Electron temperature in a weakly ionized plasma when the skin effect is nonlinear

V. D. Pis'mennyĭ, T. V. Rakhimova, and A. T. Rakhimov
Nuclear Physics Research Institute, Moscow State University
(Submitted February 29, 1972; resubmitted May 25, 1972)
Zh. Eksp. Teor. Fiz. **64**, 498-504, (February 1973)

The nonlinear skin effect of electromagnetic waves in a weakly ionized plasma is considered under conditions of nonlocal connection between the electron temperature and the intensity of a high-frequency electric field in the plasma. It is shown that the dependence of the equilibrium electron temperature in the plasma on the intensity of the incident electromagnetic wave can be nonunique. In consequence, when the electromagnetic-wave intensity attains a certain value, the coefficient of reflection of the waves from the surface of the weakly ionized plasma may decrease abruptly.

Nonlinear effects connected with electron heating become appreciable in plasmas placed in relatively weak electric fields. One of such effects is the possibility of the existence of a nonunique connection between the electron temperature and the intensity of a high-frequency electric field[1].

In the present paper we consider the nonlinear penetration of electromagnetic waves into a plasma under conditions of normal skin effect, i.e., under the conditions when the dimension δ_s of the skin layer substantially exceeds the electron mean free path l. If δ_s also substantially exceeds the electron mean free path l_e connected with energy transfer to the heavy particles, then the coupling between the electron temperature and the wave field is local. It is precisely to such a plasma that the results obtained by A. Gurevich[1] can be applied. We, on the other hand, shall consider the case when the value of δ_s being established in the plasma is small compared to the electron mean free path $l_e = l\delta^{-1/2}$ connected with energy transfer to the heavy particles[1)]:

$$l \ll \delta_s \ll l\delta^{-1/2}; \tag{1}$$

$\delta \ll 1$ is the average fraction of energy transferred in an electron-heavy particle collision.

We shall show that in the case when the connection between the electron temperature and the wave field is nonlocal, a situation is possible in which the electron temperature being established at the plasma boundary is a nonunique function of the field intensity of the incident wave, i.e., it will be shown that the surface impedance of the plasma can be a nonunique function of the amplitude of the incident wave and can, in consequence, vary discontinuously when the power of the incident wave is smoothly varied. There should then occur a sharp increase in the coefficient of absorption of electromagnetic waves by the plasma.

1. Let us consider the penetration of a strong electromagnetic wave of frequency ω into a weakly ionized plasma in thermal equilibrium. Under the action of the electric field of the wave, the plasma electrons will be heated up and will give up their energy upon colliding with neutral particles. Since the plasma is weakly ionized, the temperature of the heavy particles can be considered to be constant and independent of the effective field.

We shall consider the case of a sufficiently high electromagnetic-wave frequency, when the inequality

$$\omega \gg \delta\nu, \tag{2}$$

where ν is the electron-neutral particle collision rate, is fulfilled. We shall also assume the plasma to be quasi-neutral, i.e., we shall assume that the dimension of the skin layer is substantially larger than the Debye radius,

$$\delta_s \gg r_D. \tag{3}$$

Finally, we shall require the fulfillment of the inequality

$$l\delta^{-1/2} \ll (D_a\tau_r)^{1/2}, \tag{4}$$

where D_a is the ambipolar diffusion coefficient and τ_r is the electron-ion recombination time.

The inequality (4) implies that electron recombination can be neglected in the plasma region perturbed by the electromagnetic wave. Since, moreover, we are considering the case when $\delta_s \ll l_e$, i.e., when the time of diffusion of electrons from the skin layer is substantially less than the time of accumulation by an electron of energy corresponding to the boundary electric field intensity, and the electron temperature being established at the plasma boundary is then much smaller than the ionization potential of the neutral particles, we can assume that the electromagnetic wave does not affect the ionization-recombination balance of the plasma. (This assumption is all the more valid when the plasma is produced by some extraneous ionizing radiation.) Allowance for changes in the ionization-recombination balance of the plasma may be important in the other case of the nonlinear skin effect if $\delta_s \gg l_e$ and the time of electron diffusion from the skin layer is substantially greater than the establishment time for the electron distribution function in the region of energies corresponding to the ionization threshold for the neutral particles[3].

