Interaction between a high frequency magnetic field and potential plasma oscillations in a Q-device

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In order to reduce plasma diffusion in a Q-device, an h.f. magnetic field is applied to the plasma column. The oscillation spectra under such conditions are studied. The spectra in the presence or absence of the h.f. field are compared theoretically and experimentally. The effect of oscillations on diffusion are considered for both cases.

It is well known that drift-type oscillations can develop in Q-devices because of the radial inhomogeneity of the plasma. When the concentrations are not too high and the magnetic fields are weak, collisionless drift waves are excited in the plasma[1,2]. Since d ln T/d ln n is close to zero in a Q-device, the basic type of instability in this case is the oscillations that build up with the resonance electrons, i.e., the so-called "universal drift instability"[1-3]. Its increment is different from zero only when account is taken of the ion inertia, i.e., of the finite ion Larmor radius. At higher concentrations, when electron-ion collisions begin to play a role in the plasma, drift-dissipative oscillations are produced[4,5]. It is shown in a number of papers[6,7] that the "anomalous" drift of particles across the magnetic field is determined by the diffusion from these oscillations, and for this reason it is of considerable interest to investigate the possibility of stabilizing drift oscillations in order to decrease the transverse diffusion coefficient. One such stabilization mechanism is the application of a high-frequency azimuthal magnetic field to the plasma column[8,9]. That such stabilization is possible has been shown in earlier papers by using as an example mode suppression in the spectra of drift and drift-dissipative instabilities in a Q-device[10,11], and convection-current instability in the positive column of a gas discharge[10]. We have undertaken to study in detail the interaction of a high-frequency magnetic field with plasma oscillations and its effect on plasma diffusion across a magnetic field.

The experiments were carried out in a single-end Q-device[11] with a potassium plasma ionized by a hot tungsten electrode 3 cm in diameter. The length of the plasma column was 100 cm, and the working concentration n ranged from 2×10^8 to 2×10^{10}. The constant magnetic field was varied up to 4500 Oe. Unlike the earlier experiments, the high-frequency magnetic field was produced by 70-kHz current flowing through four busbars placed symmetrically along the plasma column boundary. The current in neighboring filaments flowed in opposite directions, and the amplitude reached 45 A in each filament. This placement of the current-carrying conductors results in more effective action of the high-frequency field on the region of maximum amplitude of the oscillations localized near the boundary of the column, without destroying the homogeneity of the plasma. To measure the radial concentration distribution and the oscillation spectra, three double probes were introduced into the plasma at distances of 20, 40 and 60 cm from the heated end. The probes were made of molybdenum wire 0.2 mm in length. The collecting surface, 5 mm in length, was oriented along the field lines so that the probe recorded the transverse component of the electric field. The construction of the probes permitted them to be warmed up before each series of experiments; this prevented a potassium film from settling on the surface of the probe and causing errors in the measurements.

The oscillations were picked off from the probe under floating-potential conditions and were fed to a differential amplifier with an differential index of about 100 and an input resistance of ~100 MΩ. The signal from the amplifier was fed to a panoramic spectrum analyzer with a frequency resolution 5 kHz. The time-averaged concentrations were determined from the double-probe saturation current by a standard method.

I. COLLISIONLESS DRIFT INSTABILITY

As noted earlier[2], at low concentrations the spectra have a distinct line character in the absence of a high-frequency magnetic field (Fig. 1a). Application of a high-frequency magnetic field causes the character of the oscillation spectra to change significantly, viz., the harmonics in the spectra are suppressed and, as shown previously[11], the higher modes are suppressed most effectively. At the same time, forced oscillations build up at the generator frequency and at the combination frequencies $m\omega + s\Omega$ (Fig. 1b); their amplitude falls off rapidly at higher values of the harmonic number s, in accord with the conclusions of[12]. The integral noise power, including that of the amplitudes of the first mode, increased; for this reason the diffusion coefficient can not be expected to decrease under these conditions. In fact, measurements of the radial concentration distribution profile show that in this case the diffusion coefficient increases 20—40% when the high-frequency field is applied.

