Stable motion and acceleration of a charged particle in a linearly polarized electromagnetic wave in a gaseous medium

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The stability mechanisms underlying the motion of a particle in a transversely polarized electromagnetic wave in a gaseous medium are considered. In particular, stable acceleration of particles occurs in the presence of a constant magnetic field.

1. INTRODUCTION

If the velocity v_x of a charged particle in the direction of propagation of a linearly polarized electromagnetic wave is sufficiently close to the phase velocity v_{ph} of the wave, then the particle can be captured by the wave. The particle then executes stable oscillations about some equilibrium phase. One of the variants of such stability was considered by Arutyunyan and Avetisyan[1].

The possible stability mechanisms that obtain in the resonant motion of the particle in the wave when $v_x \approx V_{ph} = c/n$ ($n > 1$ is the refractive index) are easily revealed by going over into the comoving reference frame K′ in which the field is time independent. In this system the electric field vanishes, $E' = 0$, whereas the magnetic field $B' = B\sqrt{n^2-1}/n$, where $B = nE$ is the field in the laboratory frame of reference K.

Let an equilibrium particle in the frame K′ be at rest (the case considered in[1]). This equilibrium state does not depend on which phase Φ of the wave $B' = B_0' \sin\Phi$ the particle is in; the equilibrium values of the velocity are equal to $v'_{xs} = 0$, $v'_{ys} = 0$. It is clear that if the initial conditions differ from the equilibrium conditions, i.e., if $\Delta v'_x = v'_x - v'_{xs} \neq 0$ and $\Delta v'_y \neq 0$, then the particle gyrates in the field B′, maintaining, on the average, its equilibrium position. In the laboratory system this corresponds to a cycloidal motion of frequency $\Omega = ecE\sqrt{n^2-1}/\mathscr{E}$, where $\mathscr{E}$ is the energy of the particle. If however, the initial deviations of the velocity from the equilibrium value are sufficiently large, then the particle jumps over the maxima of the magnetic field B′ and, on the average, moves freely. If now we vary the refractive index $n = n(x)$ along the wave vector **k**, then the mean velocity of the particles captured by the wave will vary: $\langle v_x(x)\rangle = c/n(x)$, and the wave will drag the particle along. This acceleration mechanism corresponds exactly to the acceleration of a particle in crossed fields E_y and $B_z = nE_y > E_y$, $\langle v_x\rangle = E_y c/B_z$, when the magnetic field varies sufficiently slowly either in time, or along the x-axis. Then the particle, which in static fields moves with constant velocity along the x-axis, now drifts in the direction of polarization of the electric field, acquiring a velocity $v_y \neq 0$ and increasing its energy by $\mathscr{E} - \mathscr{E}_0 = eE_y(y - y_0)$. The stability is however destroyed at too large values of v_y, and, therefore, this type of motion provides neither good acceleration nor strong stability.

A much stronger stability arises in the case of oblique (with respect to the wave) motion, when $v_y \neq 0$ from the start, but $v_{xs} = c/n$ as before. In the comoving frame $v'_{xs} = 0$, but $v'_y \neq 0$. The phase $\Phi = kx - \omega t$ of the wave is related in the frame K′ to the x′ coordinate by the relation $x' = c\Phi/\omega\sqrt{n^2-1}$. Therefore, the equations of motion $dp'/dt' = eE' + ev' \times B'/c$ can be written in this frame as equations for the phase shift;

$$\frac{\mathscr{E}'/c}{\omega\sqrt{n^2-1}}\frac{d^2\Phi}{dt'^2} = \frac{e}{c}v_y'B_z' \equiv \frac{e}{c}\frac{p_y'c}{\mathscr{E}'}\sqrt{n^2-1}\,E_{0y}\sin\Phi. \tag{1}$$

Noting further that $\mathscr{E}' = \mathscr{E}\sqrt{n^2-1}/n$, $t' = t\sqrt{n^2-1}/n$, we find that $\mathscr{E}'$ and t′ in this equation can be replaced by $\mathscr{E}$ and t. Clearly, the equilibrium state corresponds to $\sin\Phi_s = 0$, when the fields $E' = B' = 0$ (whereas in the previous case $\sin\Phi$ is arbitrary and $p_{ys} = 0$). Expanding $\sin\Phi$ in powers of the small deviations $\varphi = \Phi - \Phi_s$, we obtain the frequency of the phase oscillations:

$$\Omega = \omega\left[\frac{eE_{0y}\lambda}{2\pi\mathscr{E}_s}\frac{p_{ys}c}{\mathscr{E}_s}(n^2-1)\cos\Phi_s\right]^{1/2}, \qquad p_{ys}\neq 0, \tag{2}$$

it being clear that only one of the two equilibrium phases is stable.