Then, to describe the stationary, ω-averaged, electron temperature and concentration distributions when the conditions (1)—(3) are fulfilled, we can use the following hydrodynamic equations[4-6]:

$$\frac{n_e\nu e^2|E|^2}{2m(\omega^2+\nu^2)} - \frac{3}{2}n_e\delta\nu(T_e-T) + \frac{d}{dx}\left(\frac{n_eT_e}{m\nu}\frac{dT_e}{dx}\right) = 0, \tag{5}$$

$$n_e = n_{e\infty}\left(\frac{2T}{T_e+T}\right)^k. \tag{6}$$

We consider only normal incidence of a monochromatic, linearly polarized electromagnetic wave ($E \propto e^{i\omega t}$) on a plane plasma boundary. The x-axis is directed perpendicular to the boundary; $T_e(x)$ and T are the electron and heavy-particle temperatures, $n_{e\infty}$ is the electron concentration in the unperturbed plasma; k is a number equal to unity for a substantial interaction between the electrons ($\nu_{ee} > \nu$), and $k = 1 - (T_e/\nu)d\nu/dT_e$

for $\nu_{ee} < \nu$[5-7]. It should be noted here that a similar treatment is applicable to the weakly ionized two-component electron-hole plasma of semiconductors, when the hole mass substantially exceeds the electron mass.

In virtue of the fulfillment of (1), electronic heat conductivity is taken into account in the electron energy balance equation (5). The hydrodynamic equilibrium equation (6) for the plasma takes into account the fact that under steady-state conditions the charged-particle fluxes in the perturbed region of the plasma are equal to zero.

To the differential equation (5) must be added boundary conditions. We choose them in the following form:

$$T_e(x\to\infty)\to T,\quad dT_e/dx|_{x=0}=0. \tag{7}$$

The first boundary condition corresponds to the fact that the plasma in the unperturbed region is isothermal. The second boundary condition corresponds to the vanishing of the electron-heat flux at the plasma boundary. (We are considering the case of specular reflection of the electrons from the plasma boundary.)

The spatial dependence of the electric field in the plasma is determined from the wave equation

$$\frac{d^2E}{dx^2}=\frac{4\pi n_e e^2}{mc^2}\frac{\omega(\omega+i\nu)}{\omega^2+\nu^2}E, \tag{8}$$

in which the displacement currents have been dropped, since we are considering the case of strong skin effect,

$$\omega_{pe}^2/(\omega^2+\nu^2)\gg 1. \tag{9}$$

The system of equations (5)–(8) is completely closed, and its solution allows us to describe in the case under consideration here the distinctive features of the penetration of strong electromagnetic waves into a plasma.

2. To find the dependence $T_e(x)$ being established in the plasma, we solve the system of equations (5)–(8) in the following manner. In virtue of the fulfillment of the inequality (1), we can conclude that for $x \ll l\delta^{-1/2}$ the nature of the penetration of an electromagnetic wave into the plasma is determined by the electron temperature T_{e0} being established at the plasma boundary. Then wave equation (8) has a solution valid at $x \ll l\delta^{-1/2}$ in the form

$$E(x)=E_b\exp(-i\alpha x-x/\delta_s),$$

where

$$\alpha=\frac{\omega_{pe}(T_{e0})}{\sqrt{2}c}\left\{\frac{\omega[\omega^2+\nu^2(T_{e0})]^{1/2}-\omega^2}{\omega^2+\nu^2(T_{e0})}\right\}^{1/2},$$
$$\delta_s^{-1}=\frac{\omega_{pe}(T_{e0})}{\sqrt{2}c}\left\{\frac{\omega[\omega^2+\nu^2(T_{e0})]^{1/2}+\omega^2}{\omega^2+\nu^2(T_{e0})}\right\}^{1/2} \tag{10}$$

and $E_b \equiv E(x=0)$ is the boundary value of the electric field, which can be related through the Fresnel formulas to the amplitude of the wave $E_0\exp(i\omega t-i\omega x/c)$ incident on the plasma:

$$|E_b|^2=4\frac{\omega(\omega^2+\nu^2(T_{e0}))^{1/2}}{\omega_{pe}^2(T_{e0})}E_0^2. \tag{11}$$

The condition (9) was used in writing down this expression.