Let us consider the character of the forced oscillations in greater detail. To do this we consider the dis-

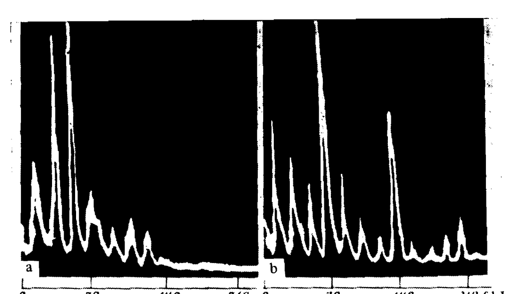

FIG. 1. Oscillation spectra taken under conditions of collisionless drift instability, a—in the absence of the high-frequency magnetic field, and b—upon application of the field (the peaks at 70, 140, and 210 KHz correspond to forced oscillations at the generator frequency Ω and the multiple frequencies 2Ω and 3Ω.

persion equation for drift universal instability, with allowance for the finite ion Larmor radius, when a high-frequency magnetic field is applied (for the derivation and notation see the Appendix):

$$\frac{\omega^*}{\omega} = 1 + i\pi^{1/2} J_0^2(x_0) \frac{\omega^* - \omega}{|k_z| v_{Te}}. \quad (1)$$

It is easily shown that no instability of this type builds up, since $\omega = \omega^*$ and $\gamma \approx 0$. However, when account is taken of the finite ion Larmor radius[3], Eq. (1) takes the form

$$\frac{\omega^*}{\omega} = 1 + k_\perp^2 \rho_i^2 + i\pi^{1/2} J_0^2(x_0) \frac{\omega^* - \omega}{|k_z| v_{Te}}, \quad (2)$$

where $k_\perp = m/r$ is the transverse wave vector of the oscillations and is the ion Larmor radius.

From Eq. (2) we find

$$\omega = \frac{\omega^*}{1 + k_\perp^2 \rho_i^2}, \quad \nu = \frac{(\omega^*)^2 J_0^2(x_0)}{|k_z| v_{Te}} \frac{k_\perp^2 \rho_i^2}{(1 + k_\perp^2 \rho_i^2)^3}. \quad (3)$$

As follows from (3), the increment of the oscillations that can build up in the absence of the high-frequency magnetic field has a maximum at

$$k_\perp^2 \rho_i^2 = 1/2, \quad (4)$$

and falls off by a factor $1/J_0^2(x_0)$ when the high-frequency field is applied, i.e.,

$$\gamma_{hf} / \gamma \approx J_0^2(x_0). \quad (5)$$

We show below the ratios of the increments of the buildup of the first, second, and third harmonics, as estimated from formula (5), and the ratios, taken from Fig. 1, of the corresponding amplitudes before and after the application of the high-frequency field.

i — mode number:	1	2	3
$\varphi_{hf}^{(i)} / \varphi^{(i)}$:	1.4	0.5	0.3
$\gamma_{hf}^{(i)} / \gamma^{(i)}$	1	0.7	0.5

We conclude from these data that the oscillation amplitude at a given frequency is proportional to the increment at that frequency, which does not contradict the experimental results[13,4] for Kelvin-Helmholtz and for drift-dissipative instabilities.

The amplitude maximum, as seen from Fig. 1, is near the third mode, for which in this case the value $k_\perp^2 \rho_i^2 \approx 0.4$ is close to the theoretically obtained maximum of the oscillation increment (see (4)).

2. DRIFT-DISSIPATIVE INSTABILITY

At higher concentrations the collision frequency increases and drift-dissipative instability is excited in the plasma. This situation is the most interesting one from the point of view of possible high-frequency stabilization, since the stabilization criterion for drift-dissipative instability is considerably weaker[9]:

$$H_1 / H_0 > 2^{3/2} (\nu_{ei} \Omega)^{1/2} / k_y v_{Te},$$

where $k_y \sim 10-100\ \text{cm}^{-1}$, considerably higher than in the collisionless case[14]. Figure 2 shows the radial concentration distribution profiles under collision conditions ($n \approx 2 \times 10^{10}\ \text{cm}^{-3}$) in the absence of a high-frequency magnetic field (plots 1a and 1b) and with the field (plots 2a and 2b). The profiles were taken at distances 20 and 60 cm from the hot electrode.

