Effect of ultrasonic surface waves on liquid crystals

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The increase of transparency of a thin oriented layer of a nematic liquid crystal (MBBA) under the action of ultrasonic surface waves is investigated. A theoretical model of the phenomenon is proposed; it is based on the effect of a rotation of the plane of polarization. An expression is obtained that relates the intensity of the light flux, averaged over a period, to the amplitude of the oscillation velocity in the wave. Experimental data, obtained at frequency 6 MHz, agree with the results of the calculations.

It is well known[1-4] that the effect of an ultrasonic wave on a thin layer of a nematic liquid crystal is to change its transparency and its color. The present paper describes a theoretical and experimental investigation of the effect of an ultrasonic surface wave on an oriented layer of a nematic liquid crystal, which in its optical properties resembles a plate of uniaxial crystal, cut perpendicularly to the optic axis (a so-called normal layer[5]).

For further discussion we choose a coordinate system with the y axis directed along the optic axis OO′ of the layer (Fig. 1). We choose as the xz plane the solid surface on which the layer lies; the x axis will correspond to the direction of propagation of the surface wave. A beam of light falls on the layer in the direction normal to it; that is, along the optic axis of the layer. With this choice of the coordinate system, the dielectric permittivity tensor can be expressed in the form

$$\varepsilon_{ik}=\varepsilon_{\parallel}l_{i1}l_{k1}+\varepsilon_{\parallel}l_{i2}l_{k2}+\varepsilon_{\perp}l_{i3}l_{k3}, \tag{1}$$

where the l's are the direction cosines of axis i with respect to axis k.

If there is a velocity gradient in the liquid crystal, then in consequence of the Maxwell effect the dielectric permittivity tensor takes the form[6]

$$\varepsilon_{ik}=\varepsilon_{\parallel}l_{i1}l_{k1}+\varepsilon_{\parallel}l_{i2}l_{k2}+\varepsilon_{\perp}l_{i3}l_{k3}+\Lambda(\partial v_i/\partial x_k+\partial v_k/\partial x_i). \tag{2}$$

Here Λ is Maxwell's constant.

In a surface wave propagated along a solid crystal (in the xz plane), there are two components of the oscillation velocity; the ratio of their amplitudes is given by the expression[7]

$$v_{0y}/v_{0x}=-(2-\xi^2)/2(1-\xi^2)^{1/2}, \tag{3}$$

where ξ is a certain constant characteristic of the given material. Since for quartz $\xi\approx 0.87$, the longitudinal and transverse components of the oscillation velocity in this material are about the same. In a layer of liquid crystal placed on a solid surface, the gradients of the components of the oscillation velocity are[7]

$$\begin{gathered}\partial v_x/\partial x=kv_{0x},\\ \partial v_x/\partial y=-v_{0x}\beta\cos(\omega t+\pi/4),\\ \partial v_y/\partial x=kv_{0y},\quad \partial v_y/\partial y\cong\alpha v_{0y},\end{gathered} \tag{4}$$

where $\omega=2\pi f$, f is the frequency of the oscillations, $\beta^2=\omega/2\gamma$, γ is the kinematic viscosity of the crystal, k is the wave number, $k=\omega/c$, c is the speed of sound in the liquid crystal, and α is the absorption coefficient of sound in the crystal, $\alpha=\omega^2\gamma/c^3$. Estimates show that within the range of ultrasonic frequencies, the following are valid:

$$\begin{gathered}\partial v_x/\partial y\gg\partial v_y/\partial x,\\ \partial v_x/\partial y\gg\partial v_y/\partial y,\\ \partial v_x/\partial y\gg\partial v_x/\partial x;\\ \varepsilon_{\parallel}\gg 2\Lambda v_{0x}\omega/c,\quad \varepsilon_{\parallel}\gg 2\Lambda v_{0y}\alpha.\end{gathered} \tag{5}$$

Thus at frequency 6 MHz, the ratio of the gradients of the components of oscillation velocity is

$$\left(\frac{\partial v_x}{\partial y}\right)\Big/\left(\frac{\partial v_y}{\partial x}\right)=10^5.$$

Thus for the range of ultrasonic frequencies, only the component of oscillation velocity in the direction of the x axis needs to be taken into account. This last circumstance simplifies the expression (2) for the dielectric tensor of the crystal in the field of the elastic deformation gradients:

$$\varepsilon_{ik}=\left|\begin{matrix}\varepsilon_{\parallel} & \Lambda\dfrac{\partial v_x}{\partial y} & 0\\ \Lambda\dfrac{\partial v_x}{\partial y} & \varepsilon_{\parallel} & 0\\ 0 & 0 & \varepsilon_{\perp}\end{matrix}\right|. \tag{6}$$