An adiabatic variation of the refractive index leads to a slow variation of the particle energy owing to the acceleration in the transverse direction: $\Delta E = eE_{0y}\sin\Phi_s\Delta y$; because of the coupling of the motions across the magnetic field, here, as in the previous case, not only the transverse momentum $p_{ys} = p_{ys}(x)$, but also the longitudinal momentum $p_{xs} = p_{xs}(x)$, $v_{xs} = c/n(x)$ varies.

Motion in a homogeneous medium, n = const, assumes a curious character upon the application of a static (more accurately, almost static if the imaginary part of the refractive index $n_2 \neq 0$: $n = n_1 + in_2$) secondary magnetic field b_z. In this case there exists in the K′ frame the electric field $E'_y = b_z/\sqrt{n^2-1}$, which now accelerates the particle, so that

$$d\mathscr{E}/dt = d\mathscr{E}'/dt' = eE_y'v_y' = e\left(\frac{b_z}{\sqrt{n^2-1}}\right)\frac{p_yc}{\mathscr{E}'} = eb_z\frac{p_yc}{\mathscr{E}}\frac{n}{n^2-1}.$$

The stability condition again corresponds to $v'_{xs} = 0$, which yields $dp'_{xs}/dt' = 0$. Writing down the Lorentz force in the x′-direction, we obtain a supplementary equilibrium condition

$$b_z' = B_z', \qquad b_z = B_{0z}\frac{n^2-1}{n^2}\sin\Phi_s. \tag{3}$$

It is further obvious that the deviations from the equilibrium position are described by the same equations (1) and (2) as when $b_z = 0$, since b_z does not depend on Φ.

Thus, by applying a static magnetic field b_z, we can accelerate obliquely incident particles in a laser beam in a gaseous medium. In this case the velocity v_{xs} does not vary, although P_{xs} varies on account of the variation of the energy $\mathscr{E}$; the momentum p_{ys} varies directly.

In practice, the acceleration of particles with the aid of the stability mechanisms described here turns out to be not very effective because of the low efficiency ratio of present-day high-power lasers. We can apparently use the stability of the motion for the inverse effect—

the amplification of the laser pulse. This possibility is analyzed in[1] with the aid of the first of the stability mechanisms described here ($p_{ys} = 0$); this mechanism is, unfortunately, not very effective.

In the present paper we consider in detail the dynamics of the motion of a single particle in a steady, neutral, gaseous medium with a refractive index $n = n_1 + in_2$ in the presence of an electromagnetic wave and an external magnetic field.

2. THE EQUILIBRIUM MOTION OF A PARTICLE IN A HOMOGENEOUS MEDIUM IN THE PRESENCE OF A WAVE AND AN EXTERNAL FIELD

Let the vector potential of the wave be directed along the y-axis: $A = A_y(x, t)$. Let the wave propagate along the x-axis, and let the external field $b = -b_z(x)$ be directed along the z-axis. The equations of motion of the particle have the form

$$\frac{dp_x}{dt} = e\beta_y\left(\frac{\partial A}{\partial x} - b\right), \qquad \beta_y = \frac{v_y}{c}, \tag{4}$$

$$\frac{dp_y}{dt} = -e\left(\frac{1}{c}\frac{dA}{dt} - \beta_x b\right), \qquad \beta_x = \frac{v_x}{c}, \tag{5}$$

from which follows the integral of the motion

$$p_y + \frac{e}{c}A(x,t) - \frac{e}{c}\int_{x_0}^{x} b(x)\,dx = \pi_y = \text{const.} \tag{6}$$

Let

$$A(x,t) = A_0\eta(x)\cos\frac{\omega}{c}(n_1x - ct),$$
$$\eta(x) = \exp\left\{-\frac{\omega}{c}n_2(x - x_0)\right\}, \tag{7}$$

then the condition for the constancy of the phase $\Phi = c^{-1}\omega(n_1x - ct)$ is

$$\beta_{xs} = 1/n_1 \tag{8}$$

(the index s denotes equilibrium), and from the integral of the motion (6) we obtain for the equilibrium particle the energy-variation law:

$$\gamma_s^2 = \left(\frac{\mathscr{E}_s}{mc^2}\right)^2 = \left(1 - \beta_{ys}^2 - \frac{1}{n_1^2}\right)^{-1}, \qquad \beta_{ys} = \frac{p_{ys}c}{\mathscr{E}},$$
$$A_s(x,t) = A_0\xi\cos\Phi_s, \quad \Phi_s = \omega x_0/c, \tag{9}$$
$$\xi = \exp(-\omega n_2 t/n_1), \quad x_s = x_0 + ct/n_1.$$

