

# Interference of nuclear and electron scatterings of 14.4 keV resonance $\gamma$ -ray in Bragg reflection from an $\alpha$ -Fe<sub>2</sub>O<sub>3</sub> crystal

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(Submitted August 17, 1972)

Zh. Eksp. Teor. Fiz. **64**, 261-272 (January 1973)

The energy dependence of the intensity of Bragg reflection of 14.4-keV Fe<sup>57</sup> resonance  $\gamma$ -rays from the (111) planes of an  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub> single crystal is investigated with a Mossbauer diffractometer for all even orders (from the second to tenth) of reflection and various directions of the external magnetic field. Polarization dependences of interference on scattering angle,  $\Delta m$  of the nuclear transition, and direction of the magnetic field at the iron nuclei are obtained for the first time. The measurements results are in agreement with calculations based on the dynamic theory of interaction of resonance  $\gamma$ -rays with perfect crystals. The shapes of the interference curves are related to the structure of the crystal and direction of internal fields on the Fe nuclei.

## 1. INTRODUCTION

Elastic resonance scattering of Mossbauer  $\gamma$  quanta by nuclei in a crystal is accompanied by elastic Rayleigh scattering by the electron shells of the atoms. These processes interfere, since the initial and final states of the crystal for them are identical. The interference should become manifest in the form of an asymmetry of the resonant dependence of the intensity  $I(E)$  of the scattered radiation on the energy of the incident  $\gamma$  rays. Such an asymmetry was first observed in the cycle of researches of the Birmingham group in 1960–1964<sup>[1-4]</sup> and became the first experimental proof of the fundamental fact that nuclear resonant scattering of  $\gamma$  rays is coherent. These investigations, performed with 14.4 keV  $\gamma$  radiation of Fe<sup>57</sup> and with single-crystal and polycrystalline iron scatterers, played an important role in the understanding of the nature of the phenomenon, but the results were semiquantitative in nature. A detailed investigation of interference in Bragg scattering of 23.9-keV  $\gamma$  rays of Sn<sup>119</sup> by tin single crystals was carried out by Voitovetskiĭ and co-workers<sup>[5,6]</sup>. Quantitative agreement with the theory of interaction of  $\gamma$  rays with mosaic crystals was obtained.

This topic is of interest also because, as noted in a number of papers, a study of interference scattering may be useful in the determination of both the crystal-lattice structure<sup>[7-9]</sup> and of the internal fields at the nuclei in the crystals<sup>[10-12]</sup>. Recently, Parak and Mossbauer attempted to determine by this method the phase shifts of the reflections of the known structure of K<sub>3</sub>Fe(CN)<sub>6</sub><sup>[13]</sup>.

In all the earlier studies, the investigated  $I(E)$  spectrum consisted of a single line, for either the source and the scatterer had unsplit Mossbauer lines<sup>[5,6,13]</sup>, or else the hyperfine splitting in the source and in the scatterer were equal and only a small region of energies near the zero-point velocity was investigated. It turns out, however, that under certain conditions the  $I(E)$  interference dependences can have a much more complicated structure. Examples are the spectra observed by us in the interference scattering of  $\gamma$  rays of Fe<sup>57</sup> from the (666) planes of single-crystal hematite<sup>[14]</sup>. To investigate the phenomenon in question more fully, it is necessary to create conditions for separation, in pure form, of the contribution made to the scattered radiation by differently polarized components of the incident beam.

This was first realized in the present study with a single-line Mossbauer sources and magnetically-ordered scatterer having a hyperfine structure. Examples are the spectra observed by us in the interference scattering of  $\gamma$  rays of Fe<sup>57</sup> from the (666) planes of single-crystal hematite<sup>[14]</sup>. To investigate the phenomenon in question more fully, it is necessary to create conditions for separation, in pure form, of the contribution made to the scattered radiation by differently polarized components of the incident beam. This was first realized in the present study with a single-line Mossbauer source and a magnetically-ordered scatterer having a hyperfine structure. The use of Bragg reflections from a single-crystal scatterer has made it possible to reduce the incoherent background considerably and, what is particularly important, to relate uniquely the obtained interference picture with the structure of the crystal<sup>[1]</sup>.