As the figure shows, when the high-frequency magnetic field is turned on the concentration increases and its radial profile becomes steeper. (The concentration near the hot end and the character of the oscillation spectra allow us to conclude that drift-dissipative instability develops in the plasma under these conditions[4,5,14].) A numerical calculation from the formulas in[15]

$$j_\perp = \left(\frac{2T_i}{M}\right)^{1/2} \frac{1}{r} \int_0^r \frac{\partial n(r,z)}{\partial z} r\, dr,$$

shows that the diffusion coefficient

$$D_\perp = \frac{j_\perp}{\partial n / \partial r},$$

averaged over the length of the column, decreases by a factor of three from 300 cm²/sec to 100 cm²/sec, which is only two or three times larger than the classical diffusion coefficient. Figures 3 and 4 show the effect of the high-frequency magnetic field on the oscillation spectra under these conditions.

In the absence of the high-frequency magnetic field and at $H_0 = 0.8$ kOe (Fig. 3a), several oscillatory modes can be seen in the spectra, but at higher values of H_0 the spectra are much contracted towards the low frequency region, which agrees with the earlier experiments[16]. We note that the higher mode oscillations ($f_2 = 40$ and $f_3 = 60$ kHz) have a "flute-like" character in this case, i.e., $k_\parallel / k_\perp \approx 0$ for them (see Figs. 3 and 4). Thus, use of the boundary conditions leads to a rapid decrease of the increment of these oscillations at higher mode num-

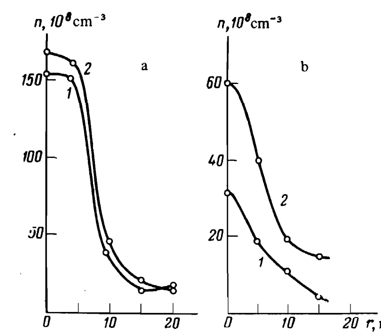

FIG. 2. Profile of radial concentration distribution taken under conditions of drift-dissipative oscillations: a—at a distance 20 cm from the hot electrode, b—at distance 60 cm. Plot 1—without the field, plot 2—with the field, H = 1.8 KOe.

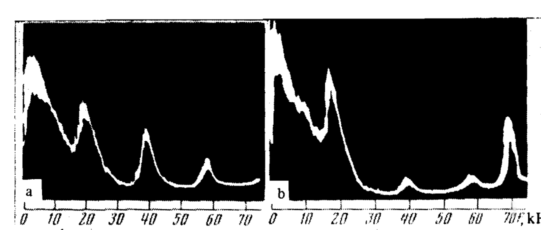

FIG. 3. Spectrum of the transverse-oscillation component obtained under drift-dissipative oscillation conditions with H_0 = 0.8 kOe: a—in the absence of the magnetic field, b—with the field.

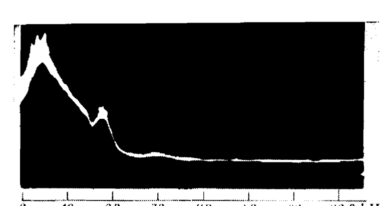

FIG. 4. Spectrum of the longitudinal component of oscillations under drift-dissipative conditions in the absence of a magnetic field.

bers because of the stabilizing effect of the finite ion Larmor radius. Application of the high-frequency field causes a sharp decrease in the integral noise level, while, as previously pointed out[11], in this case the higher modes are stabilized more than the lower ones. As seen from Fig. 3b, the high-frequency field has, in practice, no effect on the low-frequency region of the spectrum ($f \leq 20$ kHz). This is because the low-frequency oscillations have larger components k_z of the wave vector along the magnetic field.

As expected, forced oscillations appear in the spectrum at the frequency of the high-frequency magnetic field under stabilization conditions. The amplitude of the forced oscillations is, in this case, much less than the amplitude of the drift-dissipative oscillations in the absence of the high-frequency field. This is exactly the reason why no oscillations at the combination frequencies are visible in the spectrograms, unlike the low-concentration case described earlier.

