Quasilinear theory of a parametrically unstable magnetoactive plasma

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A quasilinear theory of a magnetoactive plasma irradiated by an intense beam is developed with the aid of the equation for the pair correlation function of plasma particles. The general points of the theory are illustrated by solving the quasilinear equation set for the case of the electron oscillation amplitude in the field of the pumping wave which is small compared to the perturbation wavelength of a strongly magnetized plasma at frequencies of the lower hybrid resonance and slow magentic sound. The time dependences are obtained for the parametric growth rate, spectral energy density of parametrically excited plasma perturbations, and electron distribution functions. The appearance of an appreciable number of fast electrons (about a tenth of a per cent of the total number) is observed. The turbulent conductivity of a magnetoactive plasma at the pumping wave frequency is high compared to the laminar conductivity due to collisions.

The theory of paramagnetic resonance[1-5] explains qualitatively the main processes occurring in a plasma subject to the action of powerful radiation. A quantitative comparison of the theoretical conclusions with the presently accumulating experimental data[6-15] calls for an appreciable extension of the theory. The present paper is devoted to one of the aspects of the theory of parametric resonance, namely the theory of quasilinear relaxation of a parametrically excited plasma situated in a constant and homogeneous magnetic field.

Anomalous absorption of high-power radiation by collisionless plasma[6,8,10-12] can be understood within the framework of the theory of nonlinear interaction of parametrically unstable oscillations[4,5,16] of a plasma with a specified particle distribution function. To the contrary, such phenomena as heating and acceleration of the plasma particles by radiation[7,8,9,12,15] can be explained only by studying the variation of the total distribution function. The equations for the joint evolution of the distribution function of the particles and of the spectral energy density of the parametrically unstable plasma oscillations without a magnetic field were formulated in[17]. They were used, in particular, to investigate theoretically the previously-predicted[2] rapid heating of plasma by radiation. The theory was subsequently developed[17] as applied to the case of a relatively weak pumping field[18,19]. A plasma-electron redistribution was obtained[19], whereby the number of the fast electrons increases.

The experimental studies[7,9-14] were devoted to parametric resonance in a magnetized plasma. The measurement results[7,9] make it possible to determine the distribution function of the fast plasma electrons along the external magnetic field, and in[12] it is indicated that fast ions are present in a parametrically unstable magnetized plasma. The quasilinear effects in such a plasma were first discussed theoretically in[20], where a quasilinear theory was used to obtain the consequences resulting from the hypothesis that the parametric instability becomes stabilized by raising the electron temperature in the weak pumping field. The explanation presented below will show that the quasilinear saturation of the parametrically-growing noise in a magnetoactive plasma in a weak pumping field is the result of a deformation of the electron velocity distribution function in a relatively narrow interval of above-thermal velocities, when the heating can be completely neglected.

The paper is divided into two sections. In the first we give the quasilinear equations of a parametrically excited magnetoactive plasma and discuss in detail the concrete quasilinear relaxation equations that appear in the case of a weak pumping field. In the second section we obtain the evolution of the electron distribution function, of the increment, and of the spectral energy density of parametrically excited slow magnetic sound and of the lower hybrid resonance.

1. EQUATIONS OF QUASILINEAR INTERACTION OF POWERFUL RADIATION WITH MAGNETOACTIVE PLASMA

We consider a fully ionized plasma situated in a constant and homogeneous magnetic field with intensity B. The external radiation acting on this magnetoactive plasma is assumed in the form of a high-frequency spatially homogeneous monochromatic electric pump field

$$\mathbf{E}(t) = \mathbf{E}_0 \sin \omega_0 t \tag{1.1}$$

with frequency ω_0 and intensity $\mathbf{E}_0$. The equations describing the quasilinear relaxation of a parametrically excited magnetoactive plasma can be obtained by starting from the system of equations for the distribution function f(p, t) of particles with momentum p at the instant of time t and the pair correlation function[21]. Just as in the case of an isotropic plasma[17], we can write as a consequence of such a system the relation

$$\frac{\partial f}{\partial t} + e\left\{\mathbf{E}(t) + \frac{1}{c}[\mathbf{vB}]\right\}\frac{\partial f}{\partial \mathbf{p}} = -e\frac{\partial}{\partial \mathbf{p}}\int \frac{d\mathbf{k}}{(2\pi)^3}\mathbf{E}(-\mathbf{k}, t)\,\delta f(\mathbf{k}, \mathbf{p}, t), \tag{1.2*}$$

$$\frac{\partial \delta f}{\partial t} + \mathbf{v}\frac{\partial \delta f}{\partial \mathbf{r}} + e\left\{\mathbf{E}(t) + \frac{1}{c}[\mathbf{vB}]\right\}\frac{\partial \delta f}{\partial \mathbf{p}} = -e\mathbf{E}(\mathbf{r}, t)\frac{\partial f}{\partial \mathbf{p}}, \tag{1.3}$$

$$\operatorname{div}\mathbf{E}(\mathbf{r}, t) = 4\pi e\int d\mathbf{p}\,\delta f(\mathbf{r}, \mathbf{p}, t). \tag{1.4}$$

Here e is the particle charge and c is the speed of light in vacuum; in the right-hand side of (1.4), summation over the difference source of plasma particles is implied. The Fourier transforms of the quantities are defined in the usual manner. For example, for the fluctuating electric field $\mathbf{E}(\mathbf{r}, t)$ we have

$$\mathbf{E}(\mathbf{r}, t) = \int \frac{d\mathbf{k}}{(2\pi)^3}\mathbf{E}(\mathbf{k}, t)\, e^{i\mathbf{kr}}.$$

Taking into account the explicit form of the characteristics of the differential operator in the left-hand side of (1.3) ($\mathbf{h} \equiv (\mathbf{B}/B)$ is a unit vector along the magnetic field)

$$v_i(\tau,t;\mathbf{v}) = W_{ij}(\tau)v_j + \frac{e}{m}\int_0^\tau d\tau'\, W_{ij}(\tau-\tau')E_j(\tau'+t),$$

$$\delta R_i(\tau,t;\mathbf{v}) \equiv r_i(\tau,t) - r_i = \rho_{ij}(\tau)v_j + \frac{e}{m}\int_0^\tau d\tau'\rho_{ij}(\tau-\tau')E_j(\tau'+t), \quad (1.5)$$

$$W_{ij}(\tau) \equiv h_ih_j - e_{isj}h_s\sin\Omega\tau + (\delta_{ij}-h_ih_j)\cos\Omega\tau,\ \ \rho_{ij}(\tau) \equiv \int_0^\tau d\tau' W_{ij}(\tau'),$$

we can eliminate the quantity δf from (1.2) and (1.4). As a result, the system of equations for the distribution function of the particles

$$F(p_i,t) = f\left(p_i + e\int_{-\infty}^0 d\tau W_{ij}(-\tau)E_j(t+\tau), t\right)$$

and for the field potential $\mathbf{E}(\mathbf{k}, t) = -i\mathbf{k}\varphi(\mathbf{k}, t)$ of the natural oscillations of the plasma takes the form