In the present paper we report the results of a systematic study of the interference of resonant nuclear and electronic Rayleigh scattering for the case of Bragg reflection of 14.4 keV  $\gamma$  rays of Fe<sup>57</sup> from a hematite single crystal. We obtained the polarization dependences of the interference on the scattering angle,  $\Delta m$  of the nuclear transition, and the direction of the magnetic field at the iron nuclei. The experimental spectra are compared with curve calculated by the theory of dynamic interaction of  $\gamma$  rays with ideal crystals<sup>[16]</sup>. The main features of the spectra, namely the type of the curve and the sign of the asymmetry, are related to the asymptotic expression for the reflection coefficient and to a parameter determined by the scattering and absorbing abilities of the crystal.

## 2. EXPERIMENTAL PROCEDURE AND MEASUREMENT RESULTS

The measurements were performed at room temperature with a Mossbauer diffractometer<sup>[17,18]</sup>. The experimental setup is shown in Fig. 1. The  $\gamma$  rays from the single-line Mossbauer source Co<sup>57</sup> in Cr (200  $\mu$ Ci, 2  $\times$  5 mm) were incident on single-crystal  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub> (85% Fe<sup>57</sup>, 4  $\times$  7  $\times$  0.3 mm), placed in one of the positions of the symmetrical Bragg scattering (2n 2n 2n). The  $\gamma$ -ray divergence varied with the scattering angle, from  $\sim 12'$  for the (222) reflection to  $\sim 35'$  for (10 10 10). The source moved with constant acceleration, the detector was an FEU-35A photomultiplier with NaI(Tl) crystal

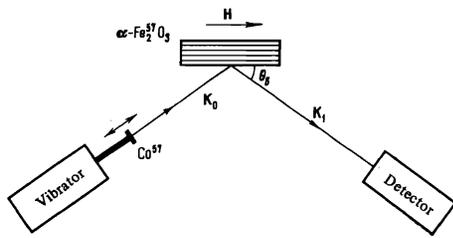


FIG. 1. Experimental setup.  $K_0$  and  $K_1$ —wave vectors of the incident and scattered radiation,  $H$ —direction of external magnetic field.

measuring  $10 \times 5 \times 0.1$  mm, and the information was accumulated in a multichannel analyzer.

The  $\alpha$ - $\text{Fe}_2\text{O}_3$  single crystal enriched with the isotope  $\text{Fe}^{57}$  was grown from the solution in a flux of molten  $\text{Bi}_2\text{O}_3 + \text{Na}_2\text{CO}_3$ <sup>[19]</sup>; the crystal surface was parallel to the (111) planes. In the measurements, the crystal was in a magnetic field of  $\sim 1$  kOe, produced by a permanent magnet. This field was always in the plane of the crystal and was directed in the different measurement runs either parallel or perpendicular to the scattering plane. Because of the weak ferromagnetism of the hematite, the magnetic sublattices were also oriented and the internal magnetic fields at the iron nuclei were respectively perpendicular or parallel to the scattering plane. The perfection of the crystal was investigated beforehand with the aid of  $\text{MoK}_{\alpha_1}$  radiation in a two-crystal geometry. The first crystal was quartz, reflection (20 $\bar{2}$ 2). The widths of the rocking curves of the reflection (222), with correction for the dispersion and divergence of the incident beam for different sections of the  $\alpha$ - $\text{Fe}_2\text{O}_3$  crystal, fluctuated between 30 and 40 seconds of angle.

To measure the angular dependences when working with the  $\gamma$  beam, the source was moved at constant velocity.

Figure 2 shows the results of the measurements of the dependence of the integrated intensity of reflection of 14.4 keV  $\gamma$  quanta on the source velocity for all existing orders of Bragg reflection<sup>2)</sup> at different directions of the magnetic field at the Fe nuclei. The time necessary to measure the curves fluctuated from  $\sim 20$  h for (666) to  $\sim 600$  h for (444). The solid lines show the results of calculations based on the dynamic theory of scattering of resonant  $\gamma$  rays by ideal crystals<sup>[16]</sup>. The abscissas of these curves are equal to  $R(v)x + y$ , where  $R(v)$  is the calculated reflection coefficient,  $x$  is a scale factor, and  $y$  is a quantity that takes into account the Bragg-scattered nonresonant 14.4 keV  $\gamma$  quanta and the background consisting of incoherently scattered  $\gamma$  quanta of other energies and the detectors own background. The quantities  $x$  and  $y$  were chosen from considerations of the best agreement between the gently-sloping sections of the theoretical curves and the experimental points. The background of most spectra was small, and its largest contribution is to the spectra of the weak reflections (444), and amounts to  $\sim 20\%$  of  $N_{\infty}$ .

