Two-component charged Fermi liquid in a magnetic field

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(Submitted April 21, 1972)
Zh. Eksp. Teor. Fiz. **64**, 273-282 (January 1973)

The electromagnetic properties of a multicomponent charged Fermi liquid in a stationary magnetic field are considered. The dielectric-constant tensor is derived and used to analyze the effect of Fermi-liquid interaction on excitation spectra and polarization. The possibility in principle of determining the zero and first expansion coefficients of the Landau function is mentioned.

1. An analysis of the properties of a one-component charged Fermi liquid shows (see, for example, the reviews[1,2]) that an interelectron correlation should become effectively manifest at frequencies ω on the order of the cyclotron frequency Ω and at perturbation phase velocities comparable with the velocity v_F of the quasiparticles on the Fermi surface (we do not concern ourselves here with spin waves). Under the experimental conditions, however, in view of the inequality $\omega \lesssim \Omega \ll kc \ll \omega_p$, where ω_p is the electron Langmuir ("plasma") frequency, the current and charge densities of the electromagnetic perturbations vanish. This eliminates from the cyclotron-wave theory the most important zeroth and firth coefficients A_0 and A_1 of the expansion of the functional kernel $\varphi(p, p')$ of the Fermi-liquid theory. As a result, experiments in the long-wave region make it possible to determine only the second and higher coefficients, which are small in magnitude; for example, $A_2 = -0.03$ for potassium.

As to the coefficient A_1, it can be determined in principle from experiments on cyclotron resonance in the short-wave region $kR \sim 30$, where $R = v_F/\Omega$, but the corresponding effects are small and are not observed in experiment[3]. One can also attempt to obtain the first coefficient A_0 and A_1 from experiments in the optical region[4-7], but this is a rather complicated matter. In addition, the high-frequency coefficients ($\omega \gg \omega_D$, where ω_D is the Debye frequency) obtained in this manner, may generally speaking differ from the low-frequency coefficients that appear in cyclotron resonance[1,7].

A different situation develops in a multicomponent liquid, for while the summary current density and charge may be small in this case, the currents and charges of the individual components can nevertheless be appreciable. As a result, even the "zeroth" coefficients A_0^{ij} begin to play a noticeable role in the low-frequency region $\omega \ll \omega_p$ even in the absence of a magnetic field[8]. As will be shown below, an experimental investigation of the oscillations and waves in a multicomponent liquid in a magnetic field could afford new possibilities for the determination of the Fermi-liquid coefficients A_0 and A_1 (as well as the higher-order coefficients), both for interparticle correlations within a single component and for the interaction between them.

It should be borne in mind that in the analysis of concrete experimental data it is necessary to take into account the symmetry of the crystal and the shape of the Fermi surface. Therefore, in the general case, especially in the presence of sections with open Fermi surface, the theoretical analysis must take anisotropy effects into account. Among the materials dealt with in the case of a multicomponent Fermi liquid are metals with several conduction bands, for example, transition-group metals, semimetals, and degenerate semiconductors. Also included is the "electron-hole condensate"[9] produced in semiconductors at low temperatures under the conditions of intense illumination.

The question of the shape of the Fermi surface of transition metals is still undecided, and all the available calculations are carried out in the isotropic-effective-mass approximation[10]. With respect to other materials, one can speak with assurance only of anisotropy. Even for these, however, by virtue of various factors (for example, the choice of the direction of the magnetic field), an effective contribution to the conductivity can be made by Fermi-surface sections with isotropic dispersion. In this case the anisotropy is imitated by the difference between the density of states, and in a number of cases one can confine oneself to the isotropic model, thereby greatly simplifying the analysis and making it possible to obtain clearer physical results.

We consider in this paper a two-component charged Fermi-liquid in a constant magnetic field. The carrier dispersion is assumed to be isotropic and in general non-quadratic. By solving the system of kinetic equations we obtain the dielectric tensor of the two-component liquid. In the long-wave limit $kR \ll 1$ we obtain the spectrum of the cyclotron and hydrodynamic waves, helicons, etc., and analyze in detail the role of the liquid interaction.

2. The system of linearized quasiclassical kinetic equations for the deviation of the spin-symmetrical part of the distribution function $\delta f_i(p, \mathbf{r}, t)$ from the equilibrium function $f_{i0}(p)$ takes the following form (cf.[8]):

$$\frac{\partial}{\partial t}\delta f_i + \left(\mathbf{v}_i\frac{\partial}{\partial \mathbf{r}_i} + \frac{e_i}{c}[\mathbf{v}_i \times \mathbf{B}]\frac{\partial}{\partial \mathbf{p}_i}\right)\left(\delta f_i - \frac{\partial f_{0i}}{\partial \varepsilon_i}\delta\varepsilon_i\right) + e_i\mathbf{E}\mathbf{v}_i\frac{\partial f_{0i}}{\partial \varepsilon_i} = J_{st}^i,$$

$$J_{st}^i = -\nu_i\left(\delta f_i - \frac{\partial f_{0i}}{\partial \varepsilon_i}\delta\varepsilon_i\right). \quad (1)$$

Here i is the index of the component, $\mathbf{v}_i = \partial\epsilon_{0i}/\partial \mathbf{p}_i$, the collisions are taken into account phenomenologically by introducing the collision frequencies ν_i, and the energy correction necessitated by the interaction with all the components j of the liquid is written in the form

$$\delta\varepsilon_i(\mathbf{p}_i, \mathbf{r}; t) = \int d\mathbf{p}_j \varphi_{ij}(\mathbf{p}_i, \mathbf{p}_j)\,\delta f_j(\mathbf{p}_j, \mathbf{r}; t). \quad (2)$$

Separating, as usual, the energy dependence $\delta f = -(\partial f_0/\partial\epsilon)g$ from δf we can then expand the functions g and φ_{ij} in eigenfunctions in accordance with the symmetry of the crystal and the shape of the Fermi surface, and obtain with the aid of (1) a system of equations for the expansion coefficients.