$$\frac{\partial F(\mathbf{p},t)}{\partial t} + \frac{e}{c}[\mathbf{vB}]\frac{\partial F(\mathbf{p},t)}{\partial \mathbf{p}} = e^2\frac{\partial}{\partial p_i}\int\frac{d\mathbf{k}}{(2\pi)^3}k_ik_j\varphi(-\mathbf{k},t)\int_{-\infty}^0 d\tau\,\varphi(\mathbf{k},t+\tau)$$
$$\times\frac{\partial F(\mathbf{p}(\tau,\mathbf{v});t+\tau)}{\partial p_j(\tau,\mathbf{v})}\exp\Big\{ik_i\rho_{ij}(\tau)v_j$$
$$+ i\frac{e}{m}\int_{-\infty}^0 d\tau'\, k_i\rho_{ij}(-\tau')\left[E_j(\tau'+\tau+t) - E_j(\tau'+t)\right]\Big\}, \quad (1.6)$$

$$\varphi(\mathbf{k},t) = i\frac{4\pi e^2\mathbf{k}}{k^2}\int_{-\infty}^0 d\tau\varphi(\mathbf{k},t+\tau)\int d\mathbf{p}\frac{\partial F(\mathbf{p},t+\tau)}{\partial \mathbf{p}} \quad (1.7)$$
$$\times\exp\Big\{ik_i\rho_{ij}(\tau)v_j + i\frac{e}{m}\int_{-\infty}^0 d\tau'\, k_i\rho_{ij}(-\tau')\left[E_j(\tau'+\tau+t) - E_j(\tau'+t)\right]\Big\}.$$

Here $\Omega = eB/mc$ is the gyroscopic frequency and δ_{ij} and e_{isj} are unit tensors of second and third rank, respectively. For the high-frequency field (1.1) we have

$$\frac{e}{m}\int_{-\infty}^0 d\tau\, k_i\rho_{ij}(-\tau)E_j(\tau+t) = a\sin(\omega_0 t+\psi),$$

where the role of the amplitude of the particle oscillations in the field is assumed by the quantity a:

$$a = \frac{|e|}{m\omega_0^2}\left\{(\mathbf{k}[\mathbf{E}_0\mathbf{h}])^2\left(\frac{\omega_0\Omega}{\omega_0^2-\Omega^2}\right)^2 + \left[(\mathbf{kh})(\mathbf{E}_0\mathbf{h}) + \frac{\omega_0^2}{\omega_0^2-\Omega^2}([\mathbf{kh}][\mathbf{E}_0\mathbf{h}])\right]^2\right\}^{1/2},$$

$$a\cos\psi \equiv -\frac{e}{m\omega_0^2}\left\{(\mathbf{kh})(\mathbf{E}_0\mathbf{h}) + \frac{\omega_0^2}{\omega_0^2-\Omega^2}([\mathbf{kh}][\mathbf{E}_0\mathbf{h}])\right\},$$

$$a\sin\psi \equiv \frac{e}{m\omega_0^2}(\mathbf{k}[\mathbf{E}_0\mathbf{h}])\frac{\omega_0\Omega}{\omega_0^2-\Omega^2}.$$

We expand the potential $\varphi(\mathbf{k}, t)$ of the fluctuation fields in the plasma and the distributions $F(\mathbf{p}, t)$ in terms of the harmonics of the frequency ω_0 of the external field

$$\varphi(\mathbf{k},t) = \sum_{n=-\infty}^{+\infty}\varphi_n(\mathbf{k},t)e^{-in\omega_0 t}, \quad (1.8)$$

$$F(\mathbf{p},t) = \sum_{n,m=-\infty}^{+\infty}F^{(n,m)}(p_z,p_\perp,t)e^{im\varphi-in\omega_0 t}. \quad (1.9)$$

In formula (1.9) we used the harmonics $F^{(n,m)}$ of the distribution $F(\mathbf{p}, t)$ also with respect to the azimuthal angle φ of the gyroscopic rotation of the particle around the magnetic field $\mathbf{B}$ (in a cylindrical coordinate system with z axis along $\mathbf{B}$); $p_z = (\mathbf{p}\cdot\mathbf{h})$, $p_\perp^2 = [\mathbf{p}\times\mathbf{h}]^2$. Equations (1.7), (1.8), and (1.9) lead (see [17,21]) to the field equation

$$\frac{\partial}{\partial t}|\varphi(\mathbf{k})|^2 = 2\gamma(\mathbf{k},t)|\varphi(\mathbf{k})|^2 \quad (1.10)$$

for low-frequency ($\omega \ll \omega_0$) almost-periodic ($\gamma \ll \omega$) magnetoactive-plasma oscillations that are parametrically excited with increment γ,

$$\varphi(\mathbf{k}) = \varphi_0(\mathbf{k},\omega(\mathbf{k},t),t) = \varphi_0(\mathbf{k},t)\exp\left\{i\int_0^t dt'\,\omega(\mathbf{k},t')\right\}$$

and are described by the zeroth harmonic ($n = 0$) of the expansion (1.8). The frequency ω and the growth increment γ are determined from the dispersion equation

$$\frac{1}{\delta\varepsilon_i(\omega+i\gamma,\mathbf{k},t)} + \sum_{n=-\infty}^{+\infty}J_n^2(a)\frac{1}{1+\delta\varepsilon_e(\omega+n\omega_0+i\gamma,\mathbf{k},t)} = 0, \quad (1.11)$$

in which $J_n(a)$ is a Bessel function of order n with argument a, and the partial longitudinal electric constants $\delta\epsilon_i$ and $\delta\epsilon_e$ of the ionic and electronic components of the plasma are determined in the usual manner [22,23].

For the zeroth harmonic $n = 0$ and $m = 0$ of the expansion (1.9), which is a slowly-varying part of the electronic distribution function $F_e^{(0,0)}(v_z, v_\perp, t) \equiv F_e(v_z, v_\perp, t)$ of the parametrically unstable plasma, we obtain from (1.6) and (1.7), taking the expansions (1.8) and (1.9) into account, the following relation (see.[20])

$$\frac{\partial F_e}{\partial t} = \frac{e^2}{m^2}\sum_{n,s=-\infty}^{+\infty}\int\frac{d\mathbf{k}}{(2\pi)^3}\left(k_z\frac{\partial}{\partial v_z} + \frac{s\Omega_e}{v_\perp}\frac{\partial}{\partial v_\perp}\right)J_s^2\left(\frac{k_\perp v_\perp}{\Omega_e}\right)|\varphi(\mathbf{k})|^2J_n^2(a)$$
$$\times\left|\frac{\delta\varepsilon_i(\omega+i\gamma,\mathbf{k},t)}{1+\delta\varepsilon_e(\omega+n\omega_0+i\gamma,\mathbf{k},t)}\right|^2\Big[\pi\delta(\omega+n\omega_0-s\Omega_e-k_zv_z) - \gamma(\mathbf{k},t)$$
$$\times\frac{\partial}{\partial\omega}\frac{\mathrm{P}}{\omega+n\omega_0-s\Omega_e-k_zv_z}\Big]\left(k_z\frac{\partial}{\partial v_z} + \frac{s\Omega_e}{v_\perp}\frac{\partial}{\partial v_\perp}\right)F_e(v_z,v_\perp,t). \quad (1.12)$$

Here e and m are the charge of the electron and its mass, Ω_e is the electron gyrofrequency, $k_z = (\mathbf{k}\cdot\mathbf{h})$, and $k_\perp^2 = [\mathbf{k}\times\mathbf{h}]^2$. The first term in the square brackets of the right-hand side of (1.12) (the δ-function) corresponds to resonant interaction of the electrons with the plasma perturbations, while the second is due to nonresonant adiabatic interaction (P is the principal-value symbol). The direct action of the radiation on the ions can be neglected in view of their large mass, $m_i \gg m$. Therefore the equation for the ion distribution function $F_i(v_z, v_\perp, t)$ takes the form of the usual quasilinear equation in a magnetoactive plasma not subject to the action of powerful radiation.