The obtained spectra contain detailed information on the polarization dependences of the nuclear resonant and electron Rayleigh scattering and their interference. In addition, they reveal clearly the dependence on the structure of the crystal lattice and on the internal magnetic field at the iron nuclei.

### 3. THEORY

The dynamic theory of Bragg scattering of resonant  $\gamma$  quanta by ideal crystals describes well all the details

of the experimental spectra. We present below the initial premises of the quantitative calculations.

A thermodynamic theory of Bragg scattering of resonant  $\gamma$  quanta in the absence of hyperfine splitting of the nuclear levels was developed in<sup>[16]</sup>. In particular, this paper cites an expression for the integrated reflection coefficient of the polarized radiation. If the scatterer has hyperfine splitting of the nuclei levels, then in the general case, when  $\gamma$  quanta of one linear polarization are incident on the crystal, quanta of another polarization can appear in the reflected beam. Then the system of dynamic equations describing the electromagnetic field in the crystal<sup>[20]</sup> does not break up into two independent subsystems, and no analytic expression can be obtained for the reflection coefficient. In our case of Bragg reflection of even orders from a single-crystal collinear antiferromagnet  $\alpha$ - $\text{Fe}_2\text{O}_3$ ,<sup>3)</sup> however, when the internal magnetic fields at the resonant nuclei are parallel or perpendicular to the scattering plane, the amplitude for the scattering of the  $\gamma$  quanta by the unit cell of the crystal with change of polarization vanishes identically. We can therefore use the expression obtained in<sup>[16]</sup> for the coefficient  $R$  of reflection of resonant  $\gamma$  quanta by a thick crystal<sup>4)</sup>

$$R = \frac{8|g_{10}|}{3 \sin 2\theta_b} \left\{ (1 - s^2 - 2q^2s^2) \frac{E(k)}{k} - \frac{3\pi}{4} s(1 - 2q^2s^2) + ks^2(1 - q^2) [3(1 - q^2s^2)\Pi(-q^2, k) - (2 - s^2 - 2q^2s^2)K(k)] \right\}, \quad (1)$$

here  $E(k)$ ,  $K(k)$ , and  $\Pi(-q^2, k)$  are complete elliptic integrals of first, second, and third kind,

$$k = [1 + s^2(1 - q^2)]^{-1/2},$$

$$s = \text{Im } g_{00}/|g_{10}|, \quad q = \text{Im } g_{10}/\text{Im } g_{00}. \quad (2)$$

Expressions (1) and (2) were written out for symmetrical reflection for allowance for the fact that in our case  $g_{01} = g_{10}$  and  $g_{11} = g_{00}$ . The quantity  $R$  is the reflection coefficient for each of the two  $\pi$  and  $\sigma$  linearly polarized components of the incident beam. The principal dynamic coefficients  $g_{\alpha\beta}$  of our problem are given by

$$g_{\alpha\beta} = \frac{\lambda^2 r_0}{\pi v_0} \left\{ -F_{\mathbf{k}_\beta - \mathbf{k}_\alpha} P_e(\alpha, \beta) - g_0 \sum_{M, M_0} \frac{1}{(E - E_{M, M_0})/\Gamma + i} \sum_j \exp(i(\mathbf{k}_\beta - \mathbf{k}_\alpha)_j \rho_j) P_{n_j M, M_0}(\alpha, \beta) \right\}. \quad (3)$$

The first and second terms in the curly brackets in (3) are respectively the coherent parts of the scattering amplitude of the  $\gamma$  quanta by the electron shells of the atom and by the resonant nuclei in the unit cell of the crystal. Here

$$F_{\mathbf{k}_\beta - \mathbf{k}_\alpha} = \sum_j \exp(i(\mathbf{k}_\beta - \mathbf{k}_\alpha)_j \rho_j) F_j(\mathbf{k}_\beta - \mathbf{k}_\alpha) \quad (4)$$

is the electronic structure factor of the unit cell. In expression (4), the summation over  $j$  is carried out over the entire unit cell, while in expression (3) the summation is only over the resonantly-scattering part.  $P_e(\alpha, \beta)$  is the polarization factor of the electron Rayleigh scattering ( $P_e^\pi = \cos(\mathbf{k}_\beta, \mathbf{k}_\alpha)$ ,  $P_e^\sigma = 1$ ).