We confine ourselves here to a two-component isotropic Fermi liquid, assuming the equilibrium distribution $f_{0i}(p_i)$ to be spherically symmetrical. Then, expanding g in spherical functions and φ_{ij} in Legendre polynomials

$$Z_j\varphi_{ij}(\mathbf{p}_i,\mathbf{p}_j)=\sum_l(2l+1)A_l^{ij}P_l(\cos\chi),\quad Z_j=\int d\mathbf{p}_j\left(-\frac{\partial f_{0j}}{\partial\varepsilon_j}\right),\tag{3}$$

we can obtain an expansion of $\delta\epsilon_i$ in spherical functions

$$\delta\varepsilon_i\Rightarrow\sum_{l,m}A_l^{ij}g_{lm}^jY_{lm}(\theta,\varphi).\tag{4}$$

Finally, integrating the system (1) with respect to φ and taking the periodicity condition into account, we obtain for the coefficients g^i_{lm} in a spherical coordinate system the following system of equations:

$$\sum_{l'm'}[(\delta_{ij}+A_{l'}^{ij})\delta_{ll'}\delta_{mm'}-A_{l'}^{ij}N_{l'm'}^{lm}(i)]g_{l'm'}^j=a_iN_{1,-\mu}^{lm}(i)E_\mu,\quad i,j=\alpha,\beta.\tag{5}$$

In the derivation of (5), the time and space dependences of the functions g were chosen in the form $\exp\{-i\omega t+i\mathbf{k}\cdot\mathbf{r}\}$, the z axis is directed along the constant magnetic field $\mathbf{B}_0$, the y axis is perpendicular to the $(\mathbf{k},\mathbf{B}_0)$ plane, and we have introduced the notation

$$N_{l'm'}^{lm}(i)=\sum_{s=-\infty}^{\infty}\int_{-1}^{1}d\cos\theta\frac{\tilde\omega_i}{\tilde\omega_i-k_\parallel v_{\parallel i}-s\Omega_i}J_{l-m}J_{l'-m'}\Theta_{lm}\Theta_{l'm'},\tag{6}$$

where $J_n(k_\perp v_{\perp i}/\Omega_i)$ is a Bessel function of order n, $\Theta_{lm}(\theta)$ is an associated Legendre polynomial normalized to unity, $a_\alpha=\sqrt{4/3\pi}ie_\alpha v_\alpha/\tilde\omega_\alpha$, $\tilde\omega_\alpha=\omega+iv_\alpha$, $\Omega_\alpha=e_\alpha B_0v_{F\alpha}/cp_{F\alpha}$, $k_\parallel$, $v_\parallel$ and $k_\perp$, $v_\perp$ are the components of the vectors along and across the magnetic field. We note also that in the spherical system $b_z=b_0$, $b_{\pm1}=(b_x\pm ib_y)/\sqrt{2}$; in particular, the velocity components are $v_\mu=v_F\sqrt{4/3\pi}Y_{1\mu}$, $\mu=0,\pm1$.

For the problem of calculating the current and the dielectric constant, which is of interest to us, the six quantities $g^i_{1\mu}$ $(i=\alpha,\beta)$ are sufficient, since, e.g., the current is given by

$$j_\mu=-\sum_ie_i\int d\mathbf{p}_iv_{i\mu}\frac{\partial f_{0i}}{\partial\varepsilon_i}(g^i+\delta\varepsilon_i)=i\sum_{i=j}\hat r_i(a_i\tilde\omega_i)^{-1}g^i_{1-\mu},$$
$$\hat r_i=r_i\left(1+A_1^{ii}+\frac{v_j}{v_i}A_1^{ij}\right),\qquad r_i={}^4/_3\pi e_i^2v_i^2Z_i.\tag{7}$$

We consider the introduction of ω_p in place of $\hat r_i$ useful in view of the already noted possible difference between the liquid coefficients in the low- and high-frequency regions. It is important that the drag flow[11] in a two-component liquid leads to an "entanglement" of the components in the total current. Therefore, to separate the current of an individual component α it is necessary to vary the total current with respect to g^α, so that

$$j_\mu^\alpha=(\delta j_\mu/\delta g^\alpha_{1-\mu})g^\alpha_{1-\mu}.\tag{8}$$

A "coupling" of the components occurs also in the continuity equation, as can be easily verified by integrating the kinetic equation with respect to the momentum.

We note also that for systems having translational invariance we can find the connection between the "bare" crystal mass m_α and the effective mass m_α^*, the difference between which is due to the liquid effects:

$$m_\alpha^*=m_\alpha(1+A_1^{\alpha\alpha}+(v_\beta/v_\alpha)A_1^{\alpha\beta}).\tag{9}$$

The presence of the second component exerts a direct influence on the effective mass (cf.[12,13]).