We note that (1.12) is only a part of the consequences of the general relations (1.8) and (1.9), which lead to a system of coupled equations for the harmonics $F_e^{(n,m)}$. We have confined ourselves here to formula (1.12) for the zeroth harmonic of the electron distribution $Fe(\mathbf{p}, t)$, since this is the only one we shall need for the concrete applications that follow below, in the case of a weak ($a \ll 1$) pump field. In a strong pump field ($a \gg 1$), the coupling between the harmonics $F^{(n,m)}$ is appreciable.

The high-frequency conductivity σ, which characterizes the rate of energy loss of the external radiation (1.1) in a magnetoactive plasma, is given by the expression

$$\frac{1}{2}\sigma E_0^2 = \omega_0\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{k^2}{4\pi}|\varphi(\mathbf{k})|^2|\delta\varepsilon_i(\omega+i\gamma,\mathbf{k},t)|^2$$
$$\times\sum_{n=-\infty}^{+\infty}nJ_n^2(a)\frac{\operatorname{Im}\delta\varepsilon_e(\omega+n\omega_0+i\gamma,\mathbf{k},t)}{|1+\delta\varepsilon_e(\omega+n\omega_0+i\gamma,\mathbf{k},t)|^2}. \quad (1.13)$$

As a concrete application of the system of Eqs. (1.10) and (1.12) derived above, let us consider the evolution of the electron distribution function F_e, assuming the ions to have a Maxwellian distribution with a thermal velocity v_{Ti} and a temperature T_i, and assuming this distribution to be constant in time, as is justified by the large inertia of the ions. Investigating F_e, we are interested only in that part $\mathscr{F}_e$ which determines the distribution of the electrons along the constant magnetic field (see the results of the experiments[7,9,10,12]), assuming the electron distribution function across the magnetic field to be Maxwellian with temperature $T_\perp$: The rationale for such an approximation of the sought function may be the experimentally established fact that in a magnetoactive plasma subject to the action of powerful radiation the rise of the electron temperature across the magnetic field is much smaller than the rise of their temperature along **B**. After integrating with respect to the transverse velocities, we can rewrite (1.12) in the form of an equation for $\mathscr{F}_e$:

$$\frac{\partial \mathscr{F}_e}{\partial t} = \frac{e^2}{m^2}\frac{\partial}{\partial v_z}\sum_{n,s=-\infty}^{\infty}\int\frac{d\mathbf{k}}{(2\pi)^3}k_z^2|\varphi(\mathbf{k})|^2 A_s(k_\perp^2\rho_e^2)\cdot$$
$$\times J_n^2(a)\left|\frac{\delta\varepsilon_i(\omega+i\gamma,\mathbf{k})}{1+\delta\varepsilon_e(\omega+n\omega_0+i\gamma,\mathbf{k},t)}\right|^2\left[\pi\delta(\omega+n\omega_0-s\Omega_e-k_zv_z)\right.$$
$$\left.-\gamma(\mathbf{k},t)\frac{\partial}{\partial\omega}\frac{P}{\omega+n\omega_0-s\Omega_e-k_zv_z}\right]\left(k_z\frac{\partial}{\partial v_z}-\frac{s\Omega_e}{v_{T\perp}^2}\right)\mathscr{F}_e(v_z,t). \quad (1.14)$$

Here $v_{T\perp} = (\kappa T_\perp/m)^{1/2}$ is the thermal velocity of the electrons across the magnetic field (κ is Boltzmann's constant), $\rho_e = v_{T\perp}/\Omega_e$ is their Larmor radius, and the function $A_s(z) = e^{-z}I_s(z)$ is determined by a modified Bessel function I_s with integer index s.

We confine ourselves to a case of experimental interest (see[7,9-14]), when the pumping-field frequency ω_0 is close to one of the hybrid resonance frequencies, $\omega_0 \approx \omega_{res}$ (θ is the angle between the wave vector **k** of the plasma oscillations and the magnetic field **B**, and $\omega_{Le} = (4\pi N_e e^2/m)^{1/2}$ is the Langmuir frequency of the electrons with density N_e):

$$\omega_{res} = \frac{1}{\sqrt{2}}\{\omega_{Le}^2+\Omega_e^2\pm[(\omega_{Le}^2+\Omega_e^2)^2-4\omega_{Le}^2\Omega_e^2\cos^2\theta]^{1/2}\}^{1/2}. \quad (1.15)$$

For the distribution function in the sum over the frequency harmonics of a relatively weak ($a \ll 1$) external field, the terms that are not small in the right-hand side of (1.14) are those with n = 0 and ±1. Retaining in the dielectric constant $\delta\epsilon_e(\omega \pm \omega_0 + i\gamma, \mathbf{k}, t)$ the terms proportional to the small detuning $\Delta\omega_0 = \omega_0 - \omega_{res}$ and proportional to the high-frequency damping decrement $\tilde{\gamma}$, we obtain ($\delta\epsilon_e'$ and $\delta\epsilon_e''$ are the real and imaginary parts of $\delta\epsilon_e$)

$$1+\delta\varepsilon_e(\omega\pm\omega_0+i\gamma,\mathbf{k},t)\approx\frac{2\delta}{\omega_0}[\Delta\omega_0\pm\omega+i\tilde{\gamma}(\mathbf{k},t)+i\gamma(\mathbf{k},t)],$$
$$\delta=\frac{1}{2}\left[\frac{\partial\omega\delta\varepsilon_e'(\omega,\mathbf{k})}{\partial\omega}\right]_{\omega=\omega_{res}},\quad \tilde{\gamma}(\mathbf{k},t)=\frac{\omega_0}{2\delta}\delta\varepsilon_e''(\omega_0,\mathbf{k},t). \quad (1.16)$$