$$g_0 = \frac{\lambda f_m \eta}{2\pi r_0(1 + \alpha)} \frac{2I + 1}{2(2I_0 + 1)},$$

and  $P_{n_j}(\alpha, \beta)$  is the nuclear-scattering polarization factor, which takes the following form for a magnetic dipole transition

$$P_{n_j}(\alpha, \beta) = 3 \left( \begin{array}{ccc} I & 1 & I_0 \\ -M & M - M_0 & M_0 \end{array} \right)^2 \left\{ \frac{1}{2} (\mathbf{h}_\alpha \mathbf{h}_\beta) + \left( -\frac{1}{2} \right)^{|M - M_0|} (\mathbf{h}_\alpha \mathbf{n}_j) (\mathbf{h}_\beta \mathbf{n}_j) \mp \frac{i}{2} [\mathbf{h}_\alpha \times \mathbf{h}_\beta] \mathbf{n}_j \right\}. \quad (5)$$

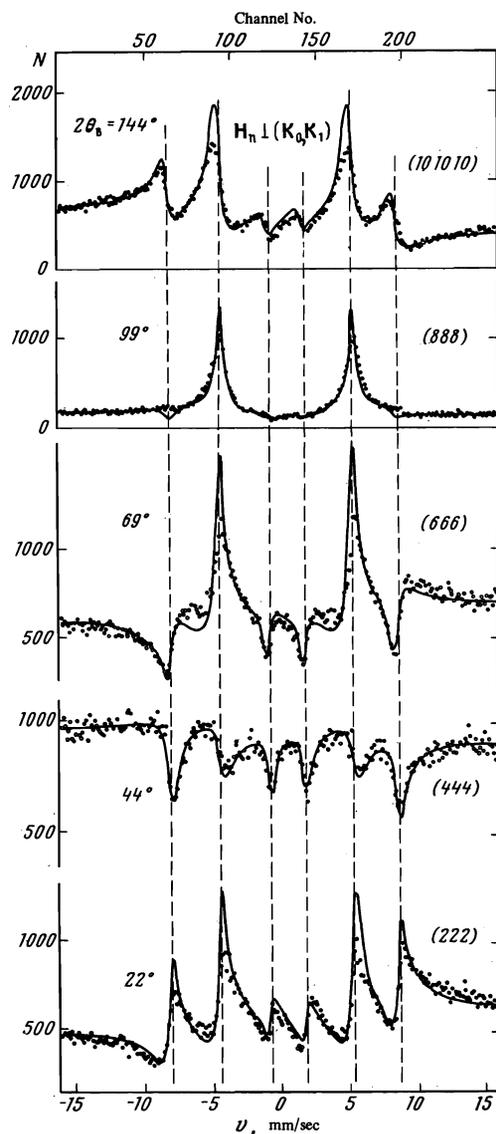
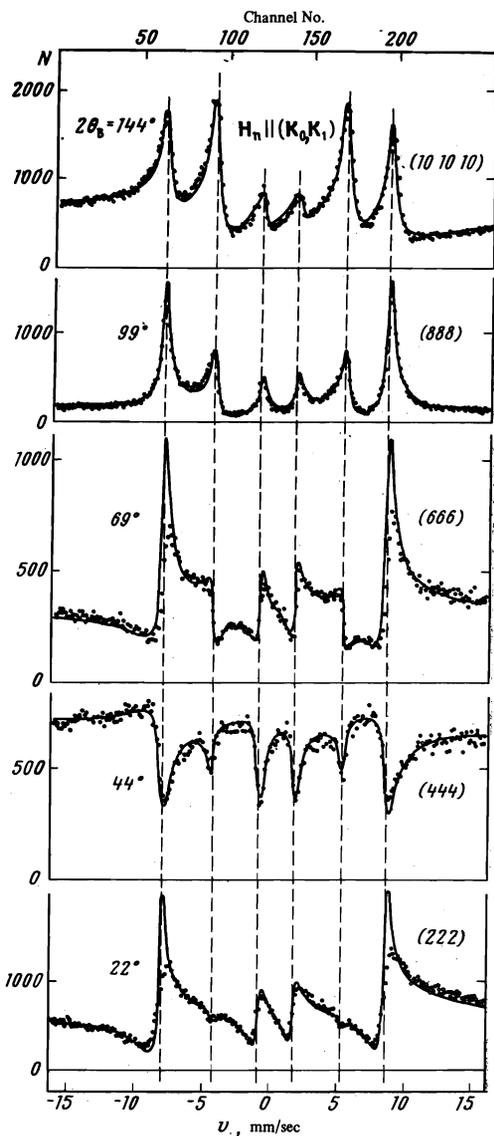