The decrease in the diffusion coefficient in the case of drift-dissipative instability is due to the effective suppression of the instability by the high-frequency magnetic field and can be estimated in the following way:

$$D_{\text{hf}} \sim \frac{\gamma_{\text{hf}}^2}{\Omega k_{\text{hf}\perp}^2} \sim D\frac{\gamma_{\text{hf}}^2}{\gamma^2}\frac{\omega}{\Omega}\left(\frac{k_\perp}{k_{\text{hf}\perp}}\right)^2, \tag{6}$$

where D_{hf}, γ_{hf}, k_{hf}, and Ω are, respectively the diffusion coefficient, the maximum instability increment, the transverse wave vector, and the oscillation frequency following application of the high-frequency magnetic field, while D, γ, k, and ω are the same quantities in the absence of the field. It is shown in[17] that the high-frequency magnetic field starts affecting the instability when this relation is satisfied:

$$\beta = \frac{H_1^2}{H_0^2}\frac{k_y v_{Te}}{\Omega \nu_{\text{cr}}} \geqslant 1. \tag{7}$$

When the equality in (7) is satisfied, the diffusion coefficient does not yet decrease upon application of the high-frequency field, and is equal to D; therefore, recalling that

$$\gamma_{\text{hf}}/\gamma = \omega/\Omega\beta^{2/3}, \tag{8}$$

we obtain from (7)

$$D_{\text{hf}} \sim D\left(\frac{\omega}{\Omega}\right)^3\left(\frac{k_\perp}{k_{\text{hf}\perp}}\right)^2\left(\frac{H_{\text{hf}}}{H_1}\right)^{4/3}. \tag{9}$$

where H_{cr} is determined by the condition $\beta = 1$.

We have determined H_{cr}/H_1 experimentally and obtained a value on the order of 1/2 under the described conditions. Estimates made by comparing the amplitude of the oscillations picked off the double and single probes yield $k_\perp/k_{\text{hf}\perp} \sim 4$ and $\omega/\Omega \sim 1/4$. From this and formula (9) we find $D_{\text{hf}}/D \sim 1/8$; in view of the approximate nature of the estimates, this is in good enough agreement with experiment.

Let us also compare the theoretical prediction with the relative decrease in the amplitude of the harmonics when the high-frequency magnetic field is applied. As follows from (8), the following relation is satisfied for the third and fourth modes:

$$\frac{\gamma_{\text{hf}}^{(2)}}{\gamma_2}\Big/\frac{\gamma_{\text{hf}}^{(3)}}{\gamma_3} = \frac{\omega_2}{\omega_3} \sim 0.7. \tag{10}$$

On the other hand, the ratio of the corresponding harmonics before and after the application of the high-frequency magnetic field is found from Fig. 3:

$$\frac{\varphi_{2\,\text{hf}}}{\varphi_2}\Big/\frac{\varphi_{3\,\text{hf}}}{\varphi_3} \sim 0.6.$$

A comparison of this result with (10) shows no conflict with the assumption that the amplitude of a given harmonic is proportional to its buildup increment.

Thus, we have shown that:

1) When a high-frequency magnetic field interacts with drift oscillations in the collisionless case the buildup increment of the oscillations falls off by a factor of $1/J_0^2$, and the oscillations with large values of k_y are the most efficiently suppressed.

2) Beats appear in the oscillation spectrum at the combination frequencies $m\omega + s\Omega$; their amplitude is commensurate with that of drift oscillations in the absence of the high-frequency magnetic field but falls off rapidly at higher values of s.

3) Under collisionless conditions the low-frequency noise level is reduced, which leads to an increase in the total anomalous diffusion coefficient.

4) Intensive drift-dissipative oscillations are effectively suppressed when the high-frequency magnetic field is turned on; then the integral noise level drops, and the diffusion coefficient decreases almost to its classical value.