For the partial dielectric constants $\delta\epsilon_{e,i}$ at low frequency ω we shall use the expressions (Ω_i is the gyrofrequency and $\omega_{Li} = (4\pi N_i e_i^2/m_i)^{1/2}$ is the Langmuir frequency of the ions with charge e_i and density n_i)

$$\delta\varepsilon_i(\omega,\mathbf{k}) = -\frac{\omega_{Li}^2}{\omega^2}\cos^2\theta-\frac{\omega_{Li}^2\sin^2\theta}{\omega^2-\Omega_i^2},$$
$$\delta\varepsilon_e(\omega,\mathbf{k},t) = -\frac{\omega_{Le}^2}{k^2}\left\{P\int_{-\infty}^{+\infty}\frac{dv_z}{v_z}\frac{\partial\mathscr{F}_e(v_z,t)}{\partial v_z}\right.$$
$$\left.+i\pi\left[\frac{\partial\mathscr{F}_e(v_z,t)}{\partial v_z}\right]_{v_z=\omega/k_z}\operatorname{sign}k_z\right\},$$

which are valid in a plasma with sufficiently cold ions $\omega \gg |k_z|v_{Ti}$ and hot electrons $\omega \ll |k_z|v_{Te}$ ($T_e \gg T_i$).[1] The magnetization of the ions and of the electrons $k_\perp^2\rho_{e,i}^2 \ll 1$ ensures one-dimensionality of the quasilinear relaxation. Under these conditions, Eq. (1.14) for $\mathscr{F}_e$ becomes much simpler:

$$\frac{\partial\mathscr{F}_e}{\partial t}=\frac{\partial}{\partial v_z}\frac{e^2}{m^2}\int\frac{d\mathbf{k}}{(2\pi)^3}k_z^2|\varphi(\mathbf{k})|^2|\delta\varepsilon_i(\omega,\mathbf{k})|^2$$
$$\times\left\{|1+\delta\varepsilon_e(\omega,\mathbf{k})|^{-2}\left[\pi\delta(\omega-k_zv_z)-\gamma(\mathbf{k},t)\frac{\partial}{\partial\omega}\frac{P}{\omega-k_zv_z}\right]\right.$$
$$+\frac{a^2}{16\delta^2}\frac{\omega_0^2}{(\omega+\Delta\omega_0)^2+(\tilde{\gamma}+\gamma)^2}\left[\pi\delta(\omega+\omega_0-k_zv_z)-\gamma(\mathbf{k},t)\frac{\partial}{\partial\omega}\frac{P}{\omega+\omega_0-k_zv_z}\right]$$
$$+\frac{a^2}{16\delta^2}\frac{\omega_0^2}{(\omega-\Delta\omega_0)^2+(\tilde{\gamma}+\gamma)^2}\left[\pi\delta(\omega-\omega_0-k_zv_z)\right.$$
$$\left.\left.-\gamma(\mathbf{k},t)\frac{\partial}{\partial\omega}\frac{P}{\omega-\omega_0-k_zv_z}\right]\frac{\partial\mathscr{F}_e}{\partial v_z}\right\} \quad (1.17)$$

The low frequencies (compared with (1.15) of the natural oscillations of the magnetoactive plasma in the case of a parametric decay instability are given by the equation

$$\omega\simeq\omega_{1,2}\equiv\frac{1}{\sqrt{2}}\{\omega_s^2+\Omega_i^2\mp[(\omega_s^2+\Omega_i^2)^2-4\omega_s^2\Omega_i^2\cos^2\theta]^{1/2}\}^{1/2}, \quad (1.18)$$

in which the subscripts 1 and 2 correspond to negative and positive values of the square root, $\omega_s = kv_s \equiv k\omega_{Li}r_{De}$ is the frequency of the long-wave ion-acoustic oscillations. The low-frequency damping decrements $\gamma_{1,2}$ in a non-isotropic plasma ($T_e \gg T_i$) are determined by the electronic distribution function $\mathscr{F}_e$ if the ion-ion collisions can be neglected ($\epsilon = 1 + \delta\epsilon_i$):

$$\gamma_{1,2}=-\frac{\pi\omega_{Le}^2}{k_2}\left[\frac{\partial\varepsilon(\omega,\mathbf{k})}{\partial\omega}\right]^{-1}_{\omega=\omega_{1,2}}\left[\frac{\partial\mathscr{F}_e}{\partial v_z}\right]_{v_z=\omega_{1,2}/k_z}\operatorname{sign}k_z. \quad (1.19)$$

We consider next the case of resonance between the pumping field and the plasma oscillation at the frequency of the lower hybrid in a magnetized plasma ($\Omega_e^2 \gg \omega_{Le}^2$)

$$\omega_0=\Delta\omega_0+\omega_{Le}|\cos\theta|, \quad (1.20)$$

when the parametric-buildup increment γ reaches its maximum value. We can then neglect in the high-frequency decrement $\tilde{\gamma}$ (see (1.16) the contribution of the small cyclotron damping. In addition, assuming the plasma to be sufficiently hot and rarefied, we neglect also the electron-ion collisions. As a result, the high-frequency damping decrement $\tilde{\gamma}$ is determined only by the inverse Cerenkov effect on the electrons (Landau damping):

$$\tilde{\gamma}(\mathbf{k},t)=-\frac{\pi}{2}\frac{\omega_0\omega_{Le}^2}{k^2}\left[\frac{\partial\mathscr{F}_e(v_z,t)}{\partial v_z}\right]_{v_z=\omega_0/k_z}\operatorname{sign}k_z. \quad (1.21)$$

Assuming the electron distribution function $\mathscr{F}_e$ to be even, we consider below the region of positive value of the velocity $v_z > 0$ along the magnetic field **B**.

An essential feature of quasilinear relaxation of a parametrically unstable plasma is the presence of not one but of many intervals of electron diffusion, due to the Cerenkov effect at the harmonics of the pumping-field frequency (cf.[19]). In particular, in the weak-field approximation ($a \ll 1$) and in a magnetized plasma, there are two such diffusion intervals (see Eq. (1.17)). Electrons with low velocities $v_z \approx \omega/k_z$ diffuse as a result of resonant interaction with low-frequency plasma perturbations, while the diffusion of the faster electrons, with velocities $v_z \approx \omega_0/k_z$, is due to the Cerenkov resonance with the high-frequency plasma oscillations (at the frequency ω_0 of the pumping field). We confine ourselves here to the study of quasilinear diffusion of relatively fast electrons $v_z \approx \omega_0/k_z$ due to their resonant interaction with the first harmonic of the weak pumping field, and show that even within the framework of this mechanism alone it becomes possible to stabilize the parametric instability[2]. Equation (1.17) for the relaxation of $\mathscr{F}_e$ reduces, with the aid of formula (1.21), by differentiating both halves, with respect to v_z, to the equation for the high-frequency damping decrement $\tilde{\gamma}$:

$$\frac{\partial}{\partial t}\tilde{\gamma}\left(\frac{\omega_0}{v_z},t\right)=v_z^2\frac{\partial^2}{\partial v_z^2}\frac{\pi e^2\omega_0^2}{16m^2v_z^2}\tilde{\gamma}\left(\frac{\omega_0}{v_z},t\right)\int\frac{d\mathbf{k}}{(2\pi)^3}k_z^2a^2|\varphi(\mathbf{k})|^2$$
$$\times|\delta\varepsilon_i(\omega,\mathbf{k})|^2\left\{\frac{\delta(\omega_0+k_zv_z)}{(\omega-\Delta\omega_0)^2+(\tilde{\gamma}+\gamma)^2}+\frac{\delta(\omega_0-k_zv_z)}{(\omega+\Delta\omega_0)^2+(\tilde{\gamma}+\gamma)^2}\right\}. \quad (1.22)$$

Integration with respect to the wave vector $\mathbf{k}$ of the plasma perturbations is conveniently carried out by choosing as the independent variables the longitudinal component k_z, the detuning $\Delta\omega_0$ connected with the total angle θ by the relation (1.20), and the azimuthal angle. Being interested later in the case when the electric-field intensity vector of the pump (1.11) is parallel to the magnetic field we see that the oscillation amplitude $a = |k_z(\mathbf{v}_E \cdot \mathbf{h})|\omega_0^{-1}$ ($\mathbf{v}_E \equiv e\mathbf{E}_0/m\omega_0$) and the integrand in the right-hand side of (1.22) do not depend on the azimuth, so that we are left only with integration with respect to the detuning $\Delta\omega_0$.