FIG. 2. Measured dependence of the reflection intensity on the velocity of the source ( $\text{Co}^{57}$  in Cr) for even orders of symmetrical Bragg reflections ( $2n\ 2n\ 2n$ ) of 14.4-keV  $\gamma$ -rays from single-crystal  $\alpha\text{-Fe}_2\text{O}_3$ . Vertical lines—positions of the nuclear resonances measured in the experiment by transmission.  $H_n$ —magnetic field at the iron nuclei,  $N$ —total number of counts in the analyzer channel. The solid curve was calculated from the dynamic theory of scattering of resonant  $\gamma$  quanta by ideal crystals.

here  $h_0$  and  $h_1$  are the polarization vectors of the magnetic field in the incident and diffracted waves, and  $n_1$  is the unit vector in the direction of the magnetic field at the nucleus. The remaining notation in formulas (3), (4), and (5) coincides with that used in [16].

In the computer numerical calculations of the reflection coefficients, we summed over the polarizations and integrated over the energy distribution of the incident radiation. The values of the parameters used in the calculations are listed in Table I.

#### 4. DISCUSSION OF RESULTS

It is seen from Fig. 2 that the lines in the spectra, in spite of the great variety in their shapes, can be arbitrarily broken up into three groups: asymmetrical peaks (this shape predominates in high orders of reflection), asymmetrical dips (fourth order), and curves predominantly of the dispersion type. For convenience in the description of the experimental spectra, we number the spectral lines from left to right, and assign to the dispersion curve a positive asymmetry if its low-velocity (energy) wing (energies) lies lower than the other wing<sup>5)</sup>. In the opposite case the curve has negative asymmetry.

It is convenient to analyze the singularities of the

individual lines with the aid of theoretical curves corresponding to individual polarizations of the incident beam (Fig. 3). It is shown in [21] that if the absorption is small in comparison with the extinction, then the form of the expression for the reflection coefficient  $R$  is determined mainly by the factor  $|g_{10}|$  (see formula (1)). This condition is satisfied in our case in velocity-dependence regions that are not too close to resonance, and therefore the main features of the observed spectra, namely the type of the curve and the sign of the asymmetry, can be easily understood on the basis of the expression for  $|g_{10}|$ . We express this quantity in the form

$$g_{10} = -\frac{\lambda^2 r_0}{\pi v_0} \left[ A_{10} + \sum_{n=1}^6 B_{10}^{(n)} \left( 2 \frac{E - E_n}{\Gamma} + i \right)^{-1} \right], \quad (6)$$

where (apart from a constant factor)  $A_0$  is the amplitude of the coherent scattering of the  $\gamma$  quanta by the electrons of all the atoms in the unit cell, and  $B_{10}^{(n)}$  is the

TABLE I. Values of the parameters used in the calculation of the reflection coefficient of single-crystal  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub> for 14.4-keV  $\gamma$ -rays

Order of reflection	$F_{Fe}$	$F_{DO}^{1/2}$	$F_O$	$f_{DO}^{1/2}$
2	19.2	0.975	5.0	0.91
4	12.6	0.905	2.6	0.71
6	9.3	0.80	1.7	0.44
8	7.1	0.66	1.4	0.23
10	6.2	0.52	1.2	0.10

Note.  $F_{Fe}$  and  $F_O$  are the atomic factors for Fe and O,  $F_{DO}^{1/2}$  and  $f_{DO}^{1/2}$  are the corresponding Debye-Waller factors. The parameters of the unit cell of  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub> are  $a = 5.42 \text{ \AA}$ ,  $u = 0.355$ ,  $x = 0.552$ ,  $\gamma = 55^\circ 17'$ . The Mossbauer factor for iron is  $f_M = 0.7$ ; the conversion coefficient is  $\alpha = 9.0$ ; the source line width is  $\Gamma_s = 2.4 \text{ \Gamma}_{nat}$ . The scatterer line width was assumed to be equal to  $\Gamma_{nat}$ . The cross sections for photoelectric absorption of 14.4-keV  $\gamma$  radiation by Fe and O are  $\sigma_{Fe} = 600 \text{ b}$  and  $\sigma_O = 40 \text{ b}$ .