3. If we break up the infinitive system (5), putting $A_l=0$ for $l>l_{max}$, then the solution for $g_{1\mu}$, as in the case of a one-component liquid[14], it can be written in the form

$$g_{1\mu}{}^\alpha=a_\alpha\frac{\Delta^\alpha_{\mu\nu}}{\Delta}E_{-\nu},\qquad\mu,\nu=0,\pm1.\tag{10}$$

Here $\Delta^\alpha_{\mu\nu}$ is the determinant obtained from the determinant Δ of the system by substituting the right-side of the system (5) in the corresponding column and dividing by the factor a_α. As a result, the conductivity tensor takes the form

$$\sigma_{\mu\nu}=\sigma_{\mu\nu}^{(\alpha)}+\sigma_{\mu\nu}^{(\beta)},\qquad\sigma_{\mu\nu}^{(\alpha)}=\frac{i\hat r_\alpha}{4\pi\tilde\omega_\alpha}\frac{\Delta_{\mu\nu}^{(\alpha)}}{\Delta}.\tag{11}$$

In a two-component system, as indicated in the introduction, the liquid interaction becomes manifest already when the coefficients A_0 and A_1 are taken into account. To reveal most clearly the role of the second component, we confine ourselves to excitations with $l_{max}=1$, i.e., to precisely those modes for which the correlation effects hardly appear in a one-component medium. In the considered model with $l_{max}=1$, it is expedient to eliminate from the system (5) the quantities g_{00} by using the continuity equation and the properties of the coefficients $N^{lm}_{l'm'}$ [14]. Then the system (5) can be written in the form

$$\sum_j[(\delta_{\alpha j}+A_1^{\alpha j})\delta_{\mu\nu}-A_1^{\alpha j}N_{1\nu}{}^{1\mu}(j)]g_{1\nu}{}^j-k_\lambda k_\nu N_{1\lambda}{}^{1\mu}(\alpha)\Big[A_1^{\alpha\beta}(A_0^{\alpha\beta}q_{\alpha\beta}g_{1\nu}{}^\alpha$$
$$+A_0^{\alpha\alpha}q_{\alpha\alpha}g_{1\nu}{}^\beta)+\sum_jA_0^{\alpha j}(1+A_1^{\alpha j})q_{\alpha j}g_{1\nu}{}^j\Big]=a_\alpha N_{1\nu}{}^{1\mu}(\alpha)E_{-\nu},\tag{12}$$
$$j=\alpha,\beta$$

(and an analogous equation with the substitution $\alpha\rightleftarrows\beta$). Here $q_{\alpha\beta}=v_{F\alpha}v_{F\beta}(3\tilde\omega_\alpha\tilde\omega_\beta)^{-1}$, summation over the tensor indices ν and λ is implied, and the sums over the components are written out in explicit form.

Since the system (12) corresponds to a sixth-order determinant, the conductivity tensor (11) (as well as the dispersion equation) is quite complicated in form. We confine ourselves therefore to the so-called "direct waves", which propagate strictly along or across the external magnetic field, writing out for this purpose the corresponding determinants in the cases $\mathbf{k}\parallel\mathbf{B}_0$ and $\mathbf{k}\perp\mathbf{B}_0$.

For a wave propagating along the magnetic field, $k_\mu=k\delta_{\mu0}$, $k_\perp=0$, $k_\parallel=k$, and the determinant Δ breaks up into a product of second-order determinants:

$$\Delta=\Delta_0\Delta_m\Delta_{-m},\quad m=\pm1;$$
$$\Delta_0=L^{\alpha\beta}L^{\beta\alpha}-M^{\alpha\beta}M^{\beta\alpha},\quad L^{\alpha\beta}=(1+A_1^{\alpha\alpha})(1-A_0^{\alpha\alpha}\eta_\alpha)-$$
$$-3A_1^{\alpha\alpha}s_\alpha^2\eta_\alpha-A_0^{\alpha\beta}A_1^{\beta\alpha}s_\beta s_\alpha^{-1}\eta_\alpha,\tag{13}$$
$$M^{\alpha\beta}=A_1^{\alpha\beta}s_\beta s_\alpha^{-1}[1-A_0^{\alpha\alpha}\eta_\alpha-3s_\alpha^2\eta_\alpha]-A_0^{\alpha\beta}(1+A_1^{\beta\beta})\eta_\alpha,$$
$$\Delta_m=G_m{}^\alpha G_m{}^\beta-A_1^{\alpha\beta}A_1^{\beta\alpha}\gamma_m{}^\alpha\gamma_m{}^\beta,\quad G_\mu{}^\alpha=1+A_1^{\alpha\alpha}\gamma_\mu{}^\alpha.\tag{14}$$

We have introduced here the notation $S_\mu=(\tilde\omega-\mu\Omega)(kv)^{-1}$, $\mu=0$, m, $\eta=-1+(2s)^{-1}\ln[(s+1)/(s-1)]$, $\gamma_\mu=1-N^{1\mu}_{1\mu}$. In the case under consideration

$$N_{1m}{}^{1m}=3\omega[2(\tilde\omega-\tilde\Omega)]^{-1}\delta_m,\ \delta_m$$
$$=1-(s_m{}^2-1)\eta(s_m),\ \tilde\Omega=m\Omega=\pm\Omega,\ s=s_0.$$