The rate of accumulation of energy is given by Eq. (4) under the condition (8):

$$\dot\gamma_s = -n_1\omega Q\beta_{ys}\xi[n_1\sin\Phi_s + n_2\cos\Phi_s] - \beta_{ys}n_1 eb(x_s)/mc, \tag{10}$$

where $Q = eA_0/mc^2$. From (6) and (7), on the other hand, follows

$$\gamma_s^2 = \frac{n_1^2}{n_1^2 - 1}\left\{1 + \left[\frac{\pi_y}{mc} - Q\xi\cos\Phi_s + \frac{e}{mc^2}\int_{x_0}^{x_s} b(x)\,dx\right]^2\right\}. \tag{11}$$

Differentiating (11) and comparing with (10), we obtain for the magnetic field $b_z(x)$ the variation law necessary for the maintainance of equilibrium:

$$\frac{eb(x)}{mc} = -\omega Q\eta(x)\left[n_2\cos\Phi_s + \frac{n_1^2 - 1}{n_1}\sin\Phi_s\right]. \tag{12}$$

Conversely, when the magnitude $b(x_0)$ of the field is given, Eq. (12) determines the equilibrium phase Φ_s. Comparing $b(x)$ with the wave field

$$c\frac{B_z(x)}{mc} = \frac{e}{mc}\frac{\partial A}{\partial x} = -\omega Q\eta(x)[n_2\cos\Phi_s + n_1\sin\Phi_s], \tag{13}$$

we obtain for the case when $n_2 = 0$ the relation (3). The electric field

$$\frac{eE_y}{mc} = -\frac{e}{mc^2}\frac{\partial A}{\partial t} = -\omega Q\eta(x)\sin\Phi_s. \tag{14}$$

In the absence of both an external magnetic field and damping, from (12) follows $\sin\Phi_s = 0$, i.e., the absence, according to (14), of acceleration. For $b_z = 0$, but $n_2 \neq 0$, there exists a small particle acceleration connected with the conservation of the integral π_y: the damping of $A(x, t)$ leads to the growth of p_y. This regime possesses stability, which we shall not analyze here.

If the distance to the nearest resonance line of the medium, $\omega_f - \omega$, is much larger than the total width of the lines, i.e., $|\omega_f - \omega| \gg \Gamma/2$, then $n_2 \ll n_1^2 - 1$. We shall assume that this condition is fulfilled in the region where the approximate estimates will be made. The exact dependence of the energy on the x-coordinate is described by the formula

$$\gamma_s^2 = \frac{n_1^2}{n_1^2 - 1}\left\{1 + \left[\frac{\pi_y}{mc} - Q\cos\Phi_s - Q\frac{n_1^2 - 1}{n_1 n_2}(1 - \eta(x))\sin\Phi_s\right]^2\right\}; \tag{15}$$

in the region where the wave does not have time to die down, approximately by

$$\left(\frac{\gamma_s}{\gamma_0}\right)^2 \approx \left\{1 + \left[\gamma_0\beta_{0y} - Q\frac{\omega}{c}(x - x_0)\frac{n_1^2 - 1}{n_1}\sin\Phi_s\right]^2\right\}(1 + \gamma_0^2\beta_{0y}^2)^{-1}, \tag{15'}$$

$$\gamma_s - \gamma_0 \approx -\frac{\omega}{c}Q(y - y_0)\sin\Phi_s. \tag{16}$$

An equilibrium relativistic particle moves at a small angle θ to the x-axis:

$$\theta^2 \approx \left(\frac{p_y}{p_x}\right)^2 = n_1^2 - 1 - \frac{n_1^2}{\gamma^2} \ll 1, \qquad \gamma \gg 1. \tag{17}$$

Resonant motion of a nonrelativistic particle would in practice be impossible, since the refractive index $n_1 > 1/\beta > 1$ would turn out in this case to be too large and the particles would be swiftly scattered out of the stable states by the residual gas.