For the parametric instability corresponding to the decay of the pumping wave into a high-frequency oscillation $\omega_{res} = \omega_{Le}|\cos\theta|$ and one of the low-frequency oscillations

$$\omega=\omega_{1,2}=\Delta\omega_0>\max\{\gamma,\tilde{\gamma},\gamma_{1,2}\}, \quad (1.23)$$

we choose from among the two branches ω_1 and ω_2 the slow magnetosonic wave $\omega = \omega_1 = |k_z|\omega_{Li}r_{De} \equiv \omega_s|\cos\theta|$, which have the maximum growth increment in a strongly magnetized plasma:

$$\gamma_{max}=-\frac{1}{2}(\tilde{\gamma}+\gamma_1)+\frac{1}{2}\left[(\tilde{\gamma}-\gamma_1)^2+\frac{1}{4}\frac{v_E^2}{v_{Te}^2}\omega_0\omega_1\right]^{1/2}. \quad (1.24)$$

After expanding the increment γ in the vicinity of the maximum value (1.24)

$$\gamma(\Delta\omega_0)\approx\gamma(\omega_1)+\frac{1}{2}(\Delta\omega_0-\omega_1)^2\frac{\partial^2\gamma}{\partial\omega_1^2},\quad \gamma(\omega_1)\equiv\gamma_{max}, \quad (1.25)$$

and integrating in the right-hand side of (1.22) with respect to the detunings $\Delta\omega_0$ with allowance for the field equation (1.10), we describe the quasilinear relaxation of the parametrically unstable magnetoactive plasma by a system of nonlinear partial differential equations:

$$\frac{\partial}{\partial\tau}\frac{\tilde{\gamma}}{\omega_0}=\frac{x^2}{2}\frac{\partial^2}{\partial x^2}\left\{e^{\psi}G(\beta)\tilde{\gamma}\left[\tilde{\gamma}+\frac{\omega_0}{2}\frac{\partial\psi}{\partial\tau}\right]^{-1}\left[1-\frac{\partial\xi}{\partial\tau}\left(\frac{\tilde{\gamma}}{\omega_0}+\frac{1}{2}\frac{\partial\psi}{\partial\tau}\right)\right]^{-1/2}\right\};$$
$$\left(\frac{\partial\psi}{\partial\tau}+2\frac{\tilde{\gamma}}{\omega_0}\right)\left(\frac{\partial\psi}{\partial\tau}+2\frac{\gamma_1}{\omega_0}\right)=\frac{2C}{x},\quad \frac{\partial\xi}{\partial\tau}\left(\frac{\partial\psi}{\partial\tau}+\frac{\gamma_1}{\omega_0}+\frac{\tilde{\gamma}}{\omega_0}\right)^3=\frac{C}{x};$$
$$C=\frac{1}{2}\frac{v_E^2}{v_{Te}^2}\frac{\omega_{Li}}{\omega_{Le}};\quad \beta=\xi\left(\frac{\tilde{\gamma}}{\omega_0}+\frac{1}{2}\frac{\partial\psi}{\partial\tau}\right)\left[1-\frac{\partial\xi}{\partial\tau}\left(\frac{\tilde{\gamma}}{\omega_0}+\frac{1}{2}\frac{\partial\psi}{\partial\tau}\right)\right]^{-1};$$
$$G(\beta)=e^{\beta}\left[1-\frac{2}{\sqrt{\pi}}\int_0^{\sqrt{\beta}}dx\,e^{-x^2}\right]. \quad (1.26)$$

The high-frequency damping decrement $\tilde{\gamma}(x,\tau)$ is here a function of the dimensionless velocity $x = (v_z/v_{Te})$ and the time $\tau = \omega_0 t$, while the function $\psi(x, \tau)$ is determined by the maximum growth increment

$$\psi(x,\tau)=-\ln Ax^5+2\int_0^{\tau/\omega_0}dt'\gamma\left(\frac{\omega_0}{xv_{Te}},\omega_1,t'\right),$$
$$A=16\frac{\omega_{Le}}{\omega_0}\frac{v_{Te}^2}{v_E^2}N_e r_{De}^3, \quad (1.27)$$

and $\xi(x, \tau)$ characterizes the second derivative of the increment in the expansion (1.25).

From the condition (1.23), which was used essentially in the derivation of the system (1.26), it follows that the latter is valid at electron velocities v_z that greatly exceed the thermal velocity ($x \gg 1$). Thus, the resonant interaction of the electrons with the first harmonic of the pumping field, which is taken by us into account here, exerts an influence on that part of the distribution function $\mathcal{F}_e(v_z, t)$ which describes the fast (above-thermal) electrons. Since the fraction of the fast electrons in the plasma is small (on the order of 1% or less), we assume the electron temperature T_e to be constant during the entire quasilinear-relaxation process. Neglecting the quasilinear diffusion in the interval of low velocities $v_z \approx \omega/k_z$, we can assume the frequency ω_1 and the low-frequency damping decrement $\gamma_1 = \sqrt{\pi/8}(\omega_{Li}\omega_1/\omega_{Le})$ to be independent of the time, thus simplifying the analysis of the system (1.26). As the initial value of the spectral energy density $W_s(\mathbf{k}, t)$ of the slow magnetosonic wave we used in (1.26) the density $W_s(\mathbf{k}) = \kappa T_e$.

The system (1.26) must be solved with the initial conditions

$$\psi(x,0)=-\ln Ax^5;\quad \xi(x,0)=0;$$
$$\left[\frac{\partial\psi(x,\tau)}{\partial\tau}\right]_{\tau=0}=\frac{2}{\omega_0}\gamma\left(\frac{\omega_0}{xv_{Te}},0\right),\ \left[\frac{\partial\xi(x,\tau)}{\partial\tau}\right]_{\tau=0}$$
$$=\frac{C}{x}\left[2\gamma\left(\frac{\omega_0}{xv_{Te}},0\right)+\gamma_1+\tilde{\gamma}\left(\frac{\omega_0}{xv_{Te}},0\right)\right]^{-3}\omega_0^3. \quad (1.28)$$

The second pair of these conditions is determined by the initial value of the increment $\gamma(\mathbf{k}, t = 0)$ of the parametric instability and by its second derivative with respect to the detuning. The formulation of the boundary conditions for the system (1.26) requires a description of the quasilinear relaxation in a wider range of velocities, and allowance for the contribution made to the relaxation by other natural oscillations of the magnetoactive plasma, which continue the slow magnetosonic wave into the region of large and small wave numbers.