In conclusion, the authors thank V. Ya. Shcherbakov and A. N. Samsonov for their great assistance in making the experiment.

APPENDIX

To obtain the dispersion equation for universal drift instability, taking account of the high-frequency magnetic field, we use the drift kinetic equation in the Fourier representation in y and x:

$$\frac{\partial f}{\partial t} + ik\mathbf{h}\frac{\partial f}{\partial z} + \frac{c}{H_0}[\mathbf{E}\times\mathbf{h}]\frac{\partial f_0}{\partial x} + i\frac{e}{m}k\mathbf{h}\frac{\partial f_0}{\partial v_\parallel}\varphi = 0; \tag{A.1}$$

$$\mathbf{h} = \mathbf{h}_0 + \mathbf{h}_1 \sin\Omega t, \quad \mathbf{h}_0 = \mathbf{H}_0/H_0, \quad \mathbf{h}_1 = \mathbf{H}_1/H_1, \tag{A.2}$$

where Ω and H_1 are the amplitude and frequency of the high-frequency magnetic field:

$$\mathbf{H}_1 = \{0, H_y, 0\}, \quad |H_y| \ll |H_0|, \quad \mathbf{E} = -\nabla\tilde{\varphi}.$$

The solution of the resulting inhomogeneous equation is found by the method of constant variation and is of the form

$$f = \frac{e\varphi}{T} + i\int_{-\infty}^{\infty}\left(\frac{c}{H_0}k_y\frac{\partial f_0}{\partial x} - \omega\frac{e}{T}\right)\exp\left[ik_z v_\parallel(t-t') + ik_y\frac{H_1}{H_0}v_\parallel(\sin\Omega t - \sin\Omega t')\varphi(t')\right]dt'. \tag{A.3}$$

Taking into account now the higher harmonics in Ω in the expansion (A.2), we seek a solution of (A.1) in the form $f_s \sim \exp\{-i(\omega + s\Omega)\}$. Then we obtain instead of (A.3)

$$f_s = \frac{e\varphi_s}{T} + \sum_{m,n,l}\left(\frac{c}{H_0}k_y\frac{\partial f_0}{\partial x} - (\omega + n\Omega)\right)\frac{e}{T} \times\delta(m-n-l+s)\frac{J_m(x)J_l(x)\varphi_n(x)f_0}{k_z v_\parallel - \omega - (n+l)\Omega}, \quad x = \frac{k_y v_{Te}}{\Omega}\frac{H_1}{H_0}, \tag{A.4}$$

where J are Bessel functions. The oscillation potential is then best represented in the form

$$\varphi = \sum_{s=0}^{\infty}\varphi_s\exp\{-i(\omega + s\Omega)t\}.$$

Summing then with respect to m and l and recognizing that the higher order terms in Ω are exponentially small, we obtain the electron-density perturbation:

$$\frac{n_s^{(e)}}{n_0}=\frac{e\varphi_s}{T}+i\frac{e}{T}\sum_n\varphi_n\frac{\omega^*-(\omega+n\Omega)}{|k_z|v_{Te}}J_{-s}(x_0)J_{-n}(x_0)\pi^{1/2},$$
$$x_0=x\left(\frac{\omega}{|k_z|}=v_\parallel=v_{Te}\right)=\frac{k_y}{k_z}\frac{\omega}{\Omega}\frac{H_1}{H_0}. \quad (A.5)$$

By virtue of the quasineutrality condition $n_s^{(e)}=n_s^{(i)}$ we set the electron density perturbation (4) equal to the ion density perturbation obtained from the continuity equation, and find

$$\varphi_s\frac{\omega^*}{\omega+s\Omega}=\varphi_s+i\pi^{1/2}\sum_n J_{-s}(x_0)J_{-n}(x_0)\varphi_n\frac{\omega^*-(\omega+n\Omega)}{|k_z|v_{Te}}. \quad (A.6)$$

Since in the experiment $x_0 \ll 1$, i.e., $J_0(x_0) \gg J_{-s}(x_0)$ when $s > 1$, we discard all terms in (A.6) containing higher order Bessel functions and obtain the dispersion equation (1).

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