NOISE EMISSION BY A RESONANT MEDIUM IN A STRONG FIELD

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The noise emission spectra of extended media in the presence of a strong monochromatic field are considered by taking into account the variation of the field in the medium. The noise sources consist of thermal and spontaneous fluctuations of the medium and also of thermal fluctuations of the entering radiation. The fine structure of the spectral lines is determined. Analytic and numerical results are obtained for the noise radiation spectra. It is shown that in the case of an active medium with inhomogeneous line broadening the strong field saturates part of the transition line and produces a dip in the spectrum. A narrow peak is located at the center of the dip. The relative depth of the dip approaches unity with increase of the medium length whereas the peak height increases insignificantly. The dependence of the spectral line shape on relations between the relaxation constants and the inhomogeneous magnitude of strong field is investigated for media with transition line broadening. The effect of extension of the medium on spectral line shape is considered and it is shown that in contrast to a medium with inhomogeneous broadening, the strong field saturates the whole transition line of the active medium. Noise emission from absorbing media in thermal equilibrium with the ambient medium is considered. It is shown that the spectral line width is determined by the strong field. The length of a passive medium for which noise emission is a maximum is estimated.

WHEN a strong field propagates in a medium, nonlinear effects are produced and influence both the propagation of the strong field itself and the shape of the spectral line of the noise emitted by the medium. The propagation of strong electromagnetic radiation in active media has been the subject of a number of papers[1-7]. Some of them deal with the influence of the strong field on the noise emission. However, these papers consider in the main only the power and width of the emission spectrum. Rautian[8] derived expressions for the spectra of the spontaneous emission of a medium in the presence of a strong monochromatic field. These expressions are valid only for a sufficiently short medium, when the variation of the field over the length of the medium can be neglected.

We consider here the spectra of noise emission of extended media in the presence of a strong monochromatic field, with allowance for the variation of the field in the medium. The analysis is valid both for active (amplifying) media and for passive (absorbing) ones. Rautian's results[8] follow from the present paper as a particular case.

1. FUNDAMENTAL EQUATIONS

We start with a closed system of equations for the field and for the density matrix of the atoms of the medium. We write the field equation in the form

$$\frac{\partial^2\mathscr{E}}{\partial t^2}+4\pi\sigma\frac{\partial\mathscr{E}}{\partial t}-c^2\frac{\partial^2\mathscr{E}}{\partial x^2}=-4\pi\frac{\partial^2\mathscr{P}}{\partial t^2}+\omega_0{}^2\mathscr{E}^{(T)}. \tag{1.1}$$

Here σ is the conductivity of the medium, $\mathscr{P}$ is the projection of the polarization vector on the field direction, $\mathscr{E}^{(T)}$ is the thermal field, and x is the coordinate along the propagation direction of the strong monochromatic field. In accordance with the Callen-Welton theorem we have

$$(\mathscr{E}^{(T)2})_{\omega,\mathbf{k}}=\frac{32\pi^2\hbar\sigma}{\omega_0}\left(\bar{n}_1+\frac{1}{2}\right) \tag{1.2}$$

where $\bar{n}_1=[\exp(\hbar\omega/kT_1)-1]^{-1}$ is the average number of photons in the equilibrium state, and T_1 is the temperature of the medium.

We specify the boundary conditions in the form

$$\mathscr{E}(t,0)=E_0\cos\omega_0 t+\mathscr{E}_0{}^{(T)}. \tag{1.3}$$

Here $E_0\cos\omega_0 t$ is the monochromatic field incident on the investigated medium and $\mathscr{E}_0^{(T)}$ is the thermal field at the input, averaged over the cross section S of the investigated medium. The spectral power density of this field is[9]

$$(\mathscr{E}_0{}^{(T)2})_\omega=\frac{4\pi\hbar\omega}{cS}\left(\bar{n}_2+\frac{1}{2}\right), \tag{1.4}$$

where $\bar{n}_2=[\exp(\hbar\omega/kT_2)-1]^{-1}$ and T_2 is the ambient temperature. We assume henceforth $\bar{n}_1=\bar{n}_2=\bar{n}$.

Owing to the presence of a strong monochromatic field, we can separate in the equations for the density matrix elements one optical transition that is resonant with this field. The upper and lower levels corresponding to these transitions will be designated a and b.

Instead of the diagonal density-matrix elements we introduce their sum and difference, $R=\rho_a+\rho_b$ and $D=\rho_a-\rho_b$. The equations for ρ_{ab}, D, and R take the form

$$\begin{gathered}\frac{\partial\rho_{ab}}{\partial t}+\mathrm{v}\frac{\partial\rho_{ab}}{\partial\mathbf{r}}+(i\omega_{ab}+\gamma_{ab})\rho_{ab}=-\frac{id_{ab}E}{\hbar}D,\\ \frac{\partial D}{\partial t}+\mathrm{v}\frac{\partial D}{\partial\mathbf{r}}+\gamma(D-D^{(0)})=-\frac{2i}{\hbar}(d_{ba}\rho_{ab}-\rho_{ba}d_{ab})E,\\ \frac{\partial R}{\partial t}+\mathrm{v}\frac{\partial R}{\partial\mathbf{r}}+\gamma(R-R^{(0)})=0.\end{gathered} \tag{1.5}$$

Here d_{ab} is the matrix element of the dipole-moment vector, $\gamma=\gamma_a=\gamma_b$ is the width of the levels a and b, and γ_{ab} is the width of the luminescence line.

We specify the field $\mathscr{E}$ and the polarization vector $\mathscr{P}$ in (1.1) in the form of traveling waves with slowly varying complex amplitudes:

$$\mathscr{E} = {}^1/_2[\hat{E}e^{-i(\omega_0 t - k_0 x)} + \text{c.c.}], \quad \mathscr{P} = {}^1/_2[Pe^{-i(\omega_0 t - k_0 x)} + \text{c.c.}]. \quad (1.6)$$

Oraevskiĭ[10] has shown that the higher harmonics of the polarization and of the population difference can be neglected subject to the following limitation on the field amplitude E:

$$aE^2 \ll \omega_0^2 / \gamma\gamma_{ab}, \quad a = |d_{ab}|^2 / 3\hbar^2\gamma\gamma_{ab}.$$

Obviously, this condition is always satisfied at real field values.