We can find the value of the electron temperature being established at the boundary from Eq. (5), assuming, in virtue of the fulfillment of the condition $\delta_s \ll l\delta^{-1/2}$, that the source of energy release in this equation is a surface source. Then the spatial dependence of the electron temperature is determined by the following equation:

$$\frac{3}{2}\delta n_e(T_e)\nu(T_e)(T_e-T)=\frac{d}{dx}\left[\frac{n_e(T_e)T_e}{m\nu(T_e)}\frac{dT_e}{dx}\right]. \tag{12}$$

The solution of Eq. (12) with the boundary condition (7) at infinity can be represented in the form

$$\frac{dT_e}{dx}=-(3m\delta)^{1/2}\frac{\nu(T_e)}{n_e(T_e)T_e}\left\{\int_T^{T_e}n_{e2}(T_e)T_e(T_e-T)\,dT_e\right\}^{1/2}. \tag{13}$$

Using this expression, we write down the heat flux carried away by the electrons from the skin layer:

$$q=\left(\frac{3\delta}{m}\right)^{1/2}\left\{\int_T^{T_{e0}}n_e^2(T_e)T_e(T_e-T)\,dT_e\right\}^{1/2}. \tag{14}$$

On the other hand, this heat flux is equal to the Joule energy released in the skin layer:

$$q=W\equiv\frac{n_e(T_{e0})\nu(T_{e0})\delta_s(T_{e0})e^2|E_b|^2}{4m[\omega^2+\nu^2(T_{e0})]}. \tag{15}$$

Therefore, the final expression determining the dependence of the boundary value of the electron temperature on the amplitude of the wave incident on the plasma can be written with the aid of (15), (14) (11), and (10) in the following form:

$$\frac{1}{2\pi}\left(\frac{m}{6\delta}\right)^{1/2}cE_0^2$$
$$=\frac{\omega_{pe}(T_{e0})}{\nu(T_{e0})}\left[1+\left(1+\frac{\nu^2(T_{e0})}{\omega^2}\right)^{1/2}\right]^{1/2}\left[\int_T^{T_{e0}}n_e^2(T_e)T_e(T_e-T)\,dT_e\right]^{1/2}. \tag{16}$$

Let us transform this equality by expressing $n_e(T_e)$ with the aid (6), assuming that the degree of ionization of the plasma is such that the interaction between the charged particles can be neglected. Then, assuming $\nu(T_{e0}) = \nu_0(T_{e0}/T)^\gamma$ and introducing the dimensionless variable $\Theta = T_{e0}/T$, we can represent Eq. (16) as follows:

$$\frac{2^{3(\gamma-1)/2}}{4\pi(6\pi\delta)^{1/2}}\frac{mc\nu_0E_0^2}{eT^{3/2}n_{e\infty}^{1/2}}=\varphi^{1/2}(\Theta)$$
$$\equiv\left[\frac{1+(1+\nu_0^2\omega^{-2}\Theta^{2\gamma})^{1/2}}{\Theta^{2\gamma}(1+\Theta)^{1-\gamma}}\int_1^\Theta\frac{\tilde\Theta(\tilde\Theta-1)}{(1+\tilde\Theta)^{2-2\gamma}}d\tilde\Theta\right]^{1/2}. \tag{17}$$

To investigate the function $T_{e0}(E_0)$ defined by the expression (17), we write down the asymptotic form of $\varphi(\Theta)$ for values of Θ close to unity:

$$\varphi(\Theta)\propto\varepsilon^2\quad\text{for}\ \Theta=1+\varepsilon\quad(\varepsilon\ll 1) \tag{18a}$$

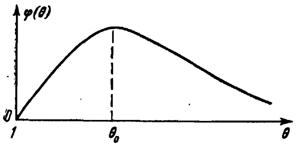
and for $\Theta \gg 1$:

$$\varphi(\Theta)\propto\Theta^{\gamma}\quad\text{for}\quad\omega\gg\nu_0,$$
$$\varphi(\Theta)\propto\Theta^{2\gamma}\quad\text{for}\quad\omega\ll\nu_0. \tag{18b}$$

