 w(\zeta, s), \quad a_2^{(2)} = \exp\left\{-\frac{i\zeta^2}{2}\right\}\frac{\partial w(\zeta, s)}{\partial\zeta}, \tag{20}$$

where

$$w(\zeta, s) = I^{(\pm)}(\zeta, s) - I^{(\pm)}(s, \zeta),$$
$$I^{(\pm)}(\zeta, s) = \frac{e^{\pm i\pi/4}}{(2\pi)^{1/2}(1-e^{2\pi z})}\int_C \exp\left\{-\frac{i\eta^2\tau^2}{2} \mp i\eta s\tau\right\}\Psi_{iz}^{(+)}(\zeta\pm\tau)\tau^{-iz-1}\,d\tau \tag{21}$$

(the integration contour comes from infinity, goes around the origin in the positive direction, and goes off to infinity). We have chosen in (19) a constant equal to $-i(\gamma)^{iz}/(1-\gamma)^{iz}$, in order that the normalization of the amplitudes at $\tau \to \infty$ ($s \to -\infty$) will coincide with the normalization of the time-independent solutions.



The integrals (21) are apparently not expressed in terms of known functions, but it is possible to obtain asymptotic formulas that are suitable at $(\zeta^2 + \tau^2)^{1/2} \gg 1$ and arbitrary complex z. The derivation of these formulas can be explained in the following manner. The function $\Psi_{iz}^{(+)}$ in (21) has no singularities at finite values of the argument, so that at large $(\zeta^2 + \tau^2)^{1/2}$ the main contribution to the integral is made, generally speaking, by contour sections on which $\Psi_{iz}^{(+)}$ can be replaced by the asymptotic expression (5). The integral (21) is calculated in this case by the stationary-phase method. At certain exceptional relations between ζ and s (for example at s close to $\eta\zeta$), the principal role is played by the section of the contour where the argument $\Psi_{iz}^{(1)}$ is of the order of unity. In this case we can take the factor τ^{-iz-1} outside the integral sign at the corresponding point, after which the remaining integral can be calculated and expressed in terms of the function $\Psi_\lambda^{(\pm)}$.

The asymptotic representation obtained in this manner for the function $w(\zeta, s)$ assumes different forms in a large number of sectors of the (ζ, s) plane; it is therefore convenient to describe it by using a special diagram, as shown in the figure. The abscissas represent the argument φ of the point on the (ζ, s) plane ($\pi/4 \le \varphi \le 5\pi/4$, since it suffices to consider the half-plane $\zeta > s$ by virtue of the relation $w(\zeta, s) = -w(s, \zeta)$). Each element of the diagram (line or vertex) corresponds to a definite function of ζ or s. At a given value of φ, the value of w is equal to the sum of the terms corresponding to the elements of the diagram intersected by the vertical line drawn through the considered point φ of the abscissa axis. The lines correspond to the functions (the subscript indicates the number of the line)

$$f_1 = f_3 = -\frac{1}{s}\left|1 - \frac{\eta\zeta}{s}\right|^{iz}, \quad f_2 = \mp e^{\pm\pi z}f_1,$$
$$f_4 = \mp\frac{(2\pi i)^{1/2}}{\Gamma(1+iz)}e^{\pm\pi z/2}|\eta s\zeta|^{iz}, \tag{22}$$
$$f_5 = \text{const}\cdot\exp\left\{\frac{i}{2}\left[s^2 - \frac{(\eta\zeta - s)^2}{1-\eta^2}\right]\right\}[(\zeta - \eta s)(\eta\zeta - s)]^{-iz-1},$$

and the vertices to the functions

$$f_a = |\eta\zeta|^{iz} \exp\left\{-\frac{\pi z \mp \pi z - is^2}{2}\right\} \Psi^{(-)}_{-iz-1}(s),$$

$$f_b = -\frac{1}{s}\left|\frac{(1-\eta^2)^{1/2}}{s}\right|^{iz} \exp\left\{\frac{is^2}{2}\right\} \times \begin{cases} \exp\left\{-\frac{\pi z \mp \pi z}{2}\right\} \Psi^{(-)}_{iz}\left(\frac{s-\eta\zeta}{(1-\eta^2)^{1/2}}\right), & |\eta| < 1 \\ \Psi^{(+)}_{iz}\left(\frac{\eta}{|\eta|}\frac{s-\eta\zeta}{(\eta^2-1)^{1/2}}\right), & |\eta| > 1 \end{cases} \tag{23}$$