The determinants $\Delta_{\mu\nu}$ are similarly transformed:

$$\Delta_{00}{}^{(\alpha)}=\tilde\Delta_{00}{}^\alpha\Delta_m\Delta_{-m},\quad\Delta_{mm}{}^{(\alpha)}=\tilde\Delta_{mm}{}^\alpha\Delta_0\Delta_{-m},$$
$$\tilde\Delta_{00}{}^\alpha=3s_\alpha{}^2\left[\eta_\alpha L^{\alpha\beta}-\frac{e_\beta}{e_\alpha}\eta_\beta M^{\alpha\beta}\right],\tag{15}$$
$$\tilde\Delta_{mm}{}^\alpha=(1-\gamma_m{}^\alpha)G_m{}^\beta-C_{\alpha\beta}\gamma_m{}^\alpha(1-\gamma_m{}^\beta)A_1^{\alpha\beta},\quad C_{\alpha\beta}=a_\beta a_\alpha{}^{-1}.$$

Formulas (13)—(15) together with (11) determine the conductivity tensor $\sigma_{\mu\nu}$, and also the dielectric tensor

$\epsilon_{\mu\nu} = 1 + 4\pi i\sigma_{\mu\nu}/\omega$. We note that the component ϵ_{00} does not depend on the magnetic field, and coincides with the corresponding expression (2) of[8] as $A_1 \to 0$.

In the case of transverse propagation we have $k_{\parallel} = 0$, $k = k_{\perp}$, and a similar separation of the determinants takes place:

$$\Delta = \Delta^{(-)}\Delta^{(+)}, \quad \Delta_{00}^{(\alpha)} = \Delta^{(+)}\tilde{\Delta}_{00}{}^{\alpha}, \quad \Delta_{mm}^{(\alpha)} = \Delta^{(-)}\Delta_{-m}\tilde{\Delta}_{mm}^{\alpha}; \tag{16}$$

$$\begin{gathered}\tilde{\Delta}_{00}{}^{\alpha} = (1-\gamma_0{}^{\alpha})G_0{}^{\beta} - C_{\alpha\beta}A_1{}^{\alpha\beta}\gamma_0{}^{\alpha}(1-\gamma_0{}^{\beta}),\\ \Delta^{(-)} = G_0{}^{\alpha}G_0{}^{\beta} - A_1{}^{\alpha\beta}A_1{}^{\beta\alpha}\gamma_0{}^{\alpha}\gamma_0{}^{\beta},\\ \tilde{\Delta}_{mm}{}^{\alpha} = (1-\gamma_m{}^{\alpha})(G_m{}^{\beta} - Q_0{}^{\beta}) - C_{\alpha\beta}(1-\gamma_m{}^{\beta})(A_1{}^{\alpha\beta}\gamma_m{}^{\alpha} - R_0{}^{\alpha}),\end{gathered}$$

$$\begin{gathered}Q_{\mu}{}^{\alpha} = {}^1/_2k^2(1-\gamma_{\mu}{}^{\alpha})[A_0{}^{\alpha\alpha}(1+A_1{}^{\alpha\alpha})q_{\alpha\alpha} + A_0{}^{\alpha\beta}A_1{}^{\beta\alpha}q_{\alpha\beta}],\\ R_{\mu}{}^{\alpha} = {}^1/_2k^2(1-\gamma_{\mu}{}^{\alpha})[A_0{}^{\alpha\beta}(1+A_1{}^{\beta\beta})q_{\alpha\beta} + A_0{}^{\alpha\alpha}A_1{}^{\alpha\beta}q_{\alpha\alpha}], \quad \mu = 0, m,\\ \Delta_m = (G_m{}^{\alpha} - Q_m{}^{\alpha})(G_m{}^{\beta} - Q_m{}^{\beta}) - (A_1{}^{\alpha\beta}\gamma_m{}^{\alpha} - R_m{}^{\alpha})(A_1{}^{\beta\alpha}\gamma_m{}^{\beta} - R_m{}^{\beta}).\end{gathered} \tag{17}$$

As to the determinant $\Delta^{(+)}$, it can be represented in the form $\Delta^{(+)} = \Delta_m\Delta_{-m} - \Delta_m^{-m}\Delta_{-m}^{m}$. In the long-wave limit $kR \ll 1$, to which we shall henceforth confine ourselves, the product $\Delta_m^{-m}\Delta_{-m}^{m}$ turns out to be of the order of $(kR)^4$ (cf.[14]), and will be omitted. In this case we obtain

$$\begin{gathered}\Delta^{(+)} = \Delta_m\Delta_{-m},\\ \gamma_0 = -k^2v^2[5(\tilde{\omega}^2 - \Omega^2)]^{-1},\end{gathered} \tag{18}$$

$$1 - \gamma_m = \frac{\omega}{\tilde{\omega} - \tilde{\Omega}}\left\{1 + \frac{2k^2v^2}{5}[(\tilde{\omega} - \tilde{\Omega})^2 - \tilde{\Omega}^2]^{-1}\right\}.$$

Formulas (11) and (16)—(18) determine the tensor $\sigma_{\mu\nu}$ in the case of transverse propagation.