2. QUASILINEAR STABILIZATION OF A PARAMETRICALLY UNSTABLE PLASMA

Assume that at the initial instant of the quasilinear relaxation the high-frequency damping decrement $\tilde{\gamma}$ exceeds γ and γ_1. We shall show that the time evolution of γ and $\tilde{\gamma}$ can be represented in this case by the curves of Fig. 1a (at $\gamma(x, 0) > \gamma_1$) and Fig. 1b (at $\gamma(x, 0) < \gamma_1$). Indeed, the system (1.26) then becomes

$$\frac{\partial}{\partial\tau}\frac{\tilde{\gamma}(x,\tau)}{\omega_0}=\frac{1}{2\sqrt{\pi}}x^2\frac{\partial^2}{\partial x^2}\frac{e^{\psi}}{\sqrt{\xi}},$$
$$\frac{\partial\psi}{\partial\tau}\frac{\tilde{\gamma}}{\omega_0}=\frac{C}{x},\quad \frac{\partial\xi}{\partial\tau}=\frac{C\omega_0^3}{x\tilde{\gamma}^3}. \quad (2.1)$$

Making the substitution $\psi = \eta + \ln(2\sqrt{\pi\xi})$ and verifying that the quantities ψ and η vary with time at approximately the same rate, $\partial\psi/\partial\tau \approx \partial\eta/\partial\tau$, we arrive, with the aid of (2.1), to one equation for $\eta(x, \tau)$:

$$\frac{\partial\eta}{\partial\tau}x^3\frac{\partial^2}{\partial x^2}xe^{\eta}=C^2. \quad (2.2)$$

Equation (2.2) admits of a particular solution in the form (C_1, C_2, C_3 are constants)

$$\eta=\ln\left\{\frac{C_2}{x}(\tau+C_1)\left(\frac{1}{2x}+C_2x+C_3\right)\right\},$$

which makes it possible to estimate the time dependence of the increment and the high-frequency damping decrement (see Fig. 1a):

$$\gamma=\frac{\omega_0}{2(\tau+C_1)},\quad \tilde{\gamma}=\frac{C\omega_0}{x}(\tau+C_1),\quad \xi=\frac{x^2}{2C^2}\left[\frac{1}{C_1^2}-\frac{1}{(\tau+C_1)^2}\right]. \quad (2.3)$$

The constant C_1 that enters in (2.3) can be estimated at $C_1 \approx \omega_0/2\gamma(0)$ by specifying the value of the increment $\gamma(x, 0)$ at the initial instant of time $\tau = 0$. We note also that the nonlinear partial differential equation (2.2) for $\eta(x, \tau)$ reduces to an ordinary equation in terms of the self-similar variable $z = x/\tau$:

$$u+z^4u'u''=0,\quad u=u(z)\equiv(x/2\sqrt{\pi\xi}C^2)e^{\psi}.$$

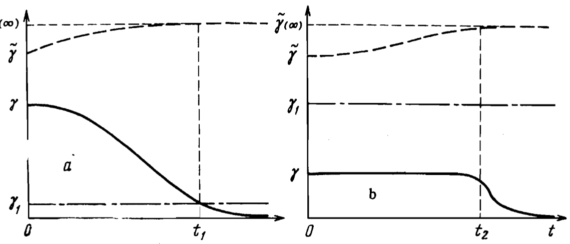

FIG. 1

Thus, during the initial stage of quasilinear relaxation, the dissipation of the high-frequency oscillations $\tilde{\gamma}$ increases, and the increment γ decreases with time. The initial stage is completed by the instant $t_1 \approx 1/2\gamma_1$, which is determined from the condition that the buildup increment be equal to the low-frequency damping decrement, $\gamma \approx \gamma_1$.

At times $t > t_1$ there sets in a second stage of quasilinear relaxation ($\tilde{\gamma} \gg \gamma_1 \gg \gamma$), when there is practically no growth of the dissipation. The high-frequency damping decrement is practically independent of the time

$$\tilde{\gamma} \approx \tilde{\gamma}(\infty) = \frac{\omega_0}{\sqrt{2\pi}} \frac{\omega_{Le}}{\omega_{Li}} \frac{v_E^2}{v_{Te}^2} \tag{2.4}$$

and the quantities ψ and ξ are determined by the second and third equations of (2.1). Making the substitution $\psi = \eta - \ln(\gamma_1\omega_0 x/\sqrt{\pi\xi}\,\gamma^2)$, we arrive at an equation for $\eta(x, \tau)$ (cf. (2.2)):

$$\frac{\partial^2\eta}{\partial\tau^2} + x\frac{\partial^2}{\partial x^2}e^{\eta} = \frac{1}{2}\left(\frac{\partial \ln \xi}{\partial\tau}\right)^2, \tag{2.5}$$

in which the right-hand side is a given function of x and τ:

$$\xi = \frac{x^2}{2C^2}\left[\frac{1}{C_1^2} - \frac{1}{(\tau_1 + C_1)^2}\right] + (\tau - \tau_1)\frac{C\omega_0^3}{x\tilde{\gamma}^3}. \tag{2.6}$$

An analysis of Eqs. (2.5) and (2.6) shows that at $t \gg t_1$ the increment decreases rapidly, $\gamma \sim \omega_0/\tau^3$, so that there is practically no growth of noise, and the second stage of the quasilinear relaxation terminates in stabilization of the parametric instability.

If the quasilinear relaxation against small values of the buildup increment $\tilde{\gamma} \gg \gamma_1 \gg \gamma$, then we have, in fact, the second stage of the relaxation, which begins under the condition $\tilde{\gamma} \gg \gamma \gg \gamma_1$, but since now $\xi(x, 0) = 0$, the solutions of (2.5) differ from those previously investigated. In this case $\xi = C\omega_0^3\tau/x\gamma^3$, the high-frequency damping decrement is given by (2.4), and when account is taken of the explicit form of ξ, Eq. (2.5) is reduced, by means of the substitution $u(x, \tau) = \eta(x, \tau) + \ln\sqrt{\tau}$, to the homogeneous equation

$$\sqrt{\tau}\frac{\partial^2 u}{\partial\tau^2} + x\frac{\partial^2}{\partial x^2}e^u = 0.$$

Introducing the self-similar variable $z = x/\tau^{3/2}$, we obtain for $u(z)$ an ordinary differential equation

$$u''(z + {}^4/_9 e^u) + {}^5/_3 u' + {}^4/_9 (u')^2 e^u = 0. \tag{2.7}$$

An estimate of the quantities u and z shows that at the initial instant $e^u \ll z$. Therefore, neglecting in (2.7) the terms $\sim e^u$, we obtain the solution $u = D_1 + 2D_2\tau/x^{2/3}$ (D_1 and D_2 are constants), which leads to a time-invariant increment $\gamma = D_2\omega_0/x^{2/3}$. Thus, during the initial stage of the quasilinear relaxation, which begins under conditions $\tilde{\gamma} \gg \gamma \gg \gamma$, the increment of the parametric buildup is practically constant and the noise increases exponentially until its value $\sim e^u$ becomes comparable with z. At larger values of the noise, Eq. (2.1) leads to an increment $\gamma = (\omega_0/\tau)\left[D_3\tau^{3/2}/x - 4/3\right]^{-1}$, which falls off in proportion to $\omega_0/\tau^{5/2}$ and corresponds to saturation of the noise. The characteristic value of the time t_2 at which the increment begins to fall off rapidly, and the noise saturates, can be estimated in the form $\gamma t_2 \lesssim 7$ (see Fig. 1b).