Physically, the change of form of the dielectric tensor in the field of the velocity gradients means the following: On unit area of a surface executing an oscillation in an incompressible viscous liquid, there acts a frictional force proportional to the gradient of oscillation velocity $\partial v_x/\partial y$ in the surface wave and directed along the x axis[7]. This force produces a rotation of the oriented molecules of the liquid crystal through an angle θ, whereby the optic axis OO′ also rotates through an angle θ.

To estimate the size of the angle θ, we introduce a coordinate system x′, y′ rotated through an angle θ with respect to the rigid system x, y (Fig. 1), so that the dielectric tensor (6) will be reduced to diagonal form. By using the method described in[8], we obtain

$$\theta=\operatorname{arctg}\left\{\Lambda\frac{\partial v_x}{\partial y}\Big/\varepsilon_{\parallel}\right\}. \tag{7}$$

We shall consider the change of transparency of a normal layer of liquid crystal in polarized light, in consequence of the rotation of the optic axis. The polaroid P_1 (Fig. 1) transmits the component E_x of the electric vector, the polaroid P_2 the component E_y. When the direction of the light beam coincides with the direction of the optic axes of the molecules in the layer, the visual field in the crossed polaroids will be dark. On rotation of the optic axis through an angle θ, caused by the surface wave, double refraction occurs, and there appears a component of the electric vector in the direction of the y axis that passes through the polaroid P_2. This component is

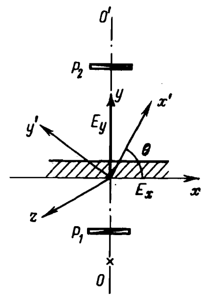

FIG. 1

$E_y = E_x \sin\theta$. By use of the expression (7) we get

$$E_y = E_x \sin\left(\operatorname{arctg}\frac{\Lambda}{\varepsilon_\parallel}\frac{\partial v_x}{\partial y}\right) = \frac{E_x a \partial v_x/\partial y}{\sqrt{1+a^2(\partial v_x/\partial y)^2}} \qquad (8)$$

Here $a = \Lambda/\epsilon_\parallel$. The intensity after passage through the polaroid P_2 is

$$I_2 = \frac{E_x^2 a^2 (\partial v_x/\partial y)^2}{1+a^2(\partial v_x/\partial y)^2}. \qquad (9)$$

By using expression (4) for the gradient of the oscillation velocity and averaging the intensity of the luminous flux over a period of the oscillations, we get

$$\langle I_2\rangle = \frac{1}{T}\int_0^T \frac{a^2\beta^2\cos^2(\omega t + {}^1/_4\pi)\, v_{0x}^2}{1+a^2\beta^2\cos^2(\omega t+{}^1/_4\pi)\, v_{0x}^2}\,dt \qquad (10)$$

Here the symbol $\langle\dots\rangle$ denotes a time average. On carrying out the integration, we obtain the following expression for the average, over the period of the oscillations, of the intensity of the luminous flux emerging from the polaroid P_2:

$$\langle I_2\rangle = A\left\{\frac{\pi}{4} - \frac{1}{\sqrt{1+a^2\beta^2 v_{0x}^2}}\operatorname{arctg}\frac{1}{\sqrt{1+a^2\beta^2 v_{0x}^2}}\right\}. \qquad (11)$$

This expression gives the dependence of the average intensity of the light flux, over a period, on the amplitude of the oscillation velocity in the wave. According to (11), the character of the changes of $\langle I_2\rangle$ as a function of the oscillation velocity is such that two limiting cases can be distinguished: namely, the value of $a^2\beta^2 v_{0x}^2$, which occurs under the radical signs in the expression (11), may be large or small in comparison with unity. In the case of small oscillation velocity ($a^2\beta^2 v_{0x}^2 \ll 1$), the mean value of the intensity changes in proportion to v_{0x}^2; in the range of values of v_{0x} that satisfy the condition $a^2\beta^2 v_{0x}^2 \gg 1$, this intensity preserves a constant value, equal to $A\pi/4$.