4. We turn to the analysis of the dispersion equation

$$|c^2(k_\mu k_\nu - k^2\delta_{\mu\nu}) - \omega^2\varepsilon_{\mu\nu}(\omega, k)| = 0 \tag{19}$$

in the low-frequency region $\omega_p^2 \gg k^2c^2 \gg \omega^2$, and pay particular attention to the determination of the conditions under which correlation effects should appear. As indicated above, we confine ourselves only to direct waves.

In the case of longitudinal propagation for the mode with $\mu = 0$, the dispersion equation coincides with the corresponding equation of a two-component Fermi liquid in the absence of a magnetic field[8]. At the indicated polarization, as shown earlier, waves of the acoustic type can propagate in the liquid in the region $\omega \ll \omega_p$. With the aid of (13)—(15) we can generalize the results of[8], taking the coefficients A_1^{ij} into account in the dispersion equation.

For a circularly-polarized wave with $m = \pm 1$ in the frequency region $\omega \lesssim \Omega \ll \omega_p$, Eq. (19) in accordance with the results of the preceding section, can be written in the form

$$\omega(1 + \hat{P}^{\alpha\beta})\frac{\hat{r}_\alpha}{\tilde{\omega}_\alpha}\frac{\tilde{\Delta}_{mm}{}^{\alpha}}{\Delta_m} + k^2c^2 = 0, \quad \hat{P}^{\alpha\beta}f_{\alpha\beta} \equiv f_{\beta\alpha}. \tag{20}$$

In a single-component degenerate gas, there can propagate under the considered conditions a low-frequency mode (helicon) with $\omega \ll \Omega$, and in a liquid, when the coefficient A_2 is taken into account, there can propagate also a cyclotron wave with $\omega \sim \Omega$[1,2]. In a two-component gas there is a low-frequency branch (a helicon), and also a hydrodynamic wave. In addition, there exists a wave with circular polarization opposite to the helicon polarization and with a frequency that depends, if the charge components have opposite signs, $e_\alpha e_\beta > 0$, on the degree of its compensation, $\omega \sim \Omega(n_\alpha - n_\beta)(n_\alpha + n_\beta)^{-1}$[15,16] (see also[17]). A similar wave with frequency $\omega \sim \Omega$ exists, as is shown below, also for components with $e_\alpha e_\beta > 0$. The reason why a wave with frequency $\omega \sim \Omega$ can propagate in a two-component medium along an external magnetic field is that its frequency is shifted relative to the cyclotron-frequency components Ω_α and Ω_β, as a result of which there may be no collisionless damping for the degenerate carriers in the long-wave limit, under certain conditions, In a liquid, the indicated wave has certain characteristic features which we shall examine in greater detail.

In a liquid, the dispersion equation (20), neglecting dispersion, takes the following form $(\nu \to 0)$:

$$\begin{gathered}(\hat{r} + k^2c^2)\omega^2 - \omega[F + k^2c^2(\hat{\Omega}_\alpha + \hat{\Omega}_\beta)] + k^2c^2\hat{\Omega}_\alpha\hat{\Omega}_\beta\varkappa = 0;\\ F = (1 + \hat{P}^{\alpha\beta})F_\alpha, \quad F_\alpha = \hat{r}_\alpha(\hat{\Omega}_\beta - C_{\alpha\beta}\hat{\Omega}_\alpha\hat{A}_1{}^{\alpha\beta}), \quad \hat{\Omega}_\alpha = \tilde{\Omega}_\alpha(1 + A_1{}^{\alpha\alpha}),\end{gathered} \tag{21}$$

$$\hat{r} = \hat{r}_\alpha + \hat{r}_\beta, \quad \varkappa = 1 - \hat{A}_1{}^{\alpha\beta}\hat{A}_1{}^{\beta\alpha}, \quad \hat{A}_1{}^{\alpha\beta} = A_1{}^{\alpha\beta}(1 + A_1{}^{\alpha\alpha})^{-1}. \tag{22}$$

The solution becomes particularly simple if the following inequality is satisfied

$$|F| \gg k^2c^2|\Omega|. \tag{23}$$

In this case we obtain for a helicon

$$\omega_{rm} = k^2c^2\hat{\Omega}_\alpha\hat{\Omega}_\beta\varkappa F^{-1} - i\nu_1, \tag{24}$$

where $\nu_1 = (\nu_\alpha p_\alpha v_\alpha^{-1}n_\alpha + \nu_\beta p_\beta v_\beta^{-1}n_\beta)(n_\alpha p_\alpha v_\alpha^{-1} + n_\beta p_\beta v_\beta^{-1})^{-1}$, and there is no Landau damping. The second solution of (21) can be expressed in the form

$$\omega_m = \omega_{0m} + \Delta\omega_m, \quad \omega_{0m} = F\hat{r}^{-1} - i\nu_2, \tag{25}$$

$$\Delta\omega_m = k^2c^2\hat{r}^{-1}[(\hat{\Omega}_\alpha + \hat{\Omega}_\beta) - F\hat{r}^{-1} - \hat{\Omega}_\alpha\hat{\Omega}_\beta\varkappa F^{-1}], \tag{26}$$

where $\nu_2 = (\nu_\alpha p_\alpha v_\alpha^{-1}n_\beta + \nu_\beta p_\beta v_\beta^{-1}n_\alpha)(n_\alpha p_\alpha v_\alpha^{-1} + n_\beta p_\beta v_\beta^{-1})^{-1}$.