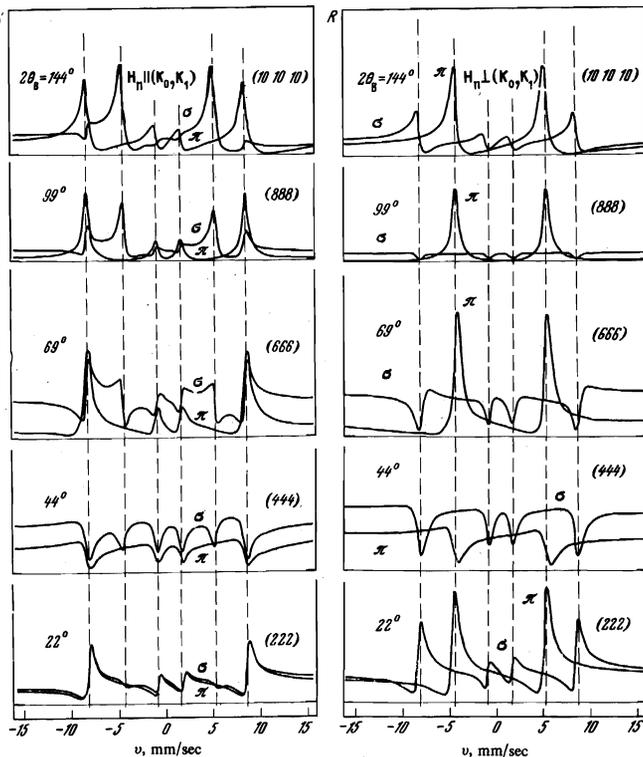


FIG. 3. Theoretical plots of the energy (velocity) dependence of the reflection coefficient of polarized resonant 14.4-keV  $\gamma$ -rays of Fe<sup>57</sup> from a single-crystal  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub>.

amplitude of the coherent scattering of the quanta by resonant nuclei of the unit cell for the  $n$ -th nuclear resonance. The calculated value of  $A_{10}$  and  $B_{10}^{(n)}$  for the investigated orders of reflection and for both polarizations of the incident radiation are given in Table II.

The presence in the resonance of a definite type of line (peak, dip, or dispersion shape) can be related quite clearly with the reflecting and absorbing abilities of the crystal. We introduce the parameters

$$C^{(n)} = \left( \frac{B_{10}^{(n)}}{A_{10}} \right)^2 \frac{A_{00}}{B_{00}^{(n)}}, \quad (7)$$

which contains the ratios of the amplitudes for scattering

and absorption of  $\gamma$  quanta by nuclei and electrons. It is easy to verify that the value  $C^{(n)} \sim 1$  corresponds to a curve with dispersion shape. At  $C^{(n)} \gg 1$  we have a peak, and at  $C^{(n)} \ll 1$  we have a dip. The values of  $C^{(n)}$  for all the components of the spectra observed by us are given in Table III.

The most interesting feature of the obtained spectra is the presence of dispersion curves with different signs of the asymmetry. The asymmetry sign can change when the order of the reflection is changed, when the magnetic field is rotated, and when  $\Delta m$  of the nuclear transition is changed. It can be shown that the sign of the asymmetry of the interference curve is determined by the ratio of the signs of  $A_{10}$  and  $B_{10}^{(n)}$ . According to our definition, the dispersion curves with positive asymmetry are those in which the signs of  $A_{10}$  and  $B_{10}^{(n)}$  are equal.

Let us consider first the orientation  $H_n \perp (K_0, K_1)$ . This case is remarkable because only one polarization of the incident beam "works" for each separately-taken nuclear transition ( $\gamma$  quanta of another polarization do not interact with the nuclei because of the selection rules, namely, the corresponding polarization factors  $P_{n_j}(\alpha, \beta)$  vanish). Thus, the second and fifth lines in all the spectra (nuclear transitions with  $\Delta m = 0$ ) are due to diffraction of  $\pi$ -polarized radiation, and the remaining four lines ( $\Delta m = \pm 1$ ) are due to  $\sigma$  polarization. It is seen from Table II that for the (222) reflection the signs of  $A_{10}$  and of all the  $B_{10}^{(n)}$  for  $\pi$  and  $\sigma$  polarization are the same, and for the (10 10 10) reflection they are opposite. Therefore the asymmetry signs of all the lines are equal within each of these spectra, but in the (222) spectrum the sign is positive and in (10 10 10) it is negative. This is connected principally with the reversal of the signs of the electron polarization factor  $P_e(\alpha, \beta)$  for the  $\pi$  polarization and of the nuclear polarization factor  $P_{n_j}(\alpha, \beta)$  for the  $\sigma$  polarization on going from (222) to (10 10 10)<sup>7</sup>. For the (444) reflection, the asymmetry of all the lines also reverses sign in comparison with (222), owing to