Equations (1.26) become much simpler in a relatively weak pumping field, when the low-frequency damping decrement γ_1 is large and the nonlinear relaxation begins under conditions $\gamma_1 \gg \tilde{\gamma}, \gamma$:

$$\frac{\tilde{\gamma}}{\omega_0} + \frac{1}{2}\frac{\partial\psi}{\partial\tau} = \frac{\tilde{\gamma}(\infty)}{\omega_0}, \qquad \xi = 2\tau\frac{\tilde{\gamma}(\infty)\omega_0}{\gamma_1^2}, \tag{2.8}$$

and the change of ψ is described by the relation

$$\frac{\partial^2\psi}{\partial\tau^2} + x^2\frac{\partial^2}{\partial x^2}\left\{\left[1 - \frac{\omega_0}{2\tilde{\gamma}(\infty)}\frac{\partial\psi}{\partial\tau}\right]e^{\psi}G(\beta)\right\} = 0, \qquad \beta = 2\tau\frac{\tilde{\gamma}^3(\infty)}{\gamma_1^2\omega_0}. \tag{2.9}$$

At sufficiently short times $\tau < [\gamma_1^2\omega_0/2\tilde{\gamma}^3(\infty)]$, Eq. (2.9) takes the form

$$\frac{\partial^2\psi}{\partial\tau^2} + x^2\frac{\partial^2}{\partial x^2}\left\{e^{\psi}\left[1 - \frac{\omega_0}{2\tilde{\gamma}(\infty)}\frac{\partial\psi}{\partial\tau}\right]\right\} = 0. \tag{2.10}$$

Its solution can be sought in self-similar form (C_0 is constant)

$$\frac{d^2\eta}{dz^2} + \left(\frac{d^2}{dz^2} - \frac{d}{dz}\right)\left\{e^{\eta} + \frac{C_0}{5}\frac{d\eta}{dz}e^{\eta}\right\} = 0,$$

$$z \equiv \ln x - \left(\frac{2}{\pi}\right)^{1/2}\frac{C_0}{5}\frac{\omega_{Le}}{\omega_{Li}}\frac{v_E^2}{v_{Te}^2}\tau; \qquad \eta \equiv \psi - \ln\left(\frac{2C_0^2}{25\pi}\frac{\omega_{Le}^2}{\omega_{Li}^2}\frac{v_E^4}{v_{Te}^4}\right). \tag{2.11}$$

When the time increases, $\tau > [\gamma_1^2\omega_0/2\tilde{\gamma}^3(\infty)]$, we obtain from (2.9), in place of (2.10),

$$\sqrt{\tau}\frac{\partial^2\psi}{\partial\tau^2} + \sqrt{\frac{\omega_0}{2\pi\tilde{\gamma}(\infty)}}\,x^2\frac{\partial^2}{\partial x^2}\left\{e^{\psi}\left[1 - \frac{\omega_0}{2\tilde{\gamma}(\infty)}\frac{\partial\psi}{\partial\tau}\right]\frac{\gamma_1}{\tilde{\gamma}(\infty)}\right\} = 0. \tag{2.12}$$

Equations (2.10) and (2.12), which describe the relaxation of γ and $\tilde{\gamma}$ in the case of $\gamma_1 \gg \tilde{\gamma}, \gamma$, are represented by the curves of Fig. 2. The characteristic relaxation time t_3 can be estimated with the aid of relation $\partial^2\psi/\partial\tau^2 \sim e^{\psi}$:

$$\gamma(x, 0)\ t_3 \approx {}^1/_2 \ln[2Ax^5\tilde{\gamma}^2(\infty)/\omega_0^2].$$

The essential features of the here-described qualitative picture of the quasilinear relaxation of a parametrically unstable plasma is the growth of the dissipation (of the high-frequency damping decrement $\tilde{\gamma}$) and the decrease of the buildup increment γ. The decrease of the increment to zero points to the stabilizing role of the quasilinear interaction. The excess of the level of the turbulent noise over the spontaneous noise is determined in order of magnitude by $\exp(2\gamma t)$ at the previously obtained instants of time $t_{1,2,3}$. A more detailed description of the evolution of the noise is obtained with the aid of the function $\psi(x, \tau)$, using the formula

$$W_e(\mathbf{k}, t) = A\varkappa T_e x^5 e^{\psi(x,\tau)}\delta(|\cos\theta| - \omega_0/\omega_{Le}).$$

Since the high-frequency damping decrement $\tilde{\gamma}$ assumes asymptotically a stationary value $\tilde{\gamma}(\infty)$ (see (2.4)), we can determine the stationary distribution function of the fast electrons in accordance with (1.21):

$$\mathcal{F}_e(v_z, \infty) = \frac{\sqrt{2}}{\pi^{1/2}}\frac{\omega_{Le}}{\omega_{Li}}\frac{v_E^2}{v_{Te}^2}\frac{1}{|v_z|} + \text{const.} \tag{2.13}$$

The resultant integration constant can be obtained by matching the distribution (2.13) to the Maxwellian distribution at the left end point v_1 of the quasilinear-diffusion velocity interval:

$$\text{const} \approx -\frac{\sqrt{2}}{\pi^{3/2}} \frac{\omega_{Le}}{\omega_{Li}} \frac{\mathbf{v}_E^2}{v_2 v_{Te}} \left\{ \ln\frac{v_2}{v_1} - \frac{v_{Te}^2}{v_1^2} \right\},$$
$$v_1 \approx v_{Te} \left\{ 2 \ln\left(\frac{\pi}{2} \frac{\omega_{Li}}{\omega_{Le}} \frac{v_1 v_{Te}}{\mathbf{v}_E^2} \right) \right\}^{1/2}. \quad (2.14)$$

We obtain here the density δN_e of the fast electrons produced in a magnetoactive plasma during the course of quasilinear relaxation of the parametric instability:

$$\delta N_e = \frac{\sqrt{2}}{\pi^{3/2}} \frac{\omega_{Le}}{\omega_{Li}} \frac{\mathbf{v}_E^2}{v_{Te}^2} N_e \ln^{-1}\left(\frac{\pi}{2} \frac{\omega_{Li}}{\omega_{Le}} \frac{v_1 v_{Te}}{\mathbf{v}_E^2} \right). \quad (2.15)$$

We note that the relative number $\delta N_e/N_e$ of the fast electrons is approximately proportional to the square of the electric field of the pumping wave and to the square root of the ion mass-to-charge ratio. In a hydrogen-like plasma with $\omega_{Le}/\omega_{Li} \approx 43$ acted upon by relatively weak radiation with $v_E^2/v_{Te}^2 \approx 5 \times 10^{-4}$, the relative number of fast electrons $\delta N_e/N_e$ reaches a value on the order of several tenths of one per cent (at $v_1 \gg v_{Te}$).