As follows from (25), the difference $(|\omega_m - \Omega_\alpha|)$ turns out to be of the order of $n_\alpha(n_\alpha + n_\beta)^{-1}|\Omega_\beta - \Omega_\alpha|$. Therefore, in the case of compounds of comparable concentration with different signs of the charge, when $|\omega_m - \Omega_\alpha| \sim \Omega$, there is actually no Landau damping in the entire long-wave region $kR \ll 1$ if the inequality (23) is satisfied. For carriers with $e_\alpha e_\beta > 0$ in the case of close cyclotron frequencies, the conditions $|\omega_m - \Omega_\alpha| > kv_\alpha$ for the absence of damping are more stringent.

Actually, in metals it is precisely the case (23) which is of interest, for otherwise strong collisionless damping appears. In semimetals and semiconductors, there is a region of applicability also for the general solutions of Eq. (21), limited only by the condition $kR \ll 1$.

For components with carriers of the same sign, and also in the case of "differently-charged" components in the absence of compensation, Eq. (25) is the high-frequency branch with $\omega_0 \sim \Omega$ and $\Delta\omega \ll \omega_0$. The correlation effects under these conditions lead to a frequency shift $\Delta\omega$ proportional to ΩA_1^{ij}, the expression for which becomes particularly simple in the linear approximation in A_1:

$$\begin{gathered}\omega_{0m} = \omega_0 + \delta\omega - i\nu_2, \quad \omega_0 = \omega_{0m}(A_1 = 0) = \pm(Z_\alpha\Omega_\beta + Z_\beta\Omega_\alpha),\\ \delta\omega = \pm(1 + P^{\hat{\alpha}\beta})Z_\alpha[A_1{}^{\beta\beta} - (v_\beta/v_\alpha)A_1{}^{\alpha\beta}] \pm\\ \pm Z_\alpha A_1{}^{\alpha\beta}(v_\beta/v_\alpha)(\Omega_\beta - e_\beta e_\alpha{}^{-1}\Omega_\alpha)(1 - e_\alpha e_\beta{}^{-1}); \quad Z_\alpha = Z_\alpha v_\alpha{}^2(Z_\alpha v_\alpha{}^2 + Z_\beta v_\beta{}^2)^{-1}.\end{gathered} \tag{27}$$

Under the same conditions we have for helicons

$$\begin{gathered}\omega_{0h}^{-1}\delta\omega = -A_1{}^{\alpha\beta}Z_\alpha v_\alpha v_\beta cB_0{}^{-1}(\Omega_\beta - e_\alpha{}^{-1}e_\beta\Omega_\alpha)(1 - e_\alpha e_\beta{}^{-1})\\ \times(Z_\alpha v_\alpha p_\alpha e_\alpha + Z_\beta v_\beta p_\beta e_\beta)^{-1},\\ \omega_{0h} = \omega_h(A_1 = 0).\end{gathered} \tag{28}$$

Under conditions close to compensation, $Z_\alpha v_\alpha p_\alpha e_\alpha + Z_\beta v_\beta p_\beta e_\beta \approx 0$, the high-frequency branch in the limit of small k and in the absence of correlation effects vanishes. In a liquid, condition (23) can still be satisfied, so that the threshold frequency ω_{0m} differs from zero and takes the following form $(A_1^2 \ll 1)$:

$$\omega_{0m} = \pm A_1{}^{\alpha\beta} Z_\alpha v_\beta v_\alpha{}^{-1} (\Omega_\beta - e_\beta e_\alpha{}^{-1} \Omega_\alpha)(1 - e_\alpha e_\beta{}^{-1}) - i\nu_2. \tag{29}$$

The existence of a wave (29) is due entirely to the correlation effects. The frequency ω_{0m} is smaller than Ω, but can still be large in comparison with ω_h and proportional to the liquid coefficients, $\omega_{0m} \sim \Omega A_1^{\alpha\beta}$. The coefficients A_1 could be estimated by determining from experiment the frequencies ω_{0m} and the polarization of the wave under compensation conditions, and also by determining the threshold for the appearance (vanishing of the high frequency mode $F \approx 0$ as $k \to 0$, if it is possible to vary the carrier density. One of the examples of a compensated sample may be an even metal.

The liquid interaction can also lead to another effect, namely the change of the polarization of the waves (for example, from left-hand polarization in the absence of correlation to right-hand polarization), this being connected with the possible reversal of the sign of the function F in (24) and (25). Neglecting the coefficients A_1, the function F is proportional to the difference $n_\alpha - n_\beta$, and its sign (and the polarization of the wave) is determined completely by the sign of this difference, which can be obtained for example, from experiments on the Hall effect. Allowance for correlation effects can lead to a reversal of the sign of the function F, which is easiest to realize in principle under conditions close to compensation. In this case, in the approximation linear in A_1, we obtain the following system of inequalities corresponding to the reversal of the sign of F as a result of the correlation effects:

$$\begin{gathered} |Z_\alpha v_\alpha p_\alpha - Z_\beta v_\beta p_\beta| < 2Z_\alpha |A_1{}^{\alpha\beta}(p_\alpha v_\beta - p_\beta v_\alpha)|, \\ A_1{}^{\alpha\beta}(Z_\alpha v_\alpha p_\alpha - Z_\beta v_\beta p_\beta)(p_\beta v_\alpha - p_\alpha v_\beta) > 0. \end{gathered} \tag{30}$$