TABLE II. Values of the coefficients  $A_{10}$  and  $B_{10}^{(n)}$  in formula (6), which determine the amplitudes of the  $\gamma$ -quantum scattering

(HKL)	Polarization	$A_{10}$	$H_n \parallel (K_0, K_1)$			$H_n \perp (K_0, K_1)$		
			$B_{10}^{(1)} = B_{10}^{(6)}$	$B_{10}^{(2)} = B_{10}^{(6)}$	$B_{10}^{(3)} = B_{10}^{(4)}$	$B_{10}^{(1)} = B_{10}^{(6)}$	$B_{10}^{(2)} = B_{10}^{(6)}$	$B_{10}^{(3)} = B_{10}^{(4)}$
(222)	$\sigma$	+23.2-3.3i	+286.1	-[3.9	+95.4	+275.7	0	+91.9
	$\pi$	+21.6-3.1i	+296.5	0	+98.8	0	+395.4	0
(444)	$\sigma$	+9.7+0.3i	-23.7	+5.1	-7.8	-19.5	0	-6.5
	$\pi$	+6.9+0.2i	-27.2	0	-9.1	0	-36.3	0
(666)	$\sigma$	-31.7+3.4i	-228.0	141.0	-76.0	-122.3	0	-40.8
	$\pi$	-11.6+1.2i	-333.8	0	-111.3	0	-445.0	0
(888)	$\sigma$	-18.5+3.8i	-187.8	+322.7	-62.6	+54.3	0	+18.1
	$\pi$	+2.3-0.5i	-429.8	0	-143.3	0	-573.0	0
(10 10 10)	$\sigma$	-12.2+2.0i	-30.6	+298.7	-10.2	+193.5	0	+64.5
	$\pi$	+9.3-1.6i	-254.6	0	-84.9	0	-339.5	0

TABLE III. Values of the parameter  $C^n$  in formula (7), which determines the shape of the interference curve.

(HKL)	Polarization	$H_n \parallel (K_0, K_1)$			$H_n \perp (K_0, K_1)$		
		$n = 1; 6$	$n = 2; 5$	$n = 3; 4$	$n = 1; 6$	$n = 2; 5$	$n = 3; 4$
(222)	$\sigma$	1.78	0.09	0.60	1.60	---	0.45
	$\pi$	2.14	---	0.71	---	2.65	---
(444)	$\sigma$	0.078	0.017	0.027	0.046	---	0.030
	$\pi$	0.176	---	0.059	---	0.24	---
(666)	$\sigma$	0.89	0.55	0.28	0.17	---	0.048
	$\pi$	9.40	---	3.23	---	126	---
(888)	$\sigma$	2.59	65.0	0.87	0.095	---	0.026
	$\pi$	384	---	131	---	517	---
(10 10 10)	$\sigma$	0.59	144	0.20	2.82	---	0.078
	$\pi$	8.48	---	0.028	---	11.2	---

the reversal in the sign of the nuclear structure factor. On going from the fourth to the sixth order of reflection, the asymmetry again changes, but this time as a result of the reversal in the sign of the electronic structure factor. The interference is weakly pronounced in eighth order of reflection, where the scattering angle is close to  $90^\circ$ .