We note that by expressing the field in the form (1.6) we neglect the influence of the noise-emission flux propagating in the opposite direction. This can be done if the noise field is weak, when

$$a\langle\delta E^2\rangle \ll 1. \quad (1.7)$$

Since the noise field can increase with increasing length of the medium, the condition (1.7) imposes a limitation on the length of the medium.

The complex amplitude of the polarization vector P is connected with the off-diagonal elements of the density matrix by the relation

$$P = \int P_v \, d\mathbf{v} = \frac{2nd_{ba}}{T} \int \int_0^T \rho_{ab} e^{i(\omega_0 t - \mathbf{k}_0 x)} \, dt \, d\mathbf{v}. \quad (1.8)$$

Substituting (1.6) and (1.8) in (1.1) and (1.5) we obtain the following system of truncated equations for the complex amplitudes of the field and the polarization

$$\frac{\partial \hat{E}}{\partial t} + c\frac{\partial \hat{E}}{\partial x} + 2\pi\sigma\hat{E} = 2\pi i\omega_0 P + i\omega_0 \hat{E}^{(T)}; \quad (1.9)$$

$$\frac{\partial P_v}{\partial t} + \mathbf{v}\frac{\partial P_v}{\partial \mathbf{r}} + [-i(\mu - \mathbf{k}_0\mathbf{v}) + \gamma_{ab}]P_v = -\frac{i|d_{ab}|^2 n}{3\hbar} D\hat{E}, \quad (1.10)$$

$$\frac{\partial D}{\partial t} + \mathbf{v}\frac{\partial D}{\partial \mathbf{r}} + \gamma(D - D^{(0)}) = \frac{i}{2n\hbar}(P_v^* \hat{E} - P_v \hat{E}^*).$$

Here $\mu = \omega_0 - \omega_{ab}$

$$\hat{E}^{(T)}(x, t) = \frac{1}{TS}\int_0^T \int_S \mathscr{E}^{(T)} e^{i(\omega_0 t - k_0 x)} \, dS \, dt.$$

Obviously, $(E^{(T)2})_{\Omega, K} = (\mathscr{E}^{(T)2})_{\omega, \mathbf{k}}/S$, where $\Omega = \omega - \omega_0$ and K is the projection of the wave vector $\mathbf{K} = \mathbf{k} - \mathbf{k}_0$ on the x axis.

The boundary conditions for (1.9) are

$$\hat{E}(t, 0) = E_0 + \hat{E}_0^{(T)},$$

where, in accord with (1.4),

$$(\hat{E}_0^{(T)2})_\Omega = \frac{8\pi\hbar\omega_0}{cS}\left(\bar{n} + \frac{1}{2}\right).$$

We represent the functions $\hat{\mathbf{E}}$, P, P_v, and D in the form of sums of the mean values and of the fluctuating components in the following manner:

$$\hat{E} = (\bar{E} + \delta E_1 - i\delta E_2)\, e^{-i\bar{\varphi}},$$

$$P = (\bar{P}_2 + i\bar{P}_1 + \delta P_2 + i\delta P_1)\, e^{-i\bar{\varphi}},$$

$$P_v = (\bar{P}_v + \delta P_{2v} + i\delta P_{1v})\, e^{-i\bar{\varphi}},$$

$$D = \bar{D} + \delta D.$$

Here $\bar{E}$ and φ are the mean values of the real amplitude and phase. Neglecting the influence of the fluctuations on the mean values, we obtain from (1.9) and (1.10) the following equations for the mean values

$$c\frac{d\bar{E}}{dx} + 2\pi\sigma\bar{E} = -2\pi\omega_0\bar{P}_1, \quad c\frac{d\bar{\varphi}}{dx} = -\frac{2\pi\omega_0}{\bar{E}}\bar{P}_2; \quad (1.11)$$

$$v\frac{d\bar{P}_v}{dx} + (\gamma_{ab} - i(\mu - k_0 v))\bar{P}_v = -\frac{i|d_{ab}|^2 n}{3\hbar}\bar{D}\bar{E}, \quad (1.12)$$

$$v\frac{d\bar{D}}{dx} + \gamma(\bar{D} - D^{(0)}) = \frac{i\bar{E}}{2\hbar n}(\bar{P}_v^* - \bar{P}_v).$$

The equations for the fluctuating parts are

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial x} + 2\pi\sigma\right)\delta E_{1,2} \pm \frac{2\pi\omega_0}{\bar{E}}\bar{P}_2\delta E_{2,1} = -2\pi\omega_0\delta P_{1,2} + \omega_0 E_{1,2}^{(T)}; \quad (1.13)$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + \gamma_{ab}\right)\delta P_{1v} - (\mu - \mathbf{k}_0\mathbf{v})\,\delta P_{2v} = -\frac{|d_{ab}|^2 n}{3\hbar}(\bar{D}\delta E_1 + \bar{E}\delta D),$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + \gamma_{ab}\right)\delta P_{2v} + (\mu - \mathbf{k}_0\mathbf{v})\,\delta P_{1v} = -\frac{|d_{ab}|^2 n}{3\hbar}\bar{D}\delta E_2, \quad (1.14)$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + \gamma\right)\delta D = \frac{1}{n\hbar}(\bar{E}\delta P_{1v} + \bar{P}_{1v}\delta E_1 + \bar{P}_{2v}\delta E_2).$$

Here $E_{1,2}^{(T)}$ are determined from the relation $\hat{E}^{(T)} = -(E_2^{(T)} + iE_1^{(T)})e^{-i\bar{\varphi}}$.