The indicated effect, according to (30), exists at any sign of the coefficient A_1 and under sufficiently relaxed restrictions on its magnitude, which is particularly clearly seen in the case of a quadratic dispersion law

$$A_1{}^{\alpha\beta}\Delta v \Delta n > 0, \quad 2|A_1{}^{\alpha\beta}\Delta v| n_\alpha > v_\alpha |\Delta n|; \quad \Delta n = n_\alpha - n_\beta, \quad \Delta v = v_\alpha - v_\beta.$$

(The conditions for the change of the polarization of the helicon have the same form, since κ is positive by virtue of the stability conditions.)

Thus, by observing experimentally, in the considered frequency region, the propagation of a wave whose polarization does not correspond to the sign of the difference $n_\alpha - n_\beta$, we can detect an appreciable influence of liquid effects. Then, by determining the cyclotron frequencies and the Fermi velocity, for example from cyclotron-resonance experiments, experiments on the de Haas-van Alphen effect, etc. (see, for example,[18]), we can obtain the signs of the coefficients A_1 and estimate their values.

The dispersion correction $\Delta\omega \sim k^2c^2r^{-1}\Omega$, determined by formula (26), takes place for semimetals and semiconductors. At metal densities, a more important role is played by allowance for spatial dispersion, which leads under at $kR \ll 1$ to the following results:

$$\begin{gathered} \Delta\omega_m = -(1 + \hat{P}^{\alpha\beta})\hat{r}_\alpha (5\hat{r})^{-1}[(\omega_{0m} - \tilde{\Omega}_\beta) s_{\alpha m}{}^{-2} - \omega_{0m} s_{\beta m}{}^{-2} A_1{}^{\beta\beta} \\ + C_{\alpha\beta} A_1{}^{\alpha\beta} (\Omega_\alpha s_{\beta m}{}^{-2} + \omega_{0m} s_{\alpha m}{}^{-2})]. \end{gathered} \tag{31}$$

We note, finally, that a solution of the type (25) exists when the inequality (23) is satisfied; in the opposite limit, a hydrodynamic wave can propagate with

$$\omega = \pm kc(-\hat{\Omega}_\alpha \hat{\Omega}_\beta \varkappa \hat{r}^{-1})^{1/2}. \tag{32}$$

We turn to the case of propagation of a wave perpendicular to the magnetic field. The dispersion equation for the ordinary wave ($\mu = 0$, odd mode) takes the form $\sigma_{00} = 0$, i.e.,

$$(1 + \hat{P}^{\alpha\beta})\hat{r}_\alpha \bar{\Delta}_{00}{}^\alpha = 0. \tag{33}$$

In the long-wave limit, accurate to terms (kR) and under the conditions $|\Omega_\alpha^2 - \Omega_\beta^2| \gg k^2v^2$, $A_1^2 \ll 1$, we obtain the solution

$$\begin{gathered} \omega_1{}^2 = \Omega_\alpha{}^2 - {}^1/_5 (kv_\alpha)^2 \psi_\alpha, \quad \omega_2{}^2 = \hat{P}^{\alpha\beta}\omega_1{}^2, \\ \psi_\alpha = Z_\alpha[1 + (v_\beta / v_\alpha) A_1{}^{\alpha\beta}(e_\beta e_\alpha{}^{-1} - 1)] - \\ -Z_\alpha{}^2[A_1{}^{\alpha\alpha} + 2(v_\beta / v_\alpha) A_1{}^{\alpha\beta}] - Z_\beta(A_1{}^{\alpha\alpha} + A_1{}^{\beta\beta}). \end{gathered} \tag{34}$$

In the case of close frequencies, ψ_1 is replaced by $(\psi_\alpha + \psi_\beta)/2$. The correlation effects are manifest by the small dispersion terms $\sim k^2$ and can lead to a reversal of the sign of the function ψ.

The dispersion equation for the extraordinary wave ($m = \pm 1$, even mode)

$$\omega(1 + \hat{P}^{\alpha\beta}) \frac{\hat{r}_\alpha}{\tilde{\omega}_\alpha} \frac{\bar{\Delta}_{mm}{}^\alpha}{\Delta_m} + k^2c^2 = 0 \tag{35}$$

neglecting the dispersion, $kR \to 0$, can be written as follows ($\nu \to 0$):

$$\hat{r}(\hat{r} + k^2c^2)\omega^2 - F^2 - k^2c^2[F(\hat{\Omega}_\alpha + \hat{\Omega}_\beta) - \hat{r}\varkappa\hat{\Omega}_\alpha\hat{\Omega}_\beta] = 0. \tag{36}$$

In the region of cyclotron frequencies $\omega \lesssim \Omega$, there is a solution ω_{0m} which coincides in form with (25). An additional analysis shows that in a liquid, as in a gas[15-17], there is actually one wave with frequency ω_{0m}, which does not depend on the propagation direction. The dependence on the angle between **k** and $\mathbf{B}_0$ becomes manifest only in the dispersion law (and in the absence of collisionless damping at $\mathbf{k} \perp \mathbf{B}_0$). When $\mathbf{k} \parallel \mathbf{B}_0$, the wave is circularly polarized; if the inequality (23) is satisfied, the circular polarization is approximately preserved also at $\mathbf{k} \perp \mathbf{B}_0$.

