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CERTAIN PROPERTIES OF AN ELECTRON PLASMA IN A STRONG ELECTROMAGNETIC FIELD

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It is shown that an electron plasma in a periodic strong electromagnetic high-frequency field (pumping field) behaves, with respect to a weak low frequency electromagnetic wave, as a single crystal. The dispersion properties of the weak electromagnetic wave are investigated and the possibility of specific resonances and of parametric amplification is noted. The electron energy spectrum in the pump field is studied.

1. The availability of powerful electromagnetic wave sources in the infrared and visible regions has made it possible to investigate properties of various media which are not manifested under ordinary conditions. In particular, this applies to plasmas.

In this report we study the behavior of a plasma in the field of a strong electromagnetic wave from a laser or microwave generator.

It turns out that for a certain spatial distribution of the field an amorphous gas-discharge plasma acquires crystalline properties in certain contexts, and a superstructure appears in the plasma of a semiconductor[1)]. The presence of this kind of superstructure can significantly alter the optical and kinetic properties of the semiconductor and lead to an amplification of the electromagnetic waves in the semiconductor[1].

It is interesting to note that the lattice period, the binding energy of the electron with the lattice point, and other parameters of the artificial "crystal" produced from an electron plasma by an electromagnetic field can be varied within wide limits by varying the amplitude and the frequency of the external field.

As was shown by Gaponov and Miller[2], the average motion of an electron acted upon by an electromagnetic wave can, under certain conditions, be described as motion in some effective potential field. This field can be extremely diverse[3]; in particular, it may have the character of an isolated potential well or of a collection of such wells. In the latter case, the electrons situated in such a field are localized in the bottom of the well and execute a finite motion with some characteristic frequency. In the simplest case, the periodic potential field is produced by three mutually perpendicular standing waves having different frequencies. A potential field of this type is periodic with a period equal to the length of the standing wave, and the oscillation frequency of the electron in this field depends on the wave ampitude.

In this report we discuss the electrodynamic and quantum-mechanical properties of an electron plasma situated in the field of three standing waves. The second section in this report concerns the electrodynamics of an artificial crystal. We assume that in the electron gas, besides the field of the three standing waves (henceforth referred to as the "pumping field"), a weak electromagnetic wave is propagated with a frequency of the order of the characteristic electron oscillation frequency near the bottom of the well. The Gaponov-Miller theory is applicable if the frequency of the finite motion is low compared with the frequency of the pumping field. The frequency of the weak electromagnetic wave is lower than that of the pumping wave. Since the wavelength of the low-frequency field is large compared with the pumping wavelength, for a low-frequency field the artificial crystal can be regarded as a continuous anisotropic medium.

In the second section we obtain the dielectric tensor for such a medium, we investigate the dispersion properties of waves propagated in an artificial crystal, and we show the presence of specific resonances associated with the finite motion of the electron in the well. If the pumping frequency is located in the visible region (laser pumping), then the electrons in the bottom of the well will oscillate at a submillimeter frequency or even lower. In the same section we discuss some analogs from nonlinear optics as well as the diffraction of short waves by the lattice points of the artificial crystal.

The third section deals with the energy spectrum of an electron in the field of the three pumping standing waves. This spectrum has a band structure, but is discrete to the first approximation of the strong-coupling method.

We note in conclusion that we are discussing a very special case of a pumping field. In principle, as we have already mentioned above, these fields can have a rather complicated structure. By varying the pumping field one can vary the electrodynamic and quantum-mechanical properties of the electron plasma. Moreover, one can stipulate a made-to-order plasma characteristic and then choose the pumping field that will bring this characteristic about.

2. Let the electron be located in a strong electromagnetic field **E** (pumping field) of the form

$$\mathbf{E}=\sum_{l=-\infty}^{\infty}\{\mathbf{E}_l^{(1)}\sin\Omega_l t+\mathbf{E}_l^{(2)}\cos\Omega_l t\},\qquad(1)$$

[1)]As L. E. Gurevich was so good to inform me, the possibility of producing superstructures in a semiconductor by using laser radiation was pointed out by Kastal'skiĭ.

where l is an integer. The effective potential energy corresponding to this field can be written as follows[2]:

$$U=\frac{e^2}{4m}\sum_{l=-\infty}^{\infty}\frac{E_l^2}{\Omega_l^2},\quad E_l^2=E_l^{(1)^2}+E_l^{(2)^2}. \tag{2}$$

Note that the Ω_l are not necessarily exact frequency multiples, so formula (1) is not, generally speaking, an expansion of the field **E** in a Fourier series with respect to time. The frequencies Ω are assumed to be large compared with the Langmuir frequencies, so that the electron gas does not affect the pumping field.

An effective potential energy can be introduced if the frequency ω_0 of the motion of the electron in the potential well is significantly less than the frequency of the electromagnetic field ($\omega_0 \ll \Omega_l$). We assume henceforth that this condition is fulfilled. In addition, we are limiting ourselves to nonrelativistic problems, so that all typical velocities are assumed to be small compared to the speed of light.

We assume here that (along with a strong electromagnetic wave of frequency Ω_l) a weak electromagnetic wave $\mathscr{E}(\mathbf{r}, t)$ with a characteristic frequency $\omega \ll \Omega_l$ is propagated in the electron gas. The equation of motion of the electron is, under these assumptions

$$\frac{d\mathbf{v}}{dt}+\nu\mathbf{v}=-\frac{1}{m}\frac{\partial U}{\partial \mathbf{r}}+\frac{e}{m}\mathscr{E}(t), \tag{3}$$

where **v** is the electron velocity and ν is the collision frequency.

For concreteness, we assume that the potential energy is produced by three mutually perpendicular standing waves:

$$E_i=E_{0i}\sin\frac{\Omega_i i}{c},\quad i=x,y,z.$$

The corresponding potential energy is of the form

$$U=\frac{e^2}{4m}\left(\frac{E_{0x}^2}{\Omega_x^2}\sin^2\frac{\Omega_x}{c}x+\frac{E_{0y}^2}{\Omega_y^2}\sin^2\frac{\Omega_y}{c}y+\frac{E_{0z}^2}{\Omega_z^2}\sin^2\frac{\Omega_z}{c}z\right). \tag{4}$$

The representation (4) is possible if the following inequalities are satisfied:

$$|\Omega_i-\Omega_k|\gg\omega_0,\quad i\neq k.$$

We expand the potential (4) near one of the minima in powers of the deviation η from this minimum

$$U={}^1/_2 m(\omega_{0x}^2\eta_x^2+\omega_{0y}^2\eta_y^2+\omega_{0z}^2\eta_z^2),$$
$$\omega_{0i}^2=(eE_{0i}/\sqrt{2}mc)^2. \tag{5}$$

It is seen from (5) that an electron in a potential well produced by three mutually perpendicular standing waves behaves, to the first order, like an anisotropic oscillator.

The criterion $\omega_0 \ll \Omega$ for the applicability of the theory imposes a limitation on the field of the high-frequency wave:

$$E_0\ll\Omega mc/|e|.$$

When $\Omega \sim 10^{14}\ \text{sec}^{-1}$ this corresponds to $E_0 \ll 10^7$ cgs esu, which is certainly satisfied; for $\Omega \sim 10^{11}$ Hz we have $E_0 \ll 10^4$ cgs esu.

The velocity **v** is connected with the deviation $\boldsymbol{\eta}$ by the relation $V = d\eta/dt$. The equation of motion (3), taking (15) into account, is written thus:

$$\frac{d^2\boldsymbol{\eta}}{dt^2}+\nu\frac{d\boldsymbol{\eta}}{dt}+\hat{\omega}_0^2\boldsymbol{\eta}=\frac{e}{m}\mathscr{E}(t). \tag{6}$$

The operator $\hat{\omega}_0^2\eta$ is determined from the relation $(\omega_0^2\eta)_i = \omega_{0i}^2\eta_i$ (the summation is not carried out with respect to repeated indices).

We assume that the field $\mathscr{E}(t)$ is monochromatic: $\mathscr{E}(t) = \mathscr{E}(\mathbf{r})e^{-i\omega t}$. Then

$$\eta_i=e\mathscr{E}_i(\mathbf{r})\,/\,[m(\omega_i^2-\omega^2-i\nu\omega)]. \tag{7}$$

The current **j** is determined in terms of η in the usual way:

$$\mathbf{j}=eNd\boldsymbol{\eta}/dt=-ie\omega N\boldsymbol{\eta}. \tag{8}$$

By substituting (7) and (8) in Maxwell's equations we obtain this expression for the dielectric tensor:

$$\varepsilon_{ii}=1+\omega_p^2/(\omega_{0i}^2-\omega^2-i\nu\omega), \tag{9}$$

Where $\omega_p^2 = 4\pi e^2N/m$ is the square of the Langmuir frequency, and N is the electron concentration.

Thus an electron gas in a pumping field behaves like a biaxial crystal in respect to a low-frequency wave. Note that in general the concentration N is a function of the coordinates. According to Al'pert et al.[4], the following relation holds for N:

$$N=N_0e^{-U/2T},$$

where T is the temperature in energy units and N_0 is the equilibrium concentration of the electron gas. In our case

$$N=N_0\exp\left\{-\frac{e^2}{8mT}\left[\frac{E_{0x}^2}{\Omega_x^2}\sin^2\frac{\Omega_x}{c}x+\frac{E_{0y}^2}{\Omega_y^2}\sin^2\frac{\Omega_y}{c}y+\frac{E_{0z}^2}{\Omega_z^2}\sin^2\frac{\Omega_z}{c}z\right]\right\}. \tag{10}$$

It follows from (10) that the electrons are localized at the minima of the pumping field, which correspond to the lattice points of a crystal lattice. Recognizing that the wavelength of the weak electromagnetic wave is much greater than the pumping wavelength, we can, as is customary in microscopic electrodynamics, average the concentration over the lattice period. We continue to designate the average concentration by N_0.

If the dielectric constant is independent of the coordinates, then $\mathscr{E}(\mathbf{r})$ can be sought in the form of a plane wave

$$\mathscr{E}(\mathbf{r})=\mathscr{E}_0e^{in\omega\mathbf{r}\boldsymbol{\gamma}/c}, \tag{11}$$

where n is the refractive index and $\boldsymbol{\gamma}$ is a unit vector in the direction of wave propagation.

We consider the isotropic case, when the amplitude of all three pump standing waves are equal. It is interesting to note that equality of the pump-wave frequencies is not required for isotropy. Maxwell's equations give the following expression for n:

$$n=[1+\omega_p^2/(\omega_0^2-\omega^2-i\omega\nu)]^{1/2}. \tag{12}$$

It is seen from (12) that there is resonance at the frequency $\omega = \omega_0$:

$$\omega_0=eE_0/\sqrt{2}mc \tag{13}$$

with the resonance value n_r determined by the formula

$$n_r=\frac{1+i}{\sqrt{2}}\frac{\omega_p}{(\omega_0\nu)^{1/2}}. \tag{14}$$

Let us estimate the resonance frequency determined by (13). Modern quantum generators emit fields of intensity up to 10^6 cgs esu. Assuming a value in the order of 10^3–10^4 cgs esu for E_0, then the resonance frequency for a gas-discharge plasma is of the order of 10^{10}–10^{11} Hz. In semiconductors, where the effective mass is as a rule, less than the free electron mass by one or two orders, the resonance frequency is higher by one or two orders. For the line to be sufficiently narrow, the relation $\omega_0 \gg \nu$ must be satisfied. The collision frequencies in a gas-discharge plasma, do not excede 10^{11}–10^{12} Hz as a rule. It is obvious that the inequality $\omega_0 \gg \nu$ at the assumed values of the pump field.

One should note, however, that the effects pointed out can be produced in semiconductor plasmas only with difficulty, since fields on the order of 10^3 cgs esu lead to breakdown of the semiconductor. Recognizing the low collision frequency in a gas-discharge plasma, the pumping can be achieved in this case by a microwave electromagnetic field. Actually, if we choose a pump frequency 10^{11} Hz, $N = 10^{12}$, and $E_0 \sim 10^2$–10^3 cgs esu, then the resonance effect is preserved and all the applicability criteria of the theory remain in effect.

A low-frequency weakly damped wave can be propagated in an electron gas located in a strong electromagnetic field when $\omega < \omega_p$, provided the following inequality is satisfied:

$$\omega,\ \nu < \omega_0. \tag{15}$$

If inequality (15) is in force, then the weakly damped wave has the linear dispersion:

$$\omega = \frac{c}{[1+\omega_p^2/\omega_0^2]^{1/2}} k, \tag{16}$$

where $k = \omega n/c$ is the wave number. It is seen from (16) that the phase velocity of a weakly damped wave is less than the phase velocity of light. If we set the concentration of the gas discharge plasma equal to 10^{13} cm^{-3}, $\nu \sim 10^8$ sec^{-1}, $E_0 \sim 10^3$ cgs esu, and $\omega \sim 10^9$ Hz, then (15) is satisfied and the phase velocity of the undamped wave decreases by a factor of ten. Analogous estimates can be made for semiconductor plasmas as well. In a gas-discharge plasma pumped by a microwave field one can, in terms of this theory, decrease the amplitude of E_0 somewhat and by the same token increase the slowing down of the wave.

It is interesting that the wave described by dispersion relation (16) has a linear spectrum, as unlike the well-known helicons, whose spectrum is quadratic.

Besides ordinary electromagnetic waves, an exciton wave with limiting frequency ω_0 and a plasma wave with limiting frequency $(\omega_p^2 + \omega_0^2)^{1/2}$[4] can also propagate in the investigated medium. Spatial dispersion according is taken into account by the method presented in[4], and the pumping wavelength c/Ω plays the role of the lattice constant.

We have assumed so far that we can limit ourselves to the first term in the expansion in powers of the deviation of the electron from the equilibrium position in formula (4). If we retain the second term we obtain the following equation of motion:

$$\frac{d^2\eta_i}{dt^2} + \omega_{0i}^2\eta_i - \beta_i\eta_i^3 = \frac{e}{m}\mathscr{E}_i, \quad \beta_i = \frac{4}{3}\frac{\Omega_i^2}{c^2}\omega_{0i}^2. \tag{17}$$

We note that even if the frequencies ω_{0i} are all equal, anisotropy of the medium manifests itself in the next higher approximation.

An equation of type (17) is used to determine the nonlinear susceptibility (see[5], for example), so that the dielectric constant can be expressed in the form

$$\varepsilon_{im}(E) = \varepsilon_{im}(\omega) + \chi_{iklm}E_kE_l \tag{18}$$

(the summation is carried out over repeated indices). The tensor $\epsilon_{im}(\omega)$ is determined from formula (9), and the expressions for χ_{iklm} are given in the book by Bloembergen[6]. It follows from (18) that in an electron gas located in a pumping field effects inherent in nonlinear optics are possible, such as frequency multiplication, parametric amplification, etc. We note that in our system considered it is easy to achieve synchronization conditions at which the waves interact effectively, since the anisotropy and dispersion of the electron gas are easily controlled by varying the parameters of the pumping field.

Up to now we have been discussing the propagation of electromagnetic waves in an electron gas situated in a pump field with the stipulation that the frequency ω of these waves is much lower than the pump frequency. We now investigate the diffraction of electromagnetic waves of a frequency $\omega > \Omega_k$ and with an amplitude that is small compared with the amplitude of the pump field, so that these waves have no effect on the electron-gas concentration. For any reasonable electron concentration we can also assume that $\omega_p \ll \omega$. So formulated, our problem is completely identical with the problem of electron diffraction by a crystal lattice. The effective scattering cross section $d\sigma$ can be obtained at once from formula (99.13) of[7]. We do not cite it here.

In our case we rewrite the Bragg-Wolf law as follows:

$$\sin(\nu/2) = (\Omega_x^2 + \Omega_y^2 + \Omega_z^2)^{1/2}/\omega. \tag{19}$$

It follows from (19) that we must have

$$\omega > (\Omega_x^2 + \Omega_y^2 + \Omega_z^2)^{1/2}.$$

3. The foregoing discussion was strictly classical. The next step is to derive the Schrödinger equation for an electron in a pump field and to study the spectrum of this electron. We begin with the Schrödinger equation in the electromagnetic field:

$$ih\frac{\partial\psi}{\partial t} = \frac{1}{2m}\left(-ih\nabla - \frac{e}{c}\mathbf{A}\right)^2\psi, \tag{20}$$

where $\mathbf{A}$ is a vector potential connected with the electric field by the relation:

$$\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}. \tag{21}$$

The gauge chosen for $\mathbf{A}$ is

$$\operatorname{div}\mathbf{A} = 0.$$

If the field is defined by (1), then the following relation holds for $\mathbf{A}$:

$$\mathbf{A} = c\sum_{l=-\infty}^{\infty}\left(\frac{\mathbf{E}_l^{(2)}}{\Omega_l}\cos\Omega_l t - \frac{\mathbf{E}_l^{(1)}}{\Omega_l}\sin\Omega_l t\right). \tag{22}$$

Using the method of averaging with respect to fast time in its usual form (see, for example,[8]), we obtain for the time-average wave function (designated ψ as before) the equation:

$$\frac{h^2}{2m}\Delta\psi + (\varepsilon - U)\psi = 0, \tag{23}$$

where ϵ is the energy of the electron (not to be confused with the dielectric constant from the preceding section and U is defined in (2). Direct estimates show that the criteria of applicability of (23) coincide with those of the averaged classical equation of motion derived by Gaponov and Miller[2].

By way of example we consider the potential produced by three standing waves. The spectrum of an electron in a periodic field has been quite thoroughly studied, and we omit the details of the calculations. In our case the variables are separable. The wave function can be represented in the product of three functions, each of which depends on only one coordinate:

$$\psi(x, y, z) = \psi_x(x)\psi_y(y)\psi_z(z). \tag{24}$$

Each of the factors in the right-hand side satisfies the equation

$$\frac{h^2}{2m}\frac{d^2\psi_i}{dx_i^2}+\left(\varepsilon_i-\frac{e^2}{4m\Omega_i^2}\sin^2\frac{\Omega_i}{c}x_i\right)\psi_i=0 \tag{25}$$

which is the well-known Mathieu equation.

By changing to dimensionless variables we can show that the quantity α is the dimensionless electron-field coupling constant:

$$\alpha_i=\frac{e^2E_{0i}^2}{4m\Omega_i^2h\Omega_i}\frac{mc^2}{h\Omega_i}.$$

The first factor in this formula is the ratio of the classical kinetic energy of an electron in a potential well to the pump-field photon energy; it can be either large or small compared with unity depending on the amplitude of the pump field. The second factor is the ratio of the electron rest energy to the pump-field photon energy and is large when $\Omega < 10^{20}$ Hz. Thus, in principle, both weak and strong coupling are possible.

It must be pointed out, however, that for any reasonable value of the pump-field amplitude and frequency the weak-coupling approximation for both gas-discharge and semiconductor plasmas leads to very narrow forbidden bands, which are difficult to observe experimentally. In comparison, strong coupling is easy to observe. If one takes a pump frequency on the order of 10^{14}, and E_0 on the order of 10^3 cgs esu, then α is of the order of 10^6; at a pump frequency on the order of 10^{11} Hz and $E_0 \sim 1$ cgs esu, α is of the order of 10^{12}.

The electron energy is equal to the sum of the partial energies:

$$\varepsilon = \varepsilon_x + \varepsilon_y + \varepsilon_z. \tag{26}$$

In the limiting case of strong coupling this formula holds for the energy:

$$\varepsilon_i = h\omega_{0i}(l_i + 1/2), \quad l_i = 1.2. \tag{27}$$

In the next higher approximation with respect to $1/\alpha$, the levels smear out into bands. Level quantization in a pump field can lead to a number of curious effects. For example, if the electron gas is degenerate, oscillations of the various physical quantities with the amplitude of the electron field, similar to de Haas-van Alphen oscillations may occur.

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