2. CALCULATION OF MEAN VALUES

In (1.12) we can neglect the derivatives with respect to the coordinate if the field relaxation distance x_{rel} is much larger than the average distance traversed by the atom during the time of establishment of the polarization, i.e.,

$$u / \gamma_{ab} \ll x_{\text{rel}}. \quad (2.1)$$

The condition (2.1) is usually well satisfied in solid and gaseous amplifiers, since $u/\gamma_{ab} \lesssim 10^{-4}$ cm and $x_{rel} \sim 1-10$ cm.

Solving (1.12) subject to the condition (2.1), we get

$$\bar{P}_v = -\frac{i|d_{ab}|^2 n}{3\hbar}\frac{\bar{E}\bar{D}}{\gamma_{ab} - i(\mu - \mathbf{k}_0\mathbf{v})} \quad (2.2)$$

$$\bar{D} = \frac{D^{(0)}(\gamma_{ab}^2 + (\mu - \mathbf{k}_0\mathbf{v})^2)}{\gamma_{ab}^2(1 + \xi) + (\mu - \mathbf{k}_0\mathbf{v})^2}, \quad \xi = aE^2.$$

For immobile atoms we therefore obtain

$$\bar{P} = \frac{g(\mu)d}{4\pi}\frac{\mu - i\gamma_{ab}}{\gamma_{ab}}\frac{\bar{E}}{1 + g(\mu)\xi}, \quad \bar{D} = \frac{D^0}{1 + \xi}, \quad (2.3)$$

$$g(\mu) = \gamma_{ab}^2 / (\mu^2 + \gamma_{ab}^2).$$

for moving atoms at $\gamma_{ab}\sqrt{1 + \xi} \ll k_0 u$ we have

$$\bar{P} = -\frac{ig(\mu)d}{4\pi}\frac{\bar{E}}{\sqrt{1 + \xi}} \quad (2.4)$$

$$g(\mu) = \frac{\pi\gamma_{ab}}{\sqrt{2\pi}\,k_0 u}\exp\left\{-\frac{\mu^2}{2(k_0 u)^2}\right\}.$$

Here $d = 4\pi|d_{ab}|^2 nD^0/3\hbar\gamma_{ab}$.

Substituting the expressions for $\bar{P}$ in (1.11), we obtain the mean values of the amplitude and of the phase. It is convenient to write down the solution of (1.11) in the form

$$\bar{E} = G(\xi) = G(\xi_0)e^{2k_0\eta x}, \quad \varphi(\xi) = F(\xi) - F(\xi_0), \quad \xi_0 = aE_0^2, \quad (2.5)$$

where $\eta = (\omega_0 g(\mu)d - 2\pi\sigma)/2\omega_0$.

The form of the functions G(z) and F(z) depends on the character of motion of the atoms. For immobile atoms

$$G(z) = z\left[1 - \frac{4\pi\sigma}{\omega_0 g(\mu)d}(1 + g(\mu)z)\right]^{-\omega_0 g(\mu)d/4\pi\sigma}, \quad (2.6)$$

$$F(z) = -\frac{\mu g(\mu)d}{4\gamma_{ab}\eta}\ln\{z[\omega_0 g(\mu)d - 4\pi\sigma(1 + g(\mu)z)]\}.$$

For moving atoms at $\gamma_{ab}\sqrt{1+\xi} \ll k_0 u$

$$G(z)=\frac{\sqrt{1+z}-1}{(\sqrt{1+z}+1)^{\alpha}}\left(1-\frac{4\pi\sigma\sqrt{1+z}}{\omega_0 g(\mu)d}\right)^{\beta}, \qquad (2.7)$$

$$F(z)=0,$$

where

$$\alpha=\frac{\omega_0 g(\mu)d-4\pi\sigma}{\omega_0 g(\mu)d+4\pi\sigma}, \quad \beta=\frac{\omega_0^2 g^2(\mu)d^2}{2\pi\sigma(\omega_0 g(\mu)d+4\pi\sigma)}.$$

The corresponding values for $\sigma = 0$ were obtained by Boĭkova[1].

3. CALCULATION OF THE POLARIZATION FLUCTUATIONS

We represent the fluctuations of the populations and of the population difference in the form of sums of spontaneous parts, which do not depend on the field fluctuations, and induced parts due to the field fluctuations.

The equations for the induced parts of the fluctuations coincide with (1.14). Since we are interested in the noise emission along the field direction, i.e., at $\mathbf{k} \parallel \mathbf{k}_0$, it follows that $K \ll k_0$. We can therefore disregard the derivatives with respect to the coordinate in (1.14) if the condition (2.1) is satisfied.

From (1.14) we can find the connection between the spectral components of the polarization fluctuations $\delta P^i_{1,2v}(\Omega)$ and the spectral components of the field fluctuations $\delta E_{1,2\Omega}$. It is convenient to write this connection in the form

$$\delta P_{iv}{}^{i}(\Omega)=\sum_{j=1}^{2}\varkappa_{ij}(\Omega,v)\,\delta E_{j\Omega}, \quad i=1,2. \qquad (3.1)$$

For $\kappa_{ij}(\Omega, v)$ we obtain the expressions

$$\varkappa_{11}(\Omega,v)=\frac{\varkappa_0\gamma_{ab}}{\gamma_{ab}^2(1+\xi)+\mu_v^2}\,\frac{(-i\Omega+\gamma)(\gamma_{ab}^2+\mu_v^2)-\gamma\gamma_{ab}^2\xi}{\Delta}, \qquad (3.2)$$

$$\varkappa_{22}(\Omega,v)=\frac{\varkappa_0\gamma_{ab}}{\gamma_{ab}^2(1+\xi)+\mu_v^2}\left[\frac{(-i\Omega+\gamma)(\gamma_{ab}^2+\mu_v^2)}{\Delta}+\frac{\gamma\gamma_{ab}^3\xi}{\Delta(-i\Omega+\gamma_{ab})}\right];$$

$$\varkappa_{12}(\Omega,v)=\frac{\varkappa_0\mu_v\gamma_{ab}}{\gamma_{ab}^2(1+\xi)+\mu_v^2}\,\frac{(-i\Omega+\gamma)(\gamma_{ab}^2+\mu_v^2)+\gamma\gamma_{ab}\xi(-i\Omega+\gamma_{ab})}{\Delta(-i\Omega+\gamma_{ab})} \qquad (3.3)$$

$$\varkappa_{21}(\Omega,v)=-\frac{\mu_v}{-i\Omega+\gamma_{ab}}\varkappa_{11}(\Omega,v).$$