It can be seen from (18) that the function $\varphi(\Theta)$ can be two-valued in nature. It is shown in the figure for the case when $\gamma < 0$, i.e., when $d\nu/dT_e < 0$. Such behavior of the electron-atom collision rate is characteristic of, for example, those gases in which the Ramsauer effect is obvserved, if the plasma temperature in this case is substantially lower than the temperature (~ 1 eV) corresponding to the Ramsauer minimum. Notice that a similar situation can arise in a semiconductor two-component plasma when the electrons and holes are scattered, for example, by charged impurities, or by piezoacoustic oscillations, since then $d\nu/dT_e < 0$ also.

The numerical computation of $\varphi(\Theta)$, which we carried out for a number of cases, showed that, for example, in xenon, for which $\gamma \approx -1/2$, the function $\varphi(\Theta)$ for $\omega < \nu_0$ attains its maximum value at $\Theta_0 = 11$.

Thus, from the derived expression (17) we can draw the conclusion that if $d\nu/dT_e < 0$, then when the intensity



of the wave incident on the weakly ionized plasma satisfies the inequality

$$E_0^2 < E_{max}^2 = 2^{3(1-\gamma)/2} \cdot 4\pi (6\pi\delta)^{1/2} e n_{e\infty}^{3/2} T^{3/2} \varphi^{1/2}(\theta_0)/mc\nu_0, \quad (19)$$

there exist two steady-state values of the electron temperature being established in the skin layer. To these two values of T_{e0} naturally correspond the two different values for the plasma surface impedance that follow from the expression (10).

It is easy to see, however, that one of the steady-state solutions found is unstable, since all the values of T_{e0} corresponding to the incident branch of $\varphi(\Theta)$ satisfy the inequality

$$\frac{d}{dT_{e0}}[W(T_{e0}) - q(T_{e0})] > 0.$$

The steady-state value of the electron temperature satisfying this inequality is unstable because as the electron temperature increases through fluctuation, the Joule energy release in the skin layer will increase more rapidly than the heat flux from the skin layer, which should lead to still further heating up of the electrons, and, thus, to the development in the skin layer of an instability due to overheating.

Therefore, the sole steady state which can be realized in the case of the nonlinear skin effect being considered here corresponds to the growth of the plasma electron temperature as the intensity of the wave incident on the plasma increases. In this case electrons and ions are expelled from the skin layer, and this leads, as can be seen from the expression (11), to the growth of the coefficient $K = |E_b|^2/E_0^2$ of penetration of electromagnetic waves into the plasma. Consequently, as the intensity of the incident electromagnetic wave is increased right up to the value $E_0 = E_{max}$, the coefficient of absorption by the plasma of electromagnetic waves gradually increases.

If, on the other hand, the amplitude of the electromagnetic wave incident on the plasma exceeds the value E_{max}, then a steady-state solution for the nonlinear skin layer in which the above-considered conditions are realized does not exist. This implies that when the intensity of the electromagnetic wave incident on the plasma is sufficiently high, the plasma electrons are heated up to such a temperature that the dimension of the skin layer exceeds the electron mean free path connected with the transfer of energy to the neutral particles. Therefore, when the amplitude of the incident wave exceeds the value E_{max}, the electron temperature in the skin layer should change abruptly from the value $T_{e1} = T\Theta_0$ (the numerical value of Θ_0 should, as is evident from the asymptotic form (18), be of the order of unity) to the value T_{e2}, whose magnitude can be estimated from the following equation:

$$\delta_s(T_{e2}) = l\delta^{-1/2}. \quad (20)$$

There occurs in this case a sharp, stepwise increase in the coefficient of absorption of electromagnetic waves by the plasma. It is natural that for this effect to appear, the inequality $T_{e2} \gg T_{e1}$ must be satisfied, i.e., the skin-layer dimension should, right up to the values $T_{e0} = T_{e1}$, be small compared to the characteristic size of the electronic thermal conductivity. Using (10) and (20), we can write this condition in the following form:

$$\left(\frac{el}{c}\right)^{4/3}\left(\frac{n_{e\infty}\omega}{\delta m\nu_0}\right)^{2/3} \gg 1 \text{ for } \omega \ll \nu_0,$$
$$\left(\frac{el}{c}\right)^{2}\frac{n_{e\infty}}{\delta m} \gg 1 \text{ for } \omega \gg \nu_0. \quad (21)$$