CONCLUSION

The foregoing analysis of the quasilinear equations describe qualitatively the relaxation of a parametrically unstable plasma, points out the stabilizing role of the quasilinear interaction, and makes it possible to determine the level of the turbulent noise, the high-frequency conductivity of the plasma, the distribution function of the fast electrons, and their number.

Let us indicate the conditions under which our results are valid. The time necessary to establish the stationary state should be much shorter than the electron free-path time for Coulomb collisions. As shown above, the quasilinear relaxation time is of the order of 5—10 reciprocal initial increments, so that the condition that the relaxation process be rapid imposes a lower bound on the pumping field. In the plasma-oscillation damping decrement $\tilde{\gamma}$ given above we have neglected a term due to collisions. The results are therefore valid if the collision damping is small in comparison with the Landau damping. At large wavelengths, when the opposite relation holds true, the quasilinear interaction between the high-frequency field and the plasma must be described in greater detail.

The upper bound on the pumping fields stems from the fact that we have investigated in the present paper the development of parametric instability only under the decay conditions (1.23). It follows from the foregoing exposition that owing to the growth of the dissipation, the high-frequency damping decrement assumes the form (2.4) by the time stabilization sets in. We therefore obtain from the condition $\tilde{\gamma}(\infty) \lesssim \omega$ the following estimate for the maximum pumping-field intensity:

$$(\mathbf{v}_E^2 / v_{Te}^2)_{max} = \sqrt{2\pi} (\omega_{Li} / \omega_{Le})^2 k r_{De},$$

which yields for a hydrogen plasma $(|\mathbf{v}_E|/v_{Te})_{max} \approx 2 \times 10^{-2}$. According to this value, the pumping field can exceed the threshold value by a factor of 10 or more. At such a maximal pumping field, the extremal number of fast electrons (2.15) produced during the course of the quasilinear relaxation of a parametrically unstable plasma is given by

$$\frac{\delta N_e}{N_e} = \frac{2}{\pi} \frac{\omega_{Li}}{\omega_{Le}} \frac{v_{Te}}{|v_z|} \ln^{-1}\left(\frac{\sqrt{\pi}}{2\sqrt{2}} \frac{\omega_{Le}}{\omega_{Li}} \frac{v_1}{v_{Te}} \right).$$

We see from this relation, in particular, that the relative number of fast electrons is independent of the plasma density, is proportional to the square root of the ion charge, and decreases with increasing fast-electron energy.

The rapid growth of the noise during the course of quasilinear interaction leads to turbulization of the plasma. The use of an expression for the electric field intensity E_l of the oscillation at the lower-hybrid-resonance frequency with spectral energy density

$$W_l(\mathbf{k}, t) = \frac{1}{16} \frac{\mathbf{v}_E^2}{v_{Te}^2} \left(\frac{\omega_0}{\tilde{\gamma}} \right)^2 W_s(\mathbf{k}, t)$$

and the use of relation (1.3) for the plasma in the stationary states, leads to the following expression:

$$\sigma_T \approx \pi^{-1} \tilde{\gamma}(\infty) (E_l / E_0)^2,$$

which determines the high-frequency turbulent conductivity of the plasma in the considered region of external pumping-field intensities. Since the amplitude of the plasma oscillations reaches a value on the order of the pumping-field amplitude, the turbulent conductivity $\sigma_T \sim \tilde{\gamma}(\infty)$, which determines the anomalous absorption of the high-frequency external field by the plasma, turns out to be proportional to the square of the pumping-field amplitude (see (2.4)) and to the ratio of the ion and electron masses, and inversely proportional to the electron temperature. It must also be noted that the obtained contribution to the conductivity $\sigma_T \sim \tilde{\gamma}(\infty)$ is large in comparison with the conductivity due to the Coulomb collisions only if the high-frequency damping decrement is determined by the Cerenkov effect, i.e., under conditions when the description proposed here is valid. At a hydrogen plasma density $N_e = 10^{11}$ cm^{-3} and an electron temperature $\kappa T_e \approx 6$ eV, the turbulent conductivity exceeds the Coulomb conductivity by two orders of magnitude ($\tilde{\gamma}(\infty) \approx 2 \times 10^7$ sec^{-1}, $\nu_{ei} \approx 2 \times 10^5$ sec^{-1}). Since we have investigated in the present paper only decay instability, the maximum conductivity is obtained in the maximum pumping field determined above, and is of the order of the sound frequency, $\sigma_T \sim \omega_1$.

In the present paper we have investigated one-dimensional quasilinear relaxation. The one-dimensional character is due to the strong external magnetic field ($\Omega_e^2 \gg \omega_{Le}^2$), which is parallel to the electric field of the pumping wave, $\mathbf{E}_0 \times \mathbf{B} = 0$ (cf. the quasilinear relaxation of a beam in a plasma[24]). The external magnetic field does not enter in the final expressions derived above. Therefore the results have, in our opinion, a wider range of applicability and describe also the simultaneous interaction of an isotropic plasma with a high-frequency pumping field at a frequency ω_0 close to the plasma frequency.

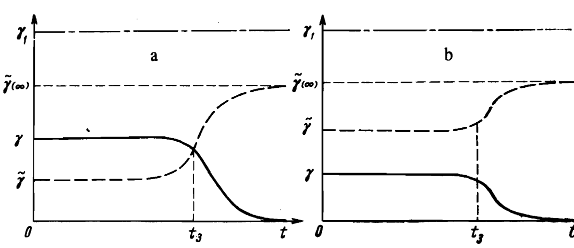

FIG. 2

*[vB] ≡ v × B.

[1]We use the temperature T_e and the thermal velocity $v_{Te} = (\kappa T_e/m)^{1/2}$ of the electrons along the magnetic field, assuming that the "gross" characteristics of the distribution function $\mathcal{F}_e(v_z, t)$ are left practically unchanged by quasilinear diffusion, so that to calculate v_{Te}, the Debye

radius r_{De}, etc. we can assume $\mathscr{F}_e$ is Maxwellian. This approximation is justified by the small contribution made to v_{Te} and r_{De} by the velocity intervals in which, owing to the quasilinear effect, there sets in the abrupt change of $\mathscr{F}_e$, which is investigated later on.

[2)]An analysis of a more general case with allowance for the diffusion of the slow ($v_z \approx \omega/k_z$) electrons show that this approximation is valid if $\gamma < \max\{\gamma, \gamma_{1,2}\}$.

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