On the other hand, if the magnetic field at the resonant nuclei is oriented in the scattering plane,  $\mathbf{H}_n \parallel (\mathbf{K}_0, \mathbf{K}_1)$ , then the first, third, fourth, and sixth lines of the spectrum receive contributions from both polarizations of the incident beam. The second and fifth lines of the spectrum are due, as before, to one of the linearly polarized components of the incident beam, but, unlike the preceding case, where the  $\pi$  polarization produced the diffraction, in this case the  $\sigma$  component takes part in the diffraction. For the second, fourth, and sixth orders of reflection, the ratio of the signs of  $A_{10}$  and  $B_{(10)}^{(1,3,4,6)}$  is the same for both polarizations as when  $\mathbf{H}_n \perp (\mathbf{K}_0, \mathbf{K}_1)$ , and therefore the signs of the asymmetries of the corresponding lines are the same. The picture of these lines ( $\Delta m = \pm 1$ ) is somewhat more complicated in the eighth and tenth orders of reflection, since the signs of the asymmetry of the interference curves for the  $\pi$  and  $\sigma$  polarizations are different. (When the scattering angle increases above  $90^\circ$ , the electronic polarization factor  $P_e^\pi(\alpha, \beta)$  reverses sign for  $\pi$  polarization.) The asymmetry of the resultant curve is determined here by their relative magnitude. For the second and fifth lines in the spectrum, the sign of the asymmetry of the interference curves for (222), (444), and (666) differs from the sign of the asymmetry of the corresponding curves at  $\mathbf{H}_n \perp (\mathbf{K}_0, \mathbf{K}_1)$ . The reason is that the signs of the nuclear polarization factor for  $\pi$  polarization at  $\mathbf{H}_n \perp (\mathbf{K}_0, \mathbf{K}_1)$  and for  $\sigma$  polarization at  $\mathbf{H}_n \parallel (\mathbf{K}_0, \mathbf{K}_1)$  are opposite. In addition, inasmuch as at the field orientation under consideration scattering takes place only for the  $\sigma$  polarization, it follows that the asymmetry sign does not change when the scattering angle is increased above  $90^\circ$ , i.e., on going to the eighth and tenth orders), since the electronic polarization factor for  $\sigma$  polarization is equal to unity.

The good agreement between the theoretical curves and the experimental results (Fig. 2) confirms the applicability of the ideal-crystal model chosen by us to the description of the investigated phenomenon. Certain quantitative discrepancies in the regions of the peaks can be attributed to the fact that the crystal is not perfectly ideal. These discrepancies should be largest where the role of the primary extinction is more significant than ordinary absorption. No dips corresponding to the first and sixth lines in the (888) spectrum were observed in the experiment at  $\mathbf{H}_n \perp (\mathbf{K}_0, \mathbf{K}_1)$ , probably because the magnetic field at the iron nuclei was not perfectly perpendicular to the scattering plane. When the angle between  $\mathbf{H}_n$  and  $(\mathbf{K}_0, \mathbf{K}_1)$  deviates from  $90^\circ$ , peaks that reached a maximum at  $\mathbf{H}_n \parallel (\mathbf{K}_0, \mathbf{K}_1)$  begin to grow rapidly at the designated points of the spectrum. This, as well as the gradual decrease of the asymmetry of the peaks when the magnetic field is rotated, could be clearly seen in the series of spectra measured at an intermediate field direction, with  $\mathbf{H}_n$  in the (111) plane of the crystal and making an angle  $\varphi = 45^\circ$  with the scattering plane.

Our investigation shows quite clearly the high sensitivity of the interference-scattering spectra to the structure of the crystal and to the directions of the internal fields at the resonant nuclei.

The authors are deeply grateful to Yu. Kagan and A. M. Afanasev for useful discussions, to K. P. Aleshin for faultless operation of the electronic apparatus, to R. A. Voskanyan for growing the  $\alpha$ - $\text{Fe}_2\text{O}_3$  single crystal, to M. A. Volkov and I. B. Filipov for help with the measurements. The material enriched with  $\text{Fe}^{57}$  was obtained from the USSR State Depository of Stable Isotopes.

- <sup>1</sup>From among earlier investigations, mention should be made of the work of Bernstein and Campbell [<sup>15</sup>], who studied the total reflection of 14.4-keV  $\gamma$ -rays of  $\text{Fe}^{57}$  by using a single-line source and an Fe film having hyperfine magnetic splitting. The experimental conditions, however did not yield the polarization dependences of the interference (they investigated only small angle scattering from a polycrystalline film, and certainly did not make it possible to connect the interference picture with the crystal structure).
- <sup>2</sup>In odd orders, the identical extinction of the electronic reflection leads to pure nuclear diffraction [<sup>11</sup>] and there is no interference between the nuclear and electronic scatterings.
- <sup>3</sup>The weak deviation from collinearity is immaterial in the present analysis.
- <sup>4</sup>We use for R the more convenient expression given in [<sup>21</sup>].
- <sup>5</sup>It should be noted that such an asymmetry corresponds to interference curve measured with an unsplit source and scatterer at a scattering angle  $2\theta < 90^\circ$ .
- <sup>6</sup>The small imaginary part of  $A_{10}$  is neglected in the discussion.
- <sup>7</sup>At the same time, an overall change in the signs of the coefficients  $A_{10}$  and  $B_{10}^{(n)}$  takes place as a result of the change in the signs of the structure factors.

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Translated by J. G. Adashko

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