Here

$$\Delta=(-i\Omega+\gamma)(-i\Omega+\gamma_{ab})+\gamma\gamma_{ab}\xi+\frac{-i\Omega+\gamma}{-i\Omega+\gamma_{ab}}\mu_v^2,$$

$$\varkappa_0=-|d_{ab}|^2 nD^{(0)}/3\hbar\gamma_{ab}, \quad \mu_v=\mu-\mathbf{k}_0\mathbf{v}.$$

Thus, the interaction of the components of weak noise fields with a strong field cause the polarizability of the medium to become a tensor of second order. The equations for the cosine and sine components (δE_1 and δE_2) of the noise field are in general interrelated.

The equations for the spontaneous parts of the fluctuations are obtained from (1.14) by discarding the terms containing δE_1 and δE_2. We solve these equations by the same method as in[11,12]. This method enables us to express the space-time spectral density of the polarization fluctuations $(\delta P_v^2)_{\Omega,K}$ in terms of the spatial spectral density $(\delta P_v^2)_K$. Under the condition $\rho_a, \rho_b \ll 1$ (see[13]) we obtain

$$(\delta P_{1v}^2)_{\Omega,K}=2(\delta P_{1v}^2)_K\,\mathrm{Re}\left(\frac{-i\Omega+\gamma}{\Delta}\right),$$

$$(\delta P_{2v}^2)_{\Omega,K}=2(\delta P_{2v}^2)_K\,\mathrm{Re}\left[\frac{-i\Omega+\gamma}{\Delta}+\frac{\gamma\gamma_{ab}\xi}{\Delta(-i\Omega+\gamma_{ab})}\right],$$

$$(\delta P_{1v}\delta P_{2v}^*)_{\Omega,K}=(\delta P_{2v}^2)_K\frac{\mu_v(-i\Omega+\gamma)}{\Delta(-i\Omega+\gamma_{ab})}. \qquad (3.4)$$

The values of $(\delta P^2_{1,2v})_K$ are given by the expression[13]

$$(\delta P_{1v}^2)_K=(\delta P_{2v}^2)_K=\frac{n|d_{ab}|^2}{3}\bar{R}\delta(v-v'). \qquad (3.5)$$

It follows from (1.5) that at $\gamma_a = \gamma_b = \gamma$ the average value of the population sum is $\bar{R} = R^{(0)}$, i.e., $(\delta P^2_{1,2v})_K$ does not depend on the field.

For a sufficiently short medium, when the field amplitude E can be regarded as constant and equal to E_0, the system (1.13) can be solved in general form. From the solution of this system we can obtain expressions for the spectral densities of the field fluctuations $(\delta E_1^2)_\Omega$, $(\delta E_2^2)_\Omega$, and $(\delta E_1 \delta E_2^*)_\Omega$, and for the noise-emission flux. The spectral density $W_\Omega(L)$ of the noise-emission flux of a medium of length L is defined as the difference between the noise fluxes at the input and at the output. Without going through the intermediate step, we present for this density the final expression:

$$W_\Omega(L)=\pi\omega_0^2c^{-1}[(\delta\mathcal{P}^2)_{\omega_0+\Omega,k}-4\pi\varkappa''(\Omega)(\bar{n}+1/2)]L.$$

Here

$$(\delta\mathcal{P}^2)_{\omega,k}=1/2[(\delta P_1^2)_{\Omega,K}+(\delta P_2^2)_{\Omega,K}-2\,\mathrm{Im}\,(\delta P_1\delta P_2^*)_{\Omega,K}]$$

is the spectral density of the polarization fluctuations,

$$\varkappa''(\Omega)=1/2(\varkappa_1'(\Omega)+\varkappa_2'(\Omega)+\varkappa_{21}''(\Omega)-\varkappa_{12}''(\Omega)),$$

$$\varkappa_i(\Omega)=\int\varkappa_{ii}(\Omega,\mathbf{v})\,d\mathbf{v},$$

$$\varkappa_{ij}(\Omega)=\int\varkappa_{ij}(\Omega,\mathbf{v})\,d\mathbf{v}$$

is the imaginary part of the polarizability of the medium.

The quantity $W_\Omega(L)$ characterizes the total thermal and spontaneous emission flux in a solid angle $\delta\vartheta = \lambda^2/S$, where λ is the wavelength[9].

4. NOISE EMISSION OF A GAS AMPLIFIER WITH INHOMOGENEOUS LINE BROADENING

It follows from (2.4) and (3.3) that in the case of inhomogeneous line broadening, when $\gamma_{ab}\sqrt{1+\xi} \ll k_0 u$ and $\mu \ll k_0 u$, we have $\bar{P}_1 = 0$ and $\kappa_{12}(\Omega) - \kappa_{21}(\Omega) = 0$. Consequently, Eqs. (1.13) are separable in this case and can be solved. By solving these equations we obtain the following expressions for the spectral densities of the fluctuations:

$$(\delta E_{1,2}^2(x))_\Omega=\frac{4\pi\hbar\omega_0}{cS}\left\{\bar{n}+\frac{1}{2}+\frac{\pi\omega_0}{c\hbar}\int_0^x\exp\left\{-\frac{4\pi}{c}\left[\sigma(x-x')+\omega_0\int_{x'}^{x}\varkappa_{1,2}'(\Omega)\,dx\right]\right\}[(\delta P_{1,2}^2)_{\Omega,K}-4\hbar\varkappa_{1,2}'(\Omega)(\bar{n}+1/2)]\,dx'\right\}. \qquad (4.1)$$