To test the correctness of the theoretical model proposed for explanation of the change of transparency of a normal layer of liquid crystal under the influence of an ultrasonic surface wave, experiments were performed on the apparatus shown in Fig. 2. The liquid crystal layer 1 is contained between the substrate 2, of quartz single crystal, and the cover glass 3. This cell is between the crossed polaroids 4 of a polarizing microscope and is illuminated by the source 5. The change of transparency of the layer was recorded by the photomultiplier 7, connected to the electrometer 8. By means of the photoattachment 6 it was possible to photograph the layer. The surface waves were excited on the optically polished surface of the Y-cut of quartz by means of a transducer in the form of a two-phase system of electrodes 9, obtained by etching of aluminum film. The transducer, containing eight pairs of electrodes, was designed for a fundamental frequency of 6 MHz (the distance between

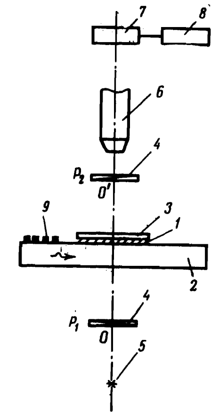

FIG. 2

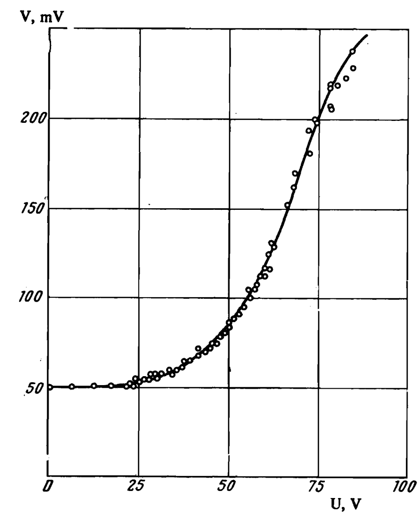

FIG. 3

neighboring electrodes was 250 microns). The distance from the transducer to the liquid crystal layer is about 1 cm. The thickness of the layer is of the order of 15 microns. The substance used was methoxybenzylidenebutylaniline, for which the temperature range of the nematic phase is 21–42°C.

In Fig. 3, the points show the experimental values of V at the output of the photomultiplier for various values of the voltage U on the surface-wave transducer. for convenience in comparison with the theoretical model, the expression (11) for the mean intensity $\langle I_2\rangle$ of the light flux was slightly modified. Since the mean light flux falling on the photomultiplier is proportional to the voltage V at its output, and since the amplitude v_{0x} of the oscillation velocity in the wave is proportional to the voltage U on the transducer ($v_{0x} = k(f)U$, where k(f) is a frequency-dependent coefficient), the expression (11) can be reduced to the following form, which is more convenient for practical use:

$$V = C_0\left\{\frac{\pi}{4} - \frac{1}{\sqrt{1+C_1^2U^2}}\operatorname{arctg}\frac{1}{\sqrt{1+C_1^2U^2}}\right\}, \qquad (12)$$

where $C_1^2 = k(f)a^2\beta^2$ depends on the frequency but C_0 is a constant. Formula (12) gives the dependence of the photomultiplier reading, which measures the intensity of the light flux, on the transducer voltage. For $C_1^2U^2 \ll 1$ and for $C_1^2U^2 \gg 1$ we have, respectively,

$$V = C_0 C_1^2 U^2/2, \qquad (13)$$

and

$$V = C_0\pi / 4. \quad (14)$$

By using two experimental points, it is possible to calculate the constants C_0 and C_1 (for frequency 6 MHz they are found to be $C_0 = 10^3$ mV and $C_1^2 = 3.2 \cdot 10^{-5}\ V^{-2}$) and to construct the theoretical dependence of the voltage V at the output of the photomultiplier on the transducer voltage U, at a given frequency. It is shown in Fig. 3 by the solid line. As is seen, all the experimental values of V fall close to the theoretical curve; this attests to the correctness of the proposed theoretical model of the phenomenon, based on the effect of rotation of the plane of polarization.

It must be mentioned that the experimental dependence shown in Fig. 3 does not reveal the tendency of the effect to saturate at large values of U, which follows from the theory. This is due to the fact that for the transducer used, as is shown by estimates, the condition $C_1^2U^2 \gg 1$ is satisfied only if $U > 500$ V, which we were not able to reach. At frequency 30 MHz, however (with the transducer operating on the fifth harmonic), the saturation effect was clearly evident and was observed at voltage ~ 150 V.

Because the intensity of the light flux is related to the oscillation velocity in the wave, the change of transparency of the layer can be used for visualization of an ultrasonic surface wave[9].

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