Just as in the case of longitudinal propagation, the influence of liquid effects comes into play most distinctly when carriers of opposite sign cancel each other. In the absence of correlation effects we have $\omega \ll \Omega$ under these conditions, and magnetic sound propagates in the medium. In a liquid we have in this case, too, a high-frequency branch whose frequency is determined by (29).

Finally, in the low-frequency region $\omega \ll \Omega$, when the condition $|F| \ll k^2c^2|\Omega|$ is satisfied, magnetic sound can propagate in the liquid. Its spectrum, neglecting dispersion, is determined by the same formula (32) as the spectrum of magnetohydrodynamic waves propagating along an external field $\mathbf{B}_0$. The dissipation is due to collisions. The liquid interaction renormalizes the Alfven velocity and leads to a frequency shift $\delta\omega$, given in the linear approximation in A_1 by

$$\omega_0{}^{-1}\delta\omega = {}^1/_2 \left(Z_\alpha A_1{}^{\beta\beta} + Z_\beta A_1{}^{\alpha\alpha} - 2Z_\alpha A_1{}^{\alpha\beta}\frac{v_\beta}{v_\alpha}\right). \tag{37}$$

Here ω_0 is determined by formula (32) in the limit as $A_1 \to 0$.

An analysis of the terms with dispersion $\sim(kR)^2$ for cyclotron waves under the considered polarization is of interest, since it makes it possible to draw conclusions concerning the sign and magnitude of the zero-order coefficients, which can play an appreciable role in this case.

In a liquid, neglecting the coefficients A_1, we have

$$\Delta\omega_m = -(1+\hat{P}^{\alpha\beta})Z_\alpha\left[\frac{2(kv_\alpha)^2(\omega_{0m}-m\Omega_\beta)}{5\omega_{0m}(\omega_{0m}-2m\Omega_\alpha)}-\frac{(kv_\beta)^2}{6\omega_{0m}}\lambda_0^{\alpha\beta}\right]. \quad (38)$$

The quantity $\lambda_0^{\alpha\beta}$ is determined by the relation $\lambda_0^{\alpha\beta} = A_0^{\beta\beta} - e_\beta e_\alpha^{-1} A_0^{\alpha\beta}$.

A simple result that reveals the role of correlation effects is obtained in the case of carriers of like charge with close cyclotron frequencies (for example, two types of electrons):

$$\omega = \pm\Omega \pm (6\Omega)^{-1}(1+\hat{P}^{\alpha\beta})Z_\alpha(kv_\beta)^2\lambda_0^{\alpha\beta} - i\nu_2, \quad (39)$$

and also near the cyclotron frequency of one of the components at a large density of states of the other, $Z_\beta v_\beta^2 \gg Z_\alpha v_\alpha^2$, $\omega \sim \Omega_\alpha$:

$$\omega = \pm\Omega_\alpha \pm Z_\alpha v_\alpha^2 (Z_\beta v_\beta^2)^{-1}(\Omega_\beta - \Omega_\alpha) \pm (6\Omega_\alpha)^{-1}(kv_\alpha)^2\lambda_0^{\beta\alpha} - i\nu_2. \quad (40)$$

Thus, as follows from the foregoing results, an investigation of the electromagnetic properties (particularly the oscillation spectrum) of a multicomponent Fermi liquid offers definite possibilities for the determination of the zeroth and first coefficients of the liquid interaction. The correlation effects lead to a shift of the lines, as shown for example by formulas (27), (28), and (37), to a change in the dispersion law and in the propagation velocity (see formulas (26), (31), and (37)–(40), and also to a change in the polarization of the wave if the conditions (30) are satisfied. Interesting results are obtained in the case of equal densities of carriers of opposite signs, for example the theory indicates that a wave due to the liquid interaction can become excited (see (29)).

Just as in a one-component liquid[1,2], all these effects can be observed experimentally only if the dissipation is low enough. If the dissipation, as above, is due to collisions, then the necessary condition can be written in the form $|\Omega A_l| \gg \nu$. At $A \sim 10^{-1}$ and $\Omega \sim 10^{11}$–10^{12} sec^{-1}, we obtain for the collision frequency the quantity $\nu \ll 10^{10} - 10^{11}$ sec^{-1}. The frequencies $\nu \sim 10^9$ sec^{-1} can be obtained in sufficiently pure metals and semiconductors at low temperatures. Therefore, for example, an analysis of the experimental data on cyclotron waves in a two-component liquid under favorable conditions would make it possible to reconstruct the zeroth and first coefficients of the expansion of the Landau function. The determination of the zeroth coefficient A_0^{ij} together with the two coefficients of expansion of the spin part of the correlation functions B_0^{ij}[19] would make it possible to reconstruct, with the aid of relations that follow from the sum rule[20], all six coefficients A_0^{ij}, B_0^{ij}, i.e., that part of the Landau function which depends only on the Fermi energy. One should bear in mind here the remarks made above concerning the anisotropy effects and the magnitude of the dissipation.

In conclusion, the authors thank V. P. Silin for interest in the work and for useful remaks.

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Translated by J. G. Adashko
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