It should be noted here that for a definite function $\nu(T_e)$ the situation can be realized in the framework of the above-considered approximations when the curve $\varphi(\Theta)$ shown in the figure will have an N-shaped form. For example, in a gas-discharge plasma this can occur when the value of T_{e2} exceeds the temperature corresponding to the Ramsauer minimum. In this case the second stable state of the plasma is also determined from the expression (17). (A similar nonunique dependence of the electron temperature in a homogeneous plasma on the electric field has been obtained by A. Gurevich[8].)

3. Thus, we have shown that under certain conditions in a weakly ionized plasma the coefficient of reflection of electromagnetic waves may decrease discontinuously as the wave intensity increases. The frequency of the electromagnetic waves should then be sufficiently high for the characteristic length of the electronic thermal conductivity to exceed the skin-layer dimension considerably.

Notice, moreover, that the results obtained can easily be generalized to the case of the penetration of electromagnetic waves into a weakly ionized nonisothermal plasma. All that changes in the above-presented expressions in this generalization is that the quantity T is replaced by the electron temperature $T_{e\infty} > T$ in the unperturbed region of the plasma.

The region of the plasma parameters and the electromagnetic-wave frequencies, where we should expect the above-considered effect, can be determined from the inequalities (1), (4), (9), and (21). Numerical estimates show that, for example, in a gas-discharge plasma with the easily attainable parameters $n_0 = 10^{15}-10^{16}$ cm^{-3}, $n_e = 10^9-10^{11}$ cm^{-3}, all the conditions of applicability of the above-considered theory can be satisfied at an electromagnetic-wave frequency $\omega = 10^9-10^{10}$ sec^{-1}. An estimate with the aid of (19) of the threshold electromagnetic-wave power then yields $\sim 10^{-1}-10^{-2}$ W/cm^2.

In conclusion, the authors express their sincere gratitude to A. V. Gurevich for a critical and useful discussion of the results obtained in the paper.

[1]Such a skin effect in the nonlinear propagation of electromagnetic waves in the one-component plasma of semiconductors has been considered by Bass and Yu. Gurevich [2].

[1] A. V. Gurevich, Zh. Eksp. Teor. Fiz. **35**, 392 (1958) [Sov. Phys.-JETP **8**, 271 (1959)].

[2] F. G. Bass and Yu. G. Gurevich, Zh. Eksp. Teor. Fiz. **51**, 536 (1966) [Sov. Phys.-JETP **24**, 360 (1967)].

[3] M. A. Liberman and A. T. Rakhimov, Zh. Eksp. Teor. Fiz. **61**, 1047 (1971) [Sov. Phys.-JETP **34**, 559 (1972)]; Zh. Tekh. Fiz. **42**, 1799 (1972).

[4] V. L. Ginzburg, Rasprostranenie élektromagnitnykh voln v plazme (The Propagation of Electromagnetic Waves in a Plasma), Nauka, 1968 (Eng. Transl., Gordon

and Breach, New York, 1972).
[5] A. V. Gurevich, Geomagn. i Aéronom. **5**, 70 (1965).
[6] F. G. Bass, Yu. G. Gurevich, and M. V. Kvimsadze, Zh. Eksp. Teor. Fiz. 60, 632 (1971) [Sov. Phys.-JETP **33**, 343 (1971)].
[7] A. V. Gurevich, Zh. Eksp. Teor. Fiz. **32**, 1237 (1957) [Sov. Phys.-JETP **5**, 1006 (1957)].
[8] A. V. Gurevich, Zh. Eksp. Teor. Fiz. **36**, 624 (1959) [Sov. Phys.-JETP **9**, 434 (1959)].

Translated by A. K. Agyei
53