The noise-emission flux of a medium of length L is represented as a sum of two terms:

$$W_\Omega(L)=W_{1\Omega}(L)+W_{2\Omega}(L),$$

where

$$W_{1,2\Omega}(L)=\frac{cS}{8\pi}(\delta E_{1,2}^2(L))_\Omega-\frac{\hbar\omega_0}{2}\left(\bar{n}+\frac{1}{2}\right).$$

It follows from (4.1) that

$$W_{1,2\Omega}(L)=\frac{\pi c^2}{2\omega_0^2}\int_0^L I_{1,2\Omega}(x')\exp\left[-\frac{4\pi}{c}\left\{\sigma(x-x')+\omega_0\int_{x'}^{x}\varkappa_{1,2}'(\Omega)\,dx\right\}\right]dx', \qquad (4.2)$$

$$I_{1,2\Omega}=\frac{\omega_0^4}{c^3}\left[(\delta P_{1,2}^2)_{\Omega,K}-4\hbar\varkappa_{1,2}'(\Omega)\left(\bar{n}+\frac{1}{2}\right)\right]. \quad (4.3)$$

An analysis of expressions (4.1) and (4.2) is in general difficult. We therefore calculate first the spectral density of the noise-emission flux at zero frequency. It follows from (3.2), (3.4), and (3.5) that at $\Omega = 0$

$$I_{10}=\frac{\omega_0^4\hbar g(\mu)d}{\pi c^3\sqrt{1+\xi}}\left[\frac{1}{1+\xi}\left(\bar{n}+\frac{1}{2}\right)+\frac{1}{2}\frac{R^0}{D^0}\right], \quad (4.4)$$
$$I_{20}=\frac{\omega_0^4\hbar g(\mu)d}{\pi c^3}\sqrt{1+\xi}\left[\frac{1}{1+\xi}\left(\bar{n}+\frac{1}{2}\right)+\frac{1}{2}\frac{R^0}{D^0}\right].$$

We assume henceforth that the variation of the field in the medium as a result of the conductivity σ is much less than the corresponding variation due to the imaginary part of the polarizability, i.e., $\sigma \ll \omega_0|\kappa_{1,2}'|$. This condition is usually well satisfied. Neglecting σ, we obtain

$$W_0(L)=\hbar\omega_0\frac{1+\xi/2}{1+\xi}\left[\left(\frac{\xi}{\xi_0}-1\right)\left(\bar{n}+\frac{1}{2}+\frac{1}{2}\frac{R^0}{D^0}\right)+\frac{1}{2}\frac{R^0}{D^0}\xi\ln\frac{\xi}{\xi_c}\right]. \quad (4.5)$$

The dependence of ξ on L is determined here by formula (2.7) at $\sigma = 0$.

On the other hand, if $|\Omega| \gg \gamma_{ab}\sqrt{\xi}$, then

$$W_\Omega(L)=\hbar\omega_0(\exp\{k_0 g(\mu+\Omega)Ld\}-1)\left(\bar{n}+\frac{1}{2}+\frac{1}{2}\frac{R^0}{D^0}\right), \quad (4.6)$$

i.e., the noise-emission spectrum does not depend on the field (see[9]). This result is physically obvious, since saturation of the working transition by the strong field comes into play only at frequencies $|\Omega| \lesssim \gamma_{ab}\sqrt{\xi}$.

In the intermediate frequency region $0 < |\Omega| < \gamma_{ab}\sqrt{\xi}$, it is difficult in general to obtain explicit expressions for the noise-emission spectrum. We consider therefore a particular case of practical importance, when $\gamma \ll \gamma_{ab}$. This condition is satisfied, for example, for a mixture of helium and neon at sufficiently high pressures.

For the frequencies Ω satisfying the condition $\sqrt{\gamma\gamma_{ab}} \ll |\Omega| \ll k_0 u$, we have

$$I_{1\Omega}=I_{2\Omega}=\frac{\omega_0^4\hbar g(\mu+\Omega)d}{\pi c^3}\left\{\left[\frac{\gamma_{ab}^2\xi}{\sqrt{1+\xi}}\frac{\gamma_{ab}^2\xi-\Omega^2}{(\Omega^2+\gamma_{ab}^2\xi)^2+4\Omega^2\gamma_{ab}^2}+\frac{\Omega^2(\Omega^2+4\gamma_{ab}^2+\gamma_{ab}^2\xi)}{(\Omega^2+\gamma_{ab}^2\xi)^2+4\Omega^2\gamma_{ab}^2}\right]\left(\bar{n}+\frac{1}{2}\right)+\frac{1}{2}\frac{R^0}{D^0}\right\}. \quad (4.7)$$

From this we obtain at $\sqrt{\gamma\gamma_{ab}} \ll |\Omega| \ll \gamma_{ab}$

$$W_\Omega(L)=\hbar\omega_0\left\{\left(\frac{\xi}{\xi_0}-1\right)\left(\bar{n}+\frac{1}{2}+\frac{1}{2}\frac{R^0}{D^0}\right)+\frac{1}{2}\frac{R^0}{D^0}\xi\cdot\left[\frac{\sqrt{1+\xi_0}-1}{\xi_0}-\frac{\sqrt{1+\xi}-1}{\xi}+\frac{1}{2}\ln\frac{(\sqrt{1+\xi}-1)(\sqrt{1+\xi_0}+1)}{(\sqrt{1+\xi}+1)(\sqrt{1+\xi_0}-1)}\right]\right\}. \quad (4.8)$$

At $|\Omega| \gg \gamma_{ab}\sqrt{\xi}$, expression (4.7) leads to (4.6).