For $\gamma \gg \gamma_0$, the particle moves at a constant angle $\theta^2 \approx n_1^2 - 1$ to the x-axis, with its energy varying linearly with $(x - x_0)$:

$$d\mathscr{E}/dx \approx -\theta eE_{0y}\sin\Phi_s, \quad \gamma \gg \gamma_0, \quad \theta \ll 1. \tag{18}$$

Let us list further the effects that limit the possibility of a prolonged acceleration of particles moving stably in a laser beam.

1) The acceleration ceases when the wave attenuates.

2) When the wave packet has a finite length l, the wave carries the particle to the leading edge of the packet, after which the particle gets ahead and is no more accelerated. The maximum length L at which the particle is still inside the packet satisfies the inequality

$$L \leqslant \frac{l}{n_1^2 - 1}\frac{\omega_f^2 - \omega^2}{\omega^2}. \tag{19}$$

3) The spreading out of a short wave packet leads to the decrease of the field and sharply depends on the proximity of a resonance line of the medium.

4) The length of the pulse is limited by the effects of the interaction between the wave and the gaseous medium—in particular, by breakdowns in the gas, as well as by the many-photon ionization effect, by nonlinear effects, etc.

5) The wave-front distortion caused by the initial angular spread of the laser beam, as well as by the inhomogeneity fluctuations in the medium, can lead to loss of stability.

6) The gas density N should be sufficiently small because of the Coulomb particle-scattering effect which takes the particles out of the stability region. This question is discussed in the following section.

3. THE REGION OF STABILITY OF THE MOTION. COULOMB SCATTERING OF THE PARTICLES IN THE GAS

Using the definition of the phase Φ, according to which

$$\dot{\Phi} = \frac{\omega}{c} n_1 v_x - \omega, \qquad \ddot{\Phi} = \omega n_1 \dot{\beta}_x, \tag{20}$$

and the equations of motion (4) and (5), as well as (7) and (12), we obtain the equation for the phase motion

$$\ddot{\Phi} = \omega^2 \beta_y \frac{n_1 e A_0}{\mathscr{E}} \eta(x) \left\{ n_2 (\cos \Phi_s - \cos \Phi) + \frac{n_1^2 - 1}{n_1^2} \sin \Phi_s - (n_1 - \beta_x) \sin \Phi \right\}. \tag{21}$$

The dominant dependence on Φ is contained in the curly brackets; neglecting the dependence of β_y and $\mathscr{E}$ on Φ, we have

$$\ddot{\Phi} = \beta_{ys} \frac{n_1 e A_0}{\mathscr{E}_s} \exp\left[-\frac{n_2}{n_1}(\Phi - \Phi_s) - \omega \frac{n_2}{n_1} t \right] \left\{ n_2 (\cos \Phi_s - \cos \Phi) + \frac{n_1^2 - 1}{n_1} (\sin \Phi_s - \sin \Phi) + \frac{\dot{\Phi}}{\omega n_1} \sin \Phi \right\} \omega^2, \tag{22}$$

Φ_0 is the phase at $t = 0$. To the equilibrium configuration corresponds the vanishing of the expression in the curly brackets; Φ_S is determined from (12) for $x = x_0$, $\dot{\Phi}_S = 0$, and $\ddot{\Phi}_S = 0$.

Expanding (22) in powers of the small deviations $\varphi = \Phi - \Phi_S$ from the equilibrium phase, we obtain the equation

$$\ddot{\varphi} + \delta \dot{\varphi} + \Omega^2 \varphi = 0. \tag{23}$$

We write out for Ω^2 and δ approximate formulas for $n_2 \ll n_1^2 - 1$:

$$\Omega^2 = \omega^2 \beta_{ys} \frac{e A_0}{\mathscr{E}_s} (n_1^2 - 1) \xi \cos \Phi_s, \tag{24}$$

$$\delta = -\omega \beta_{ys} \frac{e A_0}{\mathscr{E}_s} \xi \sin \Phi_s, \tag{25}$$

$$e A_0 = e E_{0y} \frac{\lambda}{2\pi}. \tag{26}$$

In the case of stable acceleration, when $\dot{\gamma}_S > 0$, we have $eA_0 \beta_{yS} \sin \Phi_S < 0$, $eA_0 \beta_{yS} \cos \Phi_S > 0$, $\Omega^2 > 0$, and $\delta > 0$. In the case of stable retardation, when $\dot{\gamma}_S < 0$, we have $eA_0 \beta_{yS} \sin \Phi_S > 0$, $eA_0 \beta_{yS} \cos \Phi_S > 0$, $\Omega^2 > 0$, and $\delta < 0$, i.e., we have an exponential buildup of small oscillations during retardation. The conditions for particle conservation in the phase-oscillation regime have in his case the form

$$\delta t \leqslant 1, \qquad \frac{x - x_0}{\lambda} \frac{eE\lambda}{\mathscr{E}} \frac{p_y c}{\mathscr{E}} \xi \sin \Phi_s \leqslant 1, \qquad \dot{\gamma}_s < 0. \tag{27}$$