(In accordance with the asymptotic behavior of the functions $\Psi^{(\pm)}_{\lambda}$ (5), they are represented by three-point diagrams.) The functions with the primed indices are obtained by making the substitutions $\zeta \to -s$ and $s \to -\zeta$. The positions of the vertices on the diagram are determined by the condition that the arguments of the corresponding functions $\Psi^{(\pm)}_{\lambda}$ vanish. Depending on the interval in which the parameter η is located ($\eta > 1$, $1 > \eta > 0$, $0 > \eta > -1$, $-1 > \eta$), the vertices can be arranged in four sequences, as indicated in the figure.

The presented expressions contain the principal terms of the asymptotic form of the function w for arbitrary complex z. At $\tau \gg 1$, the asymptotic formulas are suitable for all ζ. It is seen from (22)–(23) that the moduli of the functions corresponding to lines of the diagrams are slowly varying, whereas the vertices describe relatively rapid variation. It can therefore be stated that the distribution of the field at large τ is characterized by the presence of four fronts, on which the values of the amplitude change very rapidly. Three of them, a, b, and b′, move with the respective velocities u_1, u_2, and $u_1 - u_2$, while the fourth, a′, is immobile and represents the resonant layer of the stationary problem. The value of the field amplitude in the vicinity of each "front" depends on the imaginary part z″ of the parameter z. For example, at real z, the functions f_b and $f_{b'}$ each contain a small factor 1/s in comparison with f_a and $f_{a'}$.

In the region adjacent to the resonant layer $\zeta = 0$, as $\tau \to \infty$ the function w is given at $\eta < 0$, i.e., at $\gamma > 1$, in accordance with the figure, by

$$w = f_{a'} + f_3. \tag{24}$$

Recognizing that as $\tau \to \infty$ we have $f_{a'} \to \tau^{1}z$ and $f_3 \sim 1/\tau$, we find that the term $f_{a'}$ is the principal one if $z'' < 1$. At $\eta > 0$, i.e., at $\gamma < 1$, there is added to (24) an additional term $f_5 \sim \tau^{-2}z^{-2}$. In fact, the more stringent limitation on z″ is obtained in this case for the amplitude $a_2^{(2)}$, namely $z'' < 1/3$.

When the derived conditions are satisfied, the solution $\mathscr{E}^{(2)}$ tends, apart from a slowly varying factor $\tau^{1}z$, to the stationary limit (9). In complete analogy, the condition for the existence of a stationary (in the same sense) limit for the solution $\mathscr{E}^{(1)}$ takes the form $z'' > -1$ at $1/\gamma > 1$ and $z'' > -1/3$ at $1/\gamma < 1$. If both waves $\mathbf{E}_1$ and $\mathbf{E}_2$ belong to the noise spectrum of the plasma, then simultaneous satisfaction of the conditions for $\mathscr{E}^{(1)}$ and $\mathscr{E}^{(2)}$ is required for the existence of a stationary solution. If, at the same time, the group velocities have opposite directions, $\gamma < 0$, then we have simultaneously $\gamma < 1$ and $1/\gamma < 1$; on the other hand, if $\gamma > 0$, then either γ or $1/\gamma$ is smaller than unity (we do not consider here the special case $\gamma = 1$, or the case $\gamma = 2$, when $\eta^2 = 1$). Thus, the final conditions for the existence of stationary amplification are

$$\underset{\gamma<0}{-1/3 < z'' < 1/3} \qquad \underset{\gamma>1}{-1/3 < z'' < 1} \qquad \underset{0<\gamma<1}{-1 < z'' < 1/3} \tag{25}$$

The asymptotic expressions (22) and (23) also enable us to determine the field at all times at a point located far from the resonant layer. If, for example, the conditions (25) are satisfied, then the process for the establishment of the stationary amplitude $a_1^{(2)}$ is described by the passage of the front f_a, which has a structure similar to the structure of the resonant layer described in Sec. 2, and moves with velocity u_1.

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