Comparing (4.8) with (4.6) and allowing for the dependence of ξ on L, we see that at $\mu = 0$ there is a relatively broad dip of width on the order of $\gamma_{ab}\sqrt{\xi}$ at the center of the emission line. The relative depth of this dip, which we define as

$$1-W_\Omega|_{\sqrt{\gamma\gamma_{ab}}\ll|\Omega|\ll\gamma_{ab}}/W_\Omega|_{\gamma_{ab}\sqrt{\xi}\ll|\Omega|\ll k_0u},$$

tends to unity in a long amplifier, when $\xi \gg 1, \xi_0, 1/\xi_0$, and to $(\bar{n} + 1/2)/(\bar{n} + 1/2 + R^0/2D^0)$ in a short amplifier at $\xi \sim \xi_0 \gg 1$. The existence of such a dip is due to saturation of the working-level population difference by the strong field; this saturation causes the polarizability components $\kappa_{1,2}'$ to saturate.

At the center of the dip, the spectrum has a fine structure whose character depends on the length of the amplifier and on the magnitude of the input signal. In a short amplifier at $\xi \sim \xi_0 \gg 1$, there is at the center of the dip a narrow maximum of width of the order of $\sqrt{\gamma\gamma_{ab}}$ (compare (4.5) with (4.8)). The presence of this maximum is due to the increase of the spontaneous fluctuations $(\delta P_2^2)_{\Omega,K}$ at zero frequency with increasing field. The relative height of the maximum, i.e., the quantity

$$W_\Omega|_{\Omega=0}/W_\Omega|_{\sqrt{\gamma\gamma_{ab}}\ll|\Omega|\ll\gamma_{ab}}-1,$$

tends to $\sqrt{\xi_0}/2$ in a strong field.

At large amplifier lengths, when $\xi \gg 1, \xi_0, 1/\xi_0$, the shape of the noise-emission spectrum at the line center depends on the field at the input. In the case of a strong input signal ($\xi_0 \gg 1$) there is at the line center a peak whose relative height is of the order of $(1/2)\sqrt{\xi_0}\ln(\xi/\xi_0)$. In the case of a weak input signal ($\xi_0 \ll 1$), a peak appears at the line center only when the amplifier is very long and $\ln(\xi/\xi_0) \gg 1/\xi_0$. The relative height of the peak is then

$$\frac{R^0}{4D^0}\xi_0\ln\frac{\xi}{\xi_0}\Big/\left(\bar{n}+\frac{1}{2}+\frac{1}{2}\frac{R^0}{D^0}\right).$$

We note also that when the amplifier length is increased the values of the spectral density $W_\Omega(L)$ at $|\Omega| \gg \gamma_{ab}\sqrt{\xi}$ increase much more rapidly than $W_\Omega(L)$ at $\Omega = 0$.

The noise-emission spectra for the case $\gamma = \gamma_{ab}$ were calculated with a computer. The results of the calculations are shown in Fig. 1.

5. NOISE EMISSION OF ACTIVE MEDIUM WITH HOMOGENEOUS LINE BROADENING

Obviously, formulas (4.2) and (4.3) are valid also for a medium with homogeneous line broadening at $\mu = 0$. The expressions for the quantities $(\delta P_i^2)_{\Omega,K}$ and $\kappa_i'(\Omega)$ can be obtained in this case from formulas (3.2)—(3.5) by putting in them $v = 0$. The result is

$$4\pi\varkappa_1'(\Omega)=-\frac{d}{1+\xi}\frac{\gamma_{ab}^2[\Omega^2+\gamma^2(1+\xi)]+\gamma\gamma_{ab}\xi[\Omega^2-\gamma\gamma_{ab}(1+\xi)]}{\Omega^2(\gamma+\gamma_{ab})^2+[\Omega^2-\gamma\gamma_{ab}(1+\xi)]^2},$$
$$4\pi\varkappa_2'(\Omega)=-\frac{d}{1+\xi}\frac{\gamma_{ab}^2}{\gamma_{ab}^2+\Omega^2}; \quad (5.1)$$

$$(\delta P_1^2)_{\Omega,K}=\frac{d\hbar}{2\pi}\frac{R^0}{D_0}\frac{\gamma_{ab}^2[\Omega^2+\gamma^2(1+\xi)]}{\Omega^2(\gamma+\gamma_{ab})^2+[\Omega^2-\gamma\gamma_{ab}(1-\xi)]^2}, \quad (5.2)$$
$$(\delta P_2^2)_{\Omega,K}=\frac{d\hbar}{2\pi}\frac{R^0}{D^0}\frac{\gamma_{ab}^2}{\gamma_{ab}^2+\Omega^2}.$$

As follows from (5.1) and (5.2), the spectra of $\kappa_2'(\Omega)$ and $(\delta P_2^2)_{\Omega,K}$ retain a Lorentz shape in a strong field. The spectra of $\kappa_1'(\Omega)$ and $(\delta P_1^2)_{\Omega,K}$ acquire dips in fields $\xi \gtrsim \gamma/\gamma_{ab}$. In a strong field, the width of the dip is of the order of $(\gamma\gamma_{ab}\xi)^{1/2}$.

We now derive expressions for the spectral densities of the noise-emission flux components $W_{1\Omega}(L)$ and $W_{2\Omega}(L)$. An expression for $W_{2\Omega}(L)$ can be obtained at all frequencies:

$$W_{2\Omega}(L)=\frac{\hbar\omega_0}{2}\left\{\left(\bar{n}+\frac{1}{2}+\frac{1}{2}\frac{R^0}{D^0}\right)\left[\left(\frac{\xi}{\xi_0}\right)^{g(\Omega)}-1\right]\right.$$

$$+\frac{R^0}{2D^0}\frac{\gamma_{ab}^2}{\Omega^2}\xi\left[1-\left(\frac{\xi}{\xi_0}\right)^{g(\Omega)-1}\right]\Bigg\}. \tag{5.3}$$

In formula (5.3), the dependence of ξ on L is determined from (2.6). It follows from (4.3) that the maximum of the spectral density $W_{2\Omega}(L)$ is reached at $\Omega = 0$. The spectrum of $W_{2\Omega}(L)$ in a short amplifier has a width of the order of γ_{ab}.