Let us further assume (this is justified below) that $|\dot{\Phi}/\omega| \ll n_1^2 - 1$. Then Eq. (22) can be integrated approximately, considering ξ as a nearly constant (in comparison with the phase oscillations) function of the time; moreover, $\exp\{-(n_2/n_1)(\Phi - \Phi_0)\} \approx 1$ and $n_2 \ll n_1^2 - 1$, and therefore an approximate integral of the motion is

$$\mathscr{H} \approx {}^1/_2 \dot{\Phi}^2 + U(\Phi), \tag{28}$$

$$U(\Phi) = -\omega^2 \beta_{ys} \frac{e A_0}{\mathscr{E}_s} (n_1^2 - 1) \xi [\cos \Phi - \cos \Phi_s + (\Phi - \Phi_s) \sin \Phi_s]. \tag{29}$$

For the case of acceleration, when $\dot{\gamma}_S > 0$, we have $3\pi/2 < \Phi_S < 2\pi$; the depth of the potential well is equal to the smallest of the extrema of $U(\Phi)$, which is equal to

$$U_{ext}(\Phi = 3\pi - \Phi_s) = 2\omega^2 \beta_{ys} \frac{e A_0}{\mathscr{E}_s} (n_1^2 - 1) \xi \left[\cos \Phi_s + \left(\Phi_s - \frac{3\pi}{2} \right) \sin \Phi_s \right]. \tag{30}$$

The maximum value of $\dot{\Phi}$ at which the particle is still in the potential well is (setting $\alpha = \Phi_S - 3\pi/2$) equal to

$$\frac{\dot{\Phi}_{max}}{\omega} = 2 \left[\beta_{ys} \frac{e A_0}{\mathscr{E}_s} (n_1^2 - 1) \xi (\sin \alpha - \alpha \cos \alpha) \right]^{1/2}. \tag{31}$$

It can be seen from this that the condition $|\dot{\Phi}/\omega| \ll n_1^2 - 1 \approx \theta^2$ is well fulfilled if θ is not too small.

Let us now consider particle scattering in the gas. $\dot{\Phi}$ changes abruptly during scattering:

$$\Delta \dot{\Phi} / \omega = n_1 \Delta \beta_x \approx -n_1 \theta_0 \Delta \theta,$$

$$\theta_0 = c p_y / \mathscr{E} \ll 1.$$

After multiple scattering the requirement

$$\langle (\Delta \dot{\Phi})^2 \rangle < \dot{\Phi}_{max}^2$$

should be fulfilled. Assuming $d\langle \Delta \theta^2 \rangle \approx (\epsilon/\mathscr{E})^2 dx/X$, where $\epsilon \approx 21.5$ MeV, $\mathscr{E}$ is in MeV, and X is the radiation length of the gaseous medium, we obtain (for $\mathscr{E} \gg mc^2$) the condition

$$\frac{\mathscr{E}}{\mathscr{E}_0} \lesssim \frac{2}{\pi} (\sin \alpha - \alpha \cos \alpha) \xi \left(\frac{e E_0 \lambda}{\varepsilon} \right)^2 \frac{X}{\lambda} \frac{n_1^2 - 1}{n_1} \cos \alpha + 1. \tag{32}$$

It can be seen from this that in order to obtain an effective acceleration, we must let the frequency ω approach the resonance frequency, i.e., let $\omega \to \omega_f$, thereby increasing the radiation length X (with the quantity n_1 conserved).

In conclusion, let us note that the above-described stability mechanism can be used effectively to produce a laser-frequency modulated electron beam. In fact, the particles that do not fall into the stability region (and the distances between which are equal to λ) do not on the average feel the wave field and are turned around by the static magnetic field, whereas the stable particles move rectilinearly.

[1] V. M. Arutyunyan and G. K. Avetisyan, Dokl. Akad. Nauk Arm. SSR 52, 5 (1971).

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