For the component $W_{1\Omega}(L)$, we can easily obtain an expression at the frequency $\Omega = 0$:

$$W_{1\,\Omega=0}(L)=\frac{\hbar\omega_0\xi}{2(1+\xi)^2}\Bigg\{\left(\frac{1}{\xi_0}-\frac{1}{\xi}+\xi_0-\xi\right)\left(\bar{n}+\frac{1}{2}+\frac{R^0}{2D^0}\right)+\frac{R^0}{D^0}\left(\xi-\xi_0+\ln\frac{\xi}{\xi_0}\right)\Bigg\}. \tag{5.4}$$

On the other hand, if $|\Omega| \gg \sqrt{\gamma\gamma_{ab}\xi}$, then $W_{1\Omega} = W_{2\Omega}$. It follows from (5.3) and (5.4) that a dip exists in the spectrum of $W_{1\Omega}$. In a strong field, the width of the dip is of the order of $(\gamma\gamma_{ab}\xi)^{1/2}$, and its relative depth tends to unity.

The shape of the spectrum of $W_\Omega = W_{1\Omega} + W_{2\Omega}$ depends strongly on the ratio of the width of the dip in the spectrum of $W_{1\Omega}$ to the width of the dip in the spectrum of $W_{2\Omega}$ ($\sim\gamma_{ab}$). If $\gamma\xi \ll \gamma_{ab}$, then the width of the dip is much smaller than γ_{ab} and the decrease of $W_{1\Omega}$ as $|\Omega| \to 0$ is faster than the increase of $W_{2\Omega}$. The spectrum of W_Ω therefore contains at the line center a dip of width $(\gamma\gamma_{ab}\xi)^{1/2}$, the depth of which in a strong field tends to one-half. On the other hand, if $\gamma\xi \gg \gamma_{ab}$, then the width of the dip is much larger than γ_{ab}, and the decrease of $W_{1\Omega}$ as $|\Omega| \to 0$ is slower than the increase of W_2. Therefore the spectrum of W_Ω contains in this case a central maximum at $\Omega = 0$, and two side maxima at the frequencies $\Omega \approx \pm(\gamma\gamma_{ab}\xi)^{1/2}$. In a short medium, at $\xi_0 \ll 1$, the heights of the side maxima are approximately half the height of the central maximum, and the side maxima increase more slowly with increasing length of the medium than the central maximum.

Figure 2 shows plots of the noise-emission spectrum of an amplifier at $\gamma = \gamma_{ab}$ for different values of the input and output signals.

6. NOISE EMISSION OF ABSORBING MEDIUM UNDER THE INFLUENCE OF A STRONG FIELD

The results obtained in Secs. 1–3 remain in force also for an absorbing medium, i.e., when $D < 0$. We confine ourselves to a medium in thermal equilibrium with the ambient. In this case the condition $\bar{n} + 1/2 + (1/2)R^0/D^0 = 0$ is satisfied, i.e., $R^0/|D^0| = 1 + 2\bar{n}$.

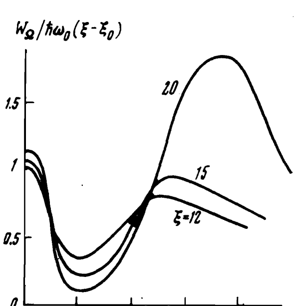

FIG. 1. Spectrum of the fluctuations of an active medium with inhomogeneously broadened transition line at $R^0/D^0 = 3$, $k_0u/\gamma_{ab} = 10$, $\gamma = \gamma_{ab}$, and $\xi_0 \equiv aE_0^2 = 10$. The curves are marked with the values of $\xi \equiv aE^2$.

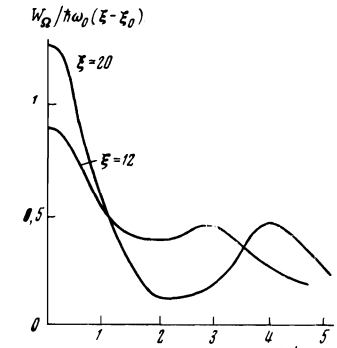

FIG. 2. Spectrum of the fluctuations of an active medium with homogeneously broadened transition line at $R^0/D^0 = 3$, $\gamma = \gamma_{ab}$, and $\xi_0 = 10$.

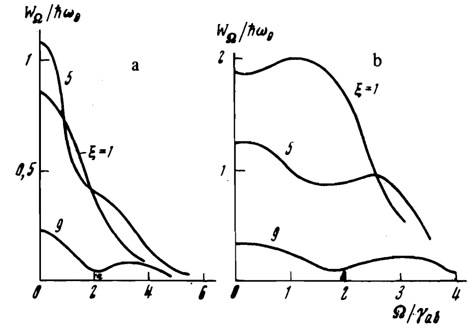

FIG. 3. Spectrum of fluctuations of an absorbing medium with inhomogeneous broadening (a) and with homogeneous broadening (b) of the transition line $\gamma = \gamma_{ab}$, $n + ½ + R^0/2D^0 = 0$, $\xi_0 = 10$; $k_0u/\gamma_{ab} = 10$ in case (a).

We analyze first the expressions for the quantities $I_{1,2\Omega}$ in the case of a gaseous medium with inhomogeneous line broadening at $\gamma = \gamma_{ab}$. After averaging over the velocities at $\mu = 0$, we obtain

$$I_1=\frac{\omega_0^4\hbar g(\Omega)d}{\pi c^3}\frac{R^0}{D^0}\frac{\xi}{2^{1/2}B(\Omega)\,[1+\xi-\Omega^2/\gamma^2+B(\Omega)]^{1/2}} \tag{6.1}$$

$$I_2=\frac{\omega_0^4\hbar g(\Omega)d}{\pi c^3}\frac{R^0}{D^0}\frac{\gamma^2\xi}{2(\gamma^2+\Omega^2)(1+\xi)^{1/2}}, \tag{6.2}$$

where

$$B(\Omega) = \{[1 + (\Omega/\gamma + \xi^{1/2})^2][1 + (\Omega/\gamma - \xi^{1/2})^2]\}^{1/4}.$$

From the form of the obtained expressions for $I_{1,2}(\Omega)$ it follows that when $\xi_0 \gg 1$ the emission spectrum of the short medium contains a central maximum of width of the order of γ, the height of which is $\sim\xi^{1/2}$, and two side maxima at the frequencies $\Omega \sim \pm\gamma\xi_0^{1/2}$, the height of which is $\sim\xi_0^{1/4}$. At $\Omega > \gamma\xi_0^{1/2}$, the spectral density of the noise emission decreases sharply. The reason for this drop is that the strong field does not influence the frequencies $\Omega \gg \gamma(1 + \xi_0)^{1/2}$, and a medium in thermal equilibrium with the ambient does not produce additional radiation. For an active medium, the emission at these frequencies is different from zero.

Figure 3a shows the results of a numerical calculation of the noise-emission spectra of passive media at $\xi_0 \equiv aE_0^2 = 10$ for different values of the output field. As seen from a comparison with the plots in Fig. 1, the spectra of short amplifying and attenuating media are similar in shape, but differ in width. The similarity in the shapes of the spectra in the frequency region $\Omega \lesssim \gamma\xi_0^{1/2}$ is due to the action of the strong field, which equalizes the populations of the working levels in all cases. With increasing length of the medium at

$\xi \sim 1$ the fine structure of the spectrum vanishes and the line width becomes of the order of γ.

The intensity of the noise emission of the passive media depends on the length of the medium. It is low for very short media, when $\xi_0 - \xi \ll \xi_0$, and for very long media, when $\xi \ll 1$. There exists an optimal length at which the noise-emission intensity is maximal. This optimal length can be estimated by considering the dependence of the spectral density of the noise emission at zero frequency on the length of the medium. It follows from (4.5) that

$$W_{\Omega=0}(L) = \frac{\hbar\omega_0}{2}\frac{R^0}{|D_0|}\xi\frac{1+\xi/2}{1+\xi}\ln\frac{\xi_0}{\xi}. \quad (6.3)$$

The maximum value of $W_{\Omega=0}$ is attained under the condition

$$\ln\frac{\xi_0}{\xi} = 1 + \frac{\xi}{2(1+\xi)+\xi^2}. \quad (6.4)$$

If $\xi_0 \gg 1$, then Eq. (6.4) has an approximate solution $\ln(\xi_0/\xi) = 1$, i.e., $\overline{E}^2 = E_0^2/e$ (here and below, e is the base of the natural logarithm). Substituting this value of $\overline{E}^2$ in (6.3), we obtain the value of $W_{\Omega=0}$ at the maximum:

$$W_{\Omega=0} = \frac{\hbar\omega_0}{4e}\frac{R^0}{|D^0|}\xi_0.$$

Thus, the maximum value of the spectral density of the noise emission at $\Omega = 0$ is determined by the input field.

We now consider the case of a medium with homogeneous line broadening. Since the quantities $I_{1,2\Omega}$ in a strong field are determined mainly by the polarization fluctuations $(\delta P^2_{1,2})_{\Omega,K}$, it follows that the emission of a short passive medium has the same character as the emission of a short active medium. Just as in the case of an active medium, the emission spectrum contains either a central maximum with two side maxima, if $\gamma\xi \gtrsim \gamma_{ab}$, or a dip at the line center, if $\gamma\xi \ll \gamma_{ab}$. With increasing length of the medium, the fine structure of the spectrum becomes smoothed out.

We determine the optimal length of the medium by investigating the dependence of the spectral density of the noise emission at $\Omega = 0$ on the length. From (5.3) and (5.4) it follows that

$$W_{\Omega=0} = \frac{\hbar\omega_0}{2}\frac{R^0}{|D^0|}\xi\left[\frac{\xi_0-\xi}{(1+\xi)^2} + \left(\frac{1}{2} + \frac{1}{(1+\xi)^2}\right)\ln\frac{\xi_0}{\xi}\right].$$

From this we get at $\xi_0 \gg 1$ that $W_{\Omega=0}$ is maximal at two field values: $\xi \approx \xi_0/e$ and $\xi \approx 1$. The first of these maxima corresponds to the maximum of $W_{2\Omega} = 0$, and the second to the maximum of $W_{1\Omega} = 0$. The appearance of the second maximum is connected with the fact that in an absorbing medium with homogeneous line broadening we have $\kappa_1'(0) \le 0$ at $\xi \ge 0$, i.e., a weak field is not weakened in the presence of a strong field, but is strengthened. In the first maximum we have

$$W_{\Omega=0} = \frac{\hbar\omega_0}{4e}\frac{R^0}{|D^0|}\xi_0,$$

and in the second

$$W_{\Omega=0} = \frac{\hbar\omega_0}{8}\frac{R^0}{|D^0|}\xi_0,$$

i.e., the two maxima are approximately equal in magnitude. The noise-emission spectra of an absorbing medium with homogeneous line broadening are shown in Fig. 3b.

In conclusion let us discuss the conditions under which the results are valid. The fine structure of the spectrum is determined by the time-dependent parameters γ and γ_{ab} and by the field $a\overline{E}^2$. In the model assumed by us, the quantities γ and γ_{ab} are assumed constant. Actually they themselves can depend on the field, and this can affect the character of the fine structure. In addition, in our model we do not take into account certain subtle effects, for example the recoil effects following emission of a photon[14] and others. These effects can lead to a hyperfine structure of the spectra, which we have not considered. We note also that to measure the noise-emission spectra it is necessary to satisfy the condition that the number of photons striking the detector during the measurement time τ be much larger than unity. If the detector bandwidth is $\Delta\omega$, this condition takes the form $W_\Omega \tau\Delta\omega/\hbar\omega_0 \gtrsim \tau\Delta\omega \gg 1$, i.e., $\tau \gg 1/\Delta